

Part I : Accelerator light source

Part II : Magnetic field, material and Magnet

Part III : Technology and conventional insertion devices

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Short Bibliography



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- And References therein

Part I : Accelerator light source

What is accelerator light source (加速器光源) ?

An electromagnetic wave emitted by a relativistic electron beam deflected by a magnetic field.

生活周遭好多光源：太陽光、燈泡、電視機、手機螢幕、LED、雷射投影筆.....

There are more than 50 light sources in the world (operational, or under construction).

- Low energy storage rings : ALS, TLS, BESSYII
- Medium energy storage rings : SOLEIL, DIAMOND, CLS, ALBA, TPS, Australian

Why the world needs accelerator light source?



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Available online at www.sciencedirect.com



Biotechnology Advances 26 (2008) 246–265

Research review paper

Biological degradation of plastics: A comprehensive review

Aamer Ali Shah *, Fariha Hasan, Abdul Hameed, Safia Ahmed

Department of Microbiology, Quaid-i-Azam University, Islamabad, Pakistan

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Available online 26 January 2008

BIOTECHNOLOGY
ADVANCES

www.elsevier.com/locate/biotchadv

Plastics are man-made and continuously improved to improve stability and durability since 1990. (2019 production: 353 million tons per year)
=> 自然界還沒微生物，可以創造出酶來分解塑膠。

我們的生存正面臨很多的危機，其中之一Plastic pollution(塑膠汙染)



Hung-Hsuan Chao, Greenpeace, Taiwan



埔里鎮垃圾問題，佟振國(自由時報)，2023
<https://news.1tn.com.tw/news/life/breakingnews/4198513>

TingYi Chung 鍾廷翊, 2026, FEL

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國家 地理



這隻狀似蝦子的微小深海端足類動物，把微小的塑膠碎片和微纖維吃下肚。PHOTOGRAPH BY DAVID SHALE, MINDEN PICTURES

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國家地理



在葉門附近的亞丁灣，一條鯨鯊在塑膠袋旁游泳。雖然鯨鯊是海洋中最大的魚類，但仍然受到吞食塑膠碎片的威脅。PHOTOGRAPH BY THOMAS P. PESCHAK, NAT GEO IMAGE COLLECTION

圖片來源：Shutterstock

文 · 黃家慧

2024-03-12

《全球塑膠公約》大事記

未來，企業出口不只得考量碳費、碳稅，可能還要考量各國的「塑膠稅」。2022年4

月，英國便

可重複使用

不僅如此，

執行進度也

為塑膠，且積極採用再生塑料。

的

產業變

針對不

目標，

僅4%

除了減塑， 我們還可以怎麼做？

另一家消費龍頭可口可樂，也宣布2030年達成至少25%包裝可重複使用；聯合利華更打算在2025年之前，做到100%塑料包材可回收或可分解，同時減半原生塑膠用量。

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RESEARCH | REPORTS

BIODEGRADATION

A bacterium that degrades and assimilates poly(ethylene terephthalate)

Shosuke Yoshida,^{1,2*} Kazumi Hiraga,¹ Toshihiko Takehana,³ Ikuo Taniguchi,⁴ Hironao Yamaji,¹ Yasuhito Maeda,⁵ Kiyotsuna Toyohara,⁵ Kenji Miyamoto,^{2†} Yoshiharu Kimura,⁴ Kohei Oda^{1†}

sciencemag.org SCIENCE 1196 11 MARCH 2016 • VOL 351 ISSUE 6278



Ideonella sakaiensis 201-F6, that is able to use PET as its major energy and carbon source.

大阪堺菌

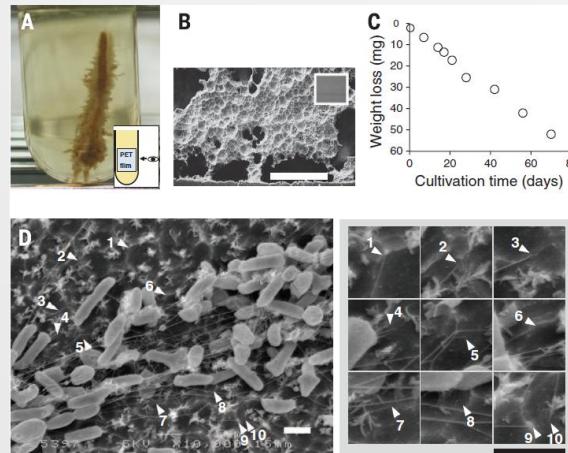
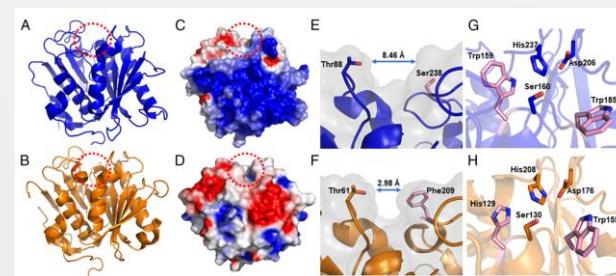


Fig. 1. Microbial growth on PET. The degradation of PET film (60 mg, 20 × 15 × 0.2 mm) by microbial consortium no. 46 at 30°C is shown in (A) to (C).

需要研究與產生更多的酶(大分子生物催化劑)來加速化學反應。



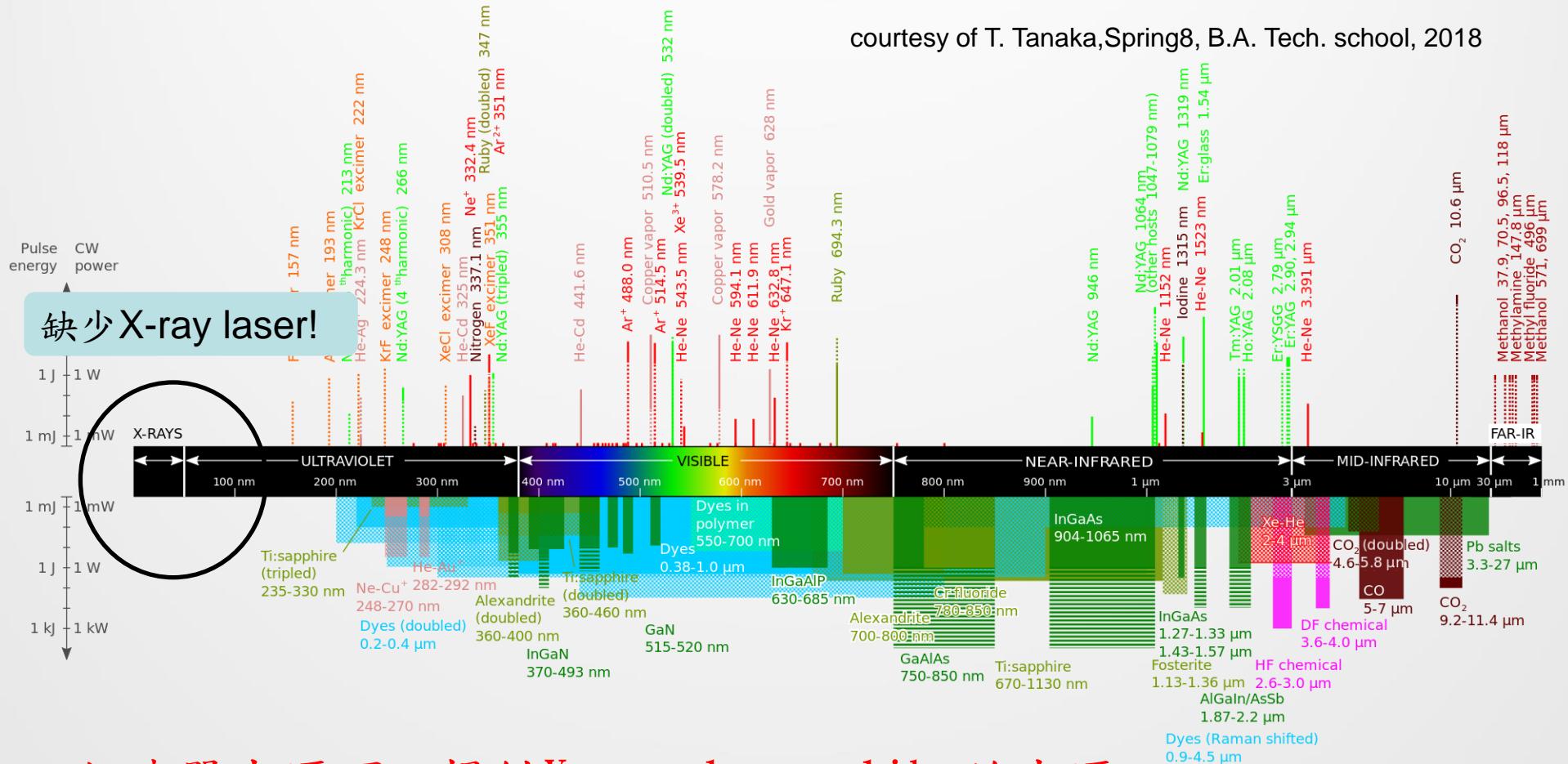
需要高亮度且短波長(\leq nm)的光源進行研究。

Lasers in X-ray Region ?



至今可取得的雷射

courtesy of T. Tanaka, Spring8, B.A. Tech. school, 2018



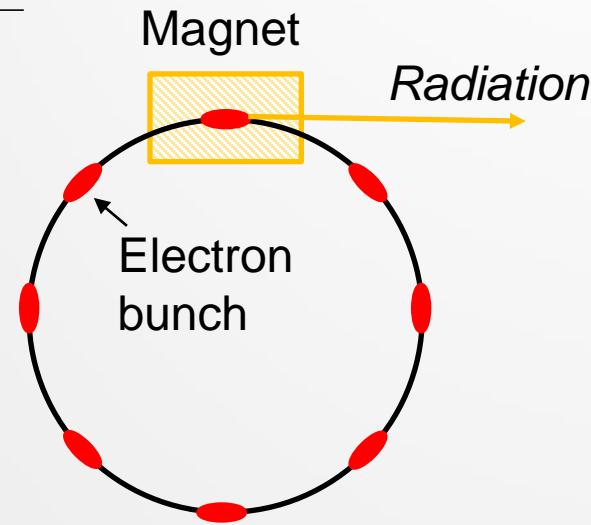
加速器光源可以提供X-ray laser-like的光源

What is accelerator light source (加速器光源) ?

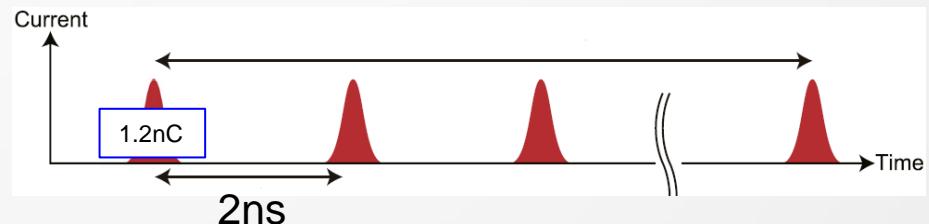
An electromagnetic wave emitted by a relativistic electron beam deflected by a magnetic field.

Light from a relativistic electron beam

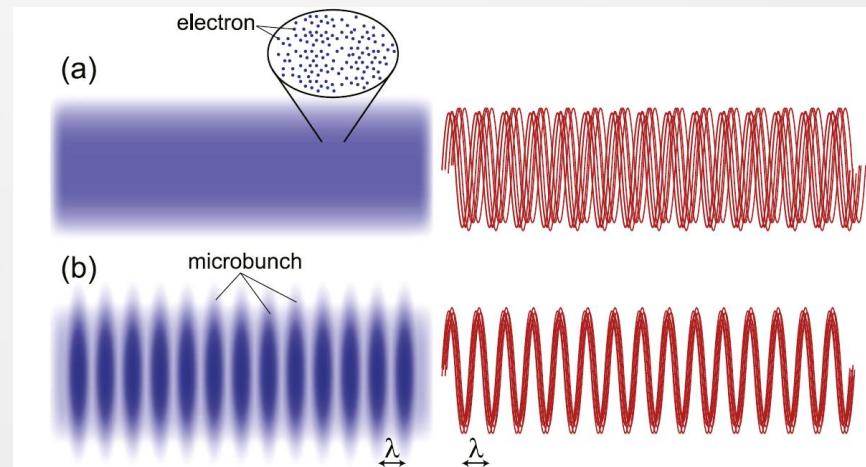
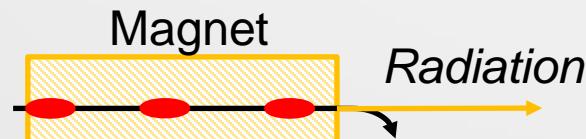
Ring_Synchrotron Radiation



TPS : One bunch has $\sim 10^{10}$ electrons



Linear_Linac, Free Electron Laser



A relativistic electron



Lorentz Factor γ

$$\text{Electron Total Energy } E = \gamma m_0 c^2 = \gamma E_0 \quad \text{Electron rest energy } E_0 = 0.511 \text{ MeV}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \beta^2}} \quad \leftrightarrow \quad \beta = \frac{v}{c} = \sqrt{1 - \frac{1}{\gamma^2}} \approx 1 - \frac{1}{2\gamma^2}$$

Electron velocity

E	γ	β
1 MeV	1.95	0.869
100 MeV	195	0.9999869
1 GeV	1956	0.999999869
3 GeV	5870	0.99999998549

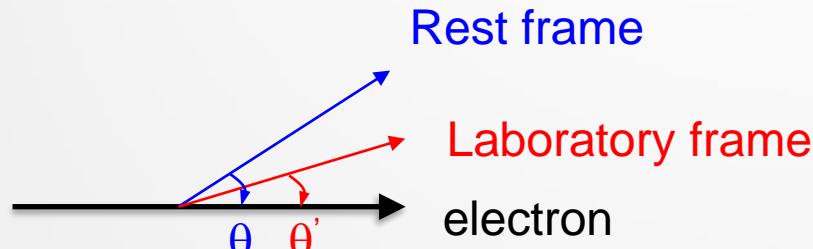
Speed of light in vacuum:
 $c = 299792.45 \text{ m/s}$

Any particle with non zero mass cannot exceed speed of light.

Light from a relativistic electron



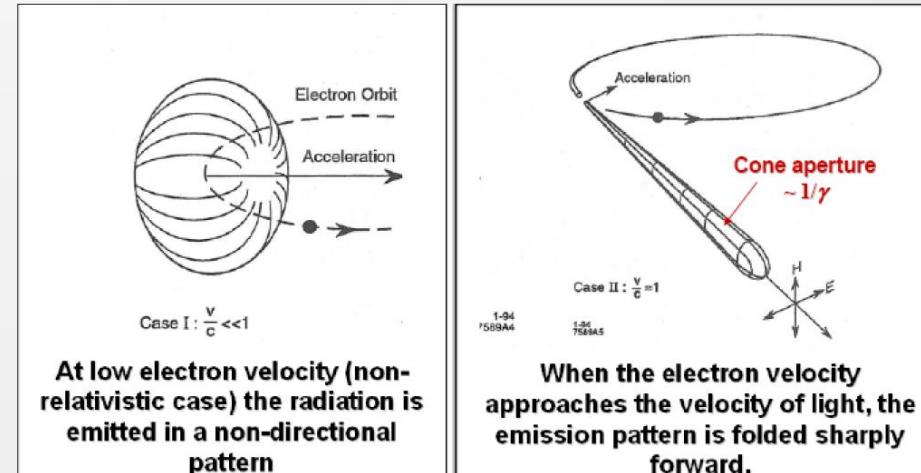
Radiation cone(空間)



Lorentz (Length) contraction

- For a rest electron, isotropic emission.
- For a relativistic electron $\beta \approx 1$, the radiation power is condensed to a narrow forward cone with a vertical angle of $\frac{1}{\gamma}$.
 $1/\gamma = 0.17\text{mrad}$ for 3GeV electron

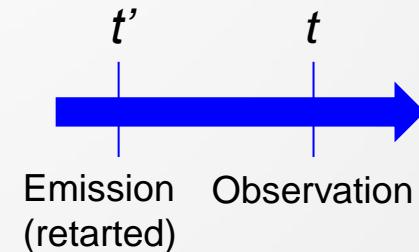
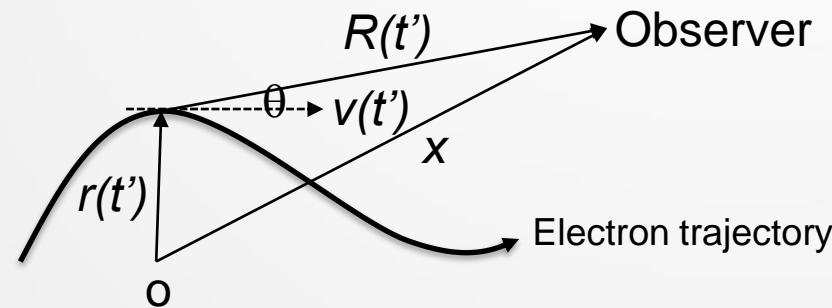
$$\tan\theta' = \frac{1}{\gamma} \frac{\sin\theta}{\cos\theta + \beta}$$



Light from a relativistic electron



Time squeezing (時間)



$$t = t' + \frac{R(t')}{c}$$

$$\frac{dt}{dt'} = 1 + \frac{1}{c} \frac{dR(t')}{dt'} = 1 + \frac{\vec{R}(t')}{R(t')} \cdot \frac{-\vec{v}(t')}{c} = 1 - \vec{n}(t') \cdot \vec{\beta}(t')$$

For $\theta = 0$ (on-axis observation),

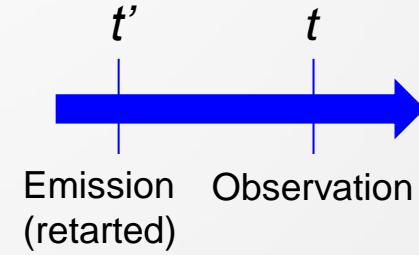
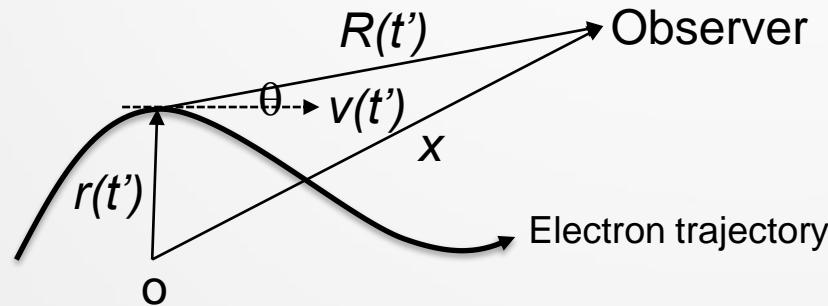
$$dt = \frac{dt'}{2\gamma^2}$$

Time squeezing

Light from a relativistic electron



Time squeezing (時間)



For $\theta = 0$ (on-axis observation),

$$dt = \frac{dt'}{2\gamma^2} \quad \text{Time squeezing}$$

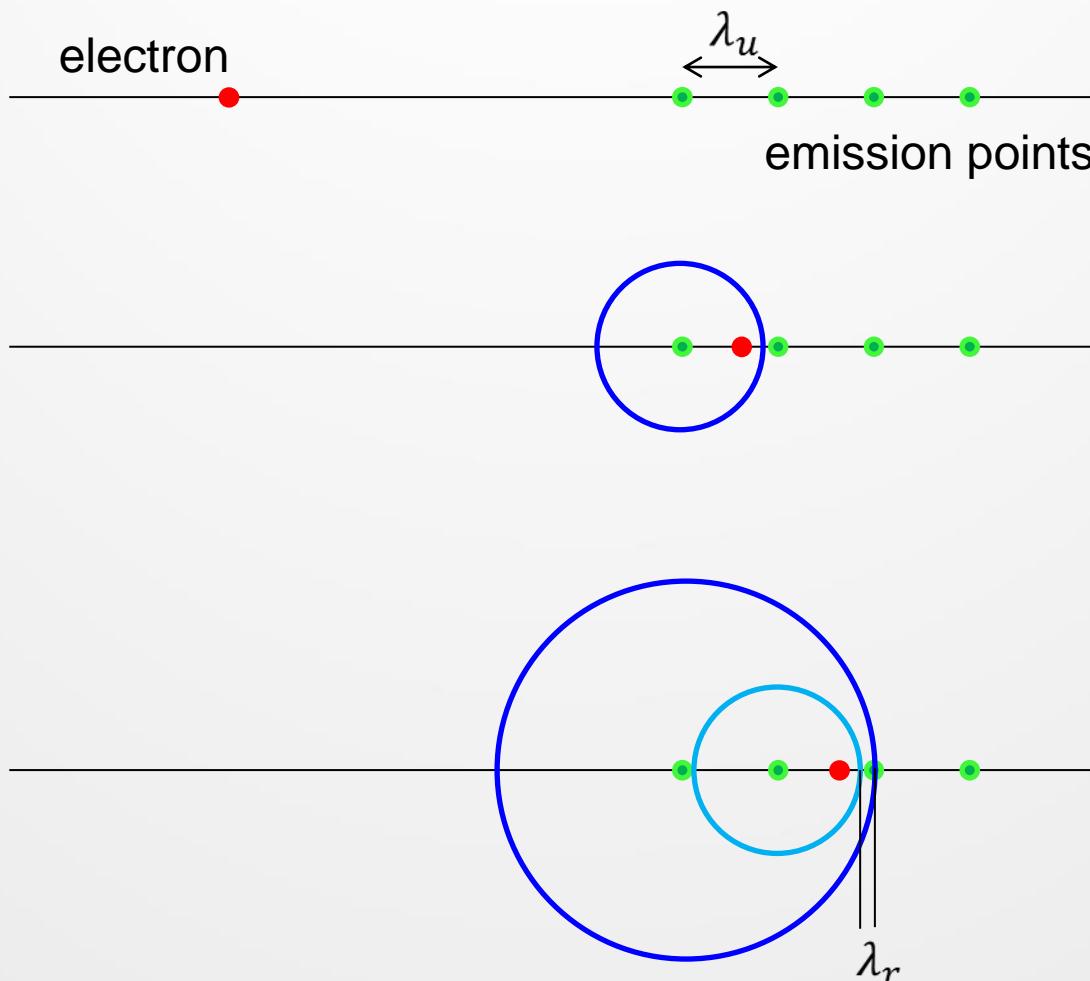
《西遊記》第四回：
孫悟空不滿玉帝所封的弼馬溫一職
重返花果山，見到猴子後發出了疑
問，而花果山的猴子回答說“天上
一天，人間一年”



$dt' = \text{one year } (3.15 \times 10^7 \text{ sec})$
 $\gamma = 5870 \text{ for } 3\text{GeV electron}$

$$dt = 0.45 \text{ sec}$$

Periodic emitter

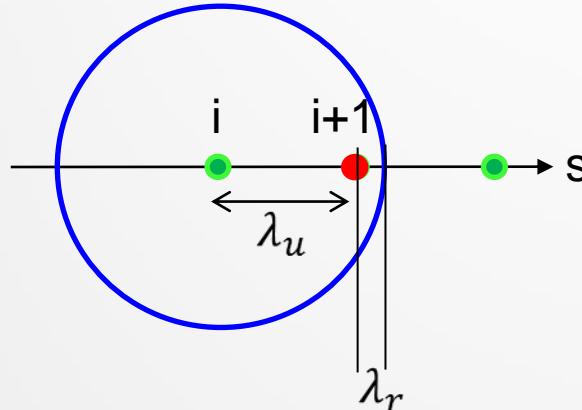


What is the relation between λ_u and λ_r ?

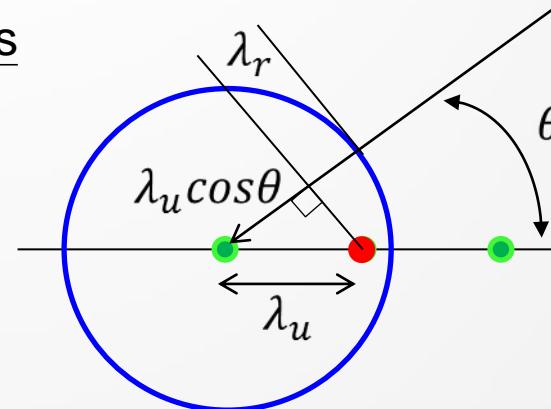
Periodic emitter_Wavelength



On axis



Off axis



Time taken by the electron to move from point i to point $i+1$: $\Delta t = \frac{\lambda_u}{\beta_s c}$
light

During this period the wavefront created at point i has expanded by $c \cdot \Delta t = \frac{\lambda_u}{\beta_s}$

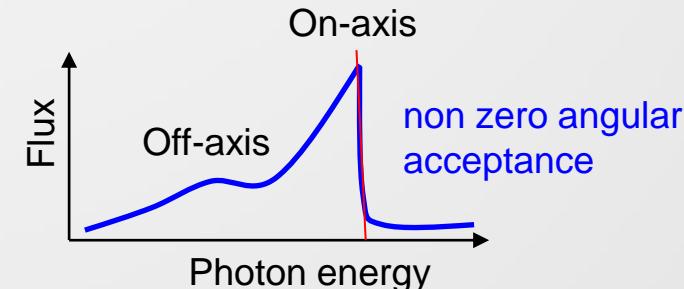
Therefore: $\lambda_r = \frac{\lambda_u}{\beta_s} - \lambda_u \approx \frac{\lambda_u}{2\gamma^2}$ ----- on axis

TPS 3 GeV, $\lambda_u=48$ mm, $\lambda_r = 7$ Å

$$\lambda_r(\theta) = \frac{\lambda_u}{\beta_s} - \lambda_u \cos\theta \approx \lambda_u \left(1 - \cos\theta + \frac{1}{2\gamma^2} \right)$$

For small angle: $\cos\theta \approx 1 - \frac{\theta^2}{2}$

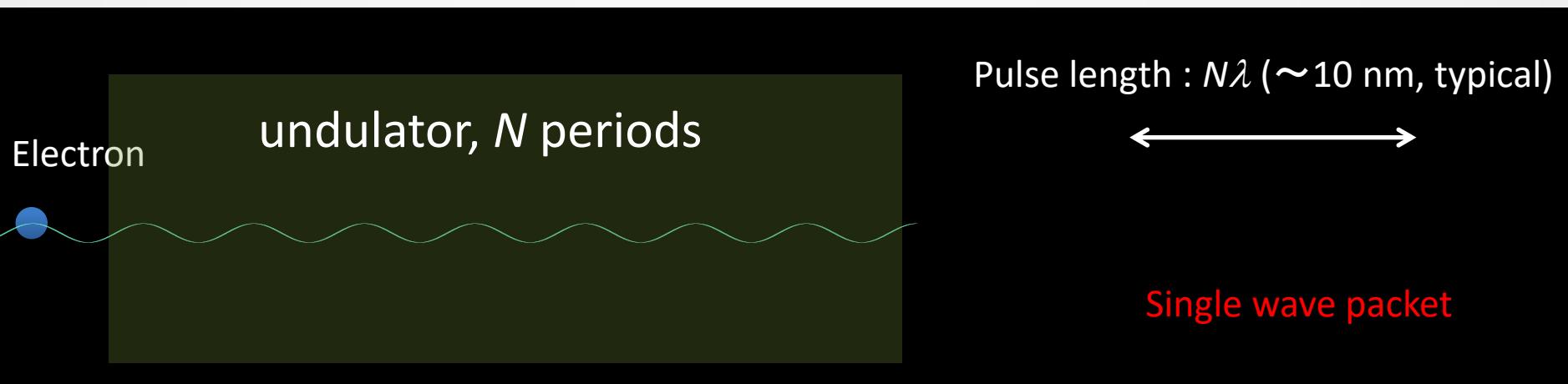
$$\lambda_r(\theta) \approx \frac{\lambda_u}{2\gamma^2} (1 + \gamma^2 \theta^2)$$
 ----- off axis



Wave packets generated from undulator



Single electron snaking in the undulator generates single wave packet.



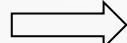
Trajectory in a planar ID



Equation of motion of an electron in a magnetic field B ,

$$\frac{d\vec{p}}{dt} = e\vec{v} \times \vec{B}$$

In the coordinate frame,
small angle approx. $\dot{x}, \dot{y} \ll 1$
electrons travelling in the z



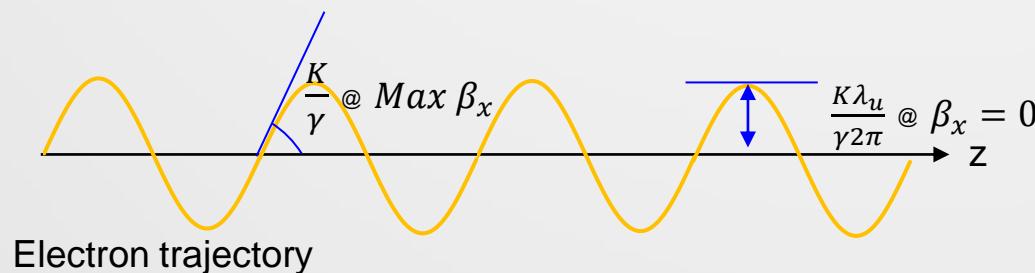
$$\ddot{x} = \frac{d^2x}{dz^2} = \frac{e}{\gamma mc} (B_y - \dot{y}B_z)$$

$$\ddot{y} = \frac{d^2y}{dz^2} = \frac{e}{\gamma mc} (\dot{x}B_z - B_x)$$

For a planar ID, the on-axis field: $(B_x, B_y, B_z) = (0, B_0 \sin\left(\frac{2\pi}{\lambda_u} z\right), 0)$

Deflection angle: $\dot{x} = \int \ddot{x} dz = \frac{B_0 e}{\gamma mc} \frac{\lambda_u}{2\pi} \cos\left(\frac{2\pi z}{\lambda_u}\right) = \frac{K}{\gamma} \cos\left(\frac{2\pi z}{\lambda_u}\right)$
(First field integral)

Position: $x = \int \dot{x} dz = \frac{K}{\gamma} \frac{\lambda_u}{2\pi} \sin\left(\frac{2\pi z}{\lambda_u}\right)$ $K = \frac{eB_0\lambda_u}{2\pi mc} = 0.9337B_0[T]\lambda_u[\text{cm}]$
(Second field integral)



Deflection parameter
(Multiple times the natural divergence angle $1/\gamma$)

TPS 3 GeV, $\lambda_u=48$ mm, $K=2$,
peak deflection =0.34 mrad, max dis.=2.6 μm .

Undulator Radiation



Transverse velocity :

$$\beta_x = \frac{dx/dt}{c} = \frac{dx}{dz} = \dot{x} = \frac{K}{\gamma} \cos\left(\frac{2\pi z}{\lambda_u}\right)$$

Longitudinal velocity :

$$\beta_z = \sqrt{\beta^2 - \beta_x^2} \approx 1 - \frac{1}{2\gamma^2} - \frac{K^2}{4\gamma^2} - \frac{K^2}{4\gamma^2} \cos\left(\frac{4\pi z}{\lambda_u}\right)$$

Oscillating term

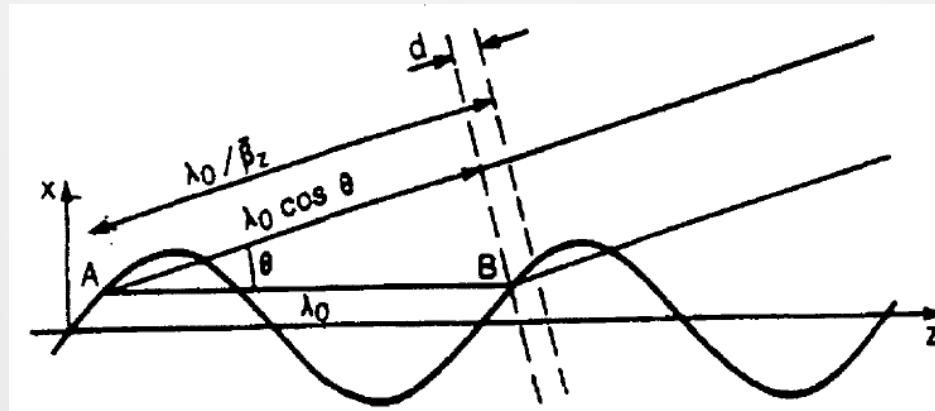
The average velocity can be adjusted by the magnetic field strength.

$\bar{\beta}_z$
Average velocity

K can be understood as how much the longitudinal velocity is slowed down due to the undulator magnetic field.

A slippage of an electron behind the wavefront over one period : $d = \frac{\lambda_u}{\bar{\beta}_z} - \lambda_u \cos\theta$

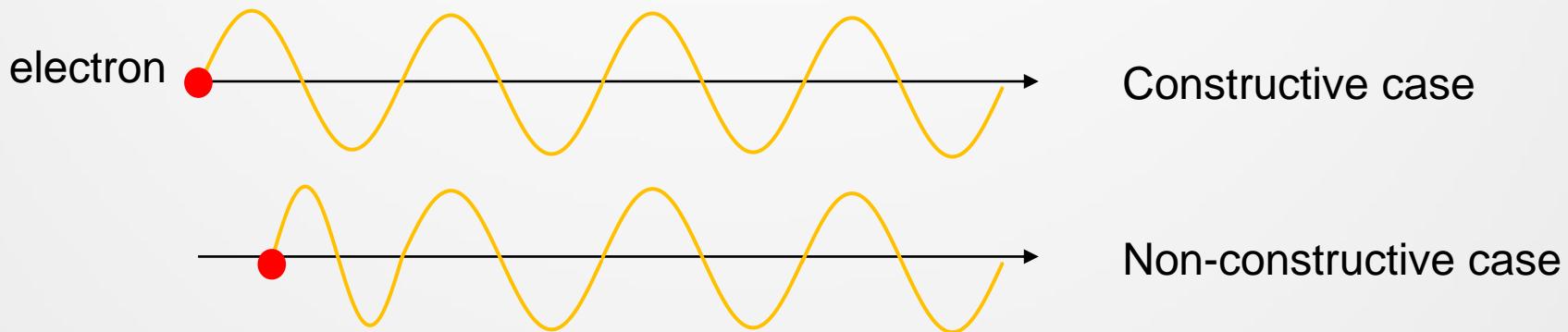
θ : observed angle



Undulator Radiation



For constructive interference between wavefronts emitted by the same electron, the slippage of **the electron must be by a whole number of wavelengths** over one period.



$$\lambda_r = \frac{\lambda_u}{n2\gamma^2} \left(1 + \frac{K^2}{2} + \gamma^2 \theta^2\right) \quad \text{Undulator eq.}$$

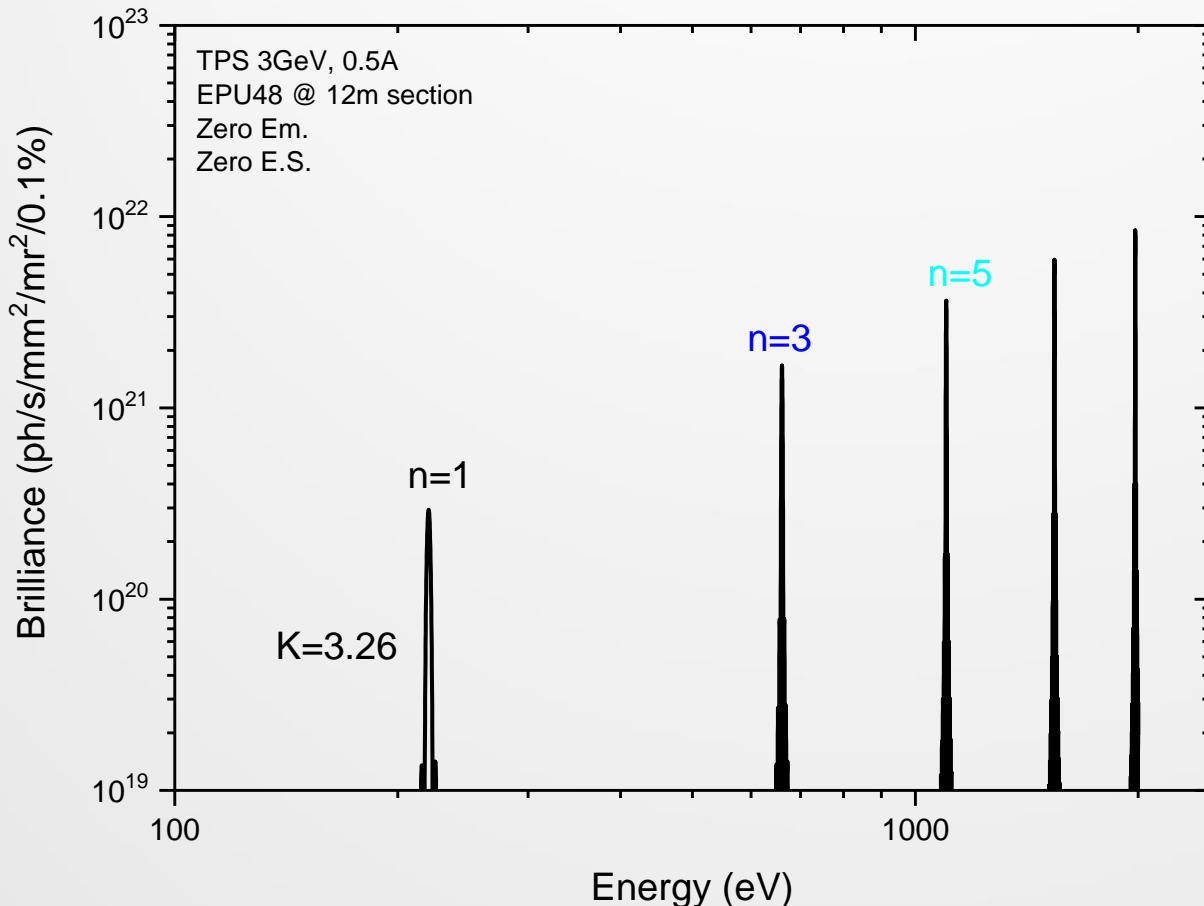
\uparrow n: harmonic number

- A magnetic field dependent wavelength.
- Increase B (also increase K), the output wavelength increase (photon energy decrease).

Spectrum of Undulator Radiation



Only wavelengths that are integer multiples and satisfy the slippage condition will result in constructive interference, producing light.

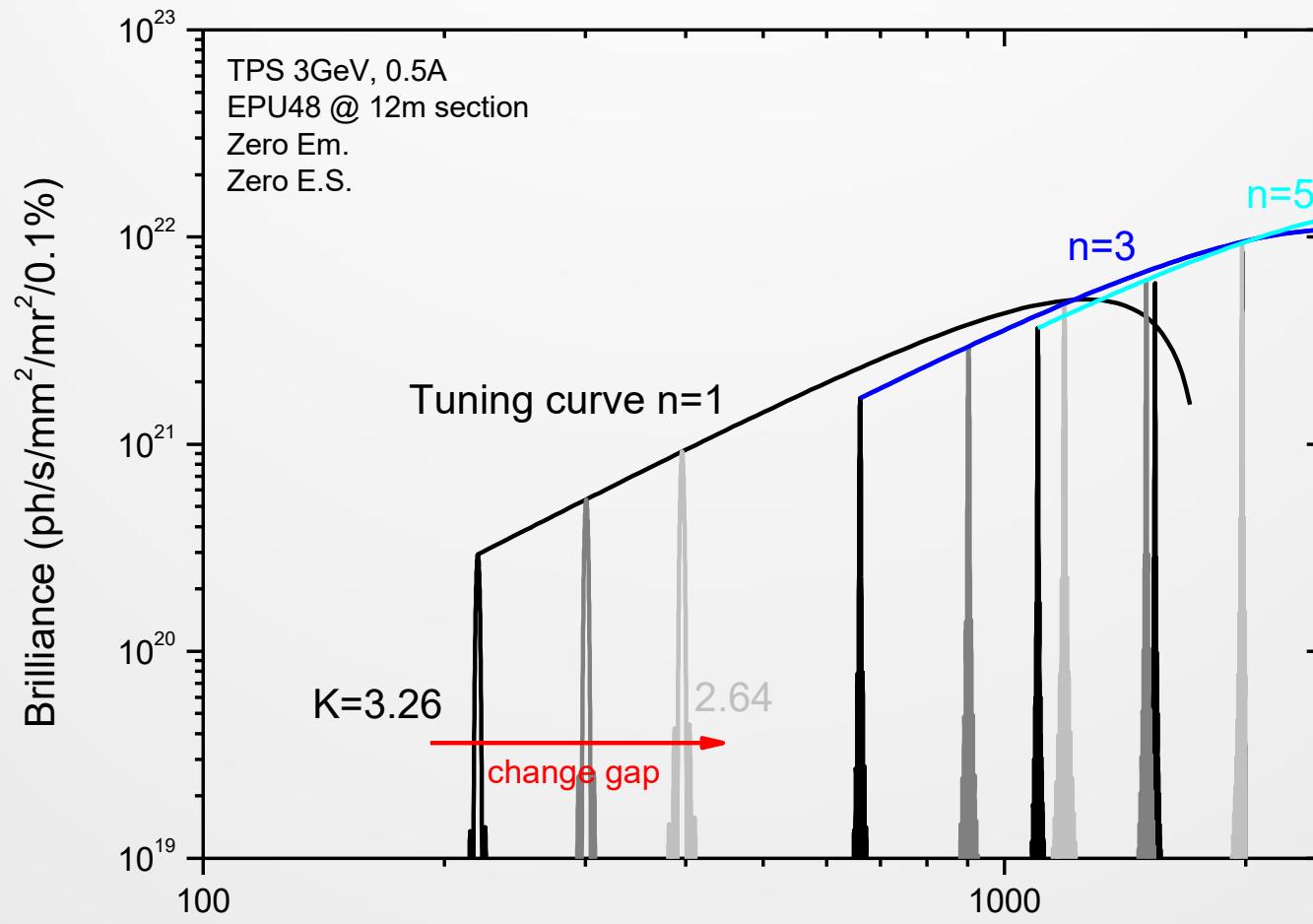


- only even harmonic?
- Peak height of $n=5 > n=1$?

Tuning curve



Scan a photon energy range => shift harmonic peak => tuning K generally via a gap change.



The tuning curve is very valuable for end users.

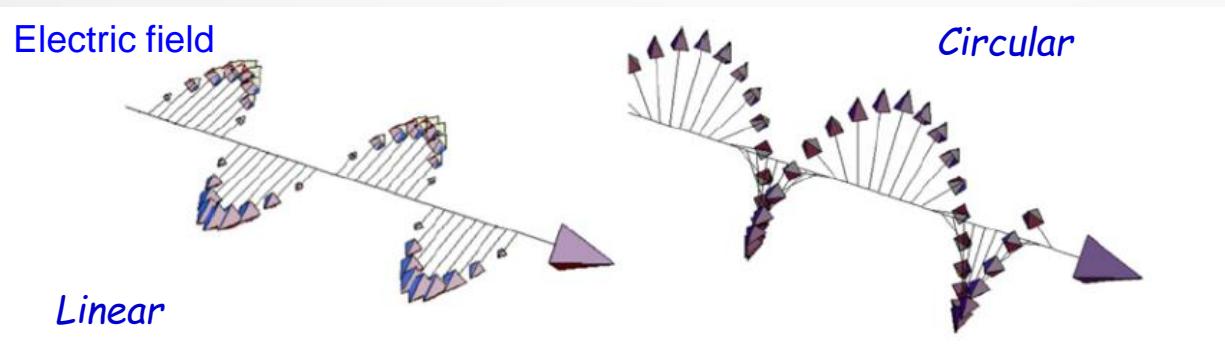
Energy (eV)

TingYi Chung 鍾廷翊, 2026, FEL

Polarization



- Polarization is described by the relationship between two orthogonal components of the Electric field.
- However, the amplitude and the phase difference cannot be measured directly.



$$\varphi = 0$$

$$E_{x_0} = E_{y_0}, \varphi = \pi/2$$

Stokes Parameters

The **intensity** can be measured for different polarization directions:

$$S_0 = I_x + I_y = I_{45^\circ} + I_{135^\circ} = I_R + I_L$$

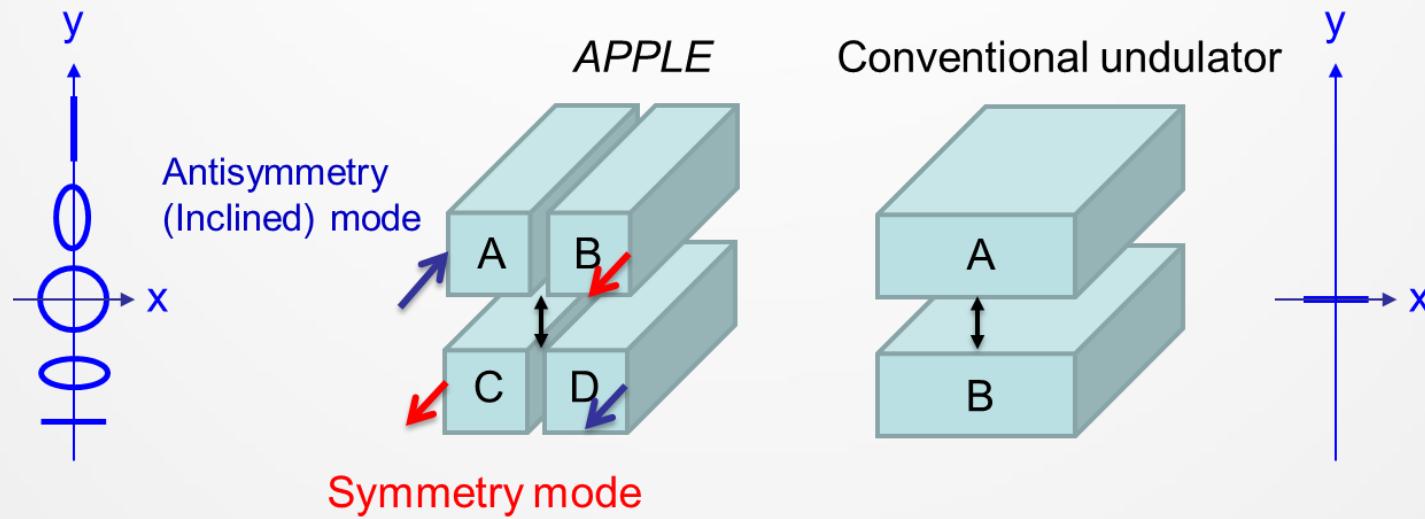
$$S_1 = I_x - I_y$$

$$S_2 = I_{45^\circ} - I_{135^\circ}$$

$$S_3 = I_R - I_L$$

Intensity of :	
Linear erect	$I_{x,y}$
Linear skew	$I_{45^\circ, 135^\circ}$
Circular	$I_{R,L}$

Elliptically polarized undulator (EPU)



Elliptically polarized undulator (EPU)



- Planar undulator (vertical field) produces only linear polarization.
- Elliptical undulator (vertical and horizontal field),

Magnetic field $(B_x, B_y, B_z) = (B_x \sin\left(\frac{2\pi}{\lambda_u}z + \varphi\right), B_y \sin\left(\frac{2\pi}{\lambda_u}z\right), 0)$

Velocity $(\beta_x, \beta_y, \beta_z) = \left(\frac{K_x}{\gamma} \cos\left(\frac{2\pi z}{\lambda_u}\right), \frac{K_y}{\gamma} \cos\left(\frac{2\pi z}{\lambda_u} + \varphi\right), 1\right)$

Longitudinal velocity : $\beta_z = \sqrt{\beta^2 - \beta_x^2 - \beta_y^2} \approx 1 - \frac{1}{2\gamma^2} - \frac{K_x^2}{4\gamma^2} - \frac{K_y^2}{4\gamma^2}$ $\overline{\beta_z}$

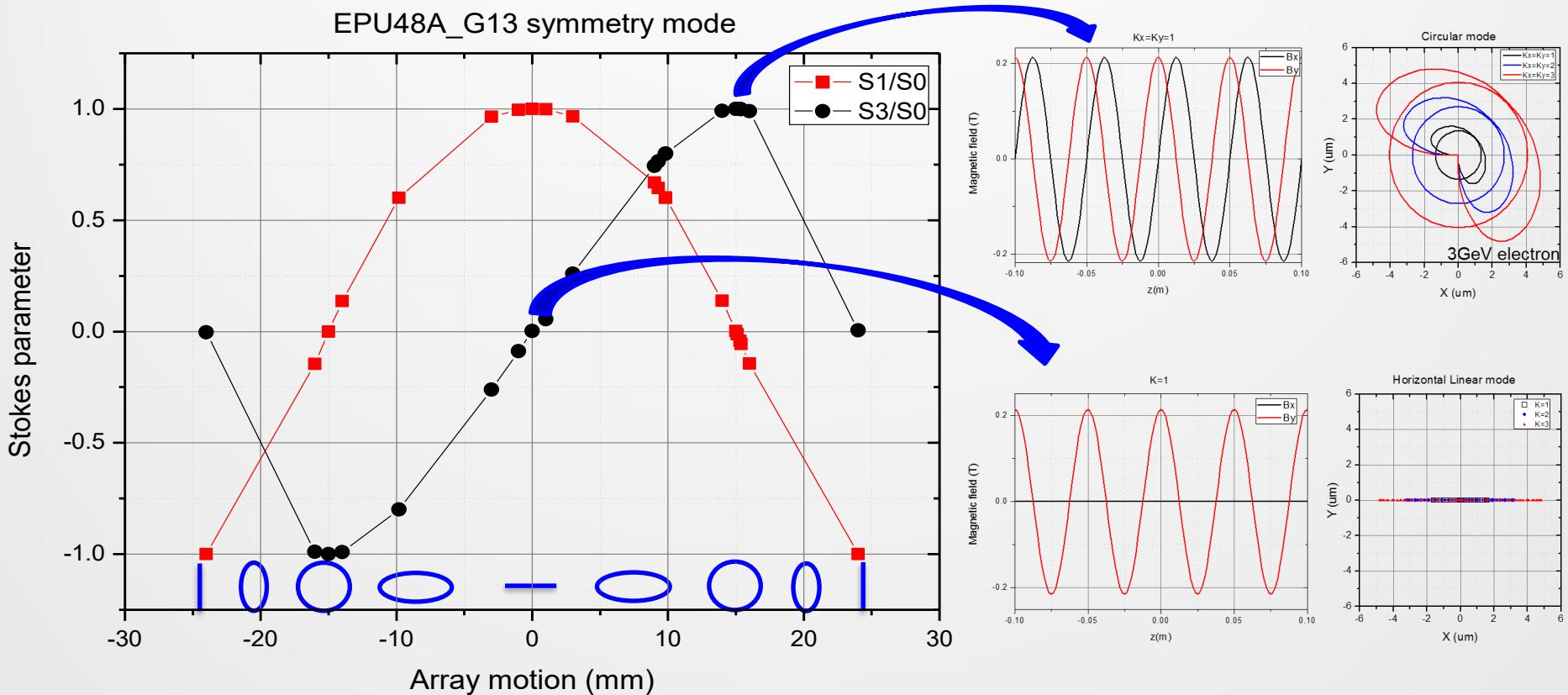
Wave length : $\lambda_r = \frac{\lambda_u}{n2\gamma^2} \left(1 + \frac{K_x^2}{2} + \frac{K_y^2}{2} + \gamma^2 \theta^2\right)$

- The wave length is independent on the phase between the field.
- Similar to the planar undulator

EPU radiation



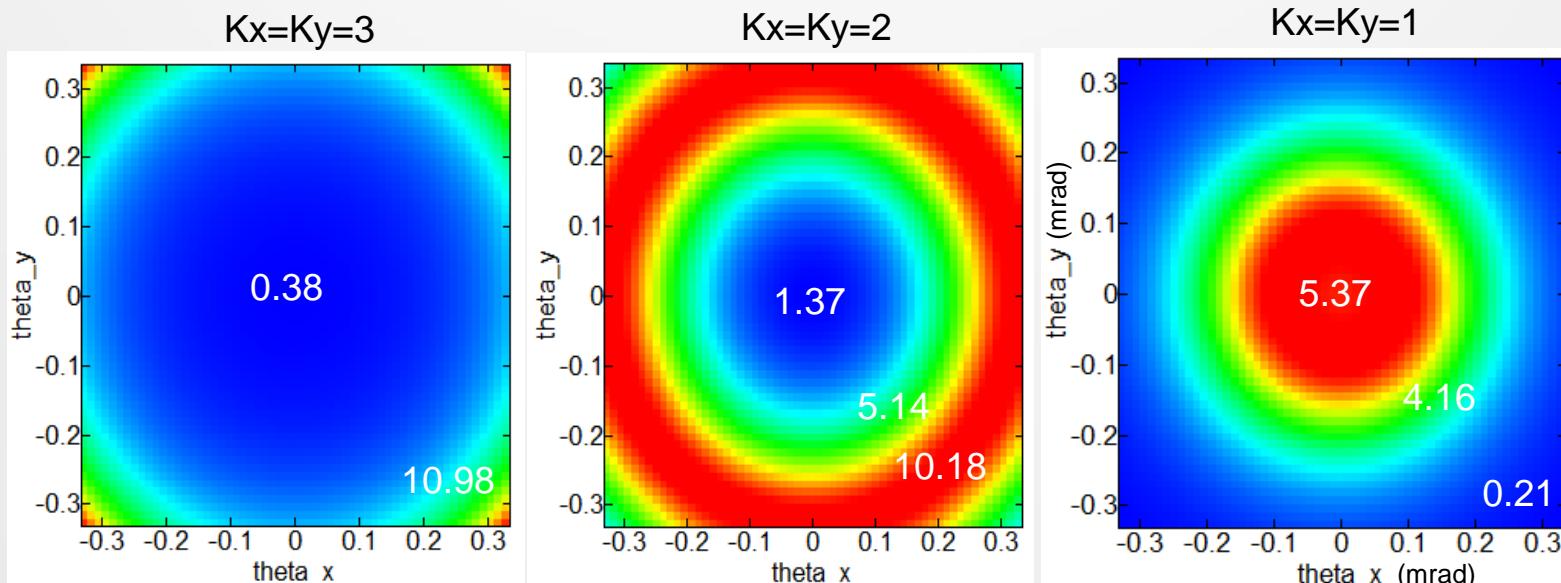
- $B_x=B_y$, the circular trajectory, pure circular polarization.
- On axis, only the first harmonic, a continuous electric field observed.



Total power and Angular power density



- Total power is **double** that produced by a planar undulator with the same magnetic field.
- The maximum power density is located at K/γ .
- The width $\sim 1/\gamma$
- larger K, lower on-axis power density but greater total power.



TPS 3GeV, 0.5A, 4m

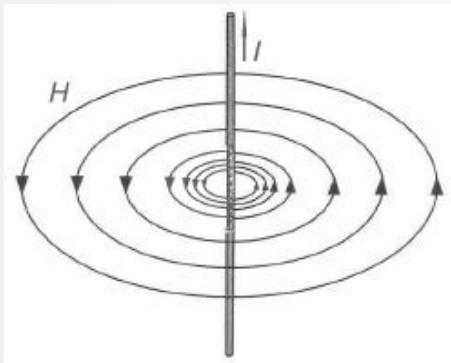
kW/mrad²

Part II : Magnetic field, material and Magnet

Magnetic field

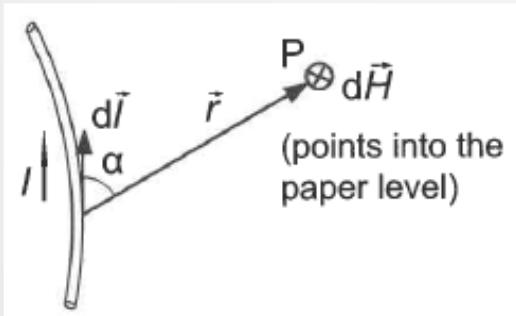


- Magnetic field are always the result of an electric charge in motion.
- Electric current flowing through a wire.



Ampere's law

$$\text{curl } \vec{H} = \vec{J} \Rightarrow \oint \vec{H} \cdot d\vec{l} = \int J \cdot dA \Rightarrow H = \frac{I}{2\pi r} \quad \text{Straight wire}$$



Biot Savart law

The magnetic field at a point P results from the superposition of the contribution from dl .

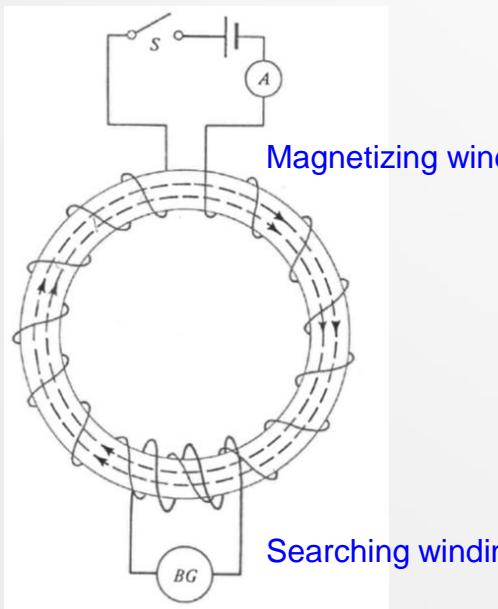
$$dH = \frac{1}{4\pi r^2} \cdot I \cdot dl \cdot \sin\alpha \Rightarrow H = \frac{I}{2r} \quad \text{Single current loop}$$

Uniform magnetic field

→

- A pair of Helmholtz coils with diameter $2r$ and arranged by r .
- A solenoid.

Magnetic material



Rowland ring method (closed loop method)

- Entirely confined flux within the coil.
- Material of a ring without forming poles.

- A ring specimen changes the flux circulation $\phi_{current}$, created by the magnetizing winding.
- The flux ϕ_{obs} is observed by the searching winding.

$\phi_{obs} < \phi_{current}$: diamagnetic

$\phi_{obs} > \phi_{current}$: paramagnetic and antiferromagnetic

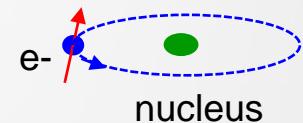
$\phi_{obs} \gg \phi_{current}$: ferromagnetic

- 每一種物質都有磁性質，只是需要區分為哪一類
- Magnetic field is always the result of an electric charge in motion.
=> Correct in matter ?

Magnetic material_微觀



- The *orbital motion* of an *electron* around the nucleus may be likened to a current in a loop of wire.
- The *spin* of an *electron*.



Calculating magnetic moment of an electron according to Bohr model

$$\mu_B = (\text{area of loop})(\text{current}) = \frac{eh}{4\pi m} = 9.274 \cdot 10^{-24} \text{ Am}^2$$

Bohr magneton: natural unit of magnetic moment

The magnetic moment of an electron in the first ($n=1$) Bohr orbit.
The magnetic moment due to its spin to $1.001 \cdot \mu_B$

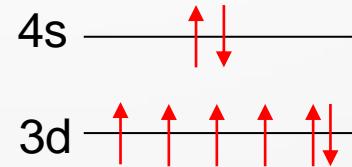
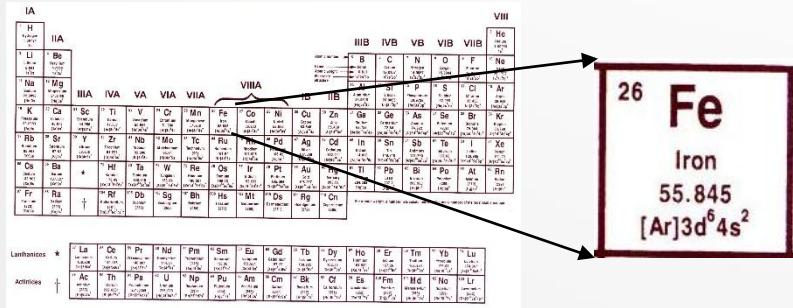
The magnetic moment of the *atom* is the vector sum of all its electronic moments.

- The magnetic moments of all the electrons cancel out, the atom no net magnetic moment: diamagnetism.
- Partial cancellation, the atom is left with a net magnetic moment: the others.

Magnetic material_微觀

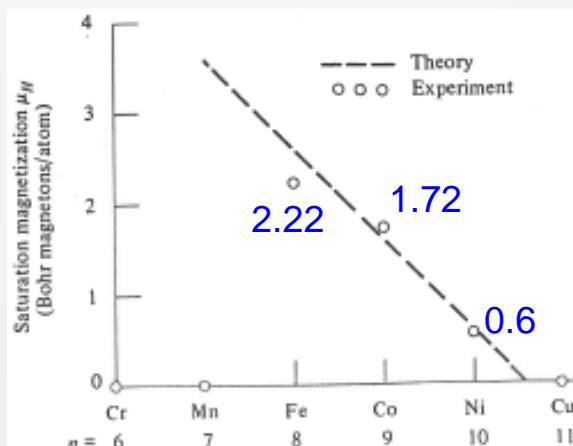


The magnetic moment of an *free* iron (Fe) atom,

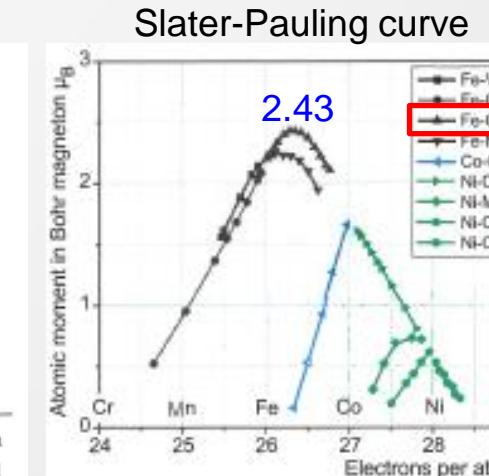


$$\mu_{free}^{Fe} = 4\mu_B$$

- In a *solid*, the energy levels of an atom are modified to a **band structure**.
- No longer an integer.
- The magnetic moment is influenced by *crystal structure*.



=> the origin of the magnetic moments is from spin and orbital moment in incomplete shells.



The magnetic moment of an atom has a maximum for a iron-cobalt alloys, reflects the max saturation magnetization of the material.

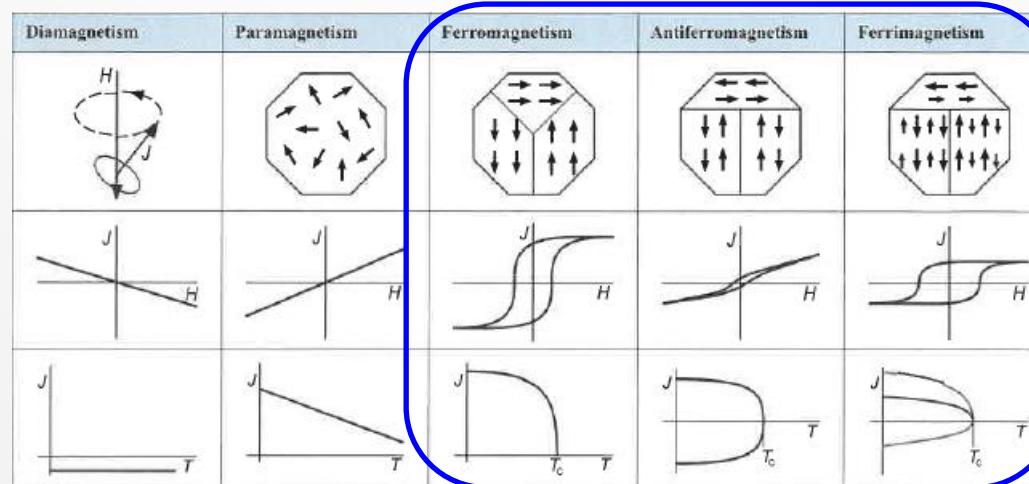
Magnetic material_巨觀



Arrangement of magnetic moments

$J(=\mu_0 M)H/MH$ curve,
Magnetic polarization J
Magnetic field strength H
Magnetization M

MT curve
Temperature T



Magnet application,

- High J at a small external field.
- Permanent magnet.

In addition to the external field, J depends on T .

At the Curie temperature T_c , the ferromagnetism becomes paramagnetism, the remanence and coercivity get zero.

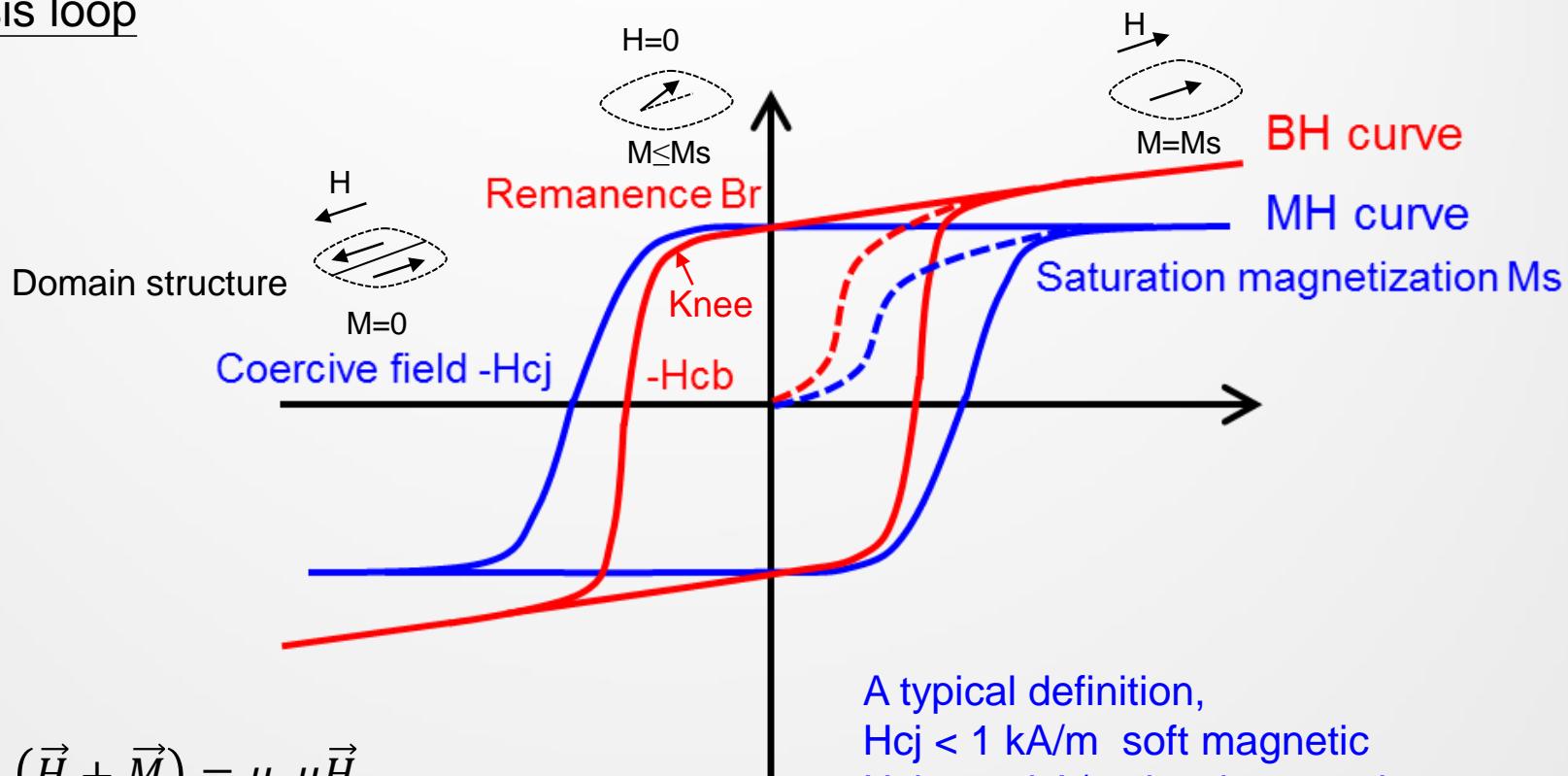
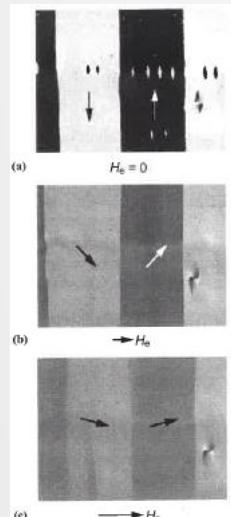
Fe, Co and Ni are the main components for the magnet application.

Material	T_c (° C)	Material	T_c (° C)
Iron (Fe)	770	$\text{Nd}_2\text{Fe}_{14}\text{B}$	310
Cobalt (Co)	1125	SmCo_5 , $\text{Sm}_2\text{Co}17$	700-800
Nickel (Ni)	360	AlNiCo	850
Gadolinium (Gd)	19	Permalloy	360-500
Terbium (Tb)	-54	Vanadium Permendur $\text{Co}_{49}\text{Fe}_{49}\text{V}_2$	950
Dysprosium (Dy)	-188	SiFe	500-750

Ferromagnetism characteristics



Hysteresis loop



$$\vec{B} = \mu_0(\vec{H} + \vec{M}) = \mu_0\mu\vec{H}$$

Flux density B , 1 T=1 Vs/m²

Field strength H , A/m

Magnetization M , A/m

Permeability in free space $\mu_0 = 4\pi \cdot 10^{-7}$ Vs/Am(=H/m)

Relative permeability μ ,

In air $\mu \approx 1$, H : 1 A/m = 1.25 μT : B

A typical definition,
 $H_{cj} < 1$ kA/m soft magnetic
 $H_{cj} > 10$ kA/m hard magnetic
 (permanent magnets)

location of the knee of B :

- 2nd quadrant for soft magnetic
- 3rd quadrant for permanent magnets

Permeability



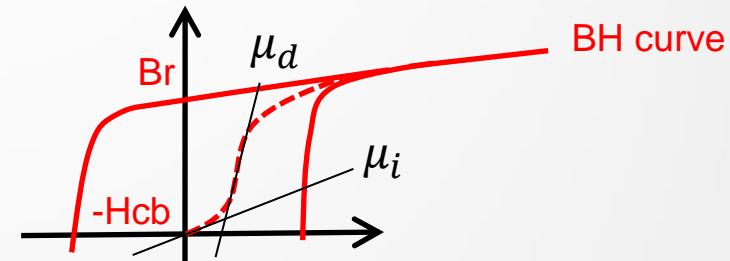
Permeability

- The ratio of B to H .
- Function of H and hence different terms in whole region.
- One of two important terms for the soft magnetic application, the other is M_s

- Hard magnetic :

NdFeB , $\mu_{\parallel} = 1.04$, $\mu_{\perp} = 1.17$

- Soft magnetic :



μ_i initial permeability

μ_d differential or maximum permeability

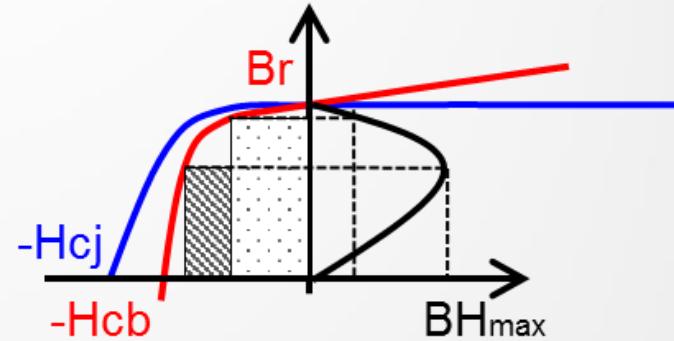
Name	Approximate composition (weight percent)			Initial permeability	Maximum permeability	Coercivity H_c (Oe)	B_s (gauss)	T_c (°C)	Resistivity (microhm-cm)
	Ni	Fe	Other						
Low-Cost Alloys									
Iron	—	100	—	150	5,000	1.0	21,500	770	10
Silicon iron	—	96	4 Si	500	7,000	0.5	19,700	690	60
Grain-oriented silicon iron	—	97	3 Si	1,500	40,000	0.1	20,000	740	47
High-Permeability Alloys									
78 Permalloy	78	22	—	8,000	100,000	0.05	10,800	580	16
Hipernik	50	50	—	4,000	70,000	0.05	16,000	500	45
4-79 Permalloy	79	17	4 Mo	20,000	100,000	0.05	8,700	460	55
Mumetal	77	16	5 Cu, 2 Cr	20,000	100,000	0.05	6,500	62	
Supermalloy	79	16	5 Mo	100,000	1,000,000	0.002	7,900	400	60
High-Saturation Alloys									
Permendur	—	50	50 Co	800	5,000	2.0	24,500	980	7
2V-Permendur	—	49	49 Co, 2 V	800	4,000	2.0	24,500	980	27
Hiperco	—	64	35 Co, 0.5 Cr	650	10,000	1.0	24,200	970	28
Supermendur	—	49	49 Co, 2 V		60,000	0.2	24,000	980	27

Energy product

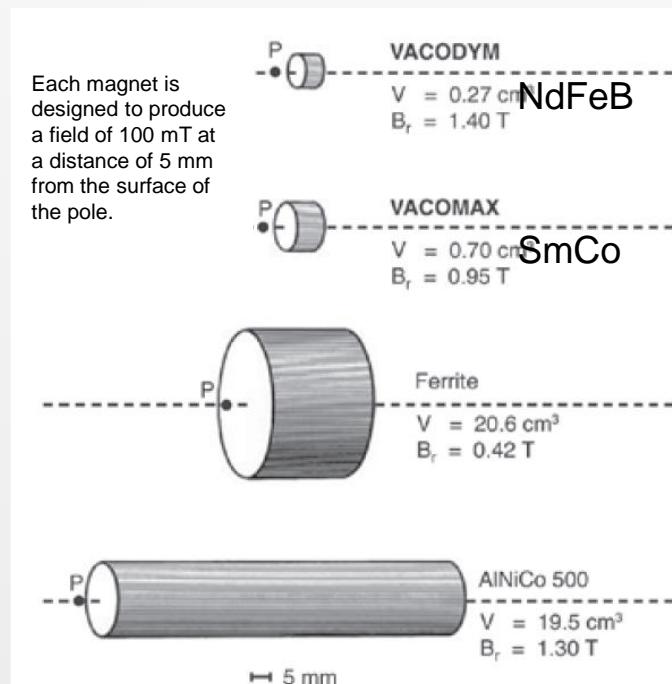


Energy product

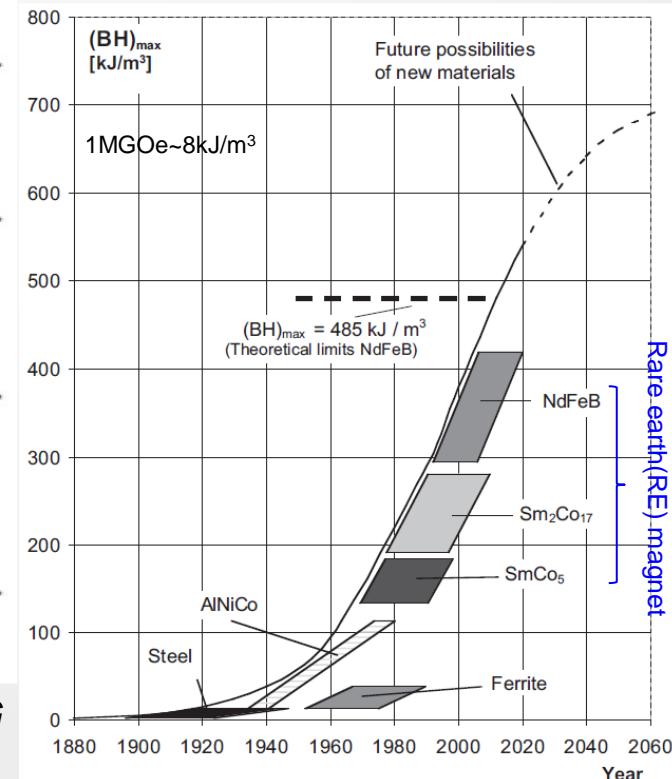
- Energy density and a volume independent magnetic characteristic.
- The largest rectangle under the BH curve.
- The main characteristic for a hard material (permanent magnet).



- ✓ Application generally in air.
- ✓ Higher energy product allows for a reduction in the magnet volume.
- ✓ Working at BH_{max} , the highest efficiency, the highest Induction level at the smallest volume, but not the highest field, larger volumes of the magnet will produce larger magnetic fields.



Vacuumschmelze GmbH & Co. KG



Hard magnetic material

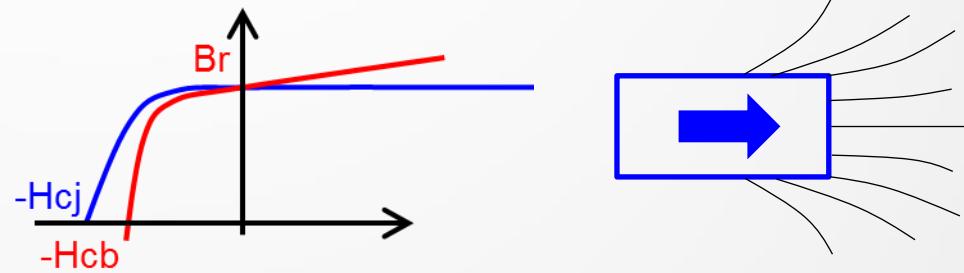


Magnet type I

- $H_{cj} < B_r$
- $\mu \gg 1$

e.g. Alnico

=> High leakage flux, much energy stored in leakage field, not usable.

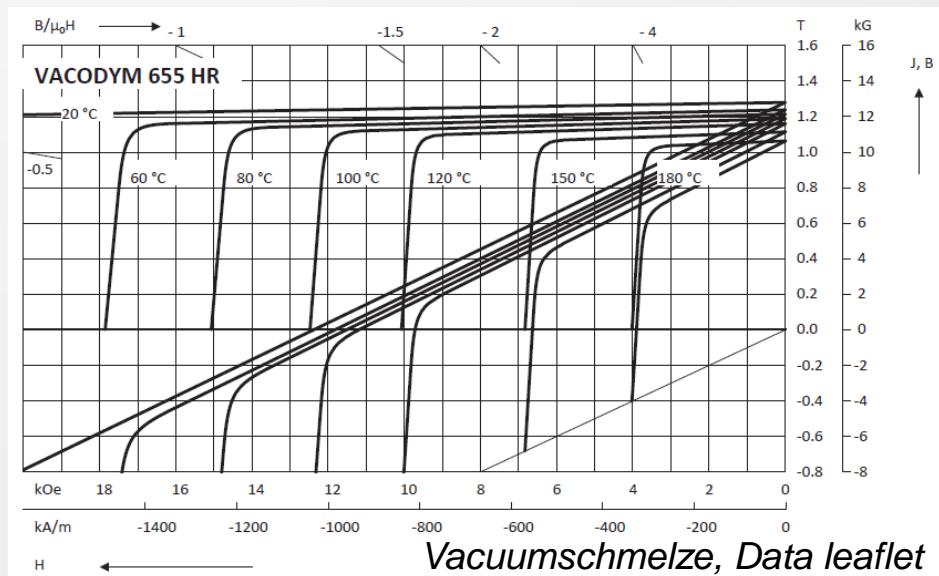
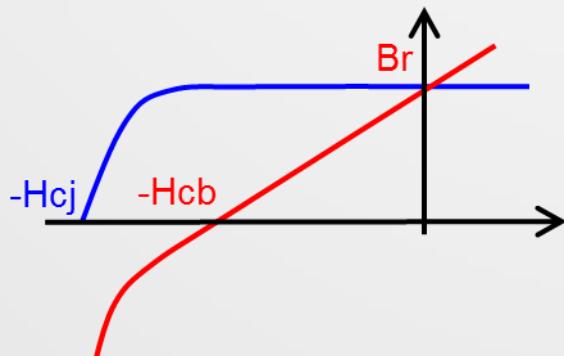
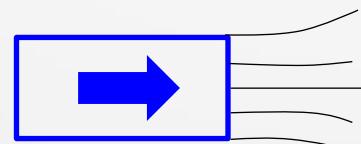


Magnet type II

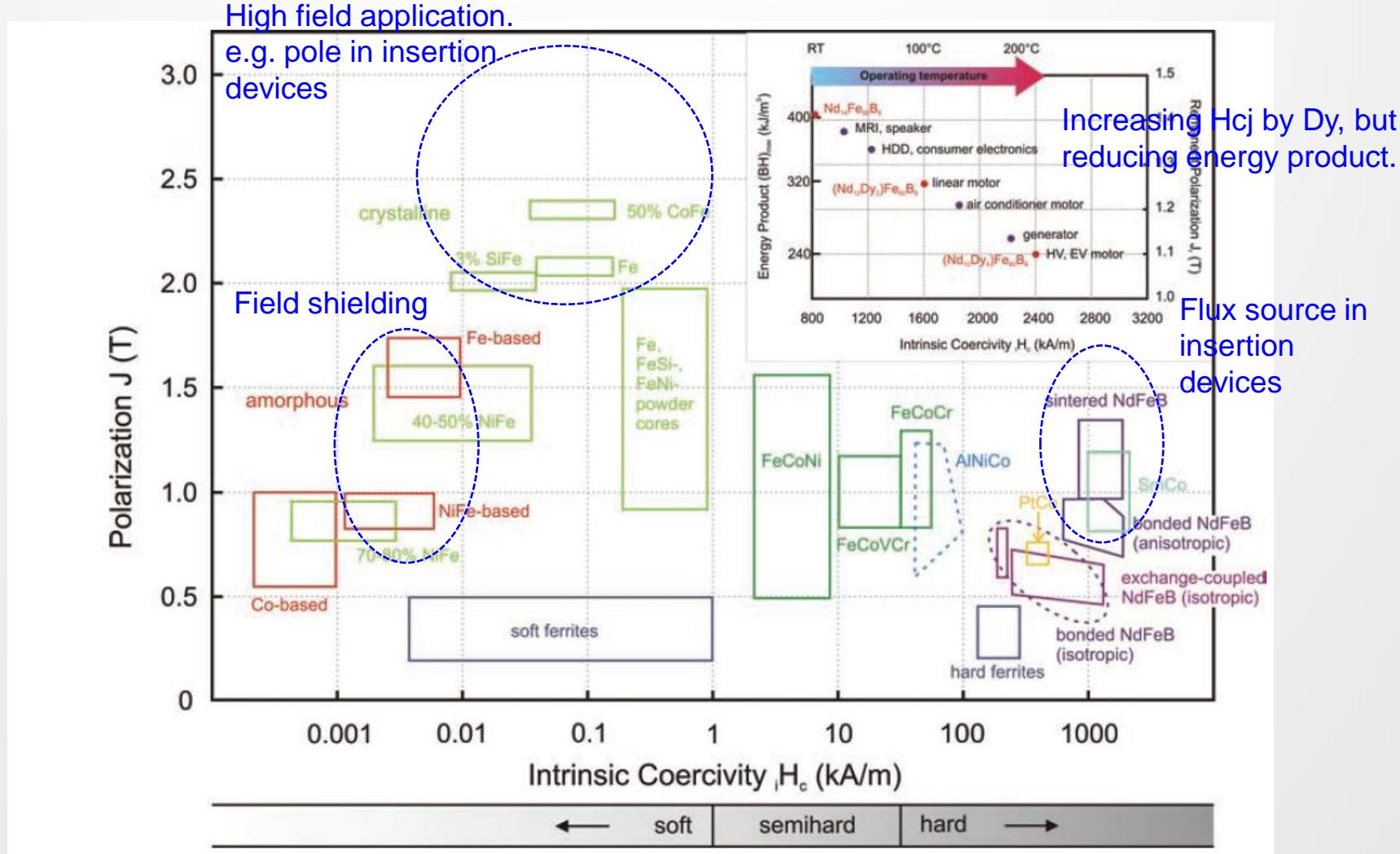
- $H_{cj} > B_r$
- $\mu \sim 1$

e.g. RE-magnet

=> Low leakage flux.



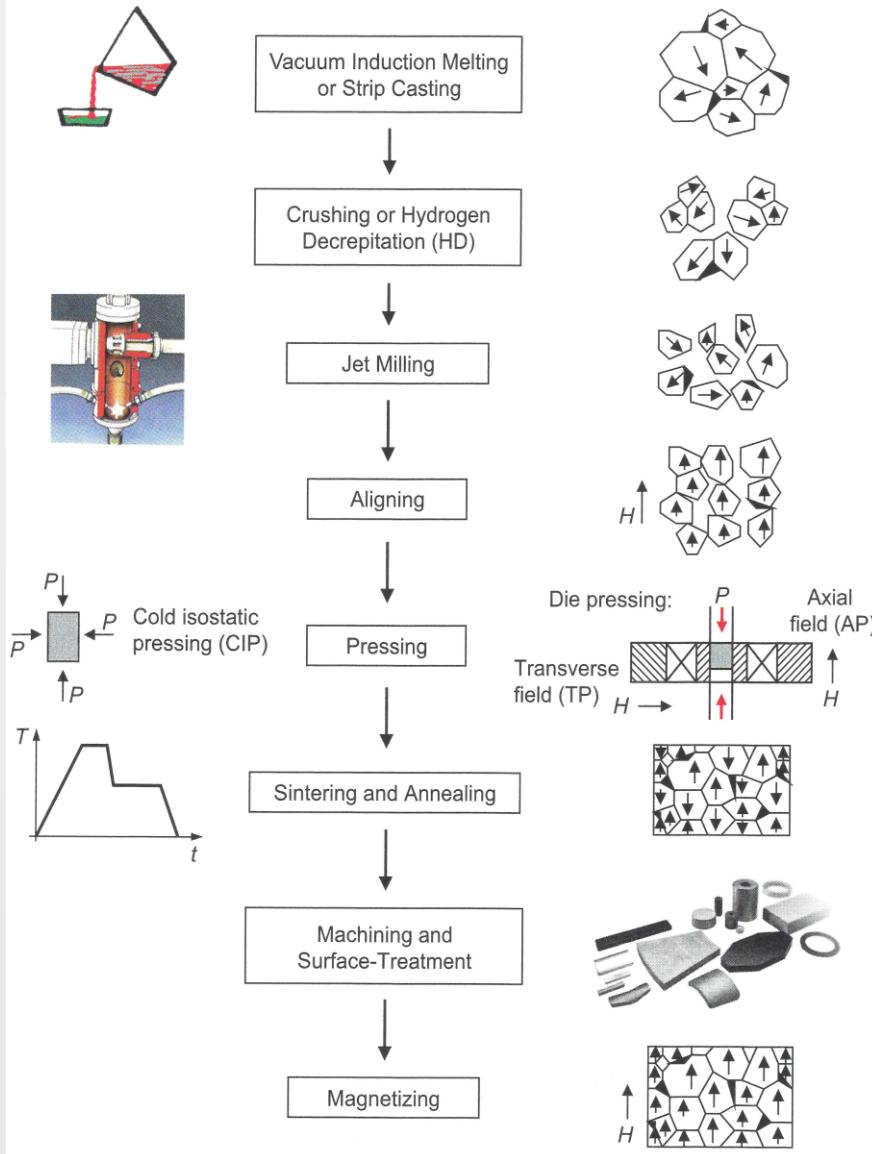
Magnetic material_Summary



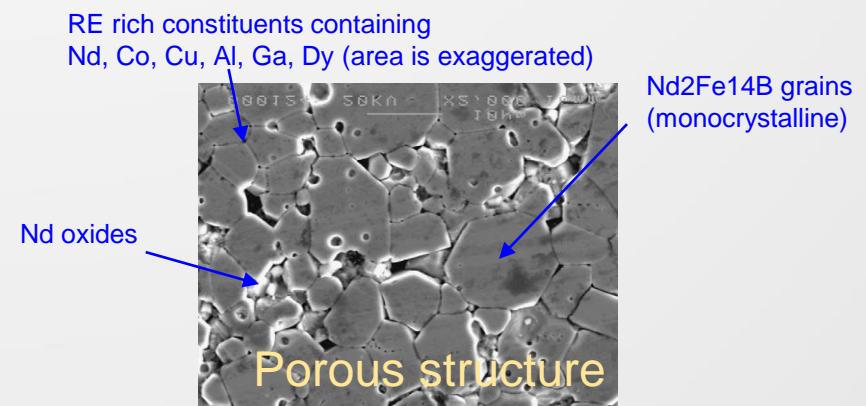
Oliver Gutfleisch et.al., Adv. Mater. 23, 821 (2011).

TingYi Chung 鍾廷翊, 2026, FEL

Sintered NdFeB



- The H_{cj} increases with smaller grain size
- Die pressing process:
 $Br : isostatic > transverse > axial$
Dipole errors: isostatic > transverse, axial
- Hydrogen decrepitation(燃燒) destroys magnetic material.
 $Nd + H_2O \gg NdOH + H$, $H + Nd \gg NdH$
appropriate chemical additions between grains and coating on the surface avoid Hydrogen decrepitation.



Courtesy of Vacuumschmelze

TingYi Chung 鍾廷翊, 2026, FEL

Part III : Technology and conventional insertion devices

ID design_Halbach type



$$\tilde{B} = B_y + iB_z$$

$$= 2B_r \sum_{n=1, p+1, 2p+1, \dots}$$

Sinusoidal along z.

$$\sin\left(2n\pi \frac{z+iy}{\lambda_0}\right)$$

$$\cdot \exp\left(-n\pi \frac{g}{\lambda_0}\right)$$

$$\cdot \frac{\sin[(n\pi/p) - (n\pi\delta/\lambda_0)]}{(n\pi/p)}$$

$$\cdot [1 - \exp(-2\pi n \frac{t}{\lambda_0})]$$

Derived by K. Halbach

t : thickness of block

δ : the air gap between block

g : gap

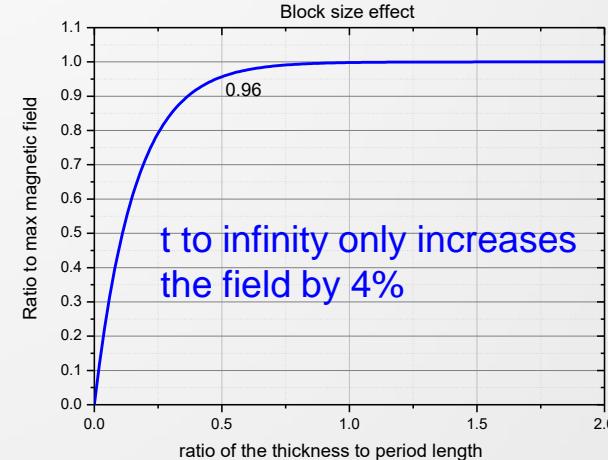
λ_0 : period length

p : blocks per period

Field decreasing with increasing gap and decreasing period

Harmonic field effect

Block size effect



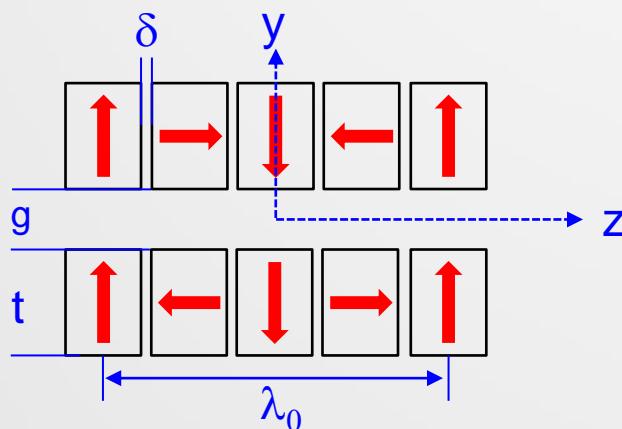
If $\delta=0$, sinc function, high harmonic decreasing.
Larger p, the less harmonics in the field.

$$p = 4,$$

The field only contains harmonics $n = 1, 5, 9, \dots$ but dominated by the fundamental harmonic ($n=1$).

If $t > \lambda_0/2 \Rightarrow \left[1 - \exp\left(-2\pi n \frac{t}{\lambda_0}\right)\right] \sim 1 \Rightarrow b_1 = 0.9, b_5 = -0.18.$
 $\Rightarrow n=1$ dominates

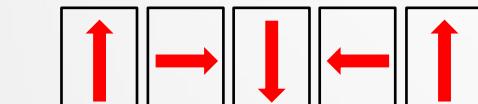
$$B_y(z, 0) = 1.8 \cdot B_r \cdot \exp\left(-\pi \frac{g}{\lambda_0}\right) \cdot \sin\left(2\pi \frac{z}{\lambda_0}\right)$$



Various technologies

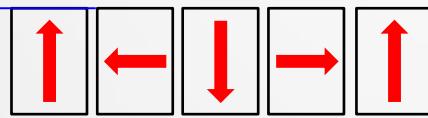


Pure permanent magnet (PPM)

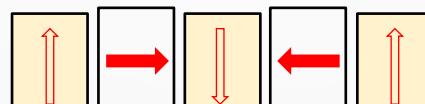


gap

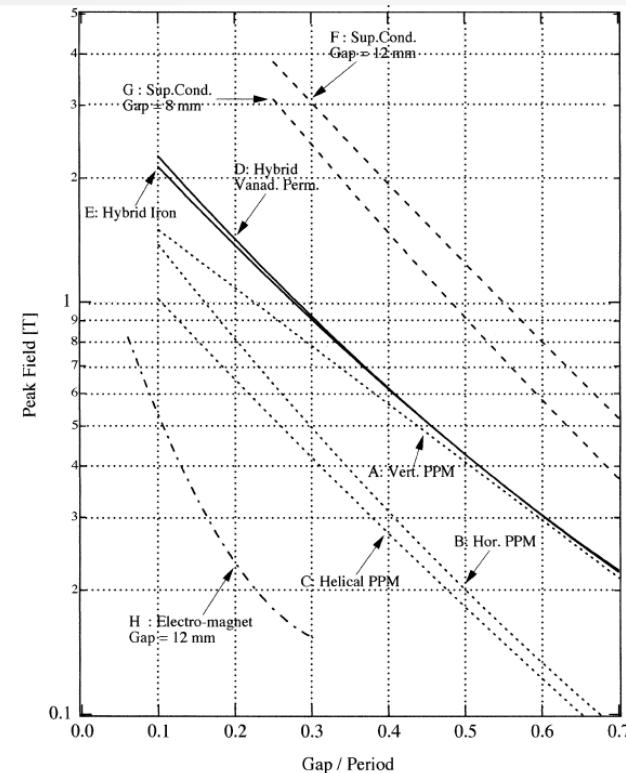
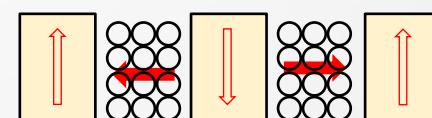
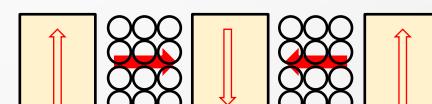
Electron beam



Hybrid, magnet + Iron



Electro-magnet (EM), wire + Iron



$$B_{peak} \approx a \cdot \exp[b \cdot \frac{g}{\lambda_0} + c \cdot (\frac{g}{\lambda_0})^2]$$

	a	b	c
PPM	2.1	-3.2	0
Hybrid	3.7	-5.1	1.52
EM	1.8	-14.3	20.3

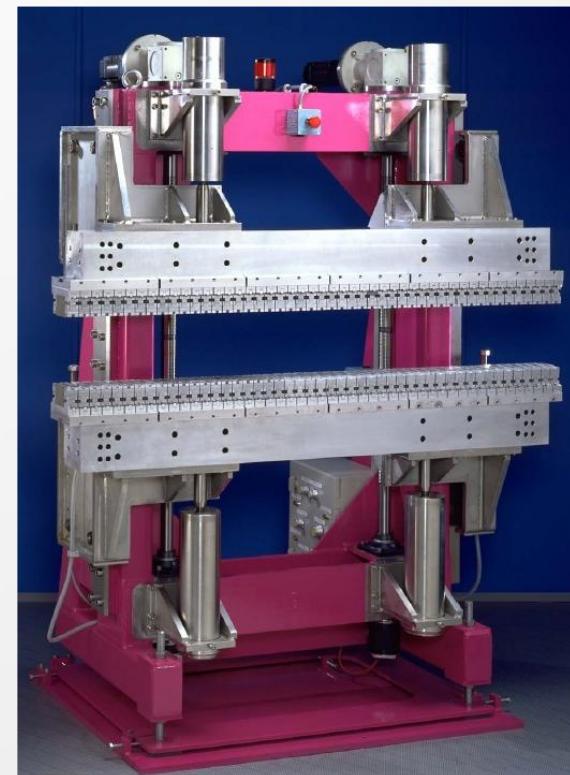
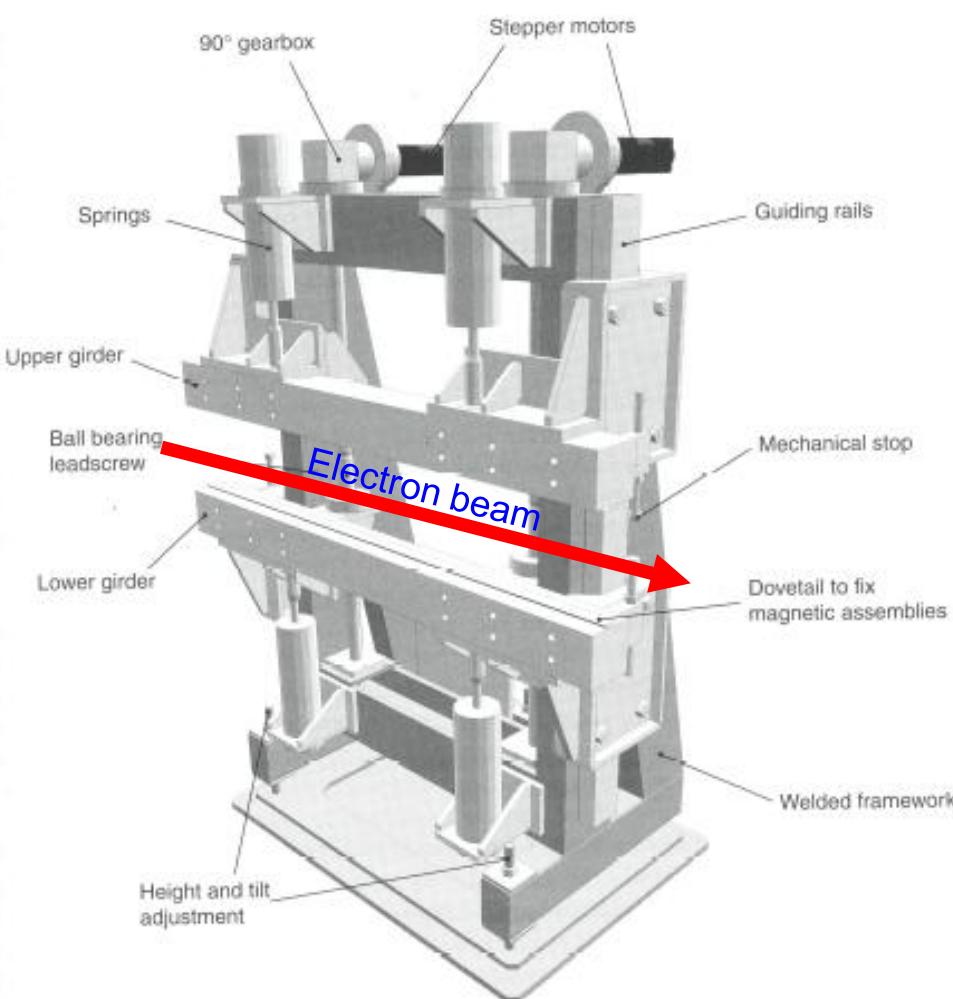
Derived by P. Elleaume

The performance of the Hybrid type is enhanced by the cryogenic technology; the EM type by the superconducting technology.

Typical structure



The ESRF standard support structure



Courtesy of J Chavanne. ESRF

Why does an insertion device need such a large and strong structure?



Magnetic force



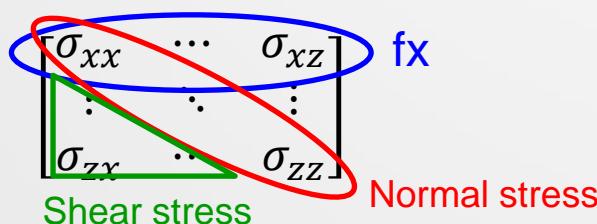
Starting from the Lorentz force law and substituting by Gauss's law and Ampère's circuital law.

$$\Rightarrow \vec{f} + \epsilon_0 \mu_0 \frac{\partial \vec{s}}{\partial t} = \vec{\nabla} \cdot \sigma$$

f : the force per unit volume
 s : Poynting vector
 σ : stress tensor

In the magnetostatic condition,

$$\Rightarrow \vec{f} = \vec{\nabla} \cdot \sigma, \quad \sigma_{ij} = \frac{1}{\mu_0} B_i B_j - \frac{1}{2} \left(\frac{1}{\mu_0} B^2 \right) \delta_{ij} \quad B : \text{magnetic field}$$



Force on the median plane(electron oscillation plane)

$$\Rightarrow F_{x,y,z} = \int \sigma_{(x,y,z)y} w dz \quad w : \text{the width of magnet block}$$

$$\Rightarrow F_y = \frac{w}{2\mu_0} \int_0^L (B_y^2 - B_z^2) dz$$

$$\Rightarrow F_y = \frac{w B_o^2 L}{4\mu_0} \quad B_z = 0, B_y = B_0 \sin(2\pi \frac{z}{\lambda}), \quad L : \text{the length of ID}$$

For a sinusoidal field with peak field B_0 , force between upper and lower girders.

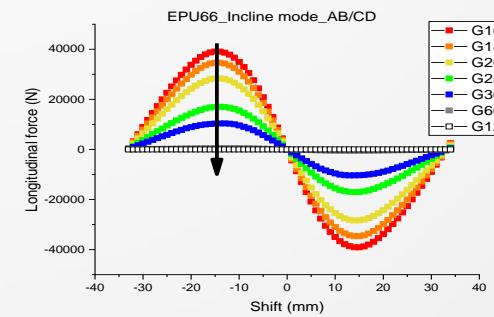
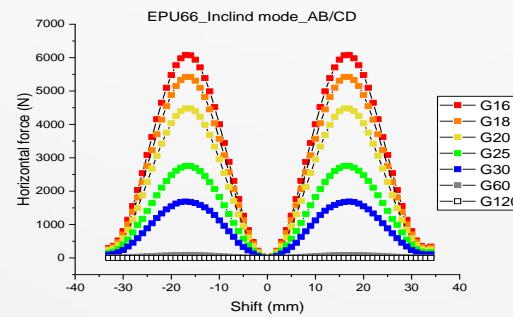
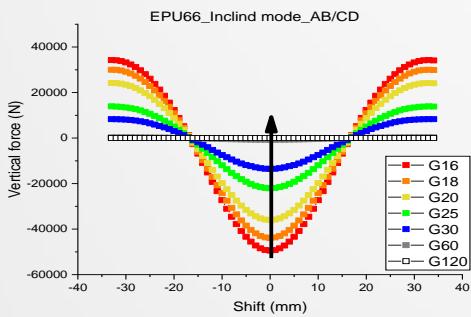
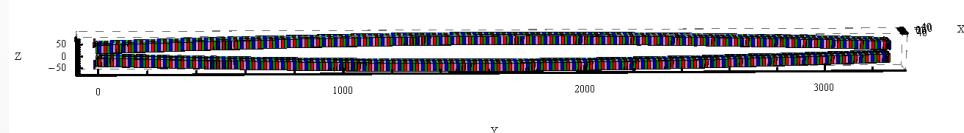
	B	W	L	F
	[T]	[mm]	[m]	[kN]
Undulator	0.8	40	1.6	8.1
Wiggler	1.5	120	1.6	85.9

Construction challenges

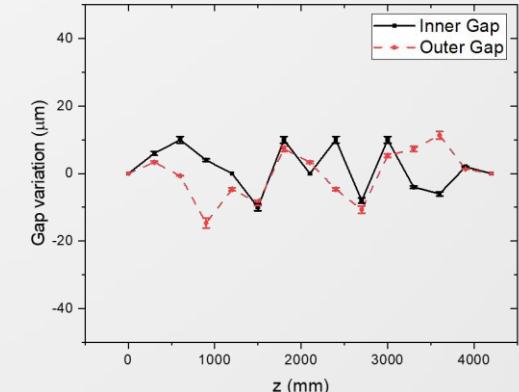
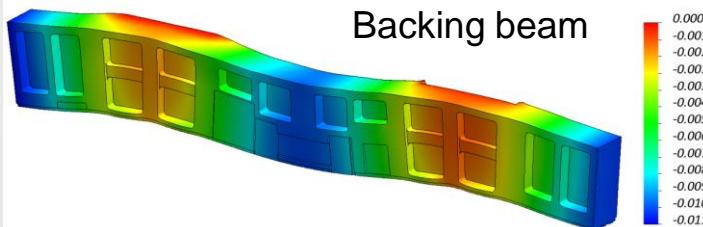
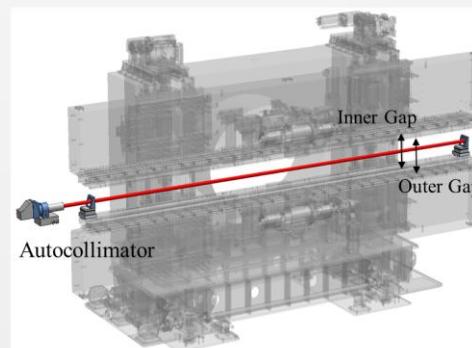
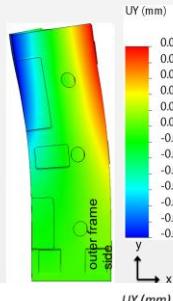
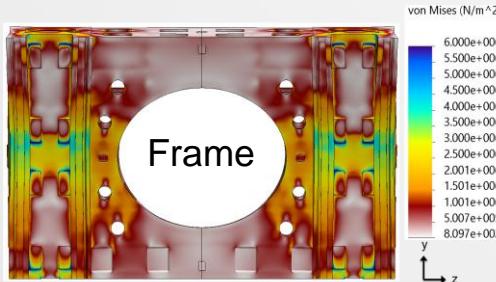


EPU suffers three dimensions of attractive or repulsive forces.

⇒ In addition to mechanical lifetime, mechanical deformation causes systematic errors in the magnetic field.

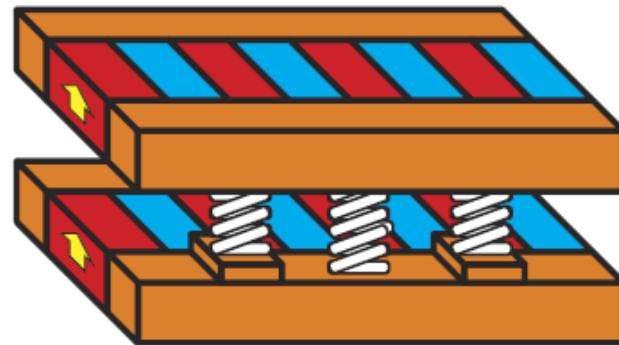


Maximum magnetic force exceeds 5 tons in TPS EPU.

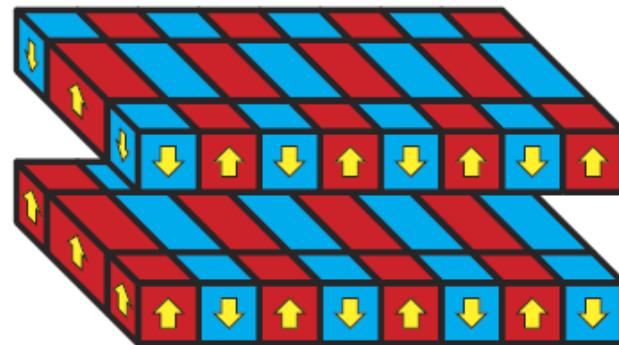


- Optimize mechanical structure: dimensions, supporting points, material..etc.
- Gap variation < 20μm.

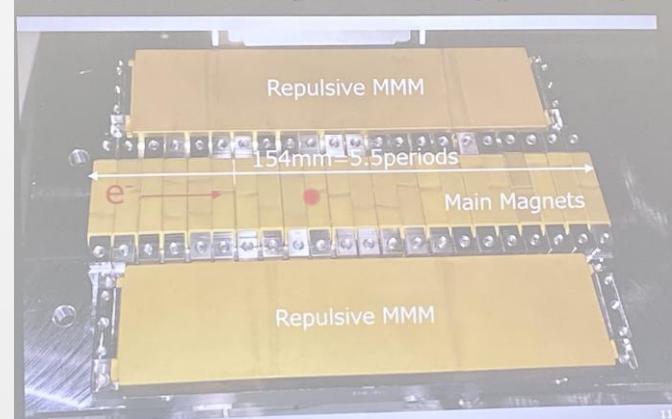
(a)



(b)



Soleil

Structure of a Magnet Module ($\lambda_u=28\text{mm}$)

Spring8

End pole design



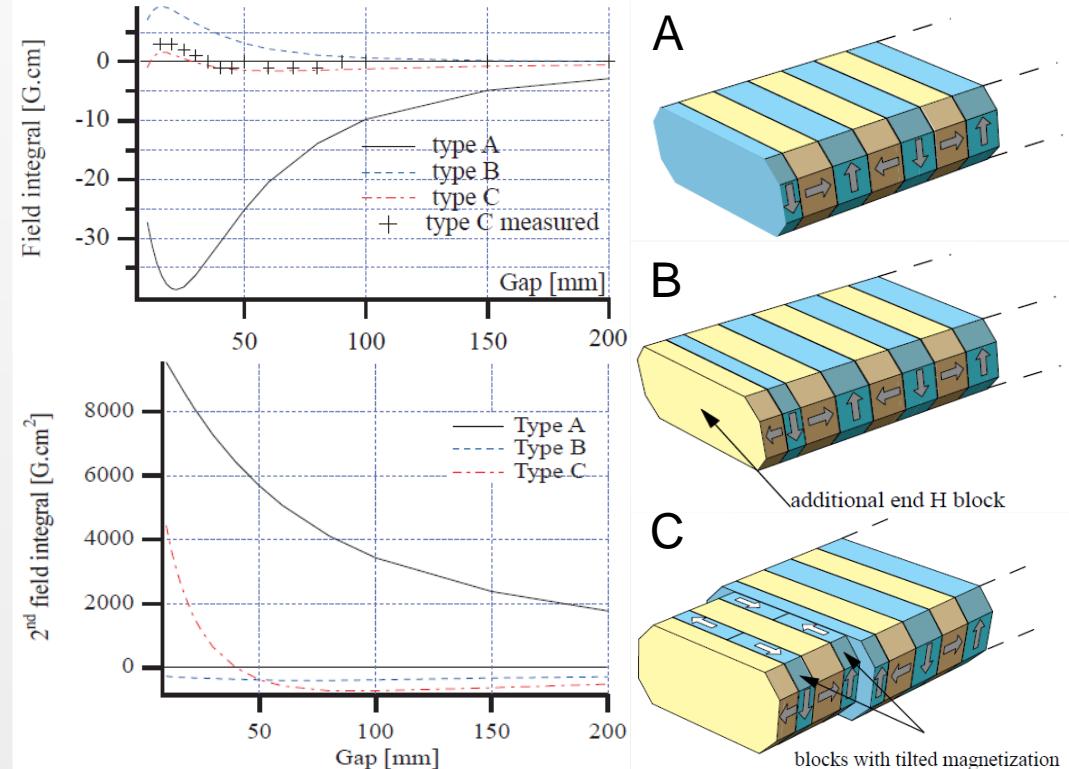
An ideal ID no perturbs the closed orbit and the dynamics of the electron beam in a storage ring for any operation.

- No net kick \Rightarrow No net First field integral
- No net offset \Rightarrow No net Second field integral

The conditions are realized by an optimized end pole design.

For an antisymmetric field with respect to the center of an ID,

- Principally, the first field integral is intrinsic zero.
- A more sophisticated entrance and exit configuration to eliminate the second field integral.



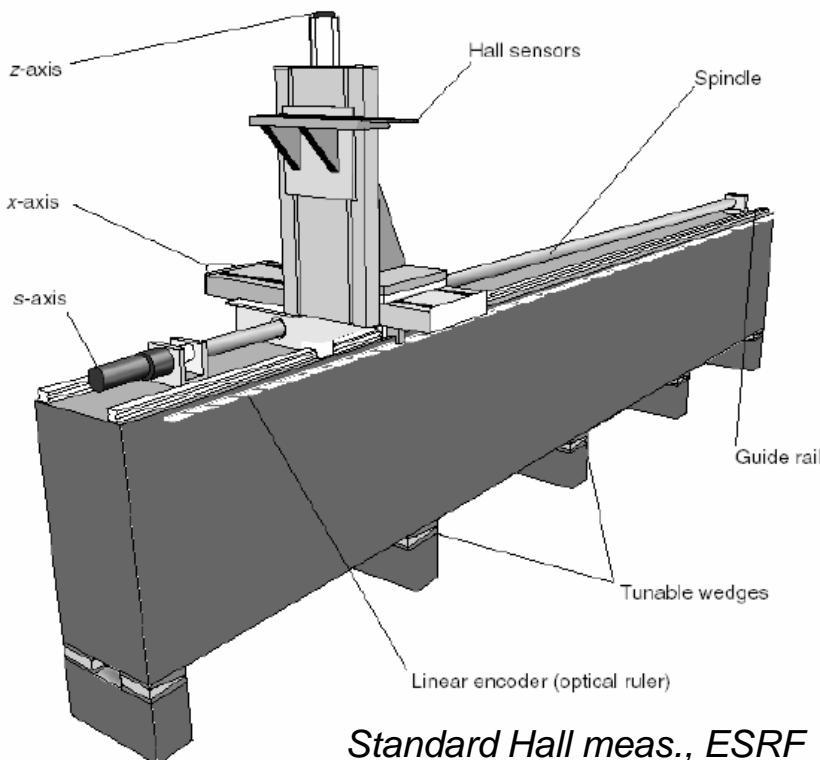
J. Chavanne, PAC(1999)

TingYi Chung 鍾廷翊, 2026, FEL

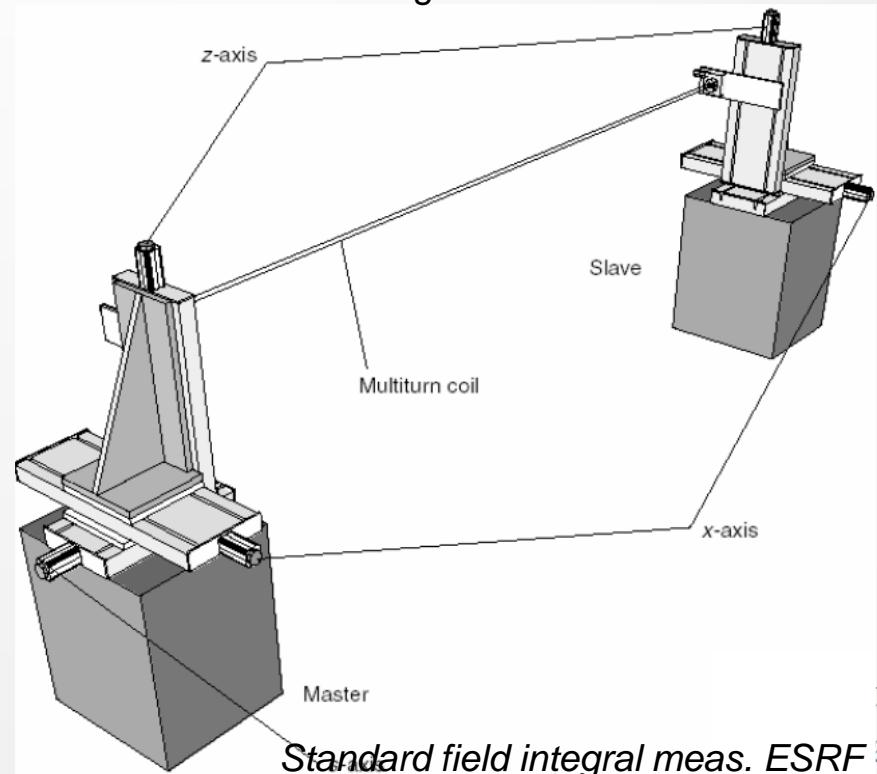
Typical magnetic field measurement system



Local field measurement



Field integral measurement



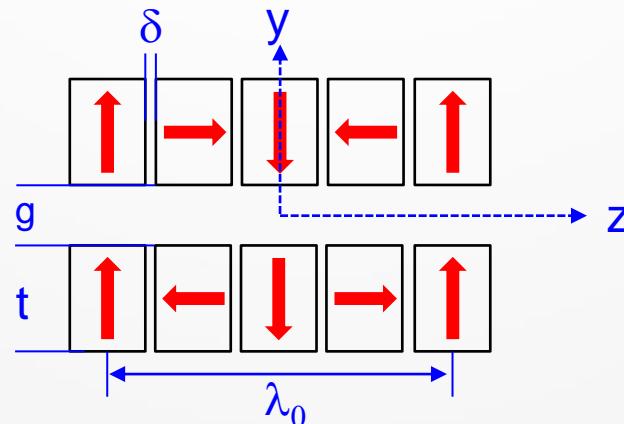
- Hall probe sensor
- On-the-fly scanning
- Laser Encoder
- Essential for phase shimming

- Rotating multturn coil or Moving stretched wire
- Horizontal and Vertical first and second field integrals
- Essential for multipole shimming



EPU48 at Hall probe and field integral measurement benches, 2015, TPS

Magnetic field error



有誤差才是正常的。
不完美才是真實的。

Error sources:

- Non uniform magnetization of the magnet blocks (poles).
- Dimensional and Positional errors of the poles and magnet blocks.
- Interaction with environmental magnetic field.

Influences:

- Disturbing the electron dynamics in the storage ring => **multipole error**
- Reduction in the SR intensity. => **phase error**

Solution:

- High quality blocks. Requiring small variation in, for example dipole error, magnetic flux density of north/south side and mechanical dimensions...
- Efficient sorting and shimming algorithms, which are more important for a mass production of IDs for FEL.

Phase error



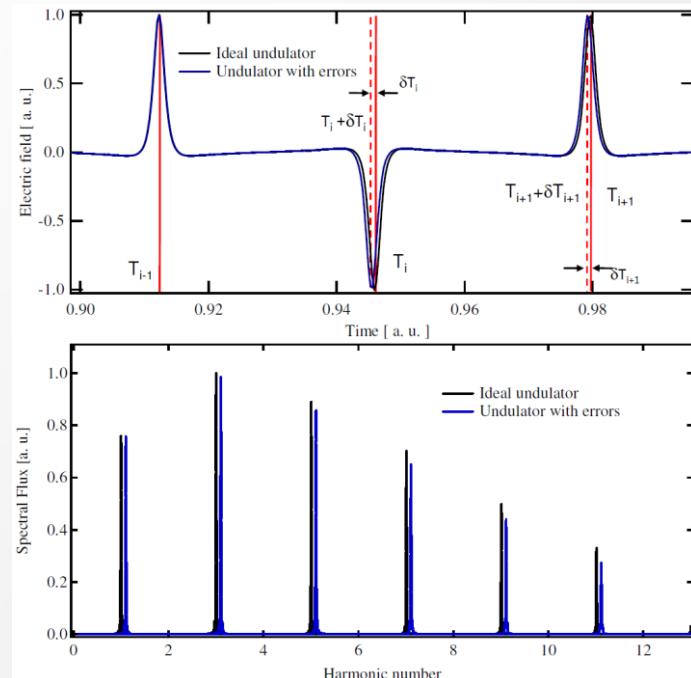
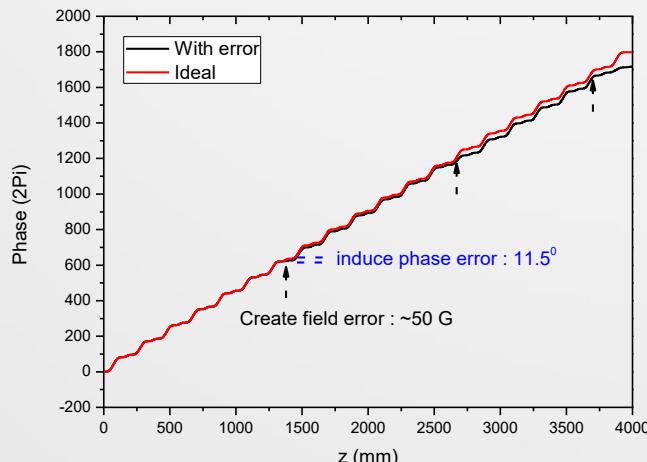
- An ideal ID, $2N$ peaks of the electric field with **equally space of T_i** go to the observer.

F.T.
⇒ Numerous harmonics.

- Magnetic field error shift T_i .

F.T.
⇒ Change the fundamental frequency and introduce destructive interference.

- Calculating the phase ϕ_i , the slippage of one optical wavelength between the electron and the light.
- Magnetic field error => change the longitudinal velocity of an electron => create phase error $\delta\phi_i$



For randomly distributed $\delta\phi_i$, the reduction of flux and brilliance on the spectrum harmonic is given by

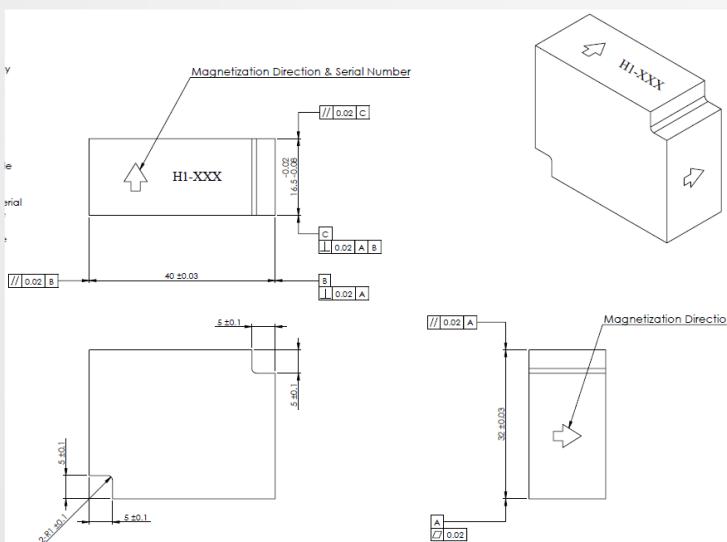
$$I_n(\sigma) = I_n(0) \exp(-n^2 \sigma^2)$$

σ : r.m.s. phase error

需求規格

Table 1. Overview of magnets for EPUT66.

Item	Magnet type	Number of Blocks
1	H1	350
2	V1	350
3	H2	350
4	V2	350
5	H1E1	20
6	H1E2	20
7	H2E1	20
8	H2E2	20
9	V1E	20
10	V2E	20
11	MF-CY	4200



實際數據

Magnet Type	H1 for BEAMLINE33 (BL33) – EPUT66				
Number of Blocks	350				
Parameter	Specification	Minimum	Average	Maximum	Accept
ΔJ_r	$\pm 1\%$	-0.37 %	1.2552 T	0.56 %	YES
$\Delta \alpha$	$\pm 1^\circ$	-0.3 $^\circ$	0.1 $^\circ$	0.4 $^\circ$	YES
$\Delta \beta$	$\pm 1^\circ$	-0.3 $^\circ$	0.1 $^\circ$	0.5 $^\circ$	YES
N/S	N/A	N/A	N/A	N/A	YES
Height	32.0 mm	31.976 mm	31.998 mm	32.010 mm	YES
Width	40.0 mm	39.976 mm	39.997 mm	40.013 mm	YES
Thickness	16.5 mm	16.422 mm	16.434 mm	16.440 mm	YES

Magnet Type	H2 for BEAMLINE33 (BL33) – EPUT66				
Number of Blocks	350				
Parameter	Specification	Minimum	Average	Maximum	Accept
ΔJ_r	$\pm 1\%$	-0.37 %	1.2540 T	0.44 %	YES
$\Delta \alpha$	$\pm 1^\circ$	-0.5 $^\circ$	-0.1 $^\circ$	0.3 $^\circ$	YES
$\Delta \beta$	$\pm 1^\circ$	-0.3 $^\circ$	0.0 $^\circ$	0.4 $^\circ$	YES
N/S	N/A	N/A	N/A	N/A	YES
Height	32.0 mm	31.982 mm	31.997 mm	32.011 mm	YES
Width	40.0 mm	39.978 mm	39.999 mm	40.008 mm	YES
Thickness	16.5 mm	16.428 mm	16.434 mm	14.440 mm	YES

Magnet Type	V1 for BEAMLINE33 (BL33) – EPUT66				
Number of Blocks	350				
Parameter	Specification	Minimum	Average	Maximum	Accept
ΔJ_r	$\pm 1\%$	-0.40 %	1.2550 T	0.42 %	YES
$\Delta \alpha$	$\pm 1^\circ$	-1.0 $^\circ$	-0.1 $^\circ$	1.0 $^\circ$	YES
$\Delta \beta$	$\pm 1^\circ$	-0.2 $^\circ$	0.2 $^\circ$	0.4 $^\circ$	YES
N/S	N/A	N/A	N/A	N/A	YES
Height	32.0 mm	31.978 mm	31.995 mm	32.020 mm	YES
Width	40.0 mm	39.982 mm	39.997 mm	40.016 mm	YES
Thickness	16.5 mm	16.432 mm	16.440 mm	16.447 mm	YES

Magnets Sorting



EPU is composed of more than 1000 “non-identical” magnets, with errors of magnetic moments.

- ⇒ To reduce random error, a strategy of magnet sorting and shimming process are developed in NSRRC.
- ⇒ To ensure good light's quality.



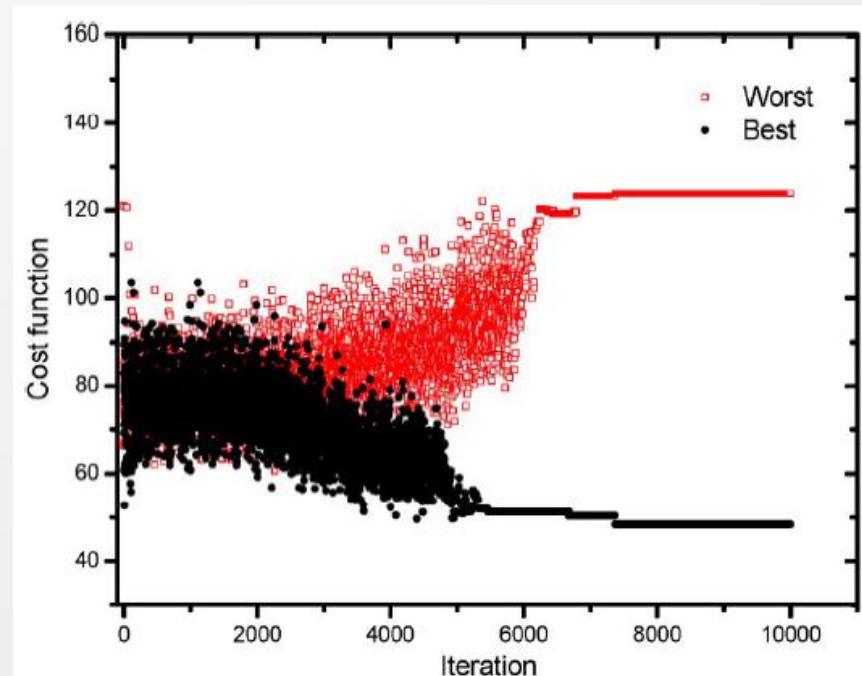
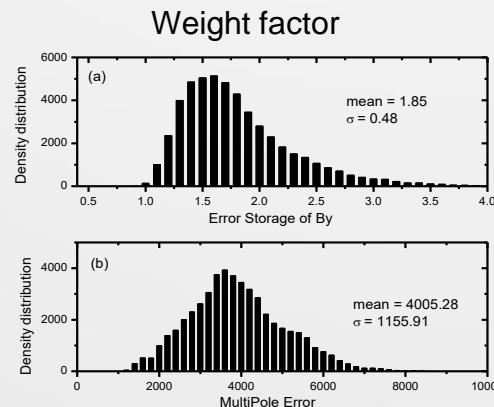
The cost function consists of terms for multipole error and phase error, which means it takes into account the major influences on the electron beam and the spectrum.

$$E = aMP + bESy + cESx + dFSy + eFSx$$

$$MP = \sum_{m=1}^N (SQ^2 + NQ^2)_m$$

$$FS = \sum_{m=1}^n \frac{I_n - (-1)^n \langle I \rangle}{\langle I \rangle}$$

$$ES = \sum_{m=1}^n \left(\frac{|I_n| - \langle I \rangle}{\langle I \rangle} \right)_m$$



The optimization of the magnet sorting is based on simulated annealing [12]. This method is similar to a gradual cooling of liquids to form a crystalline state, as opposed to an amorphous state after a sudden cooling.

 d20240627_sorting code loop.nb *

Simulated Annealing_for four rows

數量管理

3. Cost Functions (Multivariate)

1.1 项目管理师师级胜任力的核算

4. 組裝是以HL的對齊方式下，每一列由第一個M7開始，由上層往下降組裝
 5. NonUnit effect & Mechanical frame move 一組背景，方式是2/4 需要加為一列後，再加上由實體資源 Bind module 連接的資源的項目 6/6 數值

7. 研讀時，應熟讀子句 `submodule index(8) pole index(8)` 之類

note :
d0200425 HI. model得，只有把note由b->t，所以field storage的第一個note由bnote變正note，程式無錯誤。VI. model沒

```
(+1. Input data+)
(+raw data+)
(+RP066 submodule 分别读M5和M7，两者的数据pole要取10個+)
(+读数据RP043 code，所以code字节M7代表RP066 M5，M2代表RP066 M7+)
```

```
Beginning = DateString[]];
M7 = ReadList["D:\\NSRRC2016\\NSRRC\\InsertionDe

MatrixForm[4];
M8 = ReadList["D:\\NSRRC2016\\NSRRC\\InsertionDe
```

```

MatrixForm[•];
n7 = Dimensions[N7][[1]]; (*一台變形的N7數目*)
n9 = Dimensions[N9][[1]]; (*一台變形的N9數目*)
Fullrange7 = Range[n7];
Fullrange9 = Range[n9];

```

NonSubmitter = Platfrom [ReadThisList ("D:\1\NSP")]

```

NonUnitIx = Flatten[ReadList["D:\\NSR\\
Print["Nonunit effect dIy/Iy_4"];
Dimensions[NonUnitIx][1]];

```

```

ListLinePlot[NonunitIx, Mesh + Full];
Print["Nonunit effect dIx/Ix_N"]
Dimensions[NonUnitIx][[1]];
ListLinePlot[NonUnitIx, Mesh + Full]

```

```
{built initial arranges}
{submodule 整合}
Work = 1;{Worst or Best cases, 1:Best case,
```

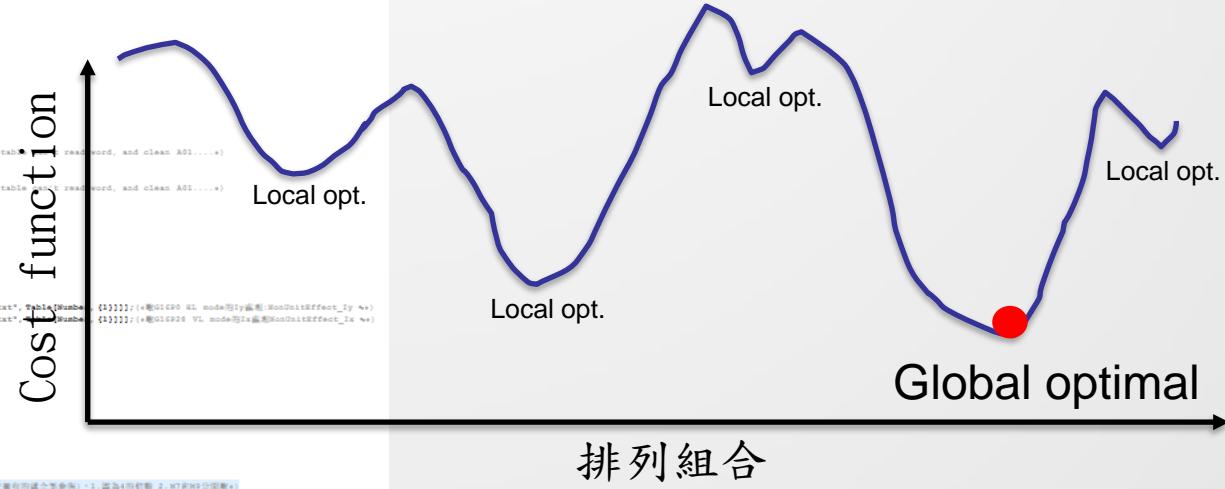
```
Pickup = 92;(*EP0556: M7有92面、M5有96面、冰点面98面)  
EX = 2;(*虚设module模块数，M7和M9是分别需要的*)  
  
(spole 盘#)  
nPub = 10;(*spole number of each submodule,EP0556: M7有10个子模块、M5有12个子模块、冰点面12个子模块*)
```

```

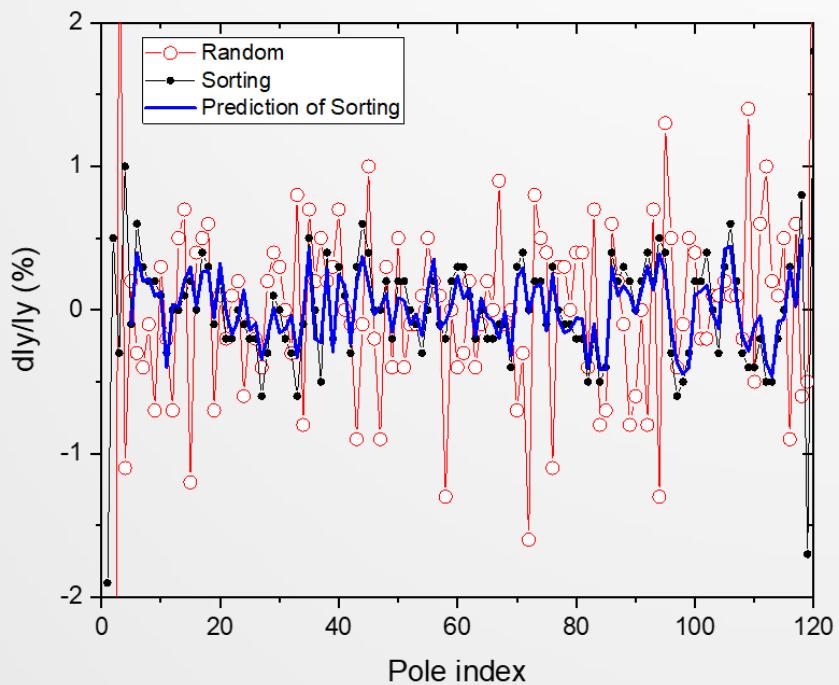
nfullPionsub = 4; /*EPOS6 case M7主要用pole num
nfullPentaub = 2; /*EPOS6 case M5主要用pole num
nPGSeq = 4; /*EPOS6: 上面由M5開始使用(*4,EPOS6;
nPGSeq = 4; /*EPOS6: 下面由M5開始使用(*4,EPOS6;
nfullPionsub = 4; /*PICKUP nfullPionsub (*4,PICKUP = 4;

```

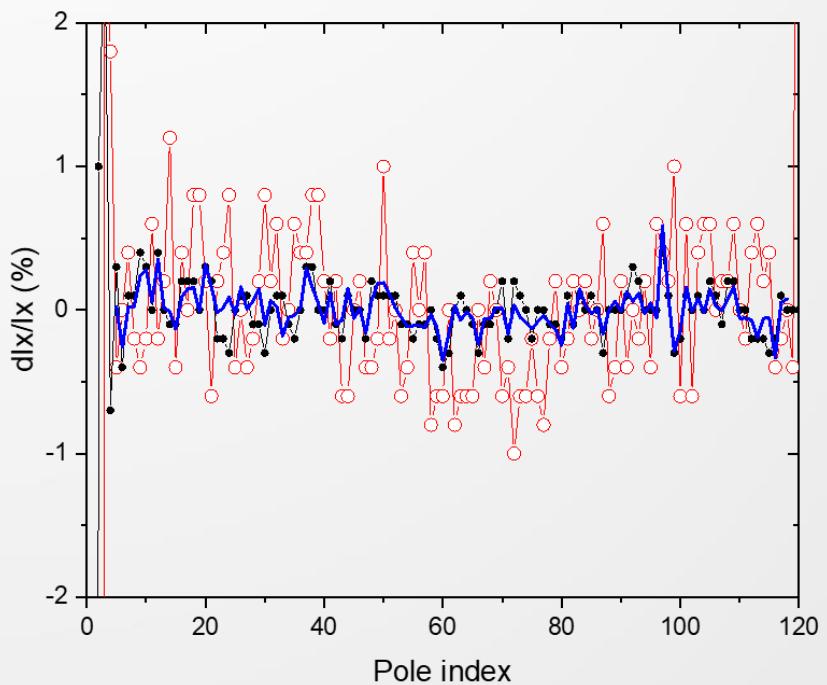
- 隨機方式:後半的最佳化沒效率，越難找到最佳解，即便是找local optimal。
- 局域尋找:很容易受限於初始條件，而只能找到鄰近的local opt.



(a)



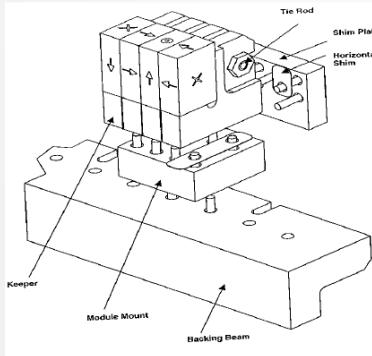
(b)



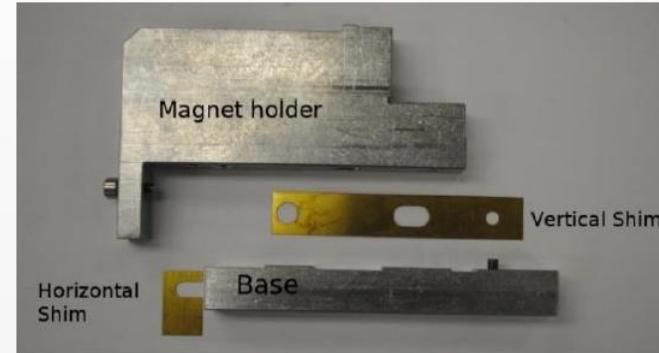
Field shimming



- **Mechanical:** Moving permanent magnet or iron pole vertically or horizontally.



S. Mark, ALS
(1998)

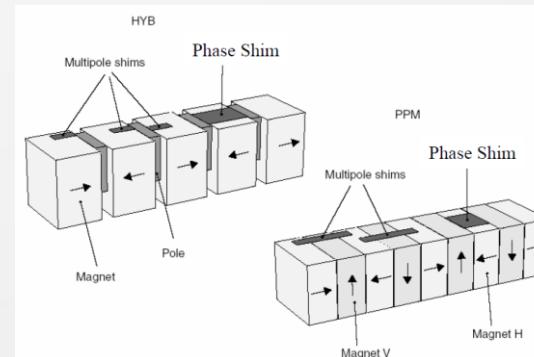


NSLSII,(2016)

- **Magnetic:** Add thin iron piece at the surface of the blocks.



LCLS,
(2004)



ESRF,
standard
Undulator

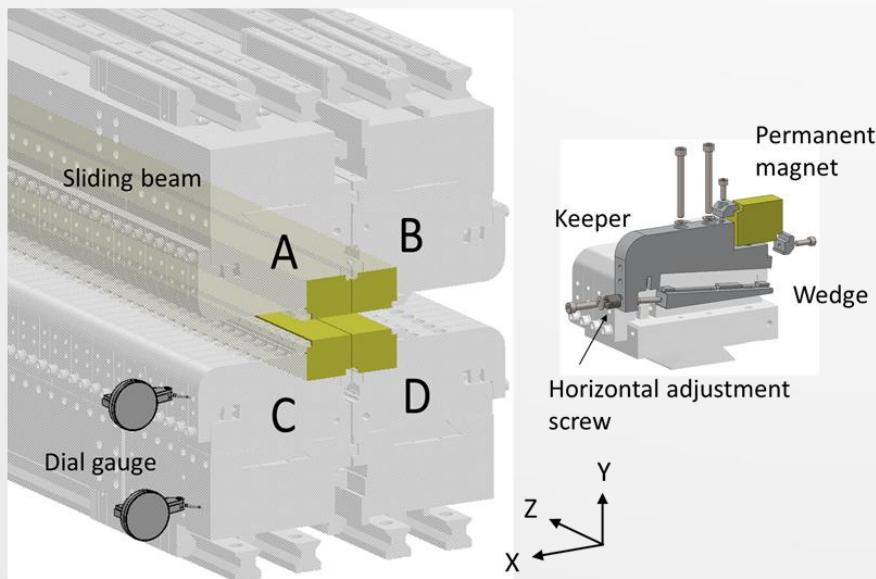
- **Swapping/flipping blocks:** Important to maintain the flat surface of a magnet array, especially in IVU.

Field shimming



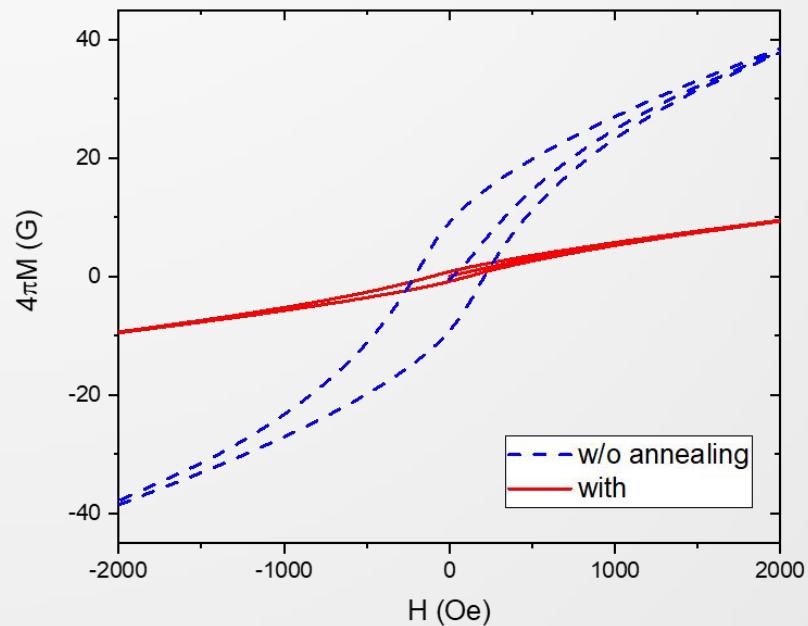
- Mechanical: Moving permanent magnet pole vertically or horizontally.

(a)



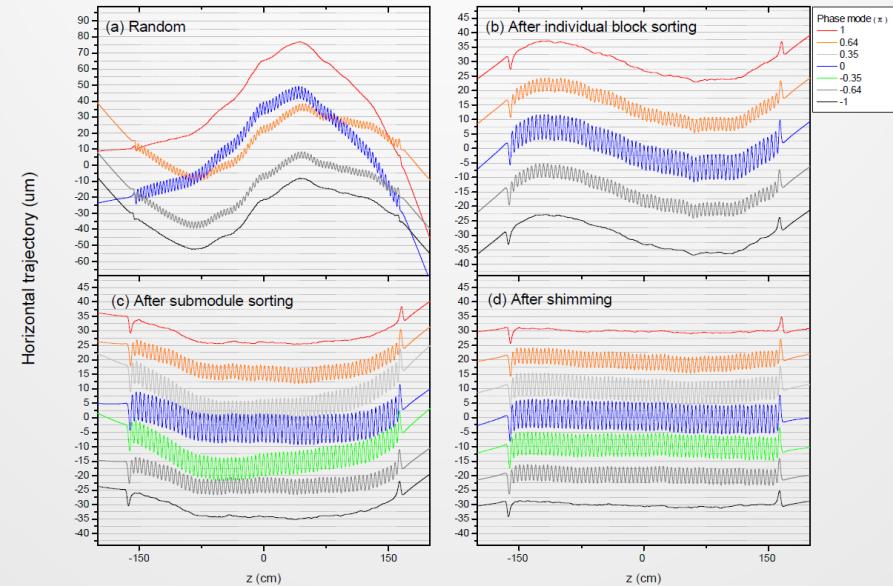
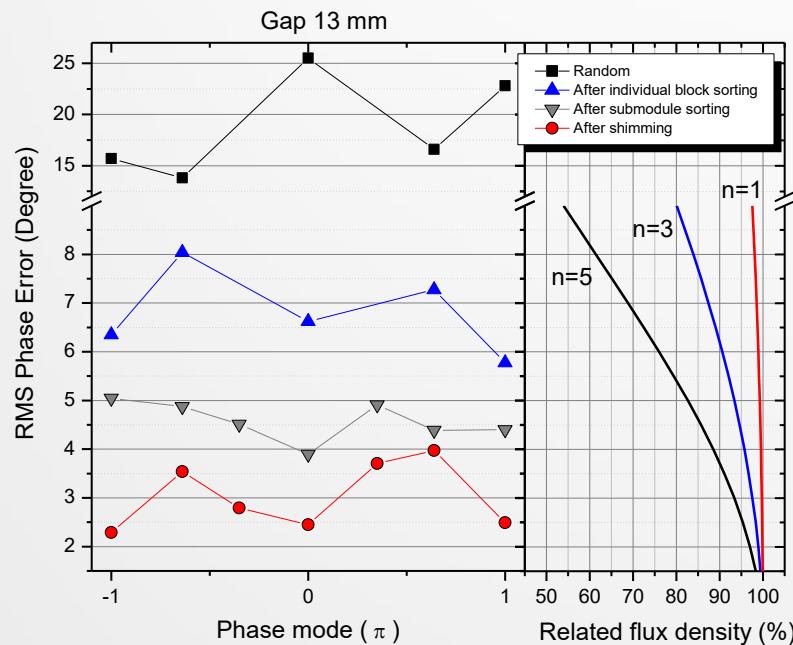
(b)

Demagnetizing by annealing



BH curve measurement by VSM

Field performance of ID at NSRRC



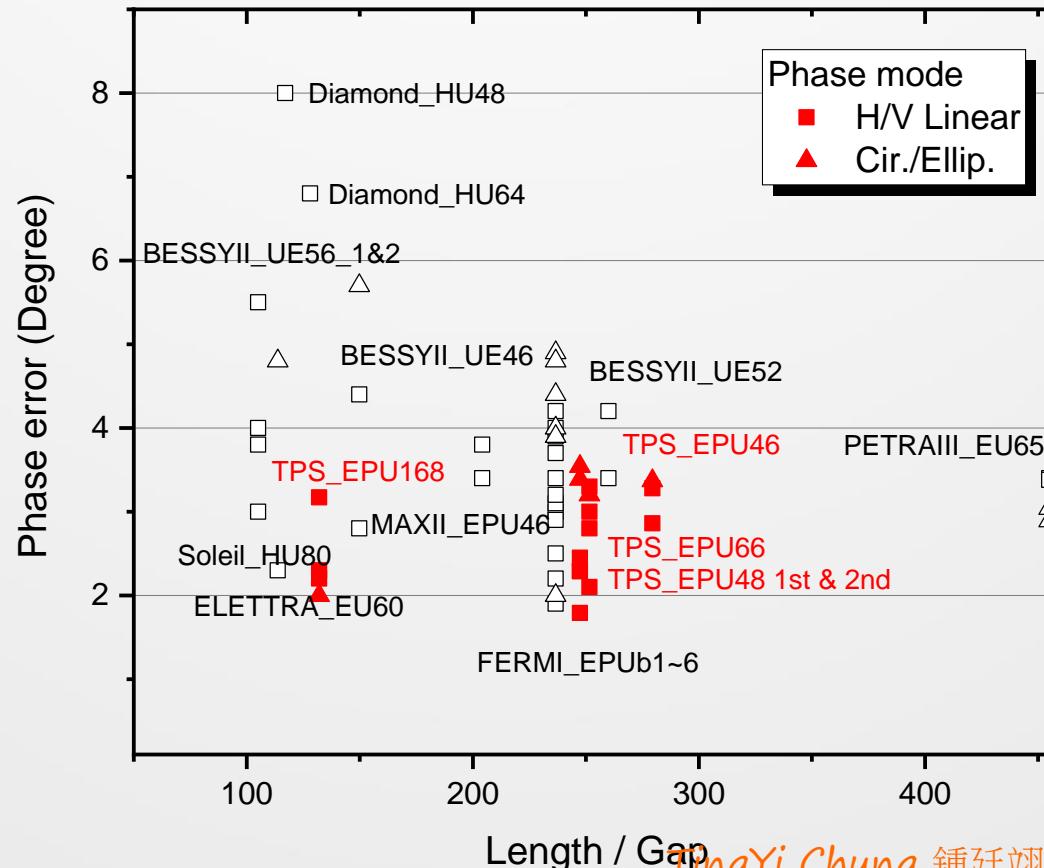
- Efficient sorting and shimming can significantly shorten the construction time.
- The flux density of high harmonics is greater than 95% of an ideal value.
- Straightness of trajectories satisfies the FEL requirement.

Field performance of ID at NSRRC



Position in EPU map:

- Constructing a longer and smaller gap of EPU is much difficult.
- Stiff mechanical structures and good quality magnets are essential.
- Mechanical arts and magnetic field treatments are equal important for a high performance EPU.



Trends in undulator for FEL and Storage ring



To reach the highest photon energy at a fixed electron energy

To increase the Brilliance for a compact device

⇒ the period should be made as short as possible.

$$\lambda_r = \frac{\lambda_u}{n2\gamma^2} \left(1 + \frac{K^2}{2} + \gamma^2 \theta^2\right)$$

In general, we can't just have short period and accept very small K, because

- The tuning range is too small, a sufficient overlap between n=1 and 3, $K \geq 2.2$.
- The FEL coupling too low (undulator length has to be very long to reach saturation),
 $K \geq 1$ (this is just a guide, not physics)

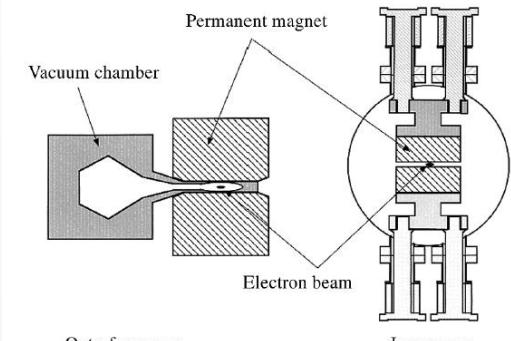
How to shorten the period length, and keep the K value?

In other words, how to create a large magnetic field in a short period?

In-vacuum Undulator



- The minimum magnet gap, which is generally set by the vacuum chamber, limits the magnetic field of an ID.
- To put the undulator magnet inside the vacuum chamber.
- Simple idea, but a lot of difficulties should be overcome to realize the IVU.



H Kitamura, Spring-8

Year	Facility	Contents	Remarks
1990	KEK	$\lambda_u = 40\text{mm}$, $G_{\min} = 10\text{mm}$, $L = 3.6\text{m}$	1st IVU that works regularly
1996	SPring-8	$\lambda_u = 32\text{mm}$, $G_{\min} = 7\text{mm}$, $L = 4.5\text{m}$	1st IVU for SPring-8
1996	SPring-8 & ESRF	$\lambda_u = 24\text{mm}$, $G_{\min} = 5\text{mm}$, $L = 1.5\text{m}$	Beam test of IVU at ESRF
1997	SPring-8	1st on-beam commissioning	4 IVUs have been installed from the beginning
1997~	SPring-8	IVUs with exotic PM configurations	Vertical, helical and figure-8
1997	SPring-8 & NSLS	$\lambda_u = 11\text{mm}$, $G_{\min} = 2\text{mm}$, $L = 0.3\text{m}$	
1999~	SPring-8 & PLS	Demagnetization test of PM material	2GeV linac (PLS), 8GeV synchrotron (SPring-8)
2000	SPring-8	$\lambda_u = 32\text{mm}$, $G_{\min} = 12\text{mm}$, $L = 25\text{m}$	IVU for the long straight section in SPring-8
2000	SPring-8 & SLS	$\lambda_u = 24\text{mm}$, $G_{\min} = 5\text{mm}$, $L = 1.5\text{m}$	Same as that tested at ESRF(magnet refreshed)
2000	SPring-8	$\lambda_u = 6, 15, 20, 24\text{mm}$, $L = 1\text{m}$	Revolver undulator (installed in PLS in 2003)
2001	ESRF	$\lambda_u = 17 \sim 23\text{mm}$, $G_{\min} = 6\text{mm}$, $L = 2\text{m}$	$\text{Sm}_2\text{Co}_{17}$ is employed (lower radiation damage)
2003	SPring-8	$\lambda_u = 15\text{mm}$, $G_{\min} = 2\text{mm}$, $L = 4.5\text{m}$	for SCSS project
2003	KEK	$\lambda_u = 40\text{mm}$, $G_{\min} = 10\text{mm}$, $L = 3.6\text{m}$	Tapered undulator
2003	SLS	$\lambda_u = 19\text{mm}$, $G_{\min} = 5\text{mm}$, $L = 1.9\text{m}$	Assembled at SPring-8
2004	ALS	$\lambda_u = 30\text{mm}$, $G_{\min} = 5\text{mm}$, $L = 1.5\text{m}$	Assembled at SPring-8
2006	SSRL	$\lambda_u = 22\text{mm}$, $G_{\min} = 5\text{mm}$, $L = 1.5\text{m}$	Assembled at SPring-8

T. Tanaka et. al. FEL (2005)

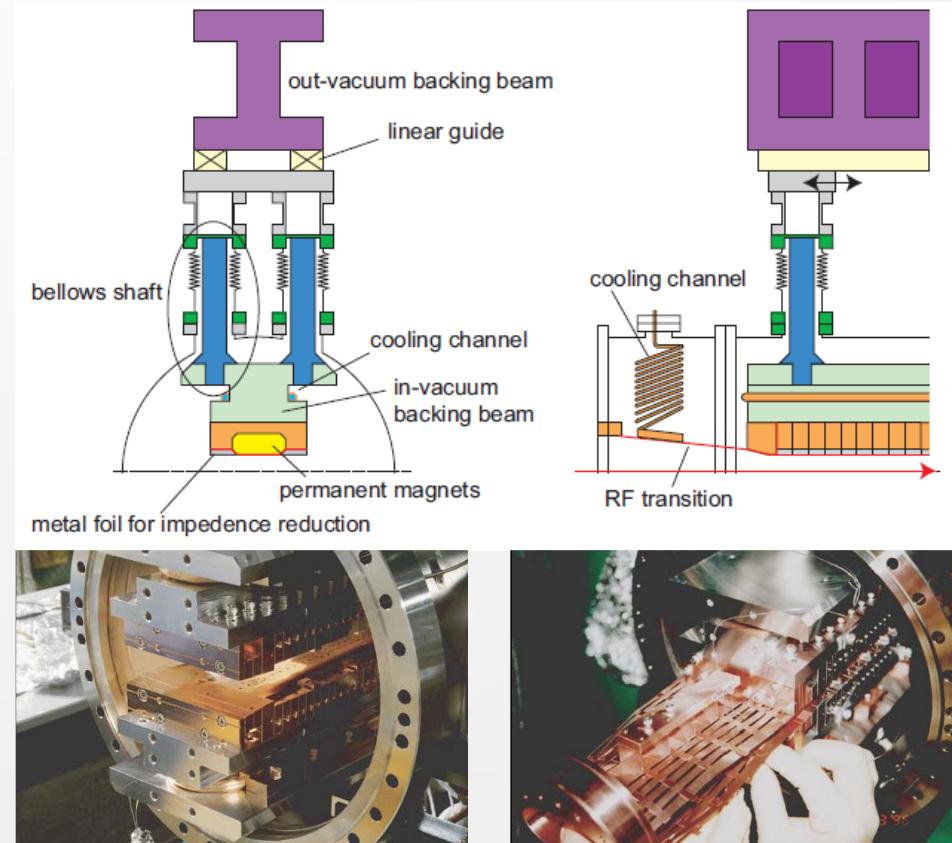
TingYi Chung 鍾廷翊, 2026, FEL

In-vacuum Undulator



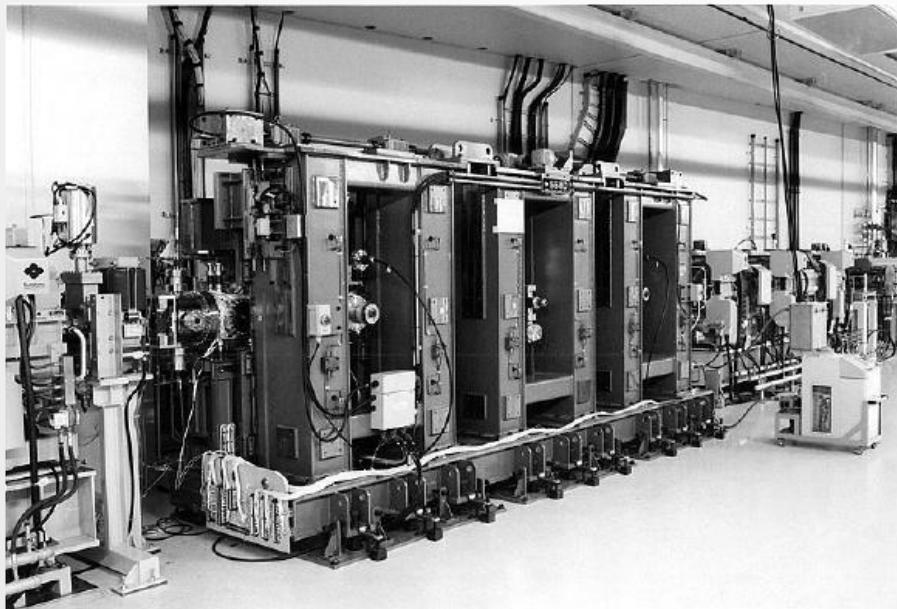
Technical challenges in IVU,

- **Vacuum condition:** In addition to sufficient pumping speed, a TiN coating on PM needs to suppress the outgassing from a porous structure of PM.
- To against the irreversible demagnetizing of PM during **bakeout**, PMs with higher coercivity (>2000 kA/m) is required.
- **Impedance reduction:** the metal sheet covers the magnet surface; the RF transition to connect the magnet end and adjacent vacuum duct smoothly.
- **Field correction, thermal expansion during bakeout**.....



T. Tanaka et. al. FEL (2005)

Installation in the Ring_ IVU



IVU32 installation, 1998, Spring8



IVU22 installation, 2015, TPS

Cryogenic Permanent Magnet Undulator



- Increasing the magnetic field by lowering the temperature of PM.
Incidental benefits:
 - ✓ H_{cj} increasing, robustness for against demagnetizing in operation.
 - ✓ Work as cryopumps.
- CPMU are a natural evolution of IVU.
- In addition to the technologies in IVU, CPMU must overcome the new challenges due to a low temp. operation.

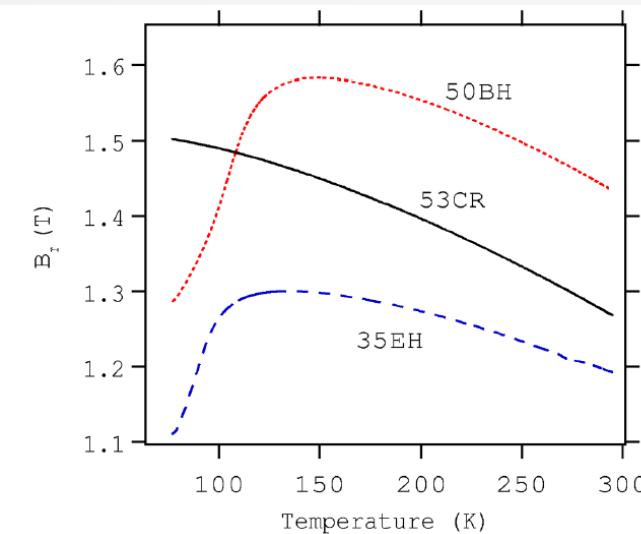


FIG. 2. (Color) Temperature dependence of the remanent fields (B_r) of sintered NdFeB magnets (35EH and 50 BH) and a PrFeB magnet (53CR).

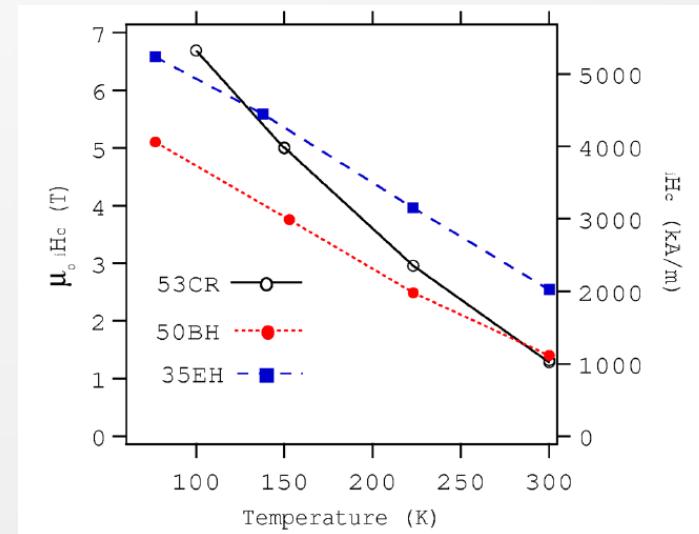
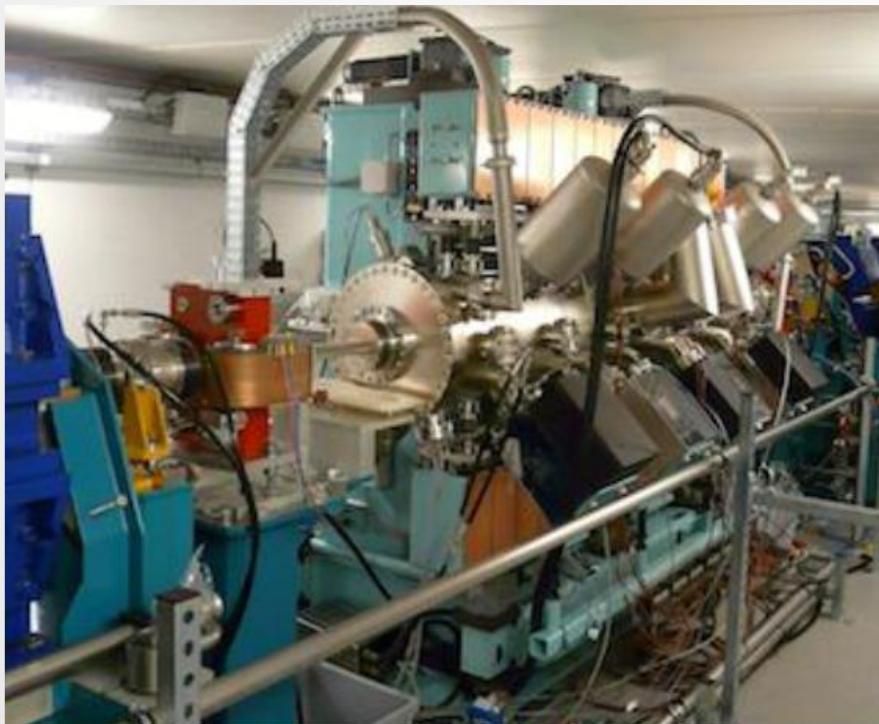


FIG. 3. (Color) Temperature dependence of the coercivity (iH_c) of sintered NdFeB magnets (35EH and 50 BH) and a PrFeB magnet (53CR).

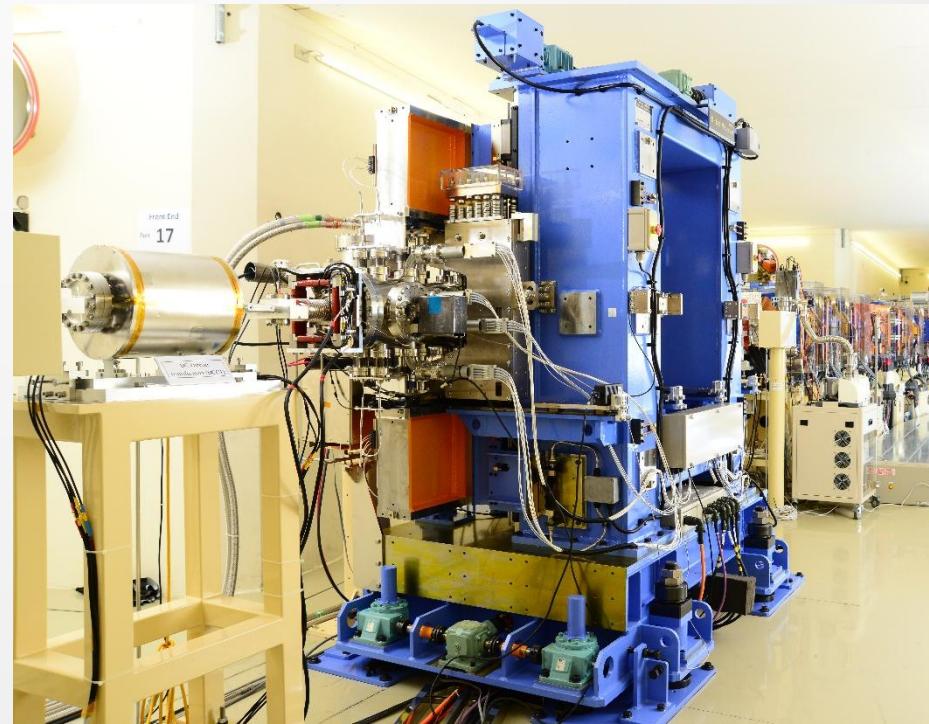
T. Hara et. al. PRSTAB (2004)

TingYi Chung 鍾廷翊, 2026, FEL

Installation in the Ring_CPMU



CPMU18, Soleil, PRSTAB (2017)



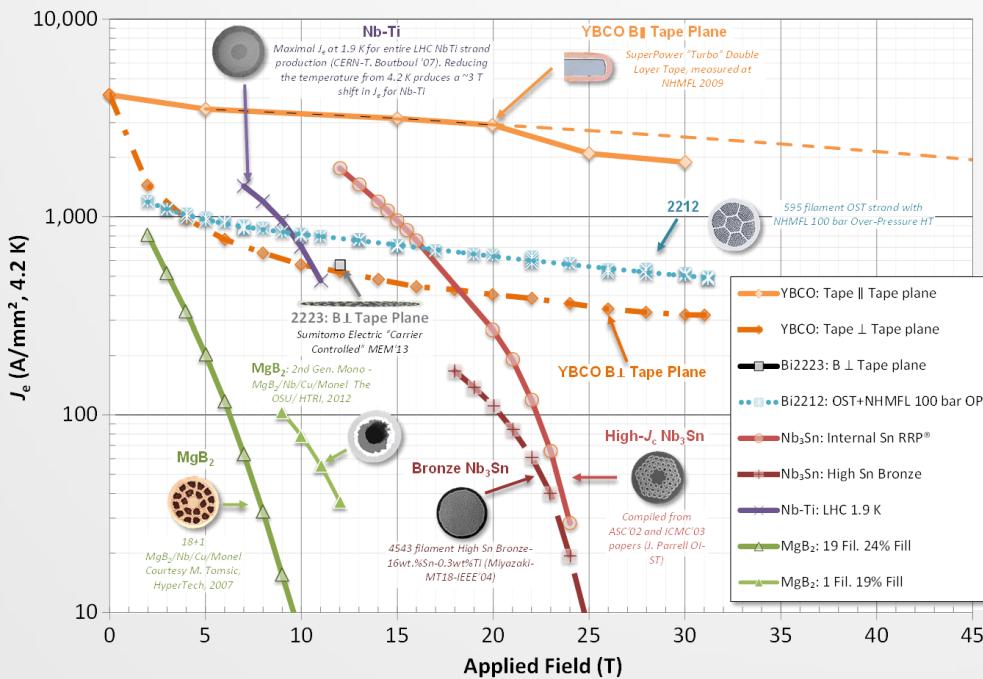
CPMU15, TPS

Superconducting undulator

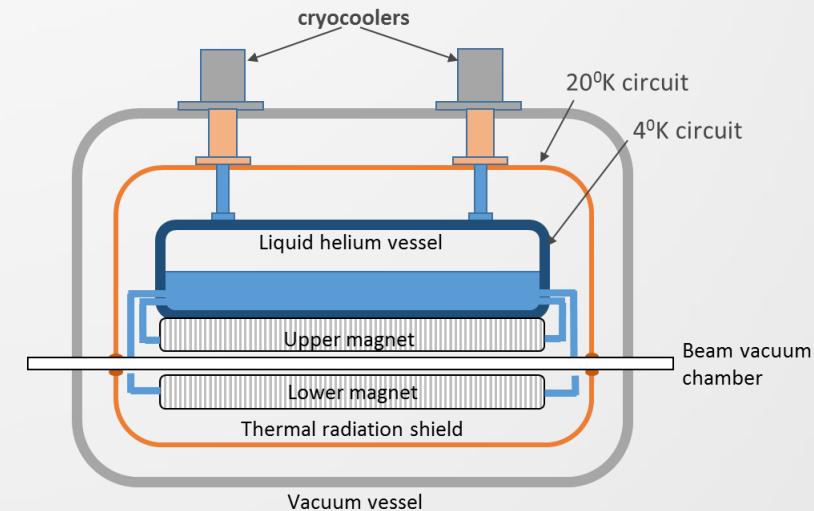


Superconductive wires winding around poles with soft ferro- material.

- Four critical parameters of wires: **critical current density, field, temperature and mini bending radius**, limit the performance of the SU.
- Cooling technology: direct, indirect or cryocooler.



National High Magnetic Field Laboratory (2011)

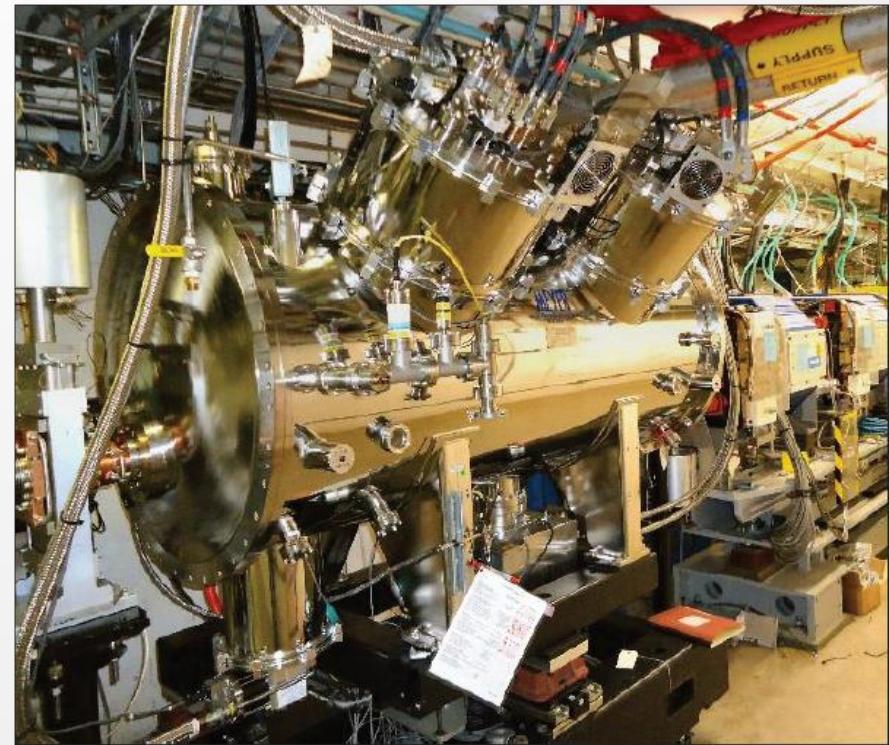


Indirect cooling, E. Gluskin, APS (2018)

Installation in the Ring_SU



Karlsruhe Institute of Technology (KIT)



Planar SU installed in the APS ring, Sector 1.

Generating various polarization



Electromagnet, superconducting magnet, asymmetric wiggler, and permanent magnet are all available to be a variably polarized device.

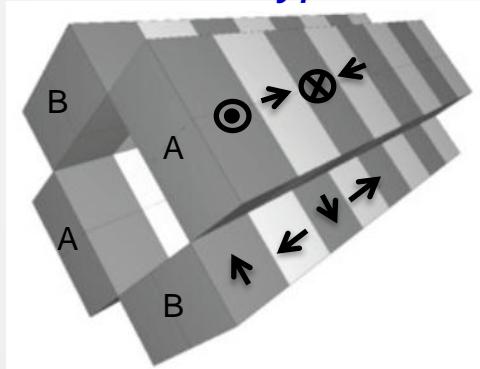
Permanent Magnet

- 1984 Crossed Undulator concept by K. J. Kim.
- 1986 Two Orthogonal Planar type by H. Onuki.
- 1990 Helios type by P. Elleaume. similar one by B. Divianco & R. Walker.
- 1992 Apple-I type was proposed by S. Sasaki.
Crossed undulator was demoed at BESSYII.
- 1993 First Apple-I type device was operated at JAERI Storage Ring.
- 1994 Apple-II type was proposed by S. Sasaki.
First Apple-II was tested at SPEAR storage ring.
Concept of QPU was proposed by Sasaki & Hashimoto.
- 1995 Six magnet arrays type was proposed by Kitamura.
- 1997 One Apple-II and Dual Helical IDs with orbit switching scheme,
were installed at SPring-8.
- 1999 One Apple-II was installed at TLS.
(Apple-II was widely used at synchrotron radiation facilities)
- 2008 Delta undulator was proposed for FEL.
- 2011 12 Apple-II total were built for Fermi@Elettra FEL project.

Permanent variable polarization devices

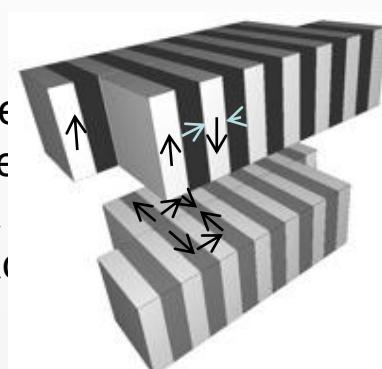


Onuki type



H.Onuki, Nucl. Instr. Meth., A246,94 (1986)

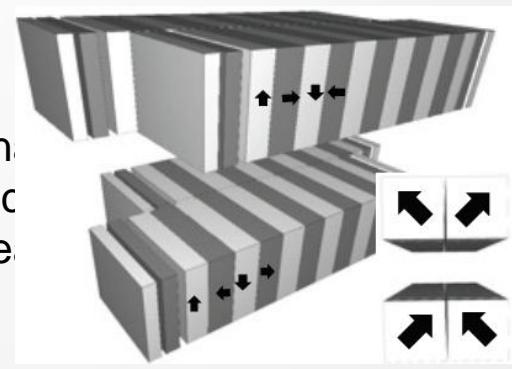
Helios



A superconductor and vertical velocity with a displacement.

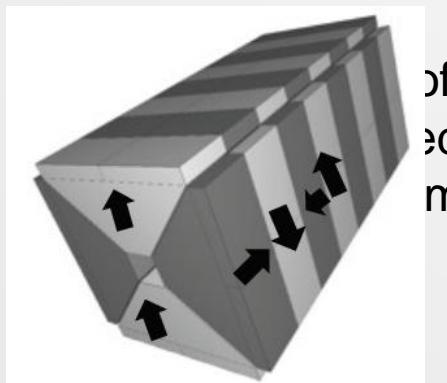
P.Elleaume, J. Synch. Rad., 1,19 (1994)

APPLE-I



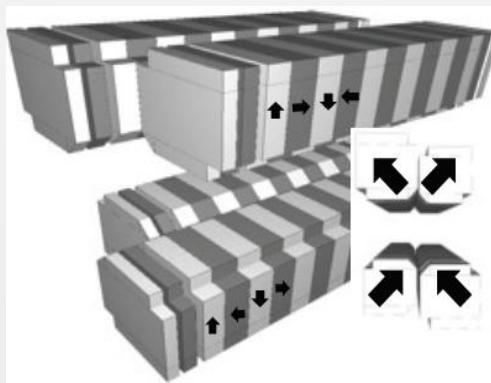
S.Sasaki et al, Jpn.J.Appl.Phys.,31,L194 (1992)

Delta



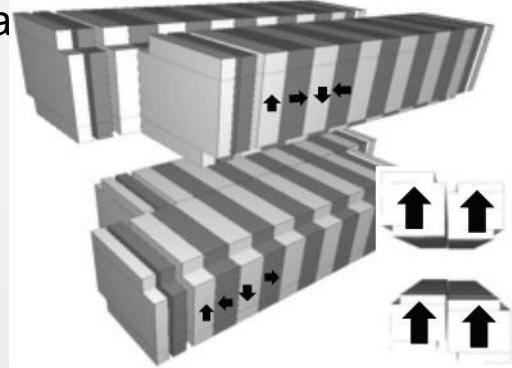
A.B. Temnykh, P.R.S.T.A,11,120702(2008)

APPLE-III



J. Bahrdt et al, 2004 FEL conference

Magnet blocks with a 45^0 magnetization. Field is symmetry **APPLE-II** has a higher a



S.Sasaki et al, Nucl. Instr. Meth., A347,87 (1994)

APPLE-II



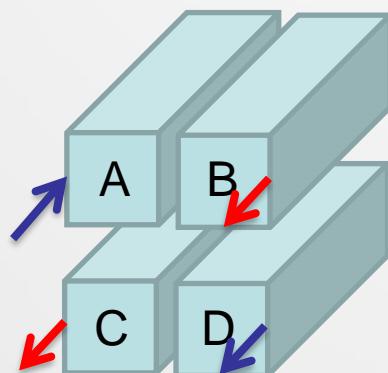
Why APPLE-II?

- Generating any polarization (linear, elliptical, circular)
- High linear/helical magnetic field
- For FEL or USR, Delta-type is the best performance to generate any polarization.

What is APPLE-II?

APPLE (Advanced Planar Polarized Light Emitter)

Antisymmetry(Inclined) mode



Symmetry mode

Symmetric Motion : $\varphi_2 = \varphi_1 = \varphi$

$$[B_z(s), B_x(s)] = \left[4B_{z0} \cos\left(\frac{\varphi}{2}\right) \cos\left(2\pi \frac{s}{\lambda_0} + \frac{\varphi}{2}\right), -4B_{x0} \sin\left(\frac{\varphi}{2}\right) \sin\left(2\pi \frac{s}{\lambda_0} + \frac{\varphi}{2}\right) \right]$$

$$\varphi = 0 \Rightarrow [B_z(s), B_x(s)] = \left[4B_{z0} \cos\left(2\pi \frac{s}{\lambda_0}\right), 0 \right]: \quad \text{Vertical}$$

$$\varphi = \pi \Rightarrow [B_z(s), B_x(s)] = \left[0, -4B_{x0} \sin\left(2\pi \frac{s}{\lambda_0}\right) \right]: \quad \text{Horizontal}$$

$$\varphi = \arctan\left(\frac{B_{z0}}{B_{x0}}\right) \Rightarrow [B_z(s), B_x(s)] = 4B \left[\cos\left(2\pi \frac{s}{\lambda_0} + \frac{\varphi}{2}\right), -\sin\left(2\pi \frac{s}{\lambda_0} + \frac{\varphi}{2}\right) \right]: \quad \text{Helical}$$

Antisymmetric Motion : $\varphi_2 = -\varphi_1 = \varphi$

$$[B_z(s), B_x(s)] = \left[4B_{z0} \cos^2\left(\frac{\varphi}{2}\right), -4B_{x0} \sin^2\left(\frac{\varphi}{2}\right) \right] \cos\left(2\pi \frac{s}{\lambda_0}\right) : \quad \text{Linear}$$

$$\varphi = 0 \Rightarrow [B_z(s), B_x(s)] = [4B_{z0}, 0] \cos\left(2\pi \frac{s}{\lambda_0}\right): \quad \text{Vertical}$$

$$\varphi = \pi \Rightarrow [B_z(s), B_x(s)] = [0, -4B_{x0}] \cos\left(2\pi \frac{s}{\lambda_0}\right): \quad \text{Horizontal}$$

Installation in the Ring_EPU



EPU56 installation, 1999, TLS



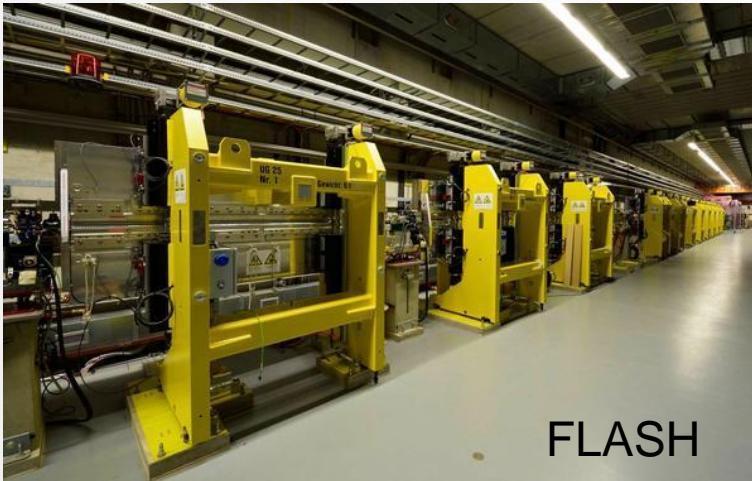
EPU48 installation, 2015, TPS

Undulator system in FEL



Hybrid/PPM undulators dominate FELs

Hybrid
9,
31.4,
2.87.



IVU
3.5,
18,
2.2



Hybrid
6.8,
30,
3.5.



APPLEII
10,
55.2/ 34.8,

Type
Gap mini,
Period,
K max



Undulator system in FEL



Trend(similar to the application in SR)

Pursuing high K (B field) at small periods to minimize electron energy E.

⇒ Narrowing gap to increase B, but compromised by

⇒ The wakefields which can disrupt lasing, increase the risk of radiation damage to the magnet, and make vacuum harder to achieve.

Specific requirements

FEL radiators are made of a number of modules.

- **Tight K variation:** all modules need to emit the same wavelength, sets tight limits on reproducibility of period and K – also E decreases along FEL so K from module to module may be adjusted (slightly) to allow for this (tapering).
- **Tight straightness of a trajectory (second field integral):** Need close control of electron trajectory within undulators to ensure constant overlap of light with electrons.
- **Tight survey alignment:** alignment of all modules to μm level tolerances over $\sim 100\text{m}$
- **Loose phase error:** phase error less important as work at first harmonics only.