國家同步輻射研究中心
National Synchrotron Radiation Research Center

Basic Accelerator Physics

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Statistical optics based study on the undulator radiation of double minimum beta-y section in TPS

Adjustable phase undulator study for TPS

Design of a new booster to storage ring transfer line in TPS

Injection simulation of nonlinear kicker in TPS

Eddy current in the Ti-coating layer on the inner surface of a nonlinear kicker

Transverse coupling issues in TPS II

Overview

Synchrotron Radiation - Basic

Goal: Accelerator as a SR source

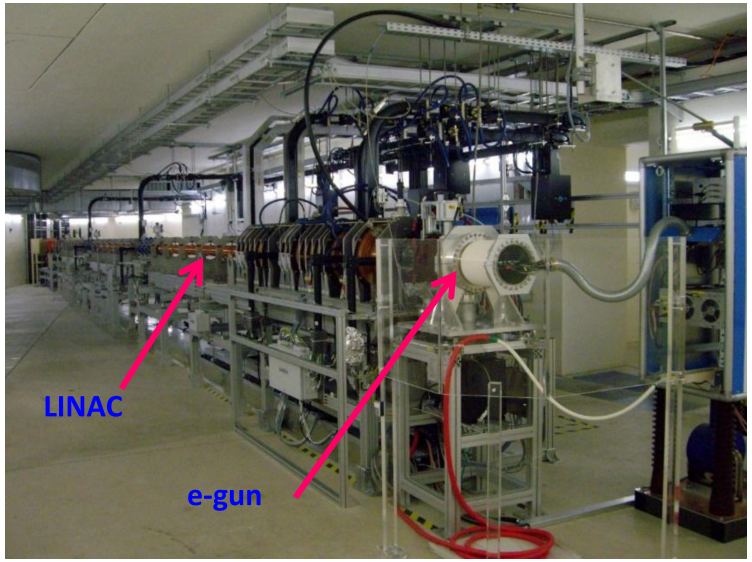
Particle, energy…

How to achieve the goal?

Generation of particle beam, acceleration, control the beam…

Overview

A particle accelerator is a machine that uses electromagnetic fields to propel charged particles to very high speeds and energies, and to contain them in well-defined beams.



➢ **Types**

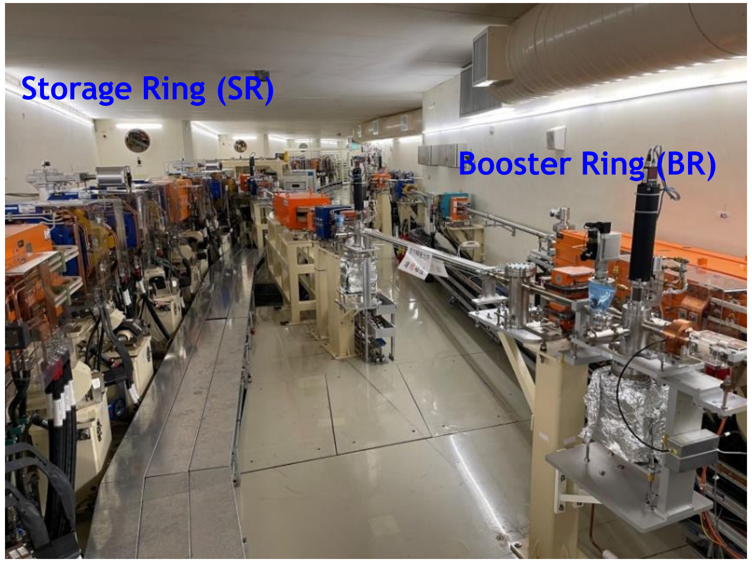
	Linear	Circular
Hadron (proton, ion …)	SNS, J-PARC	LHC
Lepton (electron, muon …)	SLC, LCLS	ALS, PF, TPS

➢ **Applications**

→ particle physics, nuclear physics (collider, neutrino source)
→ particle therapy、radio-isotope production、ion implanters
→ **Biomedical and condensed-matter physics** (neutron, muon source)
→ SR source & FEL *e.g.* 3 GeV, 518.4 m *TPS*、1.5 GeV, 120 m *TLS*

LHC @ CERN (6.8 TeV, 26.7 km)

LCLS @ SLAC



TPS @ NSRRC

https://zh.wikipedia.org/zh-tw/%E5%A4%A7%E5%9E%8B%E5%BC%B7%E5%AD%90%E5%B0%8D%E6%92%9E%E6%A9%9F
https://lcls.slac.stanford.edu/overview

Radiation Therapy

NOW健康

鼻過敏 飛蚊症 講座報名

健康養生 醫美減重 兩性關係 醫藥衛生 癌症新知 專題 影音 活動

首頁 / 醫藥衛生 / 醫藥話題 / 粒子治療儀器有如醫療院所軍備競賽 全台分布現況解析

粒子治療儀器有如醫療院所軍備競賽 全台分布現況解析

2025-05-24 18:00:00 ● 陳如頤 報導
9066

A- A+

【NOW健康 陳如頤 / 台北報導】粒子治療（質子治療與重粒子治療）為先進且昂貴的放射治療儀器，據統計，全台共有15醫療機構採購醫用粒子治療設備，數量30台。目前北醫附醫、林口長庚、中國附醫、高雄長庚等4家質子中心已正式運作，台北榮總則成立國內第1家重粒子中心。

衛福部統計目前核准30台粒子設備 密度居全球之冠

由於粒子治療儀器造價昂貴，以質子治療儀器約50億元，重粒子治療儀器造價較低，但也30億元起跳，不過，近年來台灣醫療院所有如軍備競賽，添購這些昂貴儀器，成立質子治療中心、粒子治療中心。

衛福部統計，目前核准30台粒子設備，分別為北部12台（6家醫院）、中部7台（3家醫院）、南部8台（5家醫院）、東部3台（1家醫院），如與國外相較，密度確實偏高。如果15家醫療機構均已展開營運，平均每150萬人口就有1家粒子治療中心，密度居全球之冠。

對此，衛福部醫事司副司長劉玉菁指出，我國針對粒子設備醫療設置分區合理量目標，依照人口估計，目前只有東部超過合理量，北、中、南等3區仍還有設置空間。

目前4家質子中心正式運作 9家醫院正設置採購質子設備

截至目前，已有北醫附醫、林口長庚、中國附醫、高雄長庚等4家質子中心正式運作；另有三總、台大癌醫中心分院、淡水馬偕醫院、台中榮總、彰化基督教醫院、大林慈濟醫院、奇美醫院、義大醫院、高雄醫學大學附設中和紀念醫院等9家醫院則正設置採購質子設備。

CNA

即時 政治 國際 兩岸 產經 證券 科技 生活 社會 地方 文化 運動

首頁 / 生活

北榮重粒子治癌逾5百人　硼中子設備估116年完工

2025/7/17 17:57（7/17 18:51 更新）

（中央社記者曾以寧台北17日電）台北榮總今天表示，重粒子癌症治療中心正式啟用以來，已治療516位癌症病人，其中以胰臟癌占3成最多；近期亦將進行加速器型硼中子捕獲治療設備工程，預計民國116年完工。

台北榮民總醫院重粒子癌症治療中心是日本輸出海外的3部重粒子設備中最早投入運轉，被世界粒子治療合作組織（PTCOG）列為全球第14座運轉中的重粒子中心。立法院外交及國防委員會今天前往考察國軍退除役官兵輔導委員會「精準醫療結合新型態放射治療技術」。

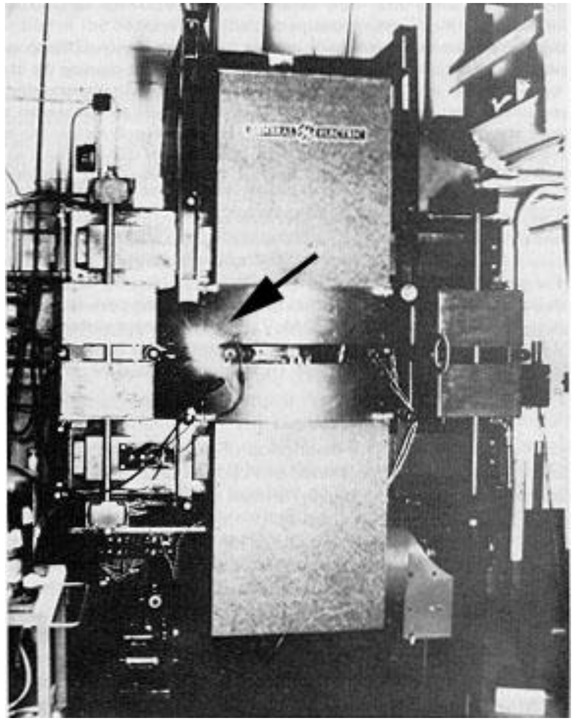
台北榮總會後透過新聞稿指出，重粒子癌症治療中心自112年5月正式啟用以來，累計已治療516名癌症病人，治療類別以胰臟癌為主（占3成），其次為攝護腺癌、肝癌與肺癌等。其中9成為國人，另有1成來自美國及東南亞等地的國際病人，顯示台北榮總在高階癌症治療領域的強大吸引力。

台北榮總表示，除了重粒子治療，今年6月16日亦啟用全新光子治療直線加速器，近期將進行加速器型硼中子捕獲治療（BNCT）設備工程，預計於2年後完工，屆時北榮將成為全球少數同時擁有重粒子與BNCT兩項尖端放射治療技術的醫療機構。

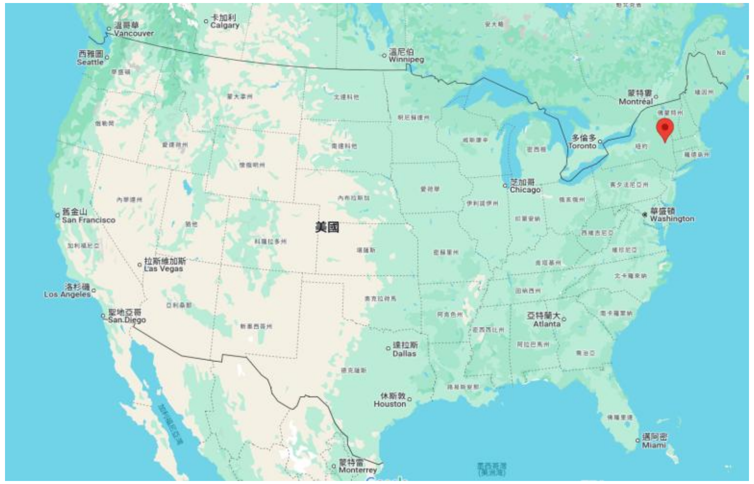
https://www.shi.co.jp/industrial/en/product/medical/proton-therapy/proton-therapy-system.html

Discovery of synchrotron radiation

General Electric Research Laboratory in Schenectady, New York, on April 24, 1947



X-Ray Data Booklet
https://xdb.lbl.gov/Section2/Sec_2-2.html



Radiation from Electrons in a Synchrotron

F. R. Elder, A. M. Gurewitsch, R. V. Langmuir, and H. C. Pollock
Research Laboratory, General Electric Company, Schenectady, New York
May 7, 1947

HIGH energy electrons which are subjected to large accelerations normal to their velocity should radiate electromagnetic energy.[1-4] The radiation from electrons in a betatron or synchrotron should be emitted in a narrow cone tangent to the electron orbit, and its spectrum should extend into the visible region. This radiation has now been observed visually in the General Electric 70-Mev synchrotron.[5] This machine has an electron orbit radius of 29.3 cm and a peak magnetic field of 8100 gausses. The radiation is seen as a small spot of brilliant white light by an observer looking into the vacuum tube tangent to the orbit and toward the approaching electrons. The light is quite bright when the x-ray output of the machine at 70 Mev is 50 roentgens per minute at one meter from the target and can still be observed in daylight at outputs as low as 0.1 roentgen.

PHYSICAL REVIEW VOLUME 74, NUMBER 1 JULY 1, 1948

Radiation from Electrons Accelerated in a Synchrotron

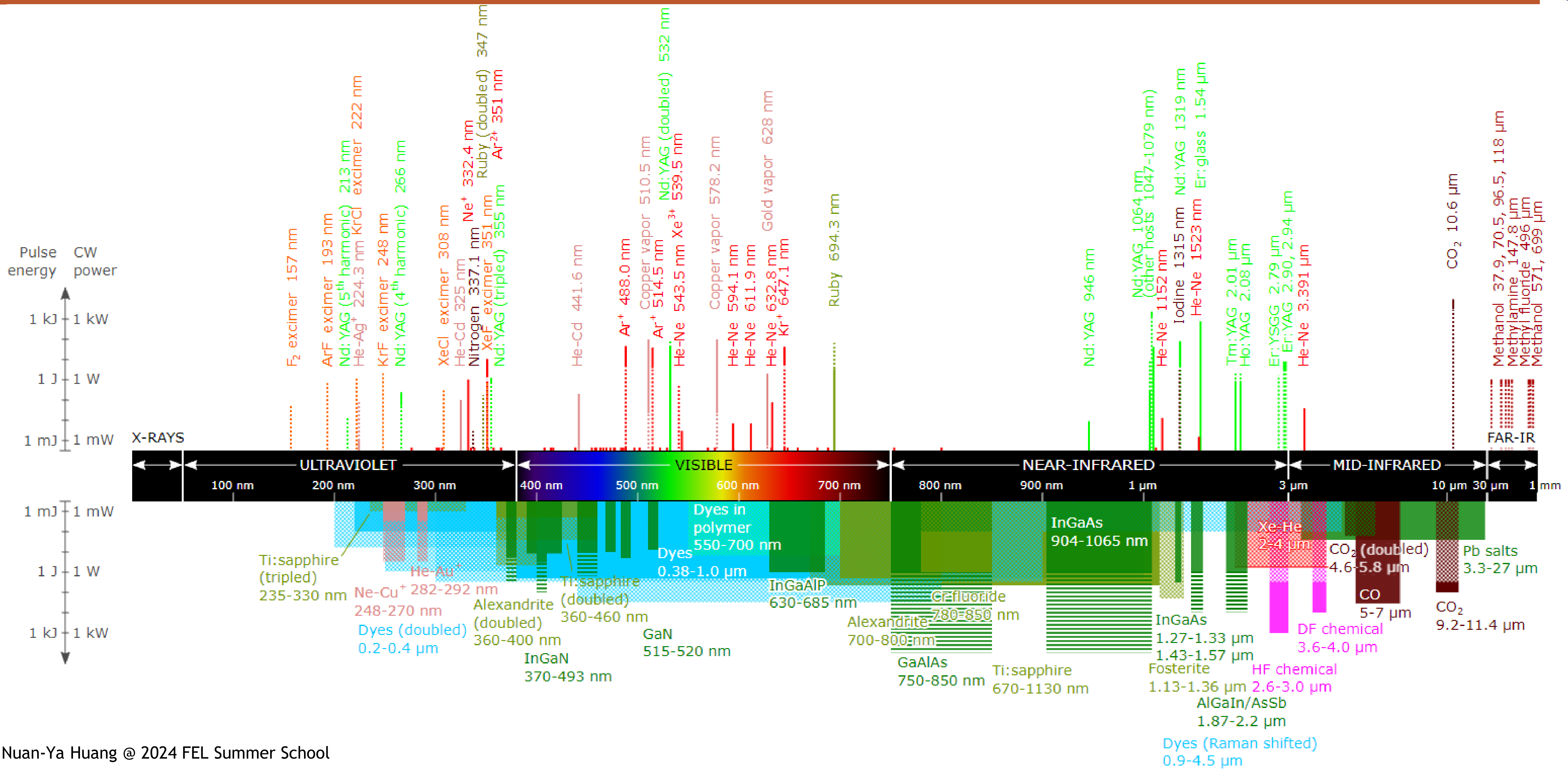
F. R. Elder, R. V. Langmuir, and H. C. Pollock
General Electric Company, Schenectady, New York
(Received March 15, 1948)

High energy electrons subjected to large radial accelerations radiate considerable energy in the optical spectrum. The distribution of energy in the light from a synchrotron beam has been measured and compared with theory at several electron energies up to 80 Mev. The results indicate reasonable agreement with theory. Measurement of total light output allowed an estimate of electron current in the beam. High speed photography of the light permitted observation of the size and motion of the beam within the accelerator tube.

1. INTRODUCTION

DURING the early operation of a 70-Mev synchrotron[1] in this laboratory, intense visible radiation from the high energy electrons was observed[2] through the glass wall of the accelerating tube. Almost fifty years ago Lienard[3] pointed out that electrons rotating in a circle should radiate energy and he gave a formula for tron orbit. In our machine the beam is first visible at about 30 Mev as a dull red spot. At 80 Mev, the present peak energy, the light is very brilliant and a bluish-white color. The spectral distribution of the radiation has been determined during the acceleration of electrons to several different levels of peak energy and in this paper is compared with the theory.

Light to Explore the extreme resolution



Nuan-Ya Huang @ 2024 FEL Summer School

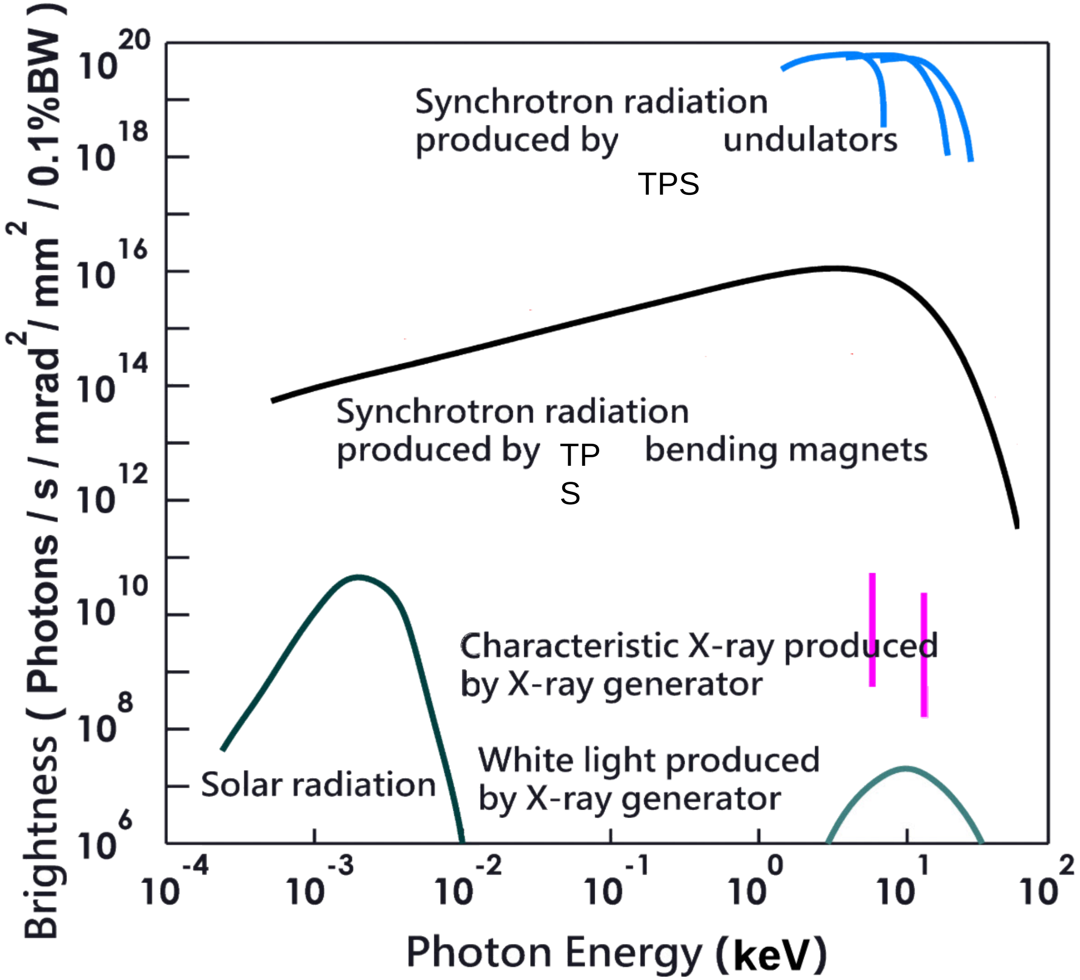
https://en.wikipedia.org/wiki/List_of_laser_types ;
Lecture of "Foundation of FEL", Takashi Tanaka, RIKEN SPring-8 Center, ISBA 2022.

Properties of Synchrotron Radiation

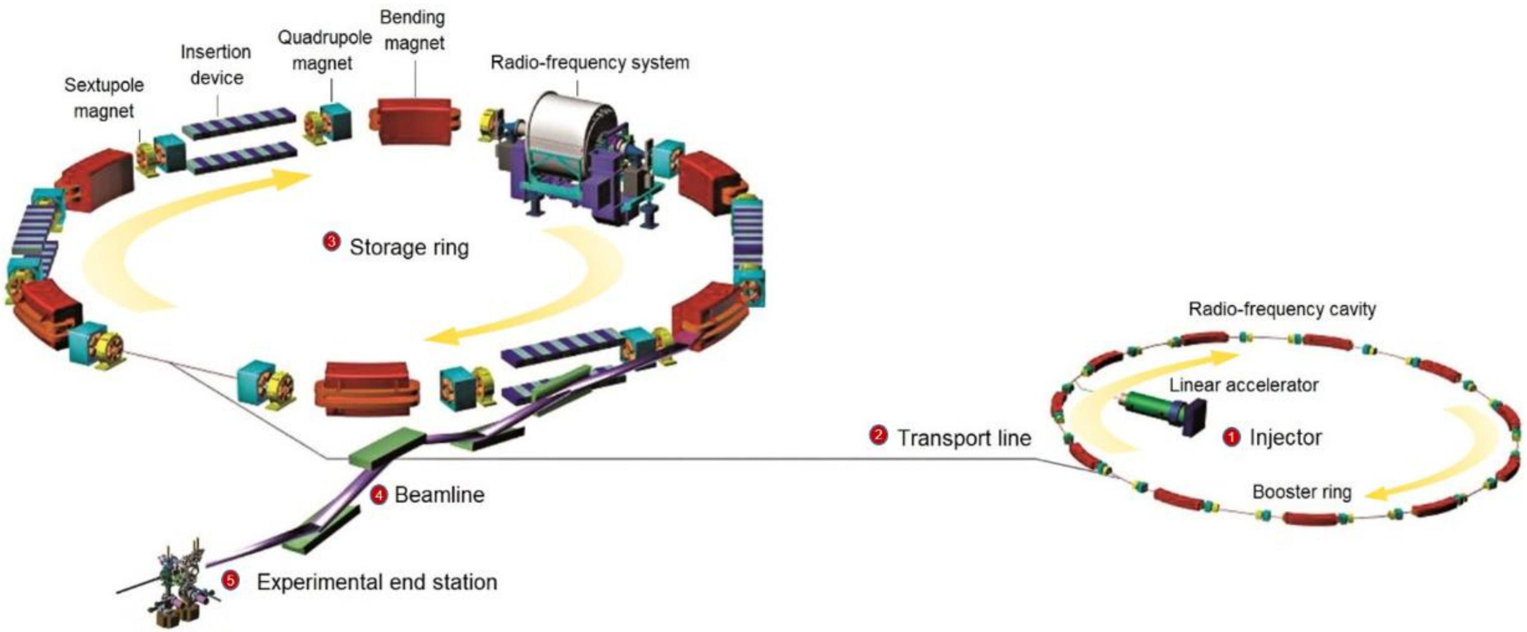
- ✓ **High brightness** (>> 10^6 x Solar radiation)
- ✓ **Tunable wavelength** (extend to hard-X-ray)
- ✓ **Excellent collimation** (~ $1/\gamma$)
- ✓ **small spot size and divergence**
- ✓ **Full polarization**
- ✓ **Pulsed time structure** (several ps with spacing of ns)

$$Brightness = \frac{flux}{(2\pi)^2 \Sigma_x \Sigma_{x'} \Sigma_y \Sigma_{y'}} \left[\frac{photons}{sec-mm^2-mrad^2-0.1\%B.W.}\right]$$

$$spectral\ flux = \frac{N_{\text{photon}}}{\Delta T \cdot \Delta\omega/\omega}$$



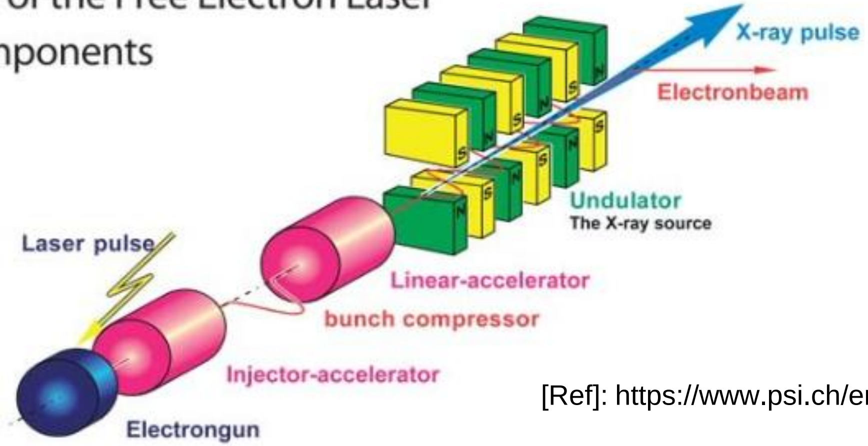
SR
(STORAGE RING)



LINAC FEL
(LINEAR ACCELERATOR FREE ELECTRON LASER)

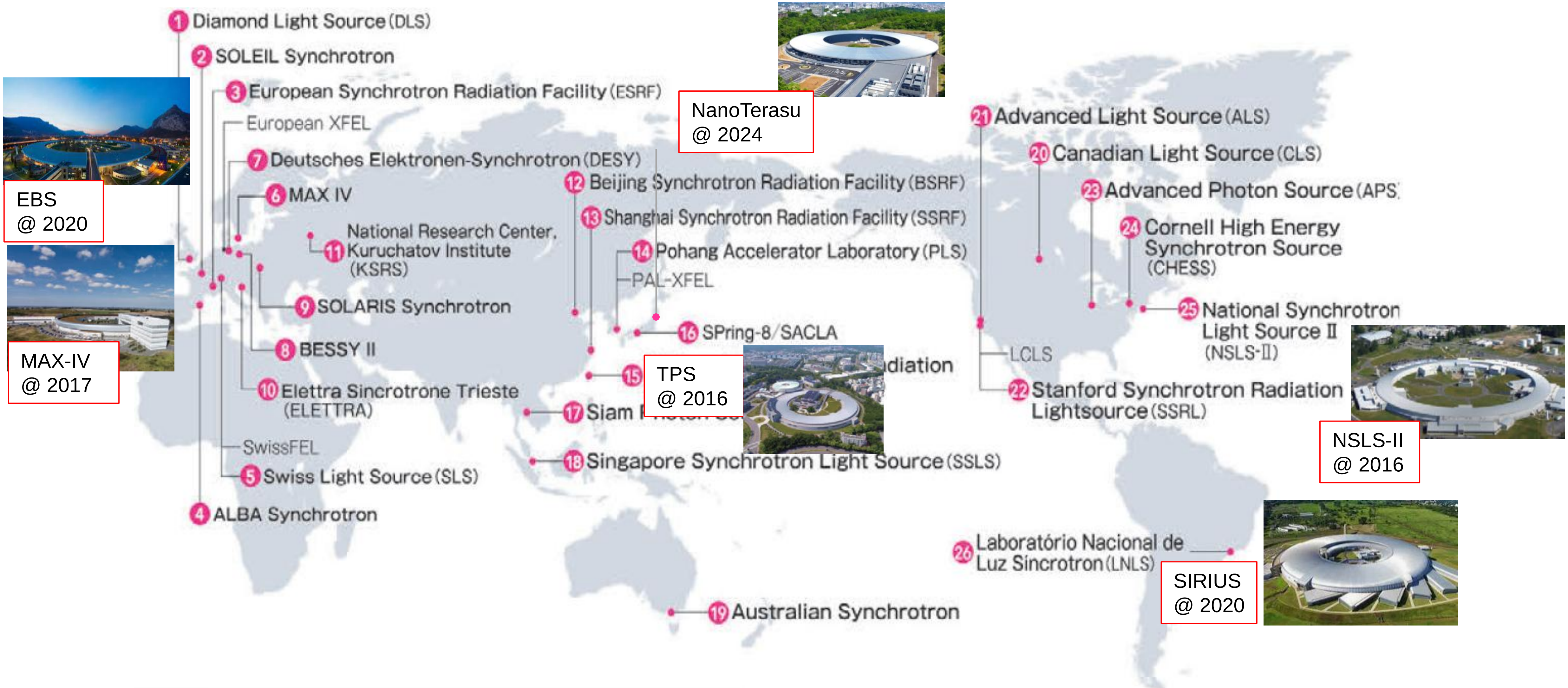
Schematic design of the Free Electron Laser with different components

1) Electrongun
2) Injector
3) Accelerator
4) Undulator



[Ref]: https://www.psi.ch/en/swissfel/how-it-works

World’s Storage Ring Facilities



Encyclopedia of the Synchrotron Radiation Facilities
https://www.aps.anl.gov/spring-8-encyclopedia-of-synchrotron-radiation-facilities%E2%80%932nd-edition

World’s Free Electron Laser Facilities

Synchrotron Radiation - Basic

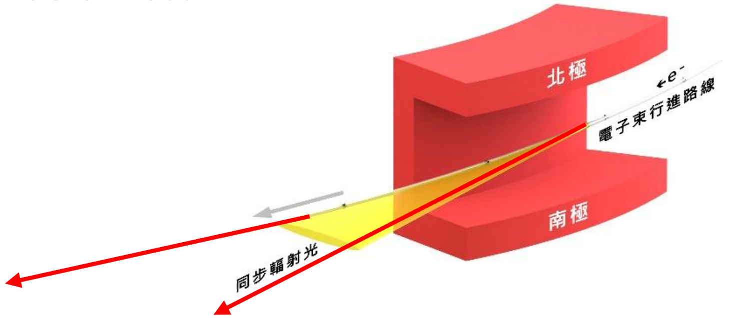
What is synchrotron radiation

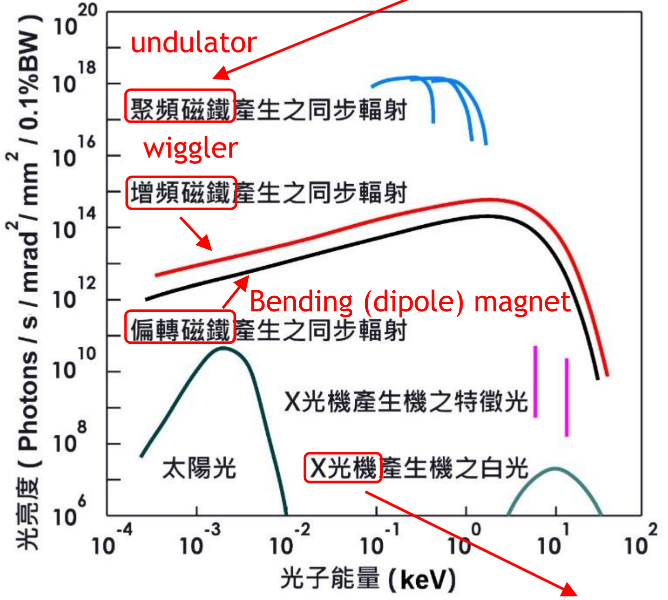
- All the radiation that generated by a synchrotron? → No
- Broad sense: Ultrarelativistic charged particle with acceleration
- Narrow sense: Ultrarelativistic charged particle with suitable quasi-constant normal acceleration

  1. Charged: electromagnetic radiation
  2. Ultrarelativistic : $\gamma \gg 1$, velocity is close (not only comparable) to c=299792458 m/s
  3. Suitable: acceleration is low enough
  4. Normal acceleration: perpendicular to velocity
  5. Quasi-constant acceleration: duration the pulse

  → **Identical** spectrum (universal function)

- Not necessary in a synchrotron or using magnetic field





https://www.nsrrc.org.tw/chinese/lightsource.aspx

Why use B but not E for deflecting beam in an accelerator

James Clerk Maxwell
1831 1879

$$\nabla \cdot \vec{B} = 0$$ Gauss's law for magnetism

$$\nabla \cdot \vec{D} = \rho$$ Gauss's law

$$\nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}$$ Faraday's law

$$\nabla \times \vec{H} = -\frac{\partial \vec{D}}{\partial t} + \vec{j}$$ Ampère's law

$$\vec{D} = \epsilon \vec{E}, \vec{B} = \mu \vec{H},$$

$\epsilon = \epsilon_0 \epsilon_r$, the permittivity
$\mu = \mu_0 \mu_r$, the permeability

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 2.99792458 \times 10^8 \text{ m/sec}$$

Why charged particle radiation?

- c is finite

  Lorentz transformation

$$\mathbf{E}' = \gamma(\mathbf{E} + \boldsymbol{\beta} \times \mathbf{B}) - \frac{\gamma^2}{\gamma + 1}\boldsymbol{\beta}(\boldsymbol{\beta} \cdot \mathbf{E})$$
$$\mathbf{B}' = \gamma(\mathbf{B} - \boldsymbol{\beta} \times \mathbf{E}) - \frac{\gamma^2}{\gamma + 1}\boldsymbol{\beta}(\boldsymbol{\beta} \cdot \mathbf{E})$$

$$c \to \infty \quad \Longrightarrow \quad \begin{matrix} \boldsymbol{\beta} = \mathbf{0} \\ \gamma = 1 \end{matrix} \quad \Longrightarrow \quad \begin{matrix} \mathbf{E}' = \mathbf{E} \\ \mathbf{B}' = \mathbf{B} \end{matrix}$$

- There is also no electromagnetic waves

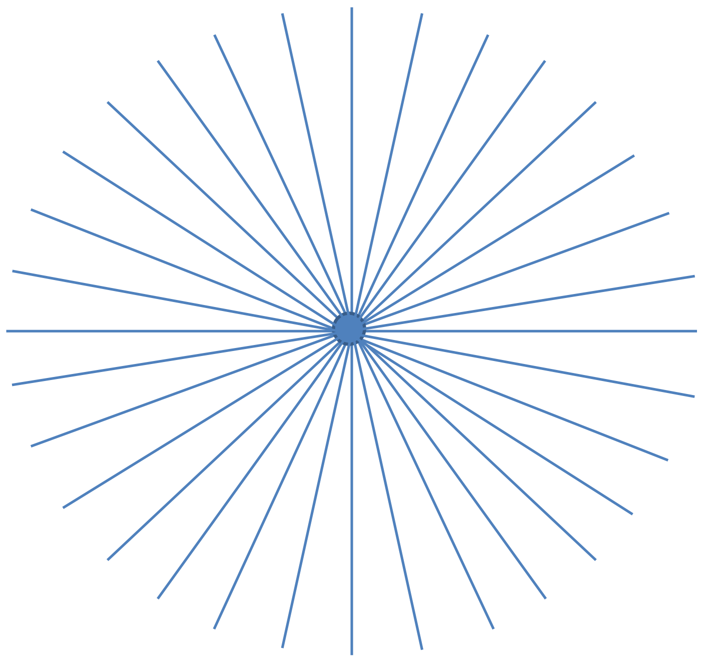
  Wave equations

$$\nabla^2 \mathbf{A} = \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2}$$
$$\nabla^2 \Phi = \frac{1}{c^2}\frac{\partial^2 \Phi}{\partial t^2}$$

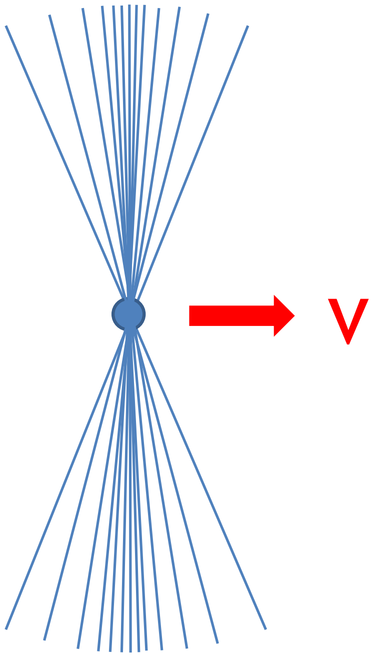
$$c \to \infty \quad \Longrightarrow \quad \begin{matrix} \nabla^2 \mathbf{A} = \mathbf{0} \\ \nabla^2 \Phi = 0 \end{matrix}$$

What is synchrotron radiation: 1. Charged

Why charged particle radiation?



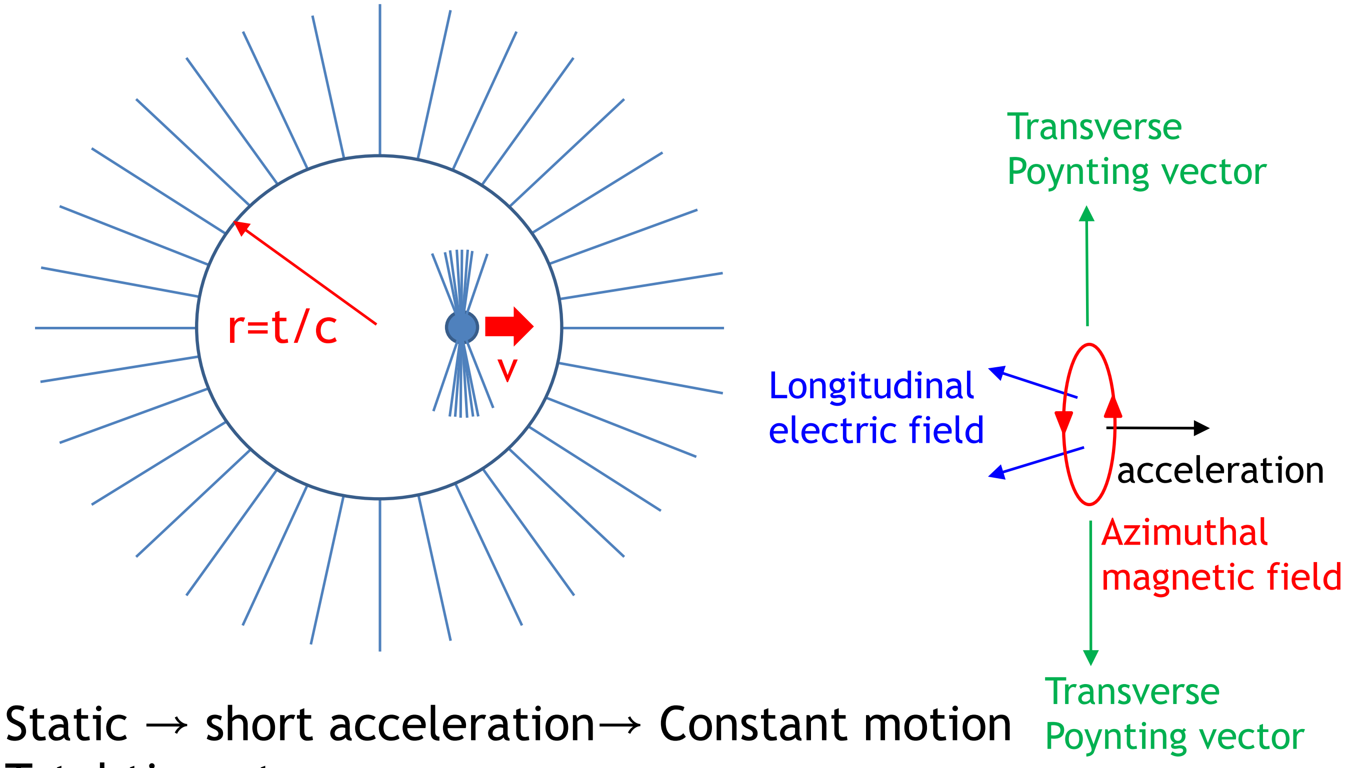
Static



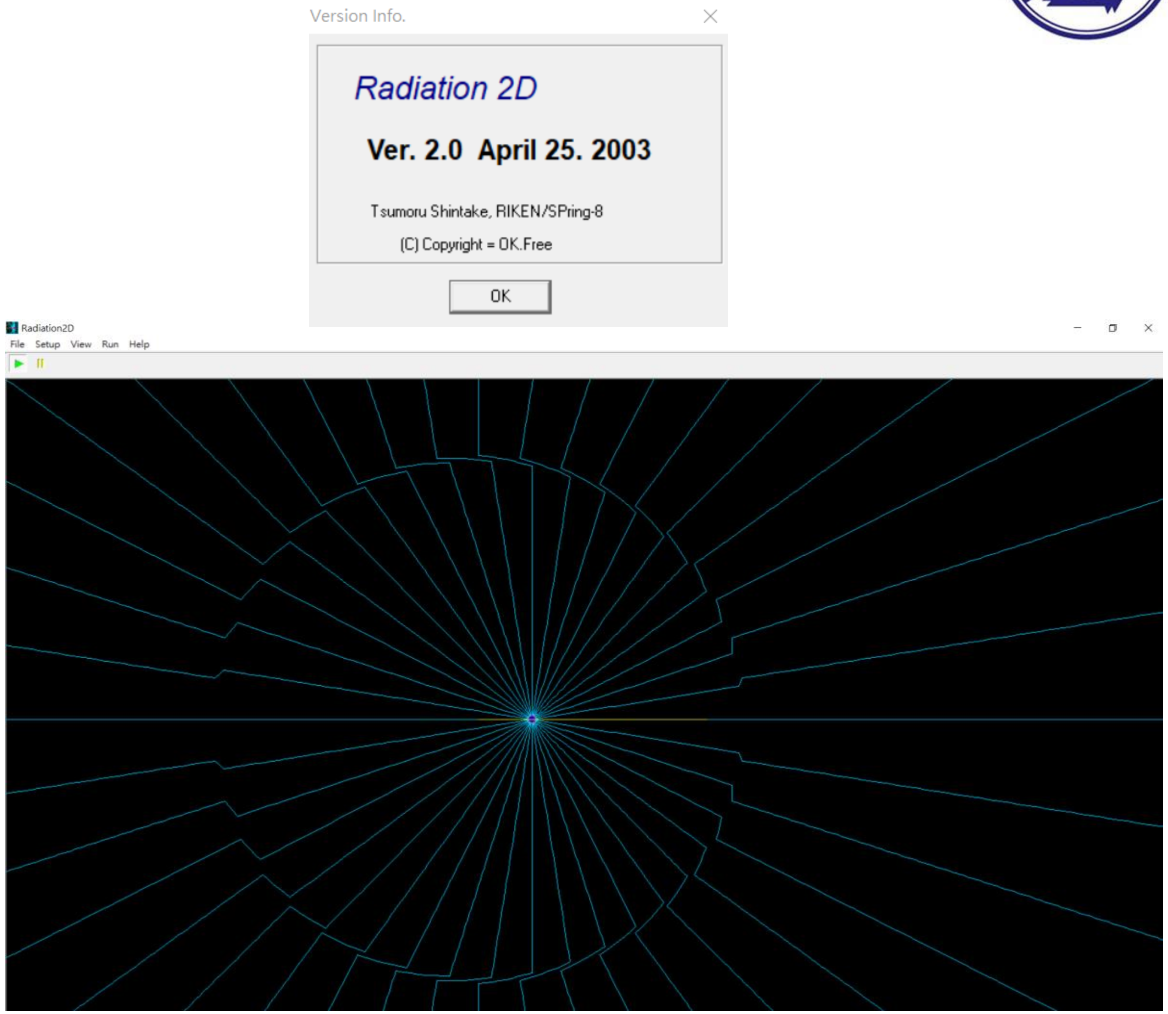
Constant motion
(by Lorentz transformation)

What is synchrotron radiation: 1. Charged

Why charged particle radiation?



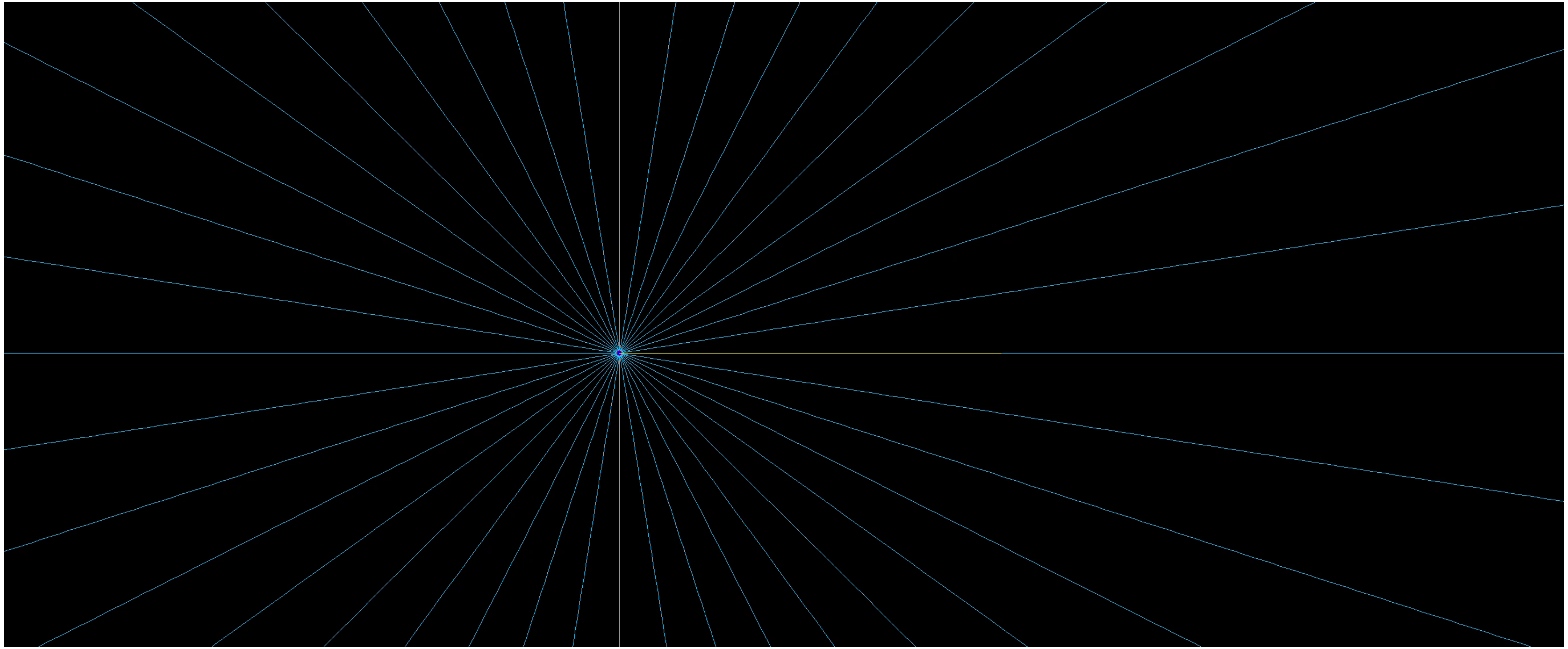
Static → short acceleration→ Constant motion
Total time: t



Tsumoru Shintake, NIM-A, 507, 1–2, 89, 2003

What is synchrotron radiation: 1. Charged

Why charged particle radiation?

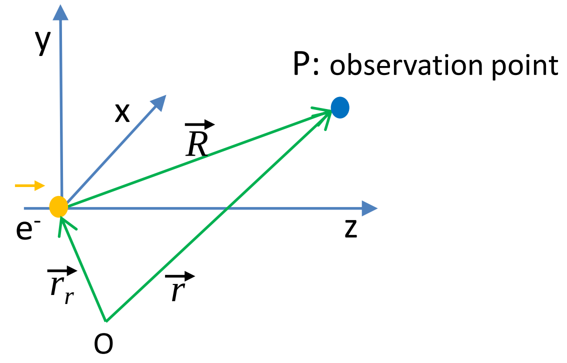


What is synchrotron radiation: 1. Charged

The wave equation in free space

$$\begin{cases} \vec{B} = \nabla \times \vec{A} \\ \vec{E} = \dfrac{-\partial \vec{A}}{\partial t} - \nabla \varphi \end{cases}$$

$$\begin{cases} \nabla^2 \vec{A} - \dfrac{1}{c^2}\dfrac{\partial \vec{A}}{\partial t^2} = -\mu_0 \vec{j} \\ \nabla^2 \varphi - \dfrac{1}{c^2}\dfrac{\partial \varphi}{\partial t^2} = -\dfrac{\rho}{\varepsilon_0} \end{cases}$$



The continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{j} = 0$$

$$\begin{cases} \vec{A}(R,t) = \dfrac{\mu_0}{4\pi} \displaystyle\int \dfrac{\vec{j}(x,y,z)}{R}\Bigg|_{t_r} dxdydz \\ \varphi(R,t) = \dfrac{1}{4\pi\varepsilon_0} \displaystyle\int \dfrac{\rho(x,y,z)}{R}\Bigg|_{t_r} dxdydz \end{cases}$$

$t_r = t - \dfrac{R(t_r)}{c}$, # Because of the finite velocity of light, the integrals must be evaluated under the retarded time.

R: the distance between the observation point and the charge element at the retarded time

$\vec{R} = \vec{r} - \vec{r}_r$, $\hat{n} = \dfrac{\vec{R}}{|\vec{R}|}$ the vector pointing from the charge element to the observation point

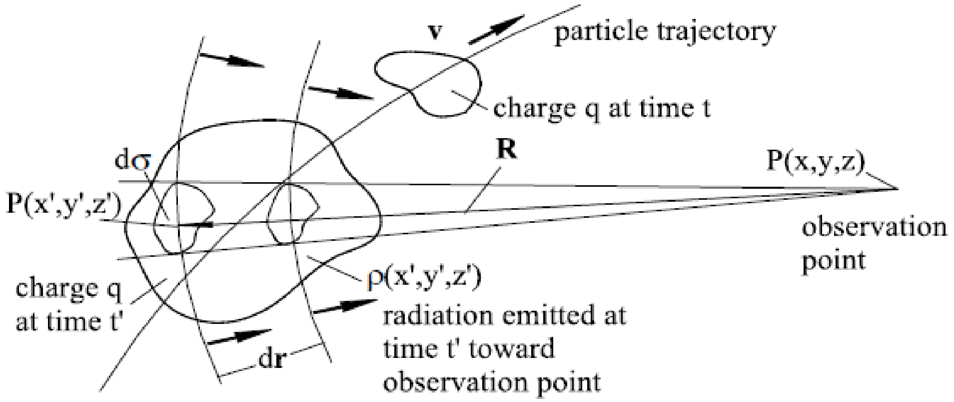
For a point charge *e* at rest

$$\begin{cases} \vec{A}(R,t) = 0 \\ \varphi(R,t) = \dfrac{1}{4\pi\varepsilon_0}\dfrac{e}{R} \end{cases}$$

For the moving charge→ Liénard-Wiechert Potential

$$\begin{cases} \vec{A}(R,t) = \dfrac{\mu_0}{4\pi}\dfrac{q}{R}\dfrac{\vec{v}}{1-\hat{n}\cdot\vec{\beta}}\Bigg|_{t_r} \\ \varphi(R,t) = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{R}\dfrac{1}{1-\hat{n}\cdot\vec{\beta}}\Bigg|_{t_r} \end{cases}$$

[ref] Helmut Wiedemann, Particle Accelerator Physics, 3rd edition, Springer.



Retarded position of a moving charge distribution

$$\begin{cases} \vec{A}(R,t) = \frac{\mu_0}{4\pi} \int \frac{\vec{j}(x,y,z)}{R}\Big|_{t_r} dxdydz \\ \varphi(R,t) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(x,y,z)}{R}\Big|_{t_r} dxdydz \end{cases}$$

The radiation observed at point P, time t is the sum of all radiation arriving simultaneously at P.

For the case of moving charge, the above integral can not be replaced by q/R directly, it must be integrated over a finite volume followed by a transition to charge. (The movement of charge much be taken into account.)

Consider the contribution from all charges contained within a spherical shell centered at P with a radius R and thickness dr to the observation point P at time t

#1 field observed at time t is originate from the fractional charge within the volume element dσdr

#2. Element of this radiation is coming from different charge elements at different time and locations. (Depending on the direction of particle velocity, there is an increase or decrease of the radiation field at P)

$$\rightarrow \quad dq = \rho d\sigma dr + \rho\vec{v}\cdot\hat{n} d\sigma dt$$
$$= \rho\left(1 - \hat{n}\cdot\vec{\beta}\right) d\sigma dr$$

$$\rightarrow \quad \begin{cases} \vec{A}(R,t) = \frac{\mu_0}{4\pi}\frac{q}{R}\frac{\vec{v}}{1-\hat{n}\cdot\vec{\beta}}\Big|_{t_r} \\ \varphi(R,t) = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}\frac{1}{1-\hat{n}\cdot\vec{\beta}}\Big|_{t_r} \end{cases}$$

[ref] Helmut Wiedemann, Particle Accelerator Physics, 3rd edition, Springer.

What is synchrotron radiation: 1. Charged

For a charge q moving with velocity v, the field can be calculated from Liénad-Wiechert potentials

$$\vec{E} = \frac{-\partial \vec{A}}{\partial t} - \nabla\varphi, \qquad \vec{B} = \nabla \times \vec{A}$$

$$\rightarrow \quad \vec{E} = \frac{q}{4\pi\varepsilon_0}\left[\frac{\hat{n}-\vec{\beta}}{\left(1-\hat{n}\cdot\vec{\beta}\right)^3\gamma^2}\frac{1}{R^2}\right]_{ret} + \frac{q}{4\pi\varepsilon_0 c}\left[\frac{\hat{n}\times\left[\left(\hat{n}-\vec{\beta}\right)\times\dot{\vec{\beta}}\right]}{\left(1-\hat{n}\cdot\vec{\beta}\right)^3}\frac{1}{R}\right]_{ret}, \quad \vec{B} = \frac{1}{c}\left[\hat{n}\times\vec{E}\right]_{ret}$$

Velocity field (coulomb regime)

Acceleration field (radiation regime)

The first term of the E field depends on the particle velocity and is inversely proportional to the square of the distance between radiation source and observer. For the particle at rest, this term will converge to the Coulomb field of a point charge q. The field is directed toward the observer for a positive charge at rest. It tilts into the direction of propagation as the velocity of the charge increases. For highly relativistic particles, this field becomes very small. For general case, we ignore this velocity field, and we are interested only in free radiation which is not anymore connected to electric charges.

The second term depends on velocity and acceleration of the charge. This term scales linearly inversely proportional to the distance so this range reaches to larger distance from the radiation source . This term is non-zero only if the particle is accelerated.
→ Charged particles radiate EM wave when accelerated.

[ref] Helmut Wiedemann, Particle Accelerator Physics, 3rd edition, Springer.

Introduction to special relativity

- Assumptions
  1. c is a constant in all reference frames, c=299792458 m/s
  2. All physics laws are the same in all inertial frames
- Properties
  1. Limit of speed (i.e. c) of all particles with finite rest mass
  2. Energy and momentum relations

Total energy — Kinetic energy — Rest mass

$$E = \gamma mc^2 = (\gamma - 1)mc^2 + mc^2 = E_k + mc^2$$

Momentum

$$p = \gamma mv$$

$$E^2 = (pc)^2 + (mc^2)^2$$

  3. Lorentz factors: γ and β

Normalized velocity

$$\beta = v/c$$

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = \frac{1}{\sqrt{1-\beta^2}}$$

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} \approx 1 - \frac{1}{2\gamma^2} \ (\gamma \gg 1)$$

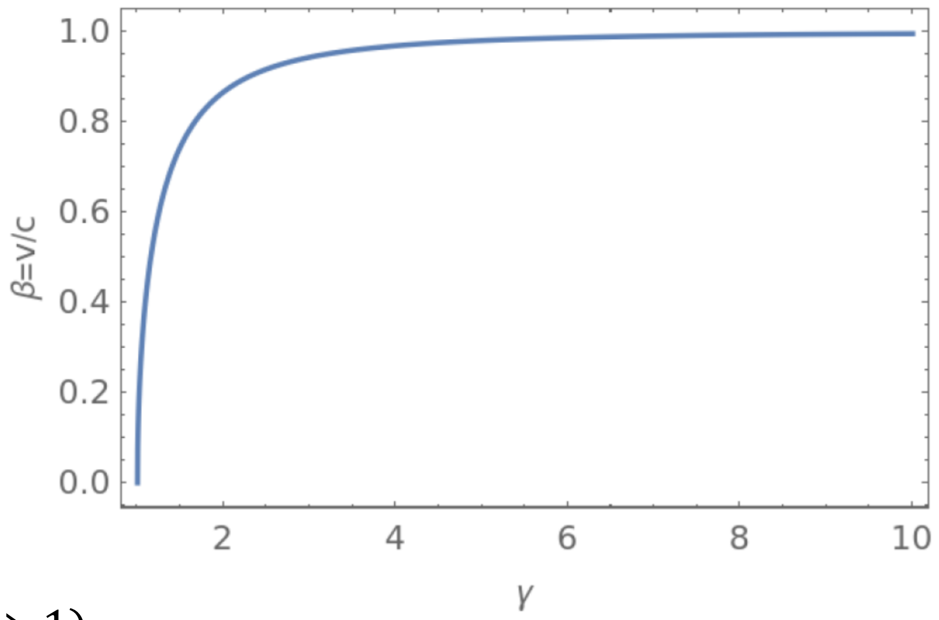
Rest mass of electron

$$m_e = 0.511 \text{ MeV/c}^2$$

Taiwan photon source (TPS)

$$E = 3 \text{ GeV} \rightarrow \gamma \approx 5870$$

$$\beta \approx 0.999999986$$

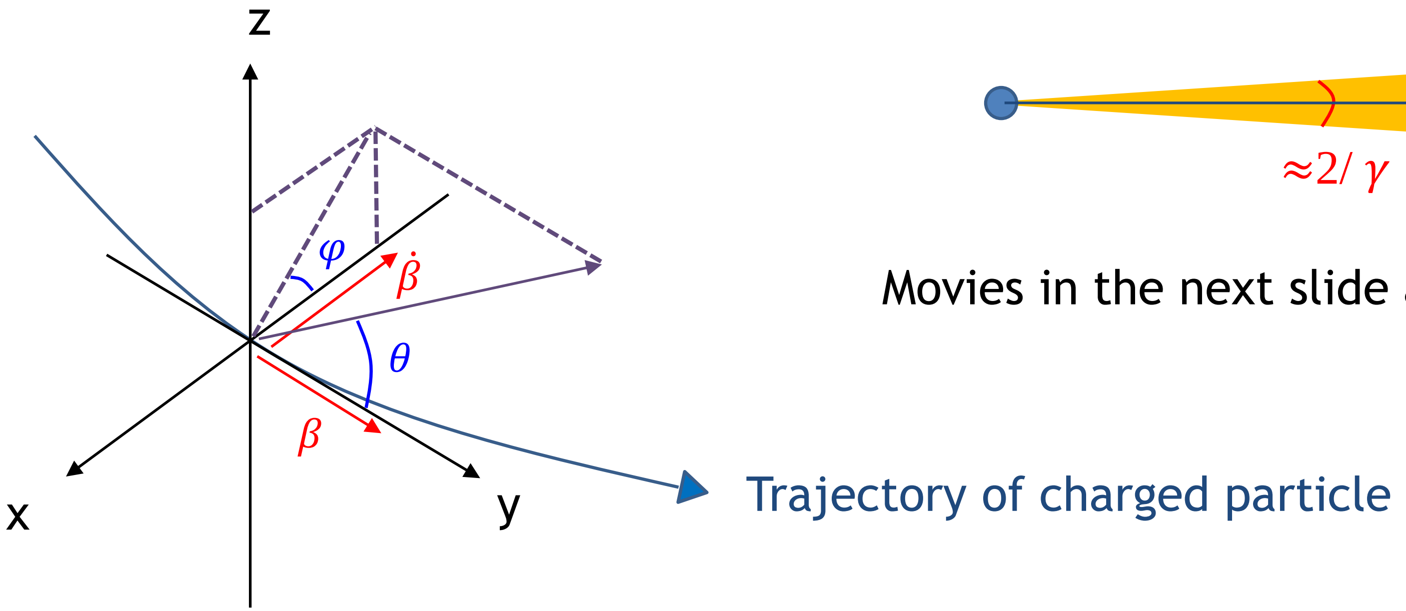


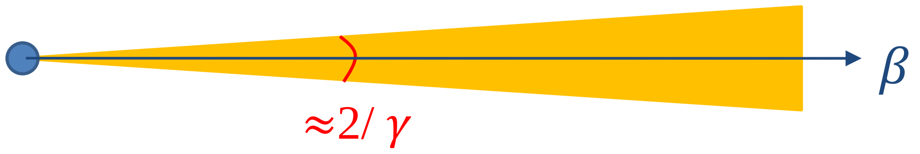
- **Ultrarelativistic:** $\gamma \gg 1$

  The divergence of power distribution decreases as kinetic energy increases

$$\frac{dP}{d\Omega} \propto \frac{1}{(1-\beta\cos\theta)^3}\left[1-\frac{\sin^2\theta\cos^2\varphi}{\gamma^2(1-\beta\cos\theta)^2}\right]$$

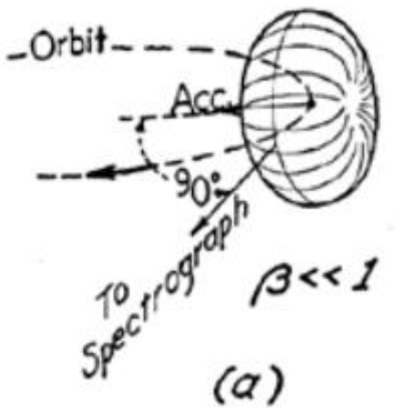
$$1-\beta\cos\theta \approx \frac{\theta^2}{2}+\frac{1}{2\gamma^2} \rightarrow \sigma_\theta \approx \frac{1}{\gamma}$$

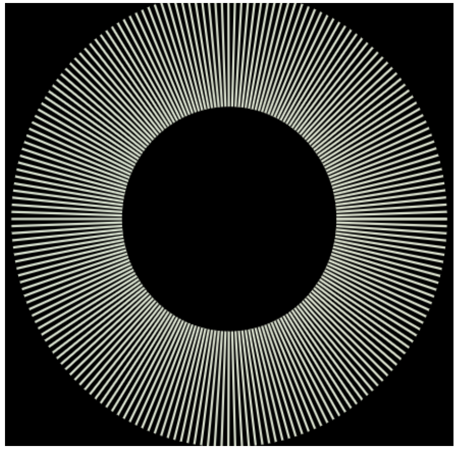




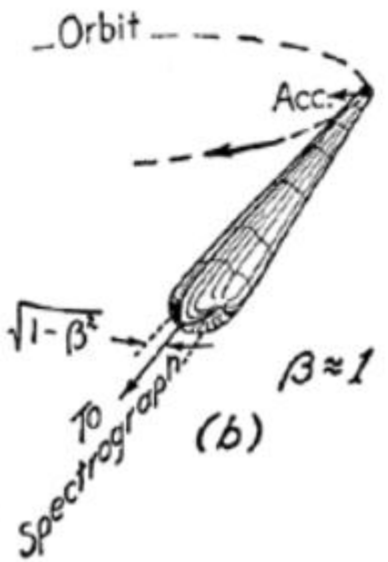
Movies in the next slide are $\frac{dP}{d\Omega}$ with averaging over φ

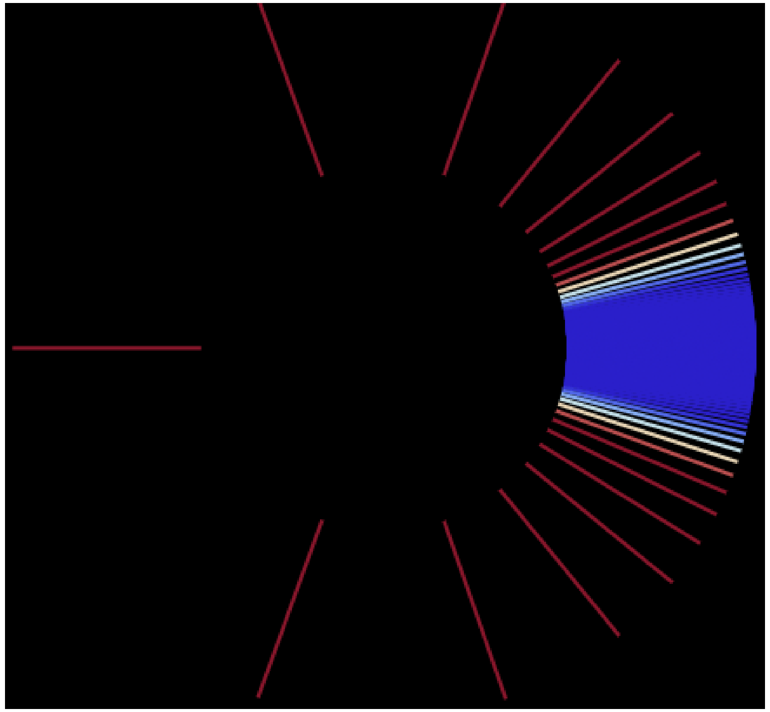
What is synchrotron radiation: 2. Ultrarelativistic





- ➢ an evenly distributed light rays incident on an observer at rest





- ➢ **Relativistic aberration effects:** → **Excellent collimation of SR light** (~ $1/\gamma$)

For a moving observer, light rays appear to be tilted in the direction of motion

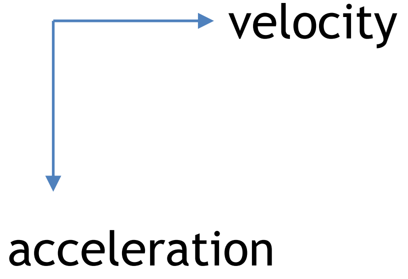
- ➢ **Doppler effect:**

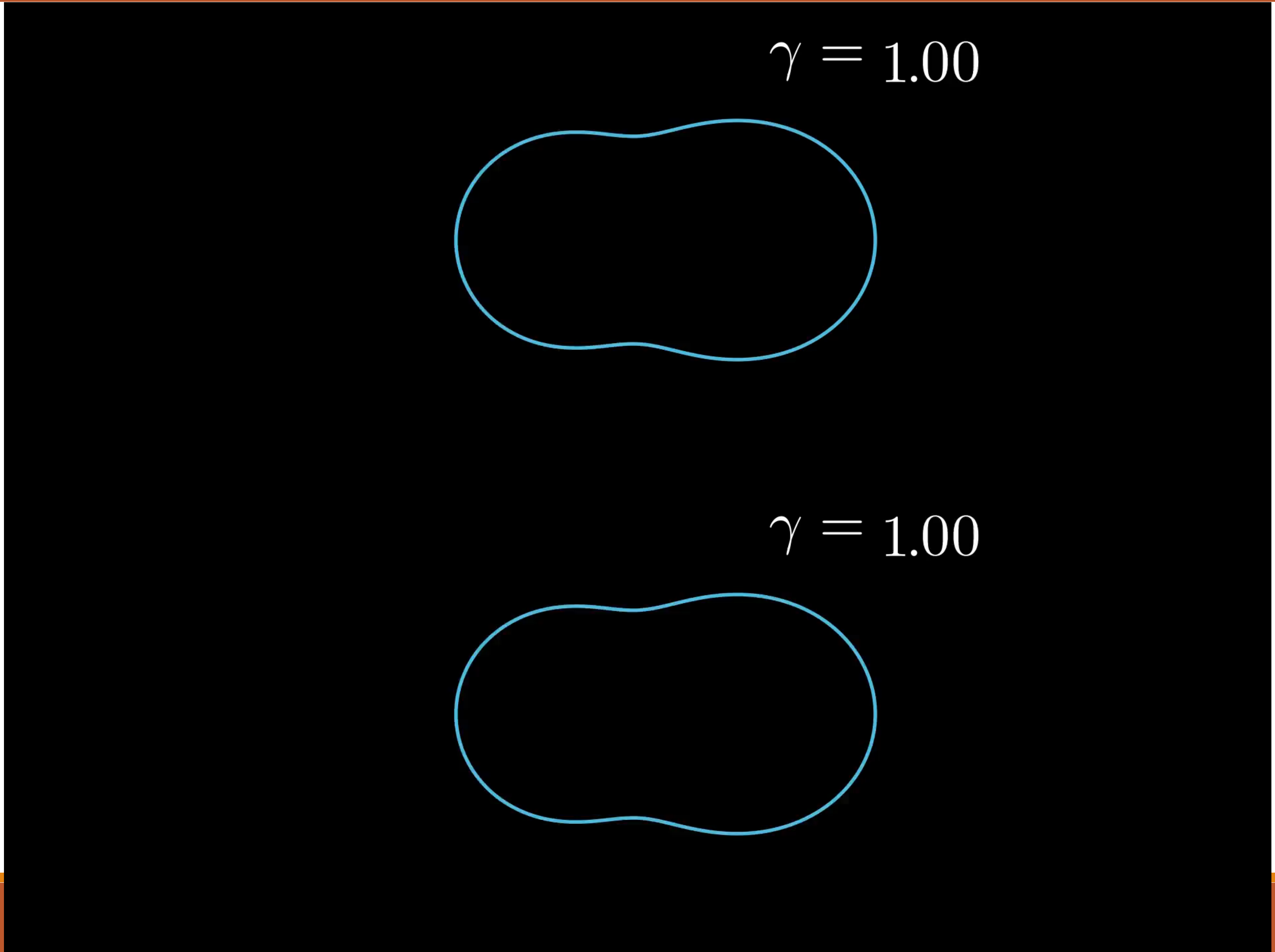
light that comes from the direction of motion is blue-shifted, while light from the opposite direction is red-shifted

Nuan-Ya Huang @ 2024 FEL Summer School

[ref] https://demonstrations.wolfram.com/RelativisticAberrationAndDopplerShift/

What is synchrotron radiation: 2. Ultrarelativistic





What is synchrotron radiation: 2. Ultrarelativistic

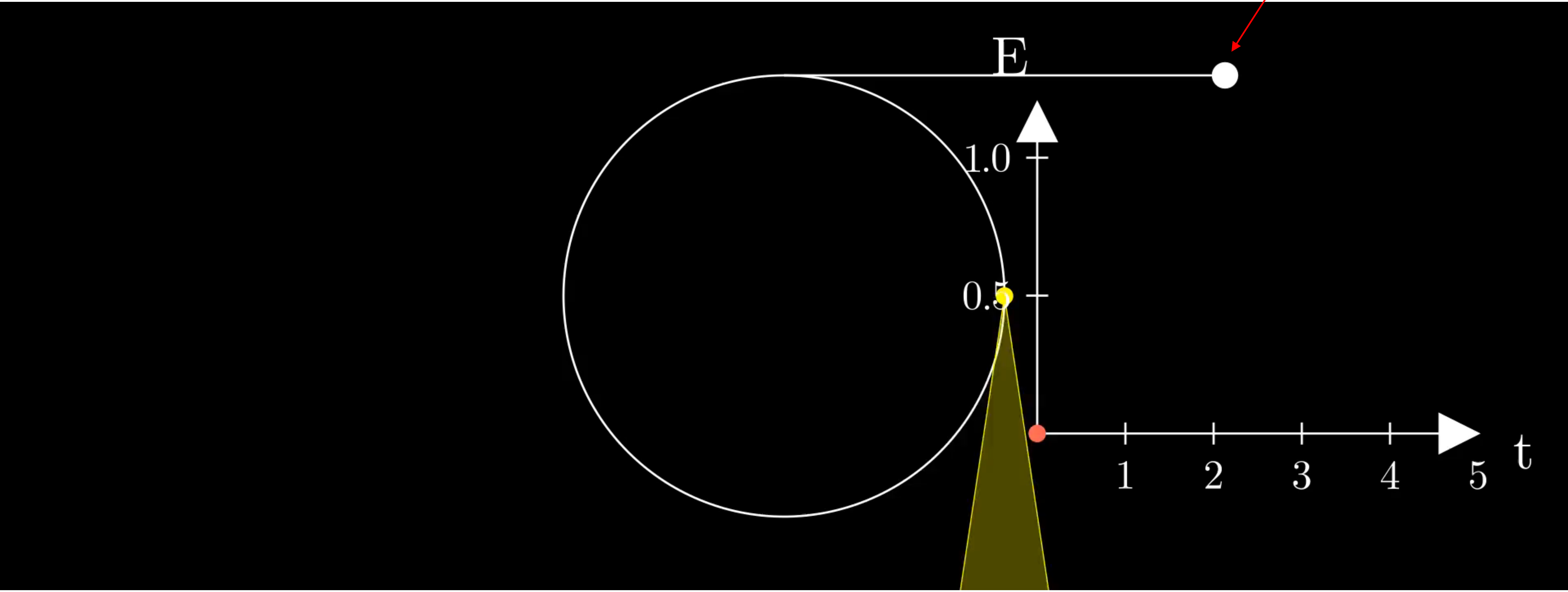
- **Ultrarelativistic: $\gamma \gg 1$ and corresponding instantaneous divergence of radiation beam**

γ	$\beta = v/c$	$1/\gamma$ (mrad)
1.005	0.1	
1.155	0.5	
2.294	0.9	
7.089	0.99	140 (~8 degree)
2935 (TLS)	0.999999942	0.34
5871 (TPS)	0.999999986	0.17 (~0.01 degree)

What is synchrotron radiation: 2. Ultrarelativistic

- High $\gamma \Longrightarrow$ short radiation pulse

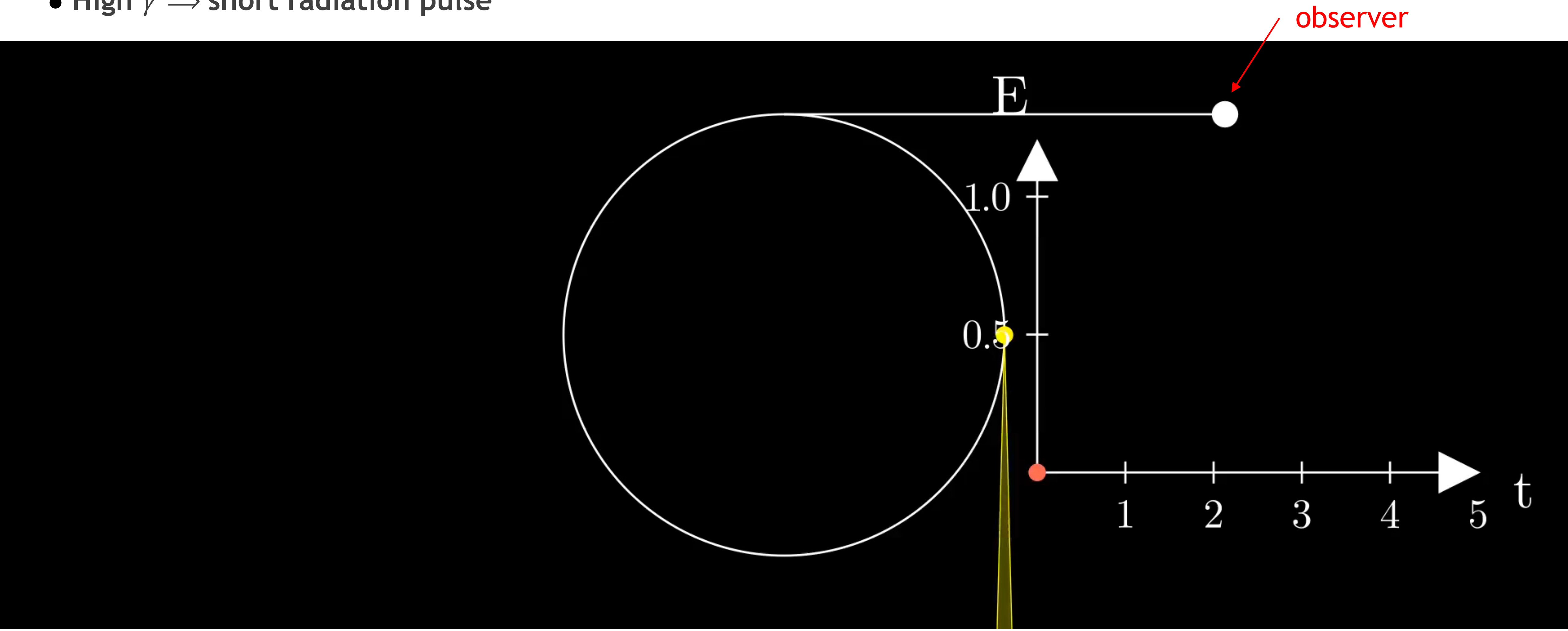
observer



Without Doppler effect

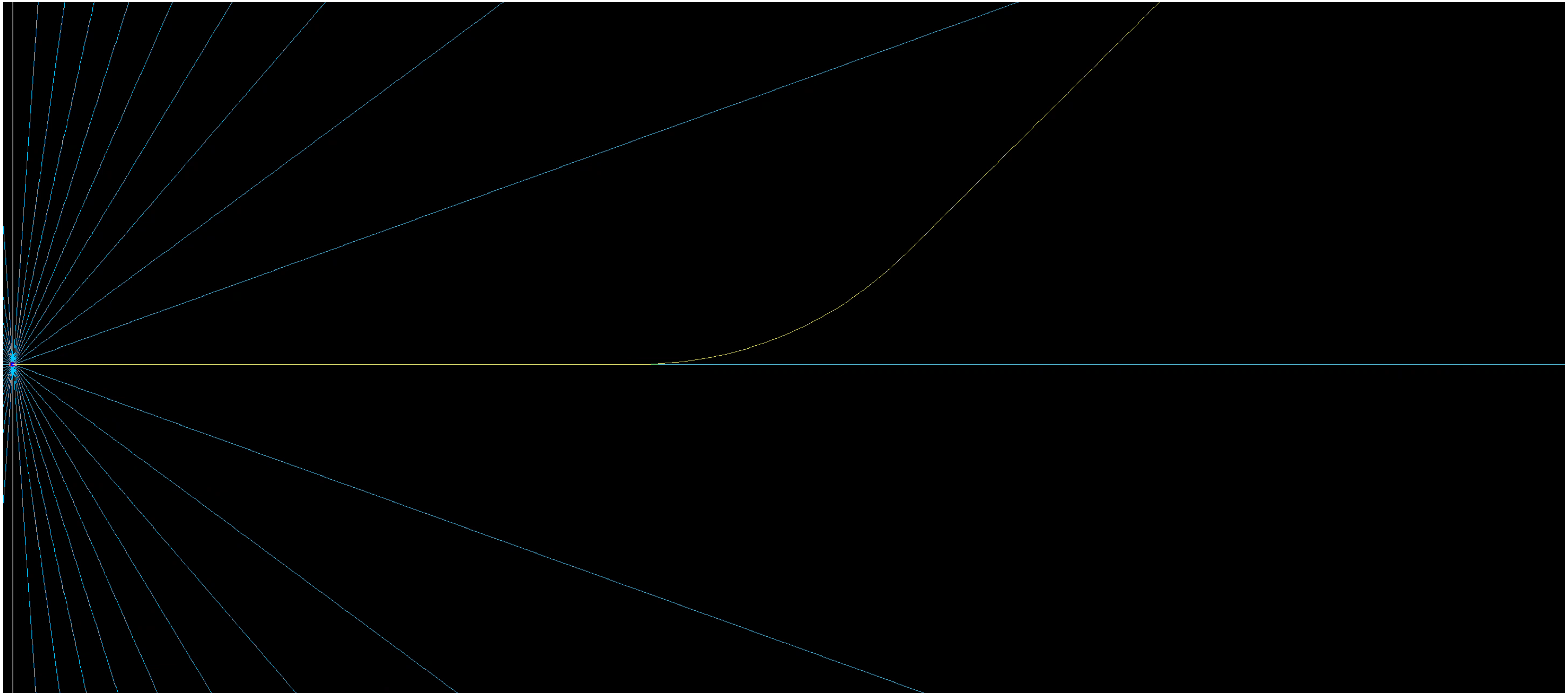
What is synchrotron radiation: 2. Ultrarelativistic

- High $\gamma \Longrightarrow$ short radiation pulse



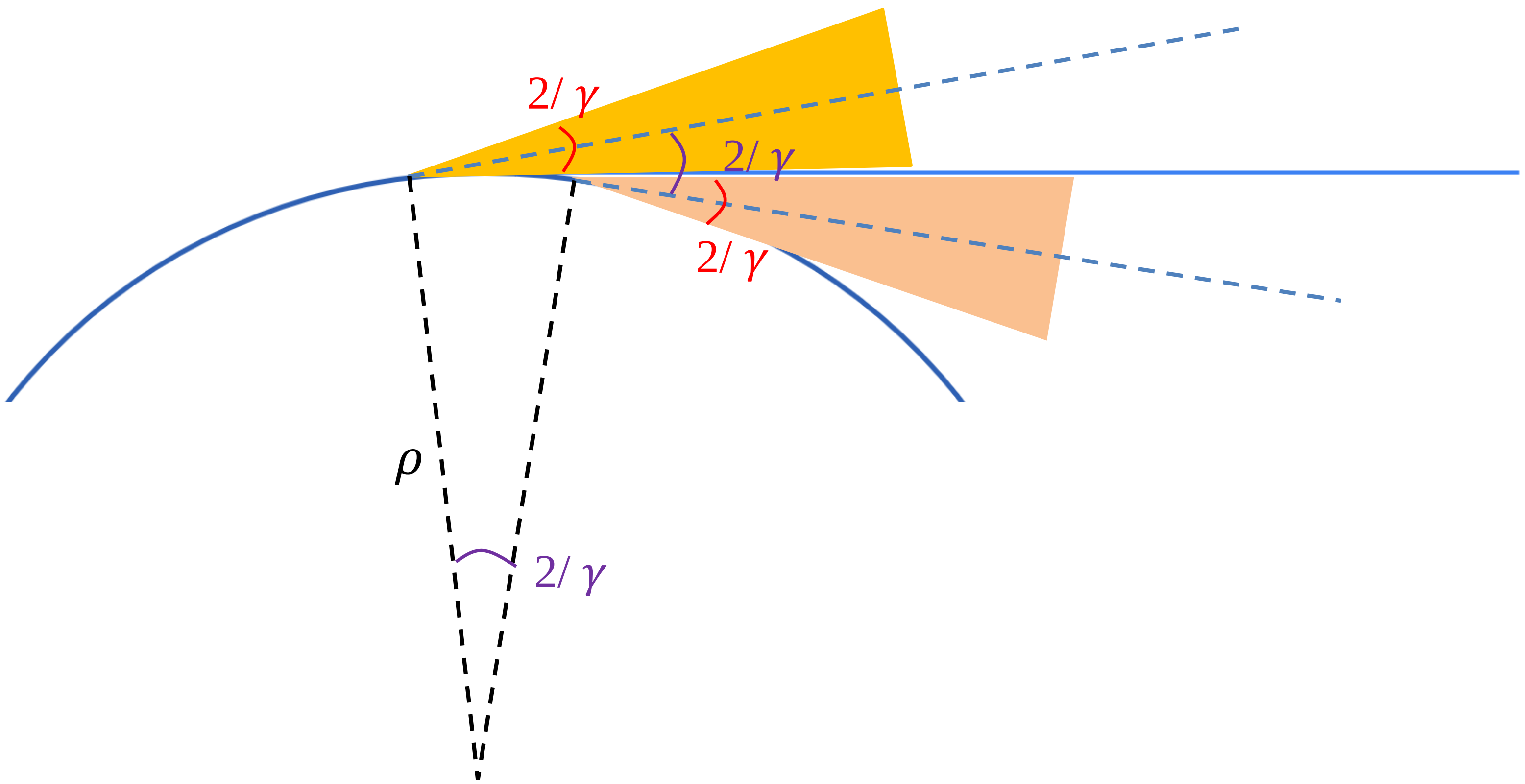
Without Doppler effect

What is synchrotron radiation: 2. Ultrarelativistic



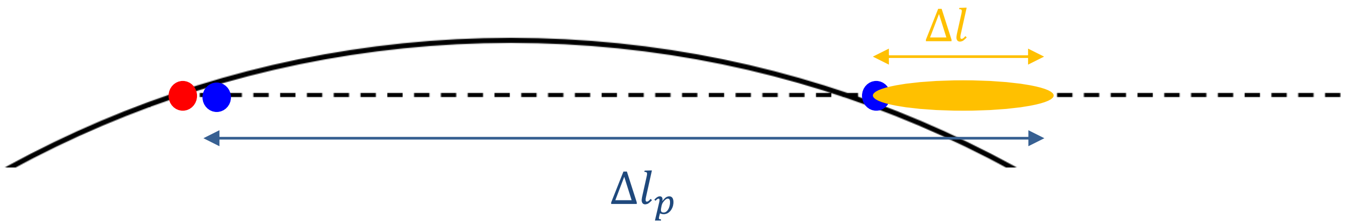
What is synchrotron radiation: 2. Ultrarelativistic

- **Ultrarelativistic:** $\gamma \gg 1$ **and corresponding instantaneous divergence of radiation beam**

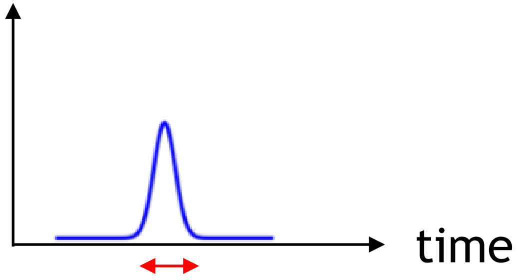


What is synchrotron radiation: 2. Ultrarelativistic

- **Ultrarelativistic: $\gamma \gg 1$ and corresponding instantaneous divergence of radiation beam**



Intensity of radiation beam seen by observer



$$\Delta t = \frac{4\rho}{3\gamma^3 c}$$

Time for electron to travel the arc $\Delta t_e = \frac{2\rho}{\gamma\beta c}$

Distance for photon to travel $\Delta l_p = \Delta t_e c = \frac{2\rho}{\gamma\beta}$

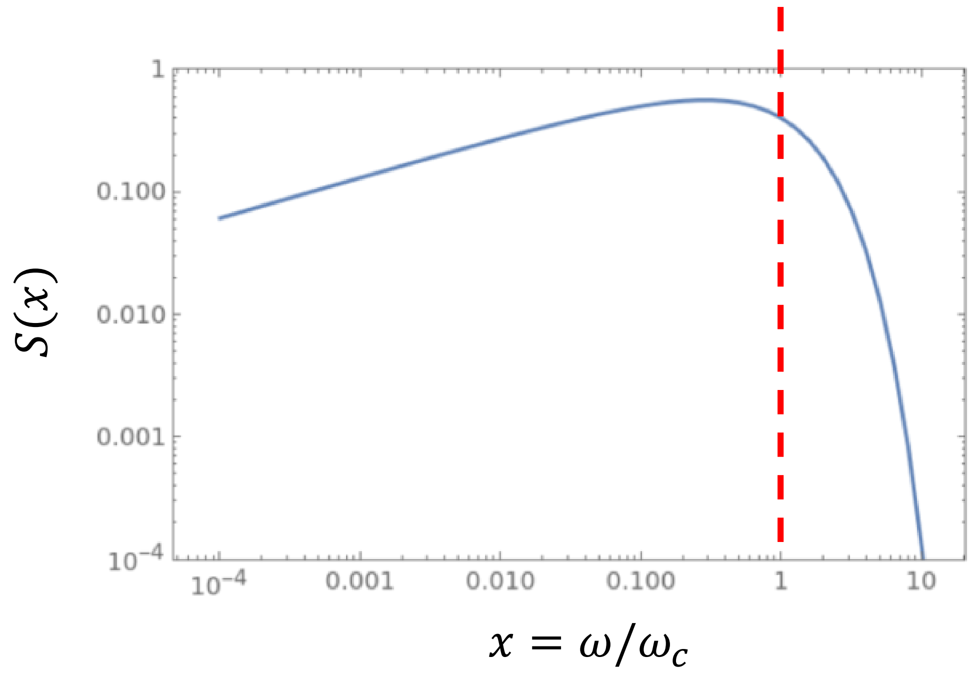
Pulse duration for an observer $\Delta t = \frac{\Delta l}{c} \approx \frac{4\rho}{3\gamma^3 c}$

Doppler effect: $O\left(\frac{\Delta t}{\Delta t_e}\right) = \frac{1}{\gamma^2}$

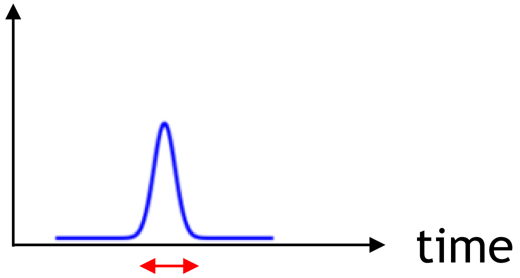
What is synchrotron radiation: 2. Ultrarelativistic

- **Critical frequency** $\omega_c = \frac{3}{2}\frac{\gamma^3 c}{\rho}$

  $\frac{3}{2}$: a factor such that the integrated power below and above ω_c are the same.



Intensity of radiation beam seen by observer



$$\Delta t = \frac{4\rho}{3\gamma^3 c} \approx \frac{\rho}{\gamma^3 c} \rightarrow \Delta\omega \approx \frac{\gamma^3 c}{\rho}$$

Frequency spectrum (integrate over polar angle θ)

$$\frac{d^2 W}{d\psi d\omega} = \frac{9\sqrt{3}}{8\pi} r_0 mc\gamma S\left(\frac{\omega}{\omega_c}\right)$$

Universal function of synchrotron radiation

$$S(x) = \frac{9\sqrt{3}}{8\pi} x \int_x^\infty K_{5/3}(x')\, dx'$$

What is synchrotron radiation: 3. low enough acceleration

- Critical photon energy

$$u = \hbar\omega$$

$$u_c[\text{keV}] = 0.665E^2[\text{GeV}^2]B[\text{T}]$$

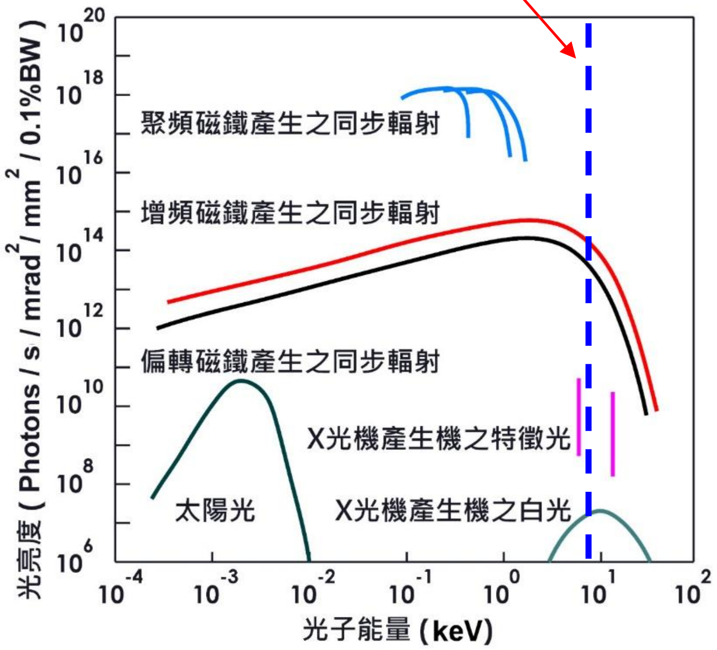
Visible light: 1.7~3.1 eV

Critical photon energy

	E[GeV]	B[T]	u_c
GE	0.07	0.81	2.64 eV
TLS	1.5	1.43	2.14 keV
TPS	3.0	1.19	7.12 keV
SPring8	8.0	0.679	28.9 keV
Dream	2000=2 TeV	1000	2.66 TeV

Soft x-ray user

Hard x-ray user



2.66 TeV> 2 TeV Problem!!

Classical electrodynamics breaks down

https://www.nsrrc.org.tw/chinese/lightsource.aspx

http://www.spring8.or.jp/en/about_us/whats_sp8/facilities/accelerators/storage_ring/

What is synchrotron radiation: 4. Normal acceleration

Longitudinal acceleration also generates radiation, and the power distribution also collimated along the velocity direction when v is close to c.

$$\frac{P_{\text{longi}}}{P_{\text{trans}}} = \frac{1}{\beta^4\gamma^2}\left(\frac{G\rho}{\gamma mc^2}\right)^2$$

G: accelerating gradient

For TPS

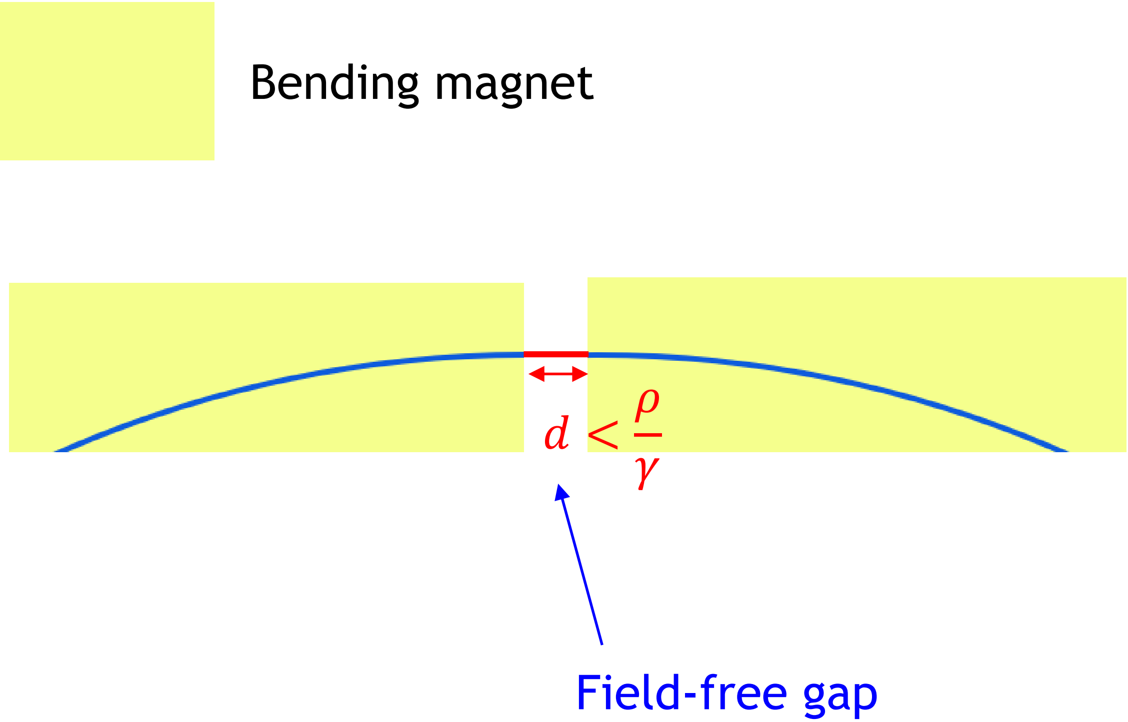
$$\gamma mc^2 = 3\ \text{GeV}$$

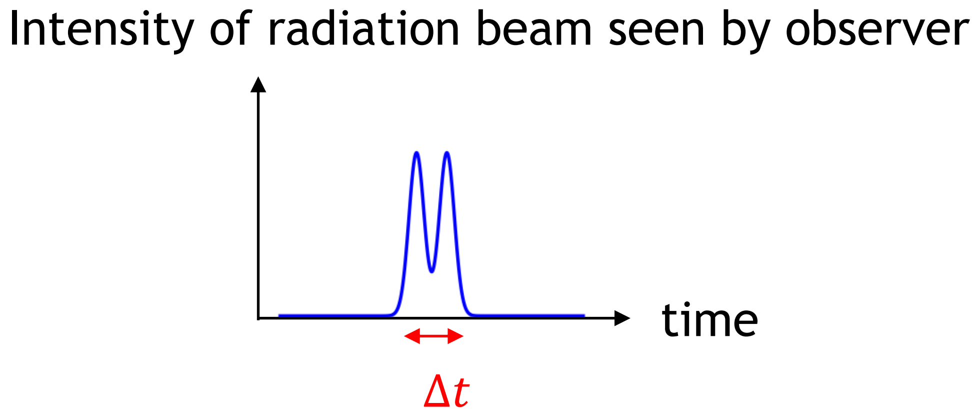
$$G \approx 10\ \text{MV/m}$$

$$\rho = 8.4\ \text{m}$$

$$\rightarrow \frac{P_{\text{longi}}}{P_{\text{trans}}} = 2.05 \times 10^{-10}$$

What is synchrotron radiation: 5. Quasi-constant acceleration





Characteristic frequency changes

Discussion in A. W. Chao's Lectures on Accelerator Physics.

Summary 1

Goals

1. Charged: electromagnetic radiation
2. Ultrarelativistic : $\gamma \gg 1$, velocity is close (not only comparable) to c=299792458 m/s
3. Suitable: acceleration is low enough
4. Normal acceleration: perpendicular to velocity
5. Quasi-constant acceleration: duration the pulse

Questions

1. Which charged particle? Electron, proton or ions?
2. How large is γ? 100, 1000, 10000 or even higher?
3. Included in 2 & 4?
4. How? Electric or magnetic field?
5. How long should the field in Q4 be?

Accelerator for Synchrotron Radiation Source

1. Which charged particle? Electron, proton or ions?

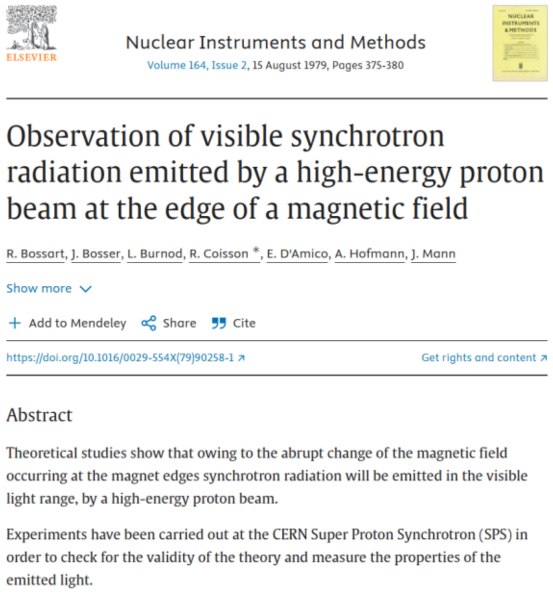
- **Total radiation power** $P = \frac{\beta^4 e^2 c^3}{2\pi} C_\gamma \frac{E^4}{\rho^2}$ ρ: bending radius

$$C_\gamma = \frac{4\pi}{3}\frac{r_0}{(mc^2)^3} = \begin{cases} 8.845 \times 10^{-5}\ \mathrm{m/(GeV)^3} & \text{Electron} \\ 4.840 \times 10^{-14}\ \mathrm{m/(GeV)^3} & \text{Muon} \\ 7.783 \times 10^{-18}\ \mathrm{m/(GeV)^3} & \text{Proton} \end{cases}$$

- **Ratio of the radiation power of electron and proton**

Mass ratio

$$8.845 \times 10^{-5}/(7.783 \times 10^{-18}) = 1.14 \times 10^{13} = 1836^4$$


Nuclear Instruments and Methods
Volume 164, Issue 2, 15 August 1979, Pages 375-380

Observation of visible synchrotron radiation emitted by a high-energy proton beam at the edge of a magnetic field

R. Bossart, J. Bosser, L. Burnod, R. Coisson *, E. D'Amico, A. Hofmann, J. Mann

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https://doi.org/10.1016/0029-554X(79)90258-1

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Abstract

Theoretical studies show that owing to the abrupt change of the magnetic field occurring at the magnet edges synchrotron radiation will be emitted in the visible light range, by a high-energy proton beam.

Experiments have been carried out at the CERN Super Proton Synchrotron (SPS) in order to check for the validity of the theory and measure the properties of the emitted light.

Synchrotron radiation from proton beam has beam observed decades ago

Energy loss per turn

	Advance photon source	Large hadron collider
Particle	Electron	Proton
E[GeV]	7	7000
ρ[m]	38.96	3096.2
$U_0 = P \times 2\pi\rho/\beta c$[MeV]	5.45	0.006
u_c [keV]	19.5	0.04=40 eV

4. How? Electric or magnetic field?

- Lorentz force

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$v \to c \Rightarrow \frac{F_E}{F_B} = \frac{\mathbf{E}}{c\mathbf{B}}$$

B=1 T → E~300 MV/m

Electrical breakdown for dry air ~3 MV/m

- Magnetic force → Normal acceleration

2. How large is γ? 100, 1000, 10000 or even higher?

TPS Beamlines

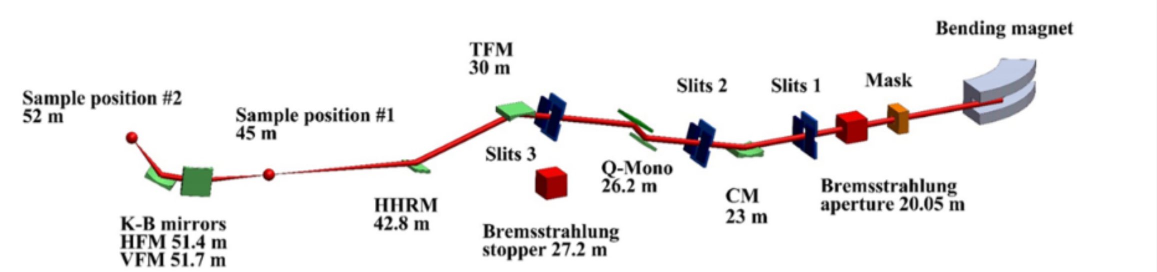
Home | Photon Source | Experimental Techniques | Beamline Directory | Apply For Beamtime

44A Quick-scanning X-ray Absorption Spectroscopy

Beamline Specification

- Energy range: 4.5 – 34 keV
- Flux (microfocused): 5×10^{11} (2×10^{10}) s^{-1} at 10 keV
- Beam size (microfocused): 60 (H) × 200 (V) (5 × 5) μm^2
- Resolving power: 7000
- High harmonic ratio: < 10^{-4}

Beamline Layout

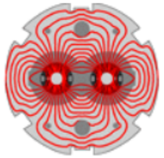


Beamline Home

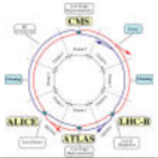
05A	07A	09A	13A
15A	19A	20A	21A
23A	24A	25A	27A
31A	32A	35A	38A
39A	41A	43A	44A
45A			

- Find a Beamline
- 44A Home
- Specifications
- Endstations
- Staff
- BL Schedule
- Optical Layout

2. How large is γ? 100, 1000, 10000 or even higher?



MAGNETS



Home	Introduction	Pictures	Favourites	Photo Gallery	Blog

There are a large variety of magnets in the LHC. However the big ones are the main dipoles.

Main Dipoles

The main budget item and a serious technological challenge are the superconducting (1.9 K) dipoles which bend the beams around the 27 km circumference of the LHC. At 7 TeV these magnets have to produce a field of around 8.4 Tesla at a current of around 11,700 A. The magnets have two apertures, one for each of the counter-rotating beams. Each one is 14.3 metres long. A total of 1232 are needed.

- Vertical B field in the dipole bends the beam round via the Lorentz force
- Need very strong magnets to get the high energy beam around the circle. Superconducting (1.9 K) dipoles producing a field of 8.3 T - current 11,850 A
- 2-in-1 magnet design.
- Bending magnets (dipoles): 14.3 metres long. Cost: ~ 0.5 million CHF each. Need 1232 of them
- Quads etc to keep beam focused and the motion stable
- Stored magnetic energy up to 1.29 GJ per sector. Total stored energy in magnets = 11GJ
- One dipole weighs around 35 tonnes.

https://lhc-machine-outreach.web.cern.ch/components/magnets.htm

Since the magnetic field is ~ few T $\rightarrow \gamma$: order of 10^3 ~ 10^4. Also, property 3 is satisfied in artificial accelerator.

What is synchrotron radiation: 3. low enough acceleration

- Critical photon energy

$$u = \hbar\omega$$

$$u_c[\text{keV}] = 0.665E^2[\text{GeV}^2]B[\text{T}]$$

	E[GeV]	B[T]	u_c
GE	0.07	0.81	2.64 eV
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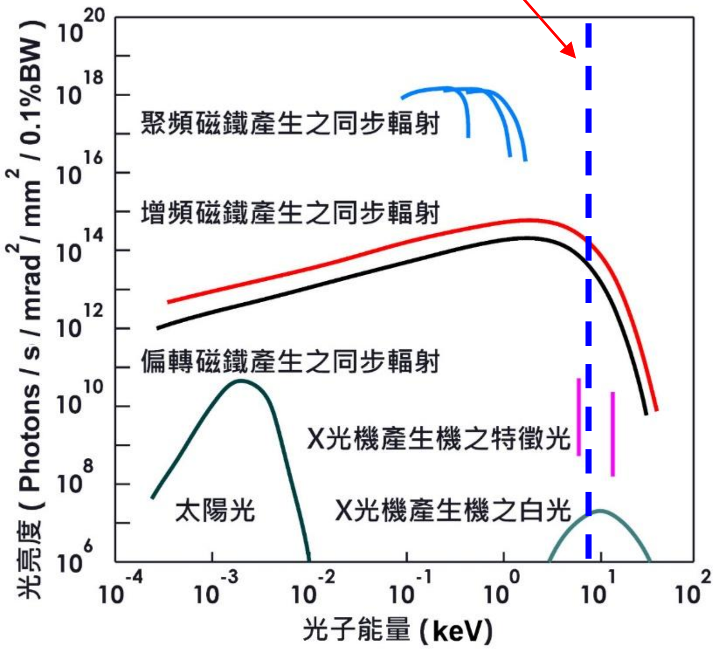
SPring8	8.0	0.679	28.9 keV

Visible light: 1.7~3.1 eV

Soft x-ray user

Hard x-ray user

Critical photon energy



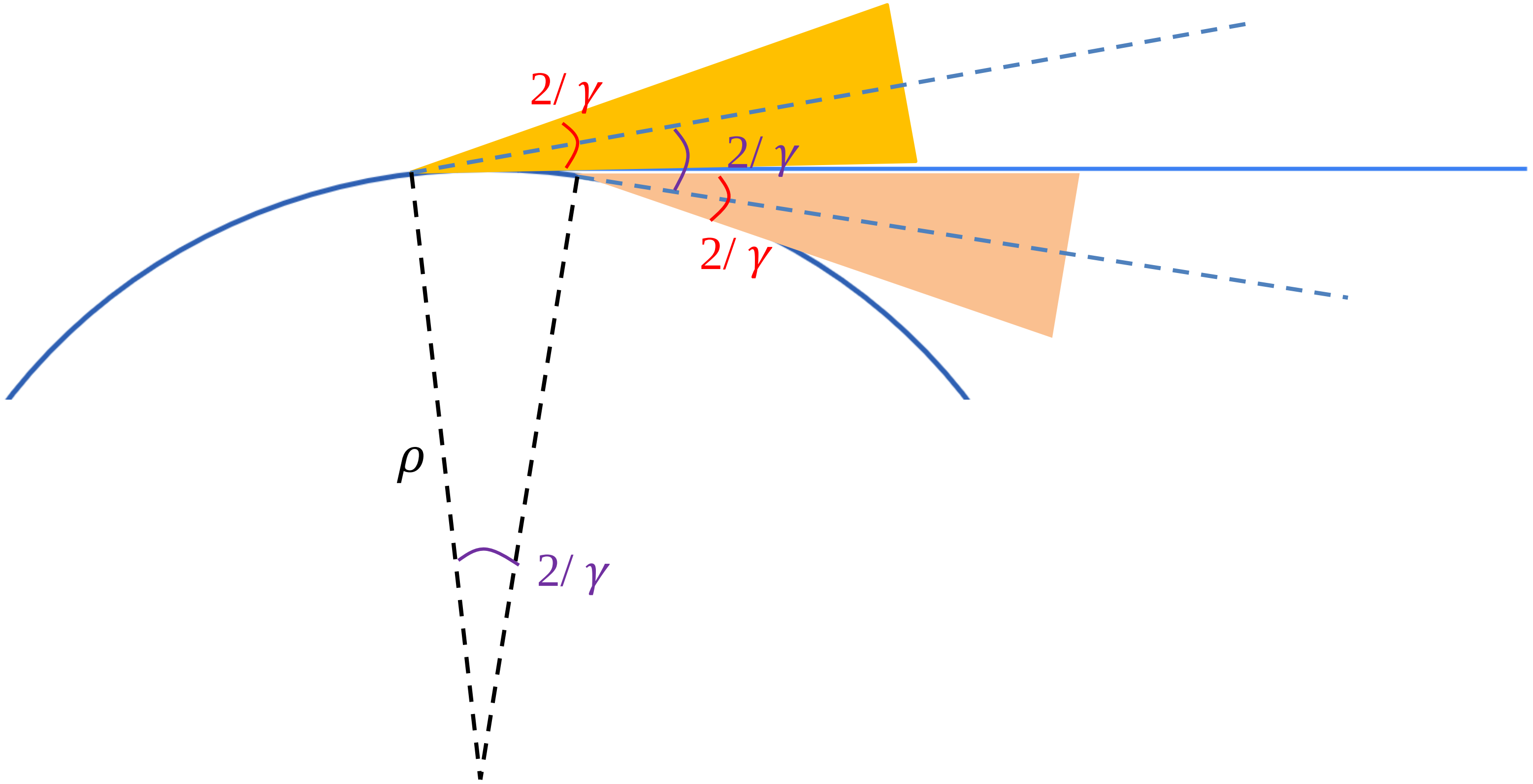
https://www.nsrrc.org.tw/chinese/lightsource.aspx

http://www.spring8.or.jp/en/about_us/whats_sp8/facilities/accelerators/storage_ring/

5. How long shold the magnetic field be?

- Constant field region > $2\rho/\gamma$

  For TPS 2*8.4/5870~ 2.86 mm (easily to be satisfied)



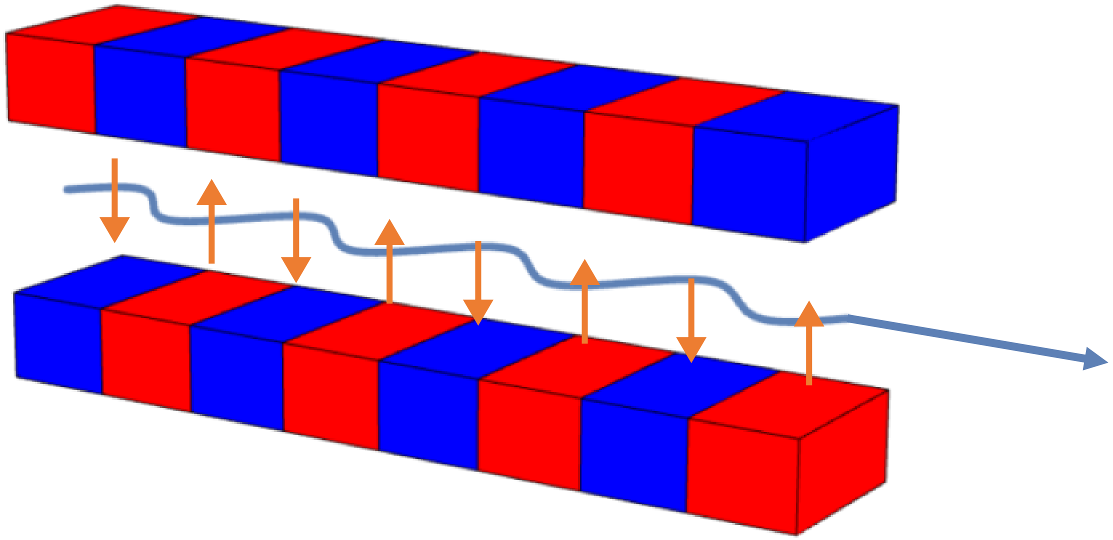
Undulator Radiation and Brilliance

Undulator radiation

If property 5 is relaxed.

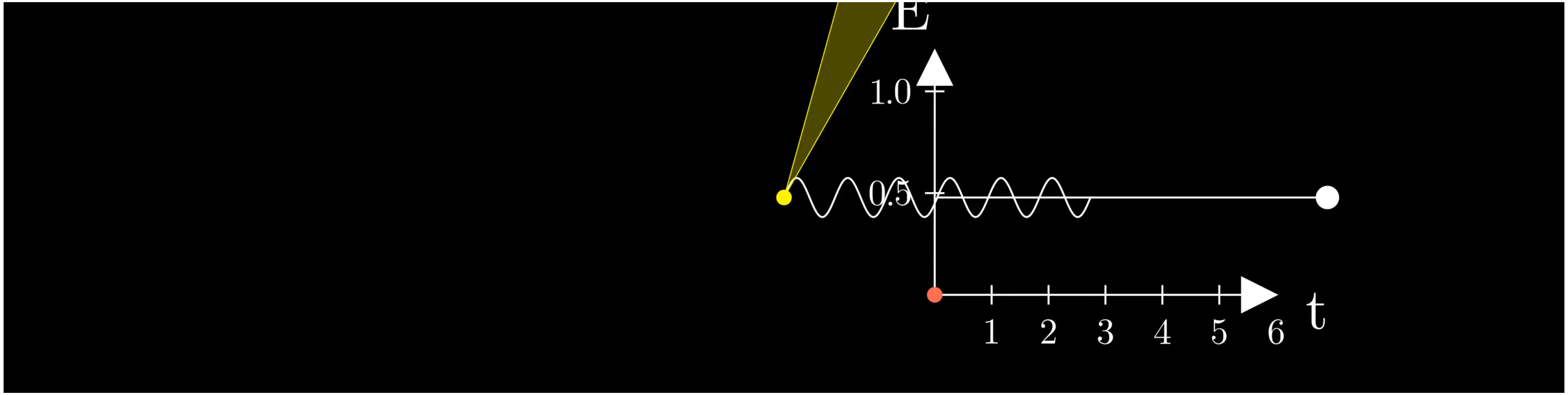
Insertion devices: special magnets that are installed in the straight sections

Wiggler and undulator: periodic magnetic field

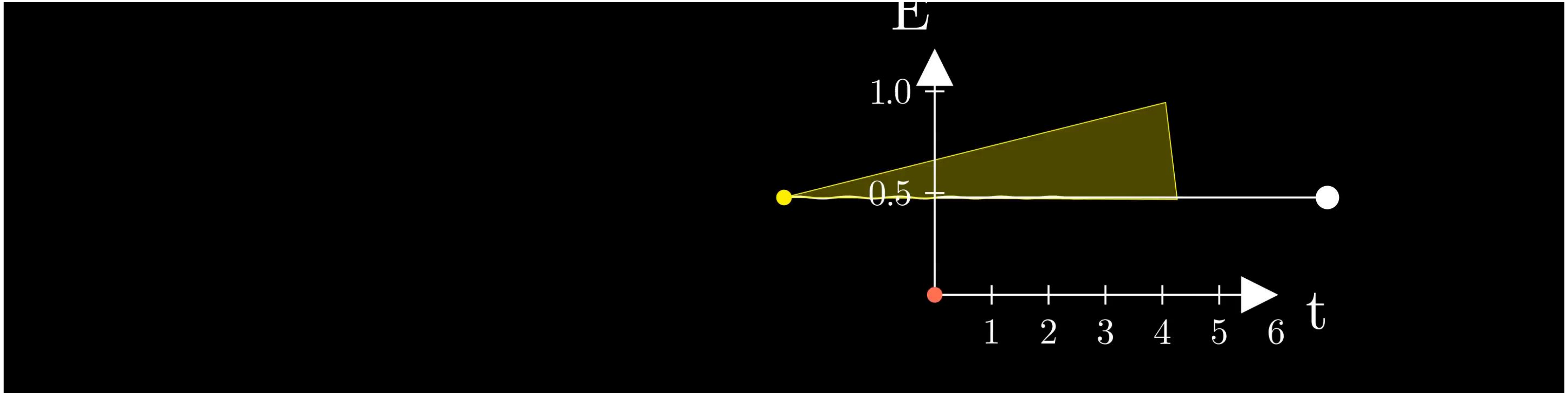


Undulator radiation

Wiggler



Undulator

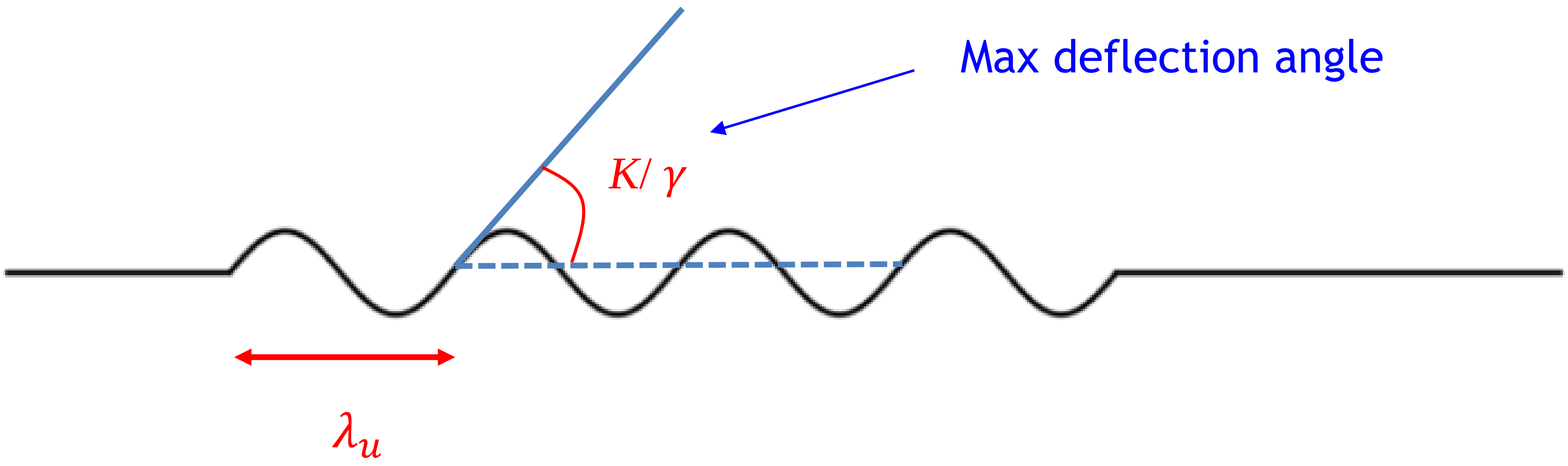


Without Doppler effect

Undulator radiation

Deflection parameter: K

$$K = \frac{eB_0\lambda_u}{2\pi mc} = 0.934\, B(\mathrm{T})\lambda_u(\mathrm{cm})$$

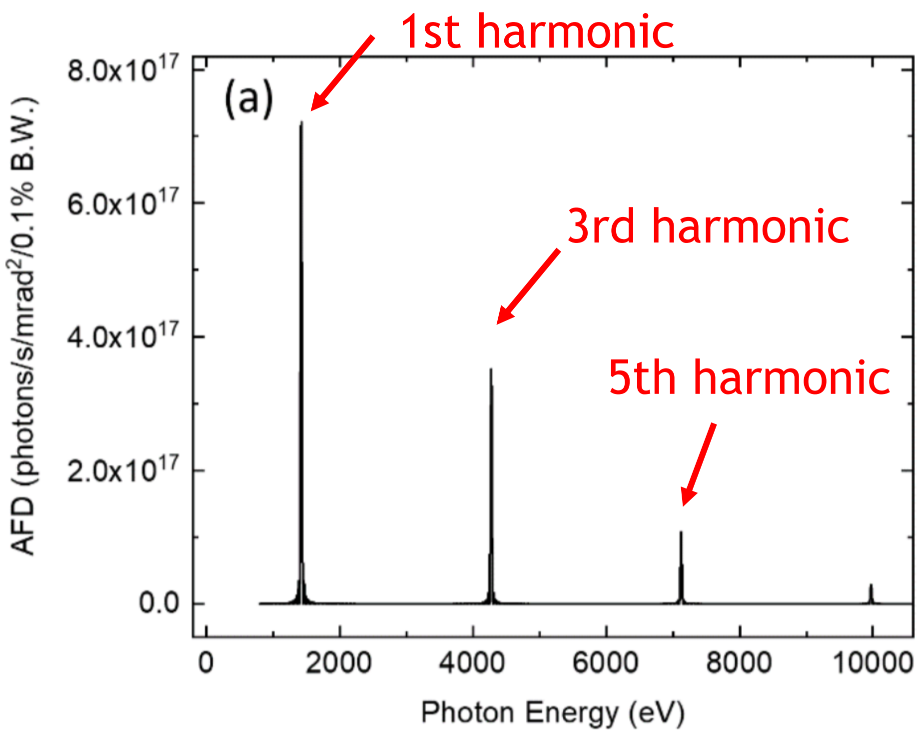


Period length of a wiggler or an undulator

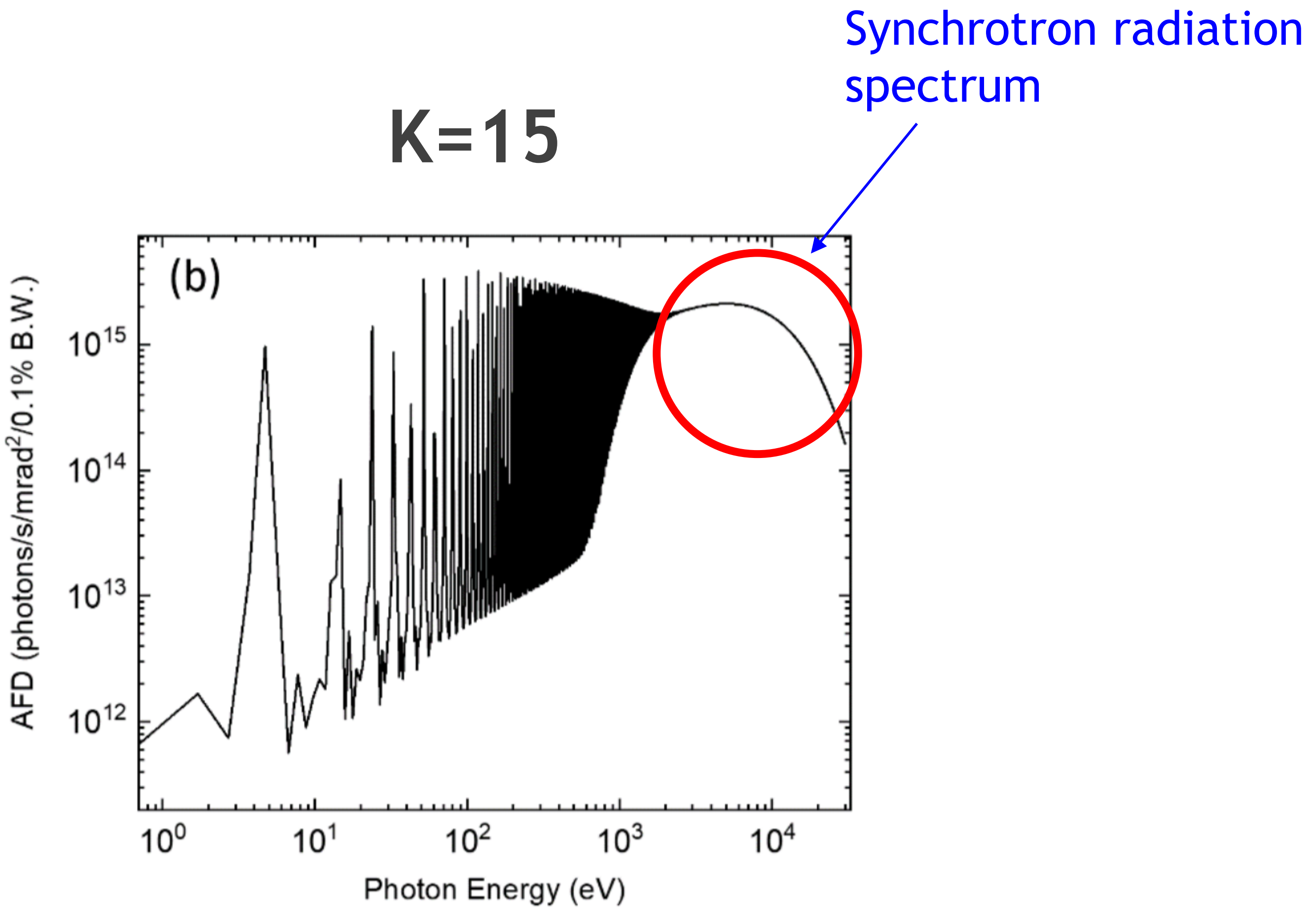
Undulator radiation

Typical on-axis spectrum of Wiggler(K>>1) and undulator(K~1)

K=1



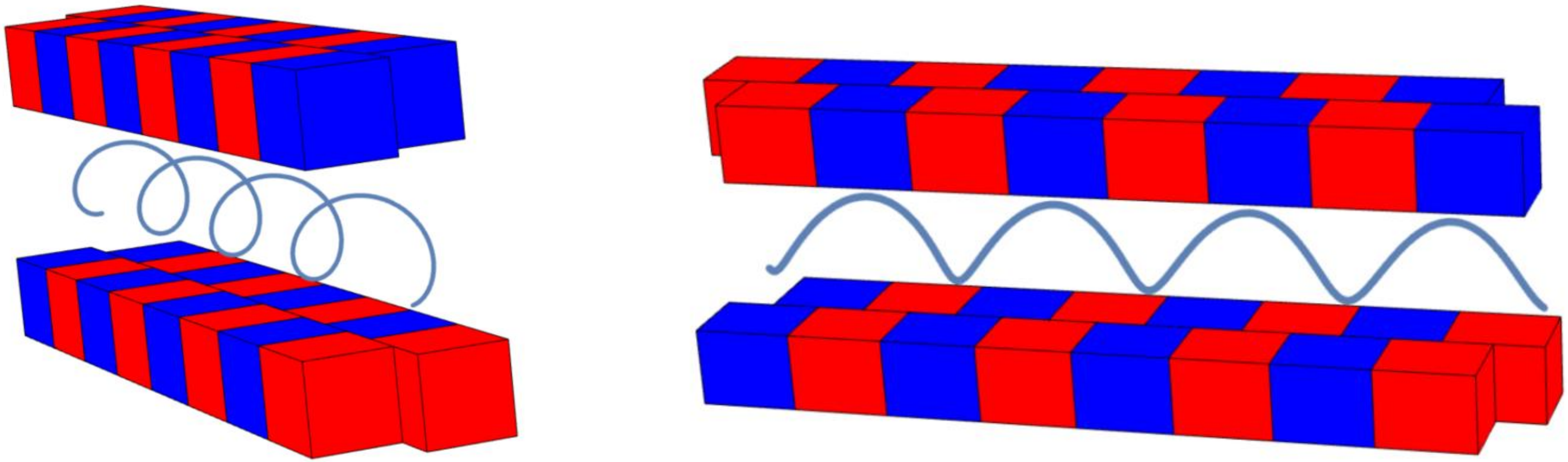
K=15



Calculated by SPECTRA (Takashi Tanaka)

Undulator radiation

Helical undulator: only fundamental harmonics for on-axis observer: constant longitudinal speed



By adjusting the relative longitudinal positions of 4 arrays of magnets

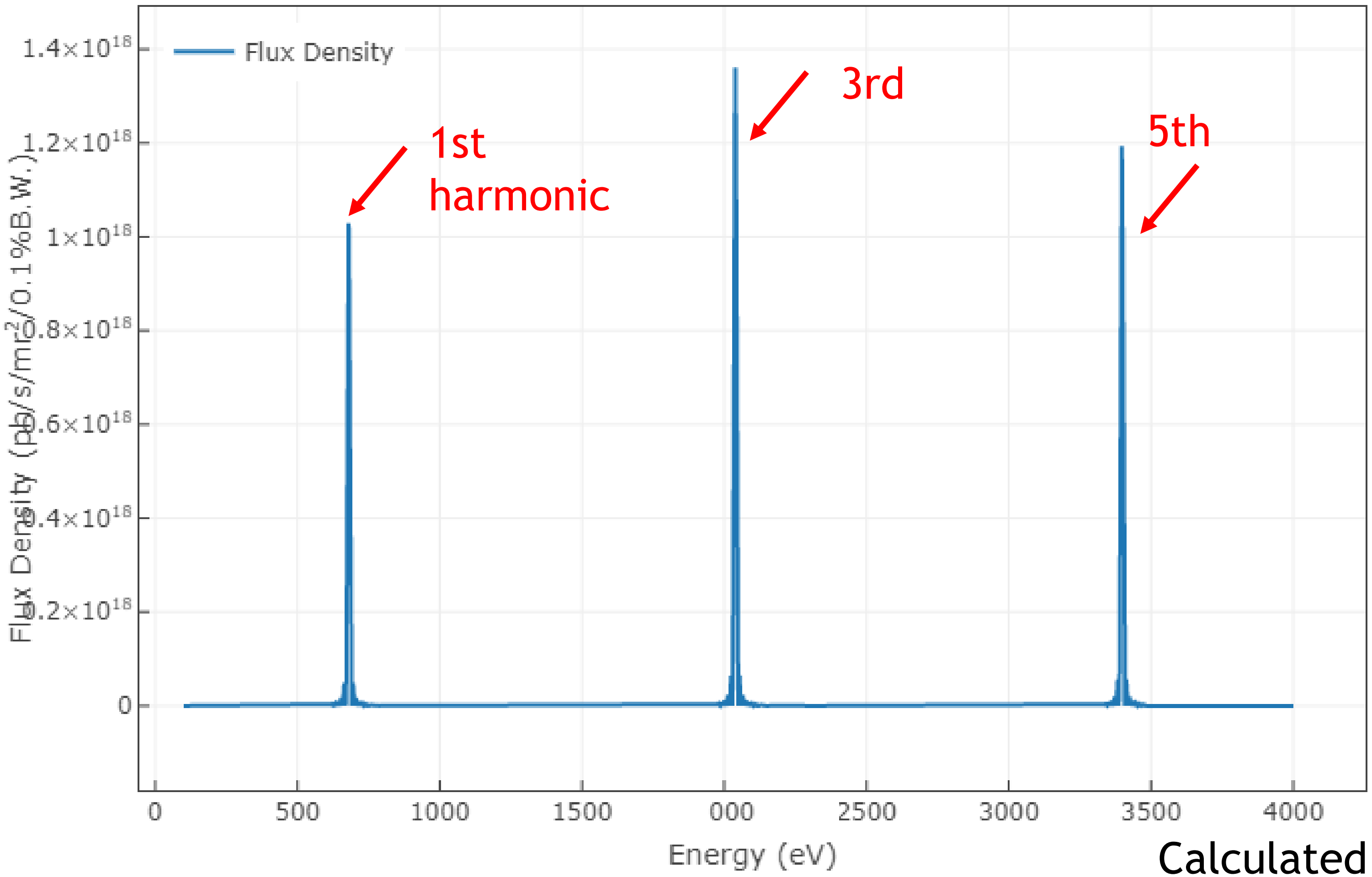
Any elliptical and inclined polarized radiation could be generated

→ Elliptical polarized undulator (EPU)

Shigemi Sasaki, Analysis of Advanced Planar Polarized Light Emitter
https://inis.iaea.org/collection/NCLCollectionStore/_Public/25/022/25022400.pdf?r=1

Undulator radiation

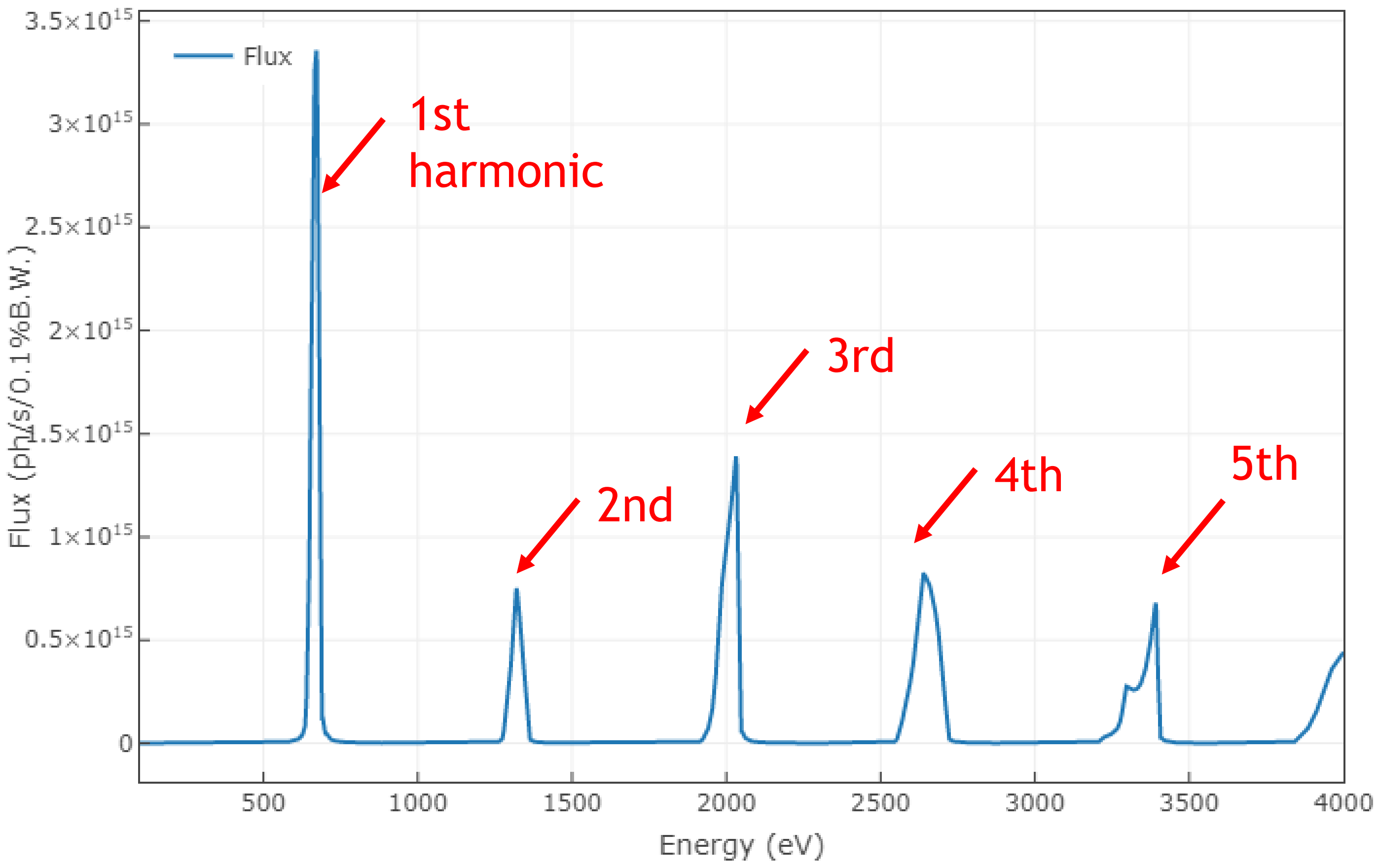
On-axis spectrum



Calculated by SPECTRA (Takashi Tanaka)

Undulator radiation

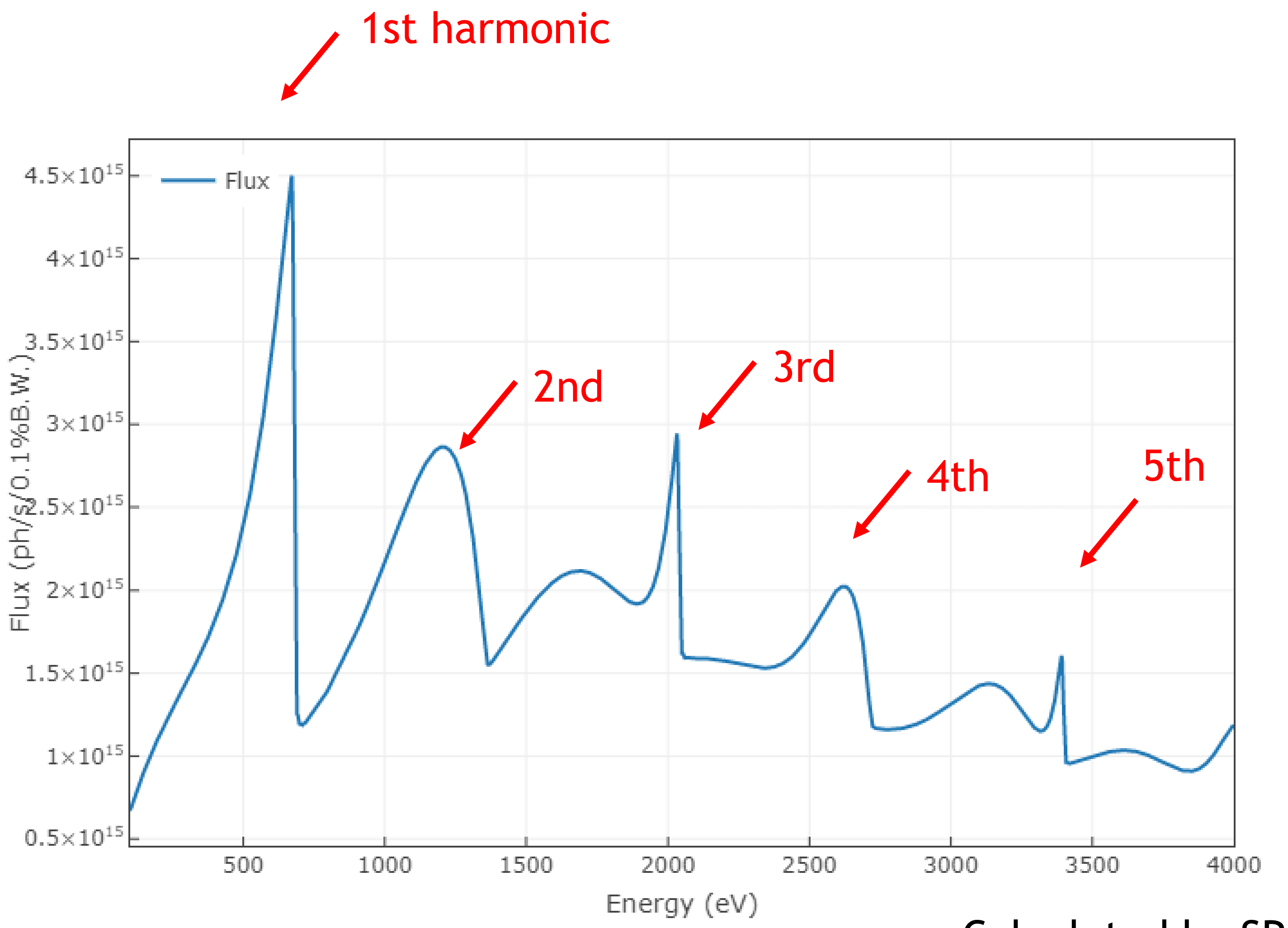
Spectrum of a small pinhole (center on the axis) at downstream of the undulator



Calculated by SPECTRA (Takashi Tanaka)

Undulator radiation

Overall spectrum



Calculated by SPECTRA (Takashi Tanaka)

Undulator radiation (1st harmonic)

Virtual source point (center of undulator)

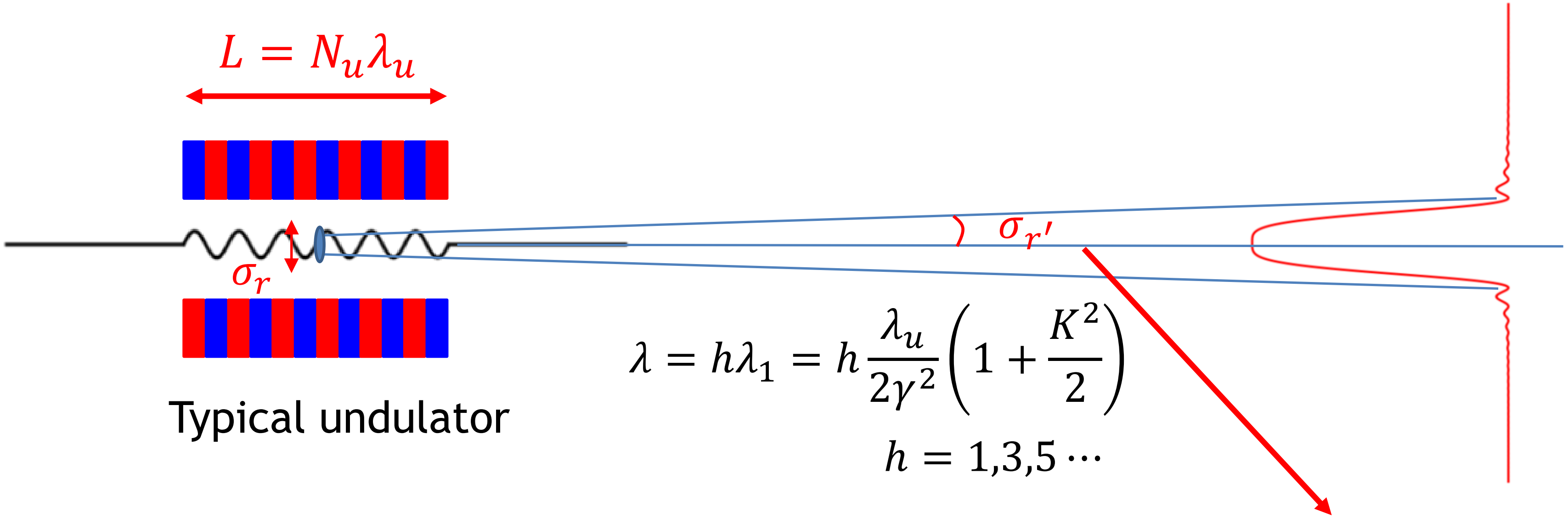
$$P(r) \propto \left[\pi - 2\mathrm{Si}\left(\frac{2\pi}{L\lambda} r^2\right)\right]^2$$

Far-field region

$$P(\theta) \propto \mathrm{sinc}^2\left[\frac{\pi L}{2\lambda}\theta^2\right]$$

Gaussian approximation

$$P(\theta) \approx e^{-\frac{\theta^2}{\sigma_{r'}^2}}$$



Typical undulator

$$\lambda = h\lambda_1 = h\frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

$$h = 1,3,5\cdots$$

$$\sigma_r = \frac{\sqrt{2\lambda L}}{4\pi}$$

Property of Fourier transform

$$\sigma_r\sigma_{r'} = \frac{\lambda}{4\pi}$$

$$\sigma_{r'} = \sqrt{\frac{\lambda}{2L}}$$

Estimation

$$F(\omega) \propto \mathrm{sinc}^2\left(\pi N_u\frac{\Delta\omega}{\omega}\right)$$

$$\omega_1 = \omega_u\frac{2\gamma^2}{1 + \frac{K^2}{2} + \gamma^2\theta^2}$$

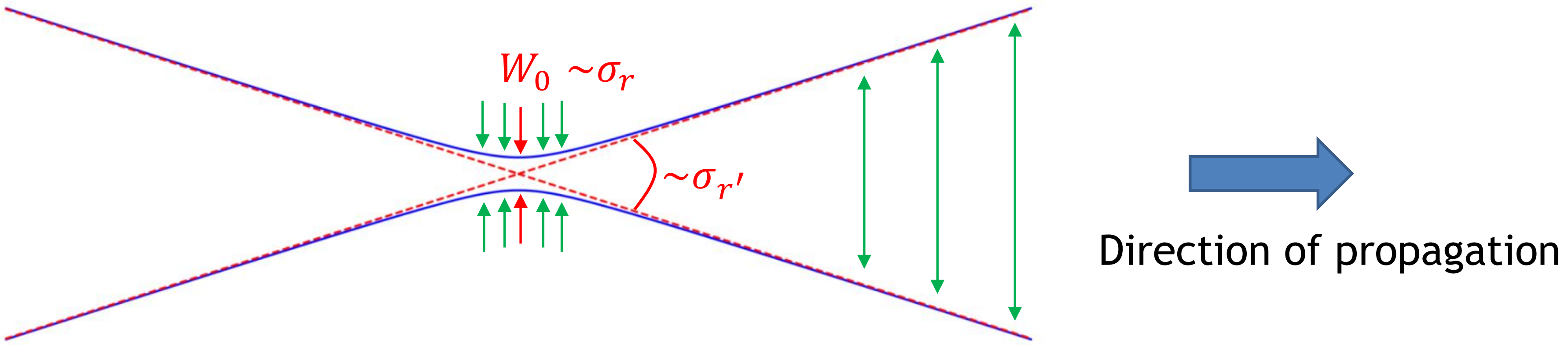
$$N_u\frac{\Delta\omega}{\omega} \approx 1,\ hN_u \gg 1$$

$$\rightarrow \sigma_{r'}^2 \approx \frac{1 + \frac{K^2}{2}}{\gamma^2 hN_u} = \sqrt{\frac{\lambda}{L}}$$

Brilliance (brightness, spectral brightness)

- **Spatial (transverse) coherence**
- **M^2 factor**

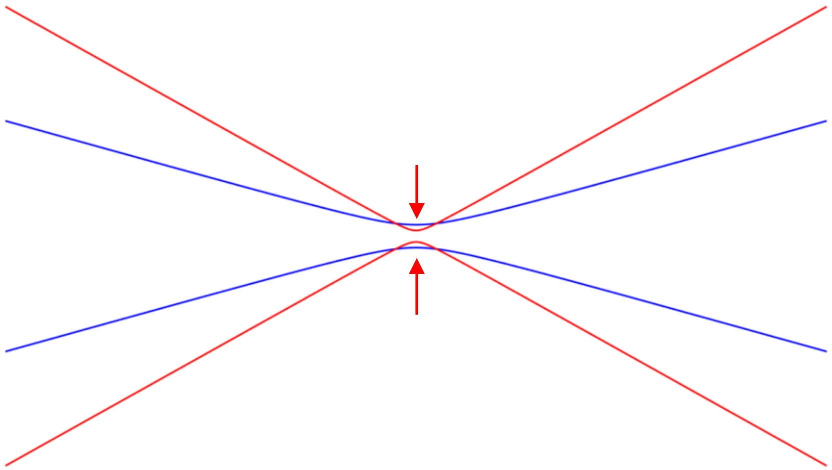
M^2: measure of beam quality ~ (divergence) × (size beam waist)



Blue: Longitudinal beam distribution
Red: Asymptote for beam divergence
Green: divergence and size are determined by fitting data that measured at different longitudinal positions.

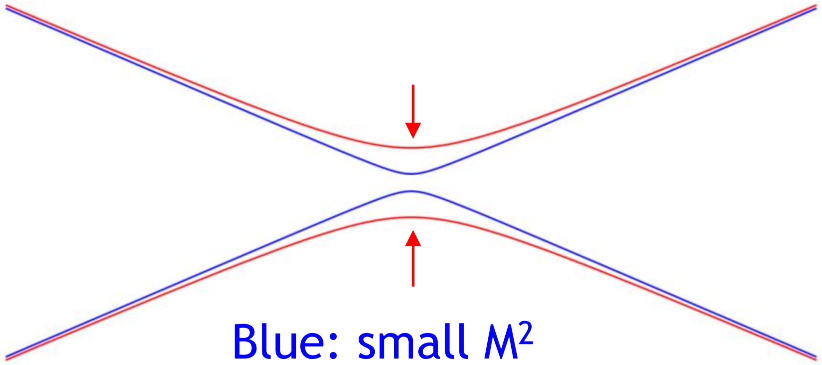
Brilliance: M^2 factor

Two beam with the same M^2



Blue: low divergence
Red: large divergence

Two beam with different M^2 but the same divergence



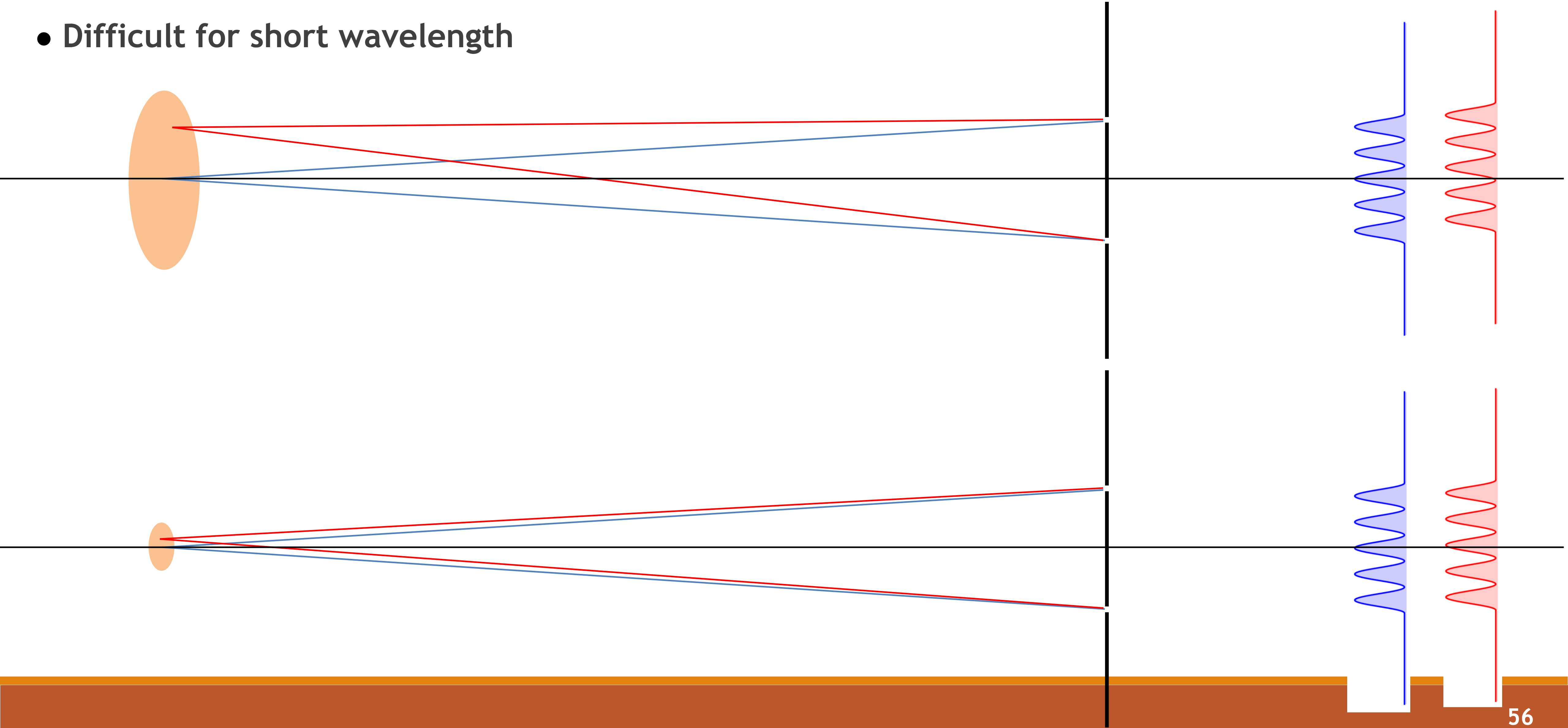
Blue: small M^2
Red: large M^2

Minimum beam waist: Gaussian beam
→M^2 =1

$$\sigma_r \sigma_{r'} = \frac{\lambda}{4\pi}$$

Brilliance: Spatial (transverse) coherence

- Visibility (V) of interference $\propto$ degree of coherence $\gamma(\boldsymbol{r}_1, \boldsymbol{r}_2)$ at slits or pinholes
- Difficult for short wavelength



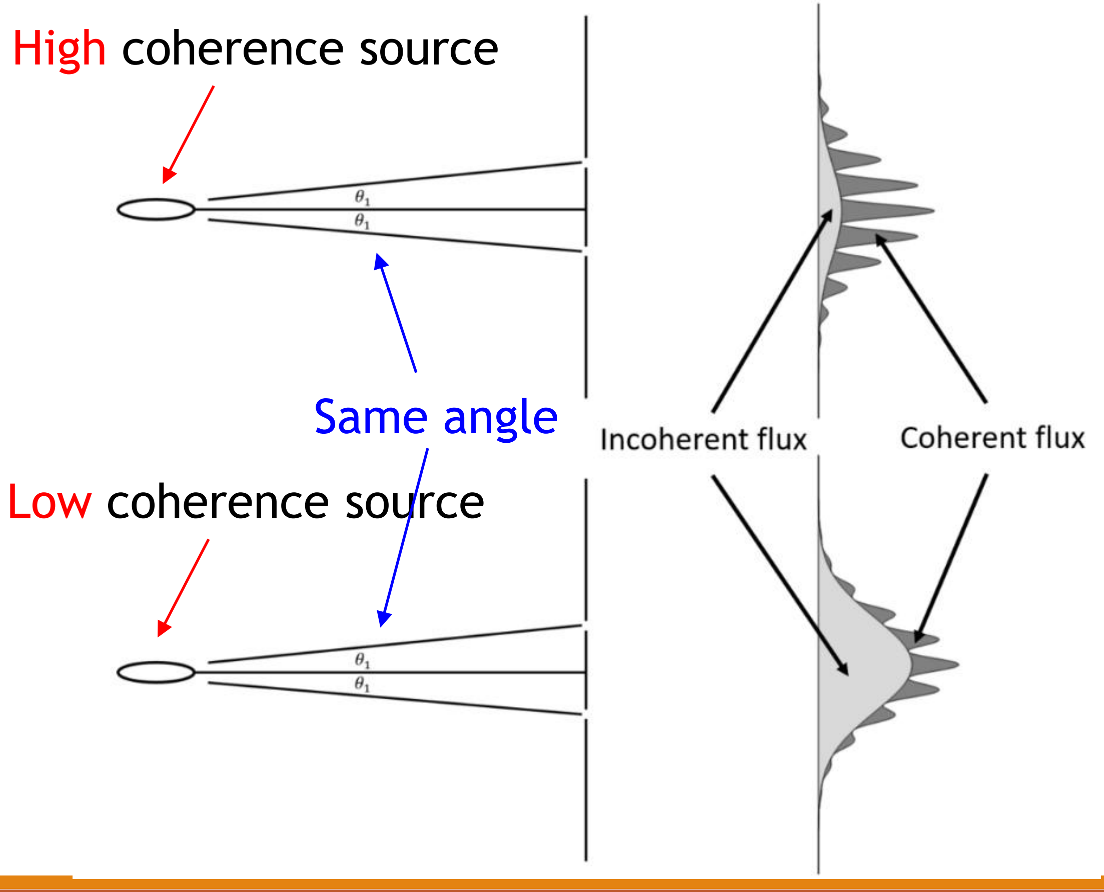
Brilliance: Spatial (transverse) coherence

- Correlation of the phase and amplitude at two positions
- Visibility (V) of interference $\propto$ degree of coherence $\gamma(\boldsymbol{r}_1, \boldsymbol{r}_2)$ at slits or pinholes

$$\gamma(\boldsymbol{r}_1, \boldsymbol{r}_2) = \frac{\langle \boldsymbol{E}(\boldsymbol{r}_1)\boldsymbol{E}^*(\boldsymbol{r}_2)\rangle}{\sqrt{\langle |\boldsymbol{E}(\boldsymbol{r}_1)|^2\rangle}\sqrt{\langle |\boldsymbol{E}(\boldsymbol{r}_2)|^2\rangle}}$$

$$V = 2\frac{\sqrt{\langle |\boldsymbol{E}(\boldsymbol{r}_1)|^2\rangle}\sqrt{\langle |\boldsymbol{E}(\boldsymbol{r}_2)|^2\rangle}}{\langle |\boldsymbol{E}(\boldsymbol{r}_1)|^2\rangle + \langle |\boldsymbol{E}(\boldsymbol{r}_2)|^2\rangle}|\gamma(\boldsymbol{r}_1, \boldsymbol{r}_2)|$$

$\langle \cdots \rangle$: emsemble average over electron beam



- Application: measurement of electron beam size

Brilliance:

- In general, spatial coherence and M² factor could be

  e.g. bending radiation that generated by filament electron beam (emittance→0)

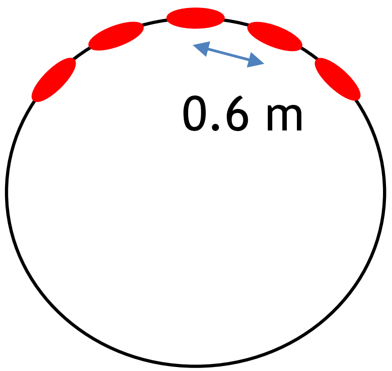
- Undulator radiation:

  brilliance↑ ⇒ transverse coherence ↑, M² ↓

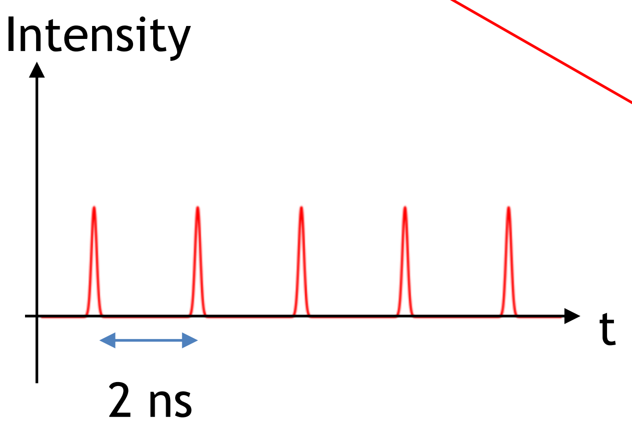
- Average brilliance v.s. Peak brilliance (FEL)

Note: for undulator
Angular flux density $\propto N_u^2$
Brilliance: $\propto N_u$ (without phase space matching)

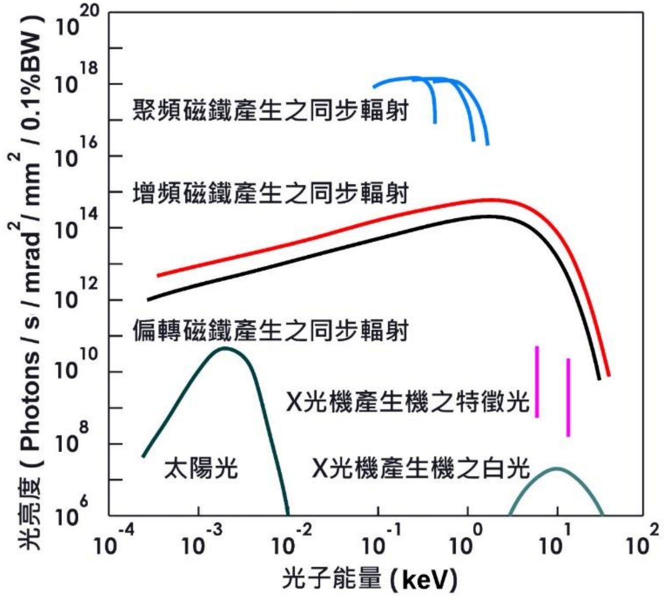
TPS



518.4 m/0.6m =864 bunches



Average brilliance



https://www.nsrrc.org.tw/chinese/lightsource.aspx

Phase space

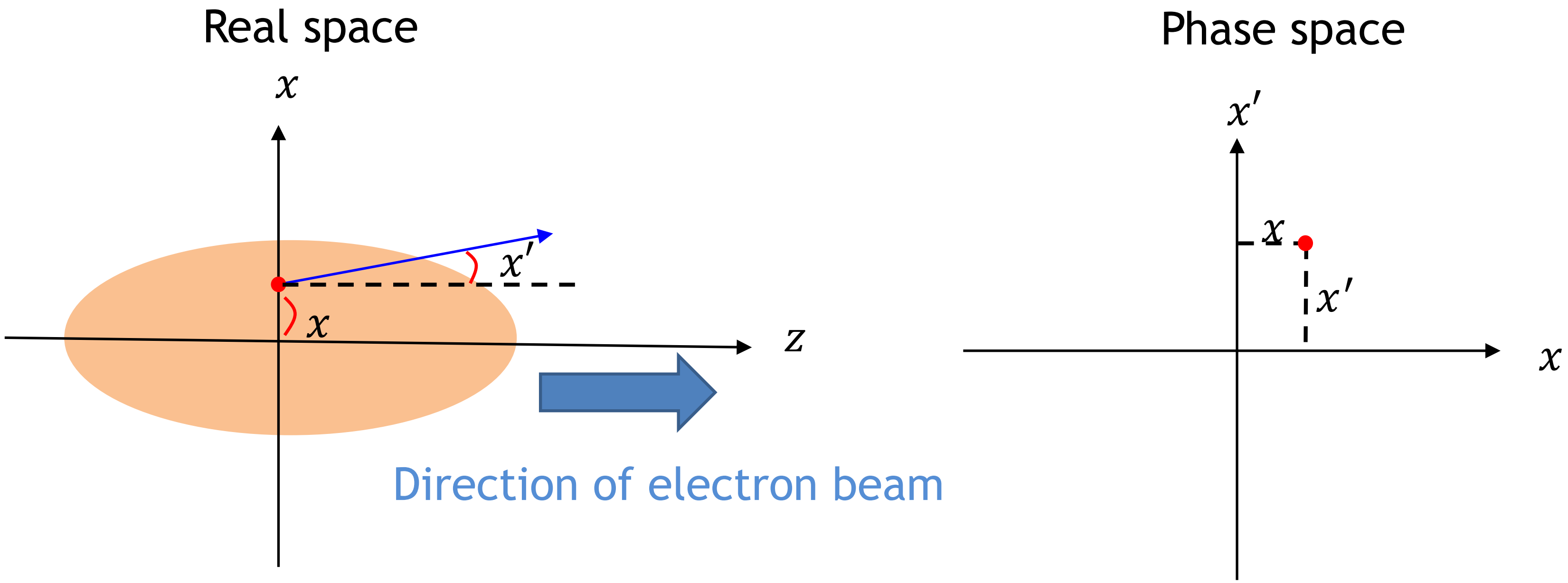
In Hamiltonian mechanics

Generalized coordinate: q

Conjugate momentum: $p = \frac{\partial \mathcal{L}}{\partial \dot{q}}$

(position, momentum, time) → canonical transforms → (position, angle, longitudinal position)

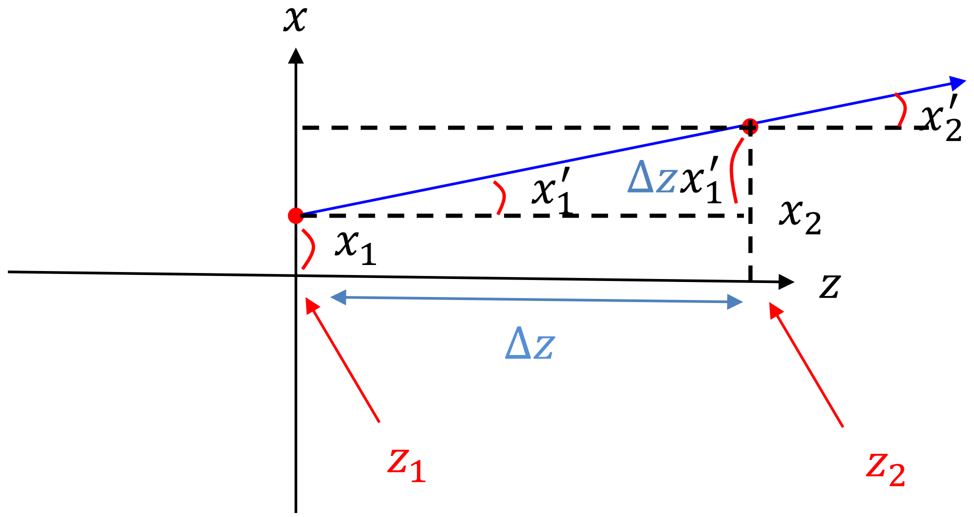
(q, p, t) → (x, x’, z)



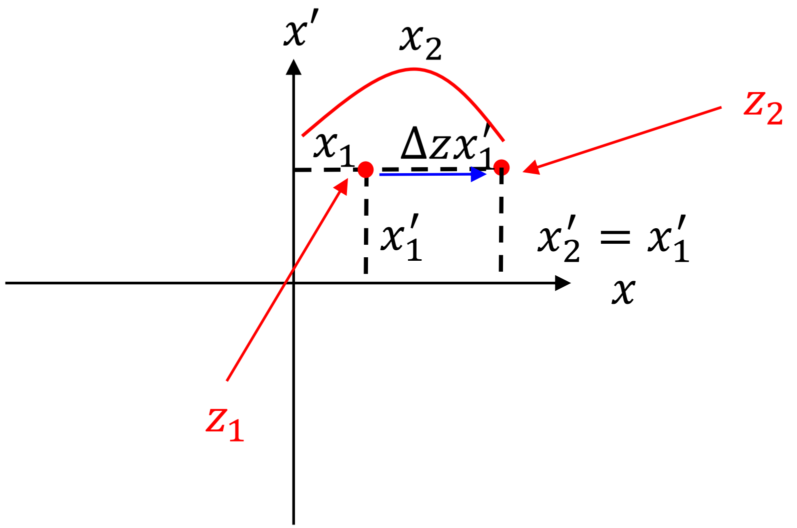
Phase space

In straight section, particle moves horizontally in space with velocity proportional to the angle

Real space



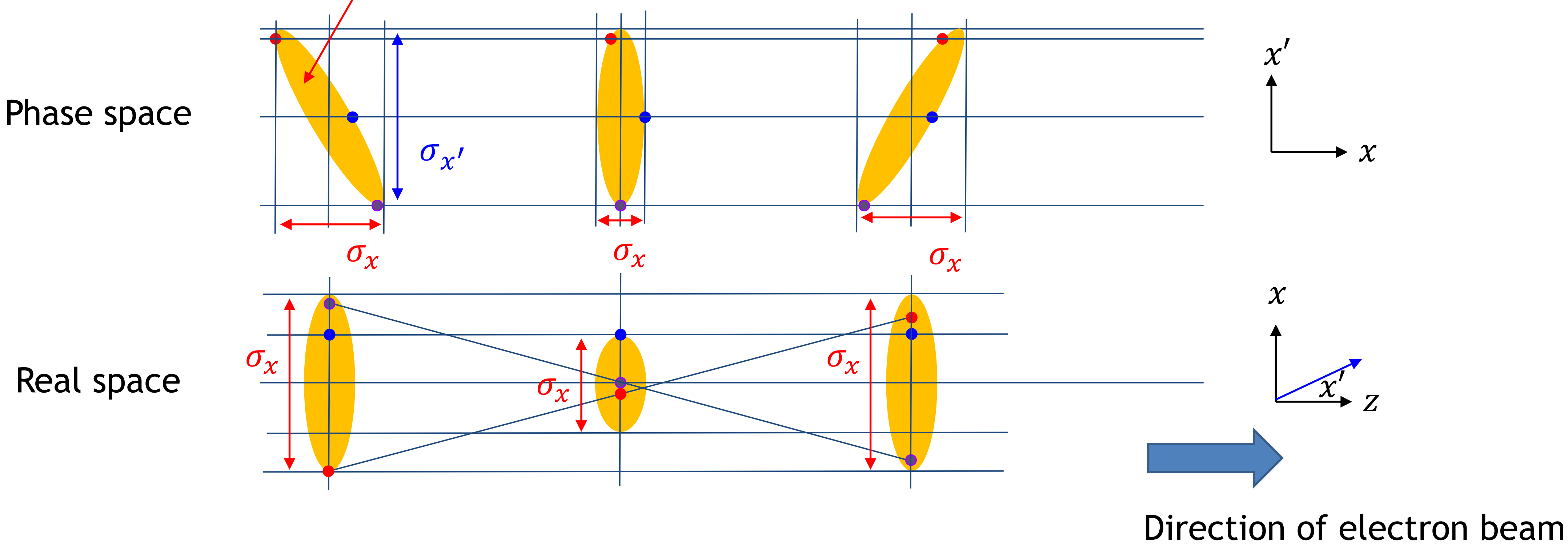
Phase space



Phase space

Area in phase space: emittance ϵ_x

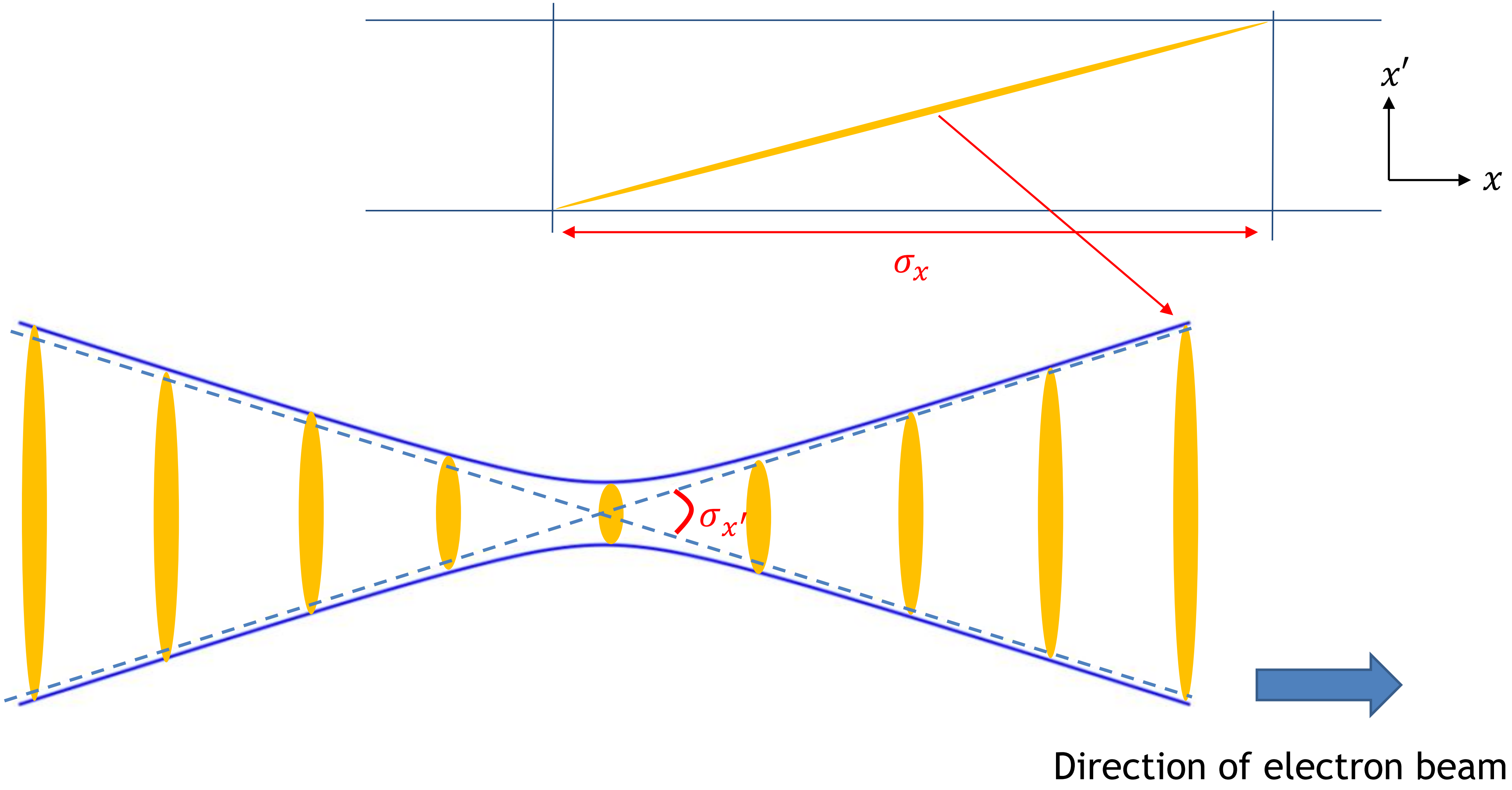
Emittance is conserved by Liouville theorem (symplectic condition)



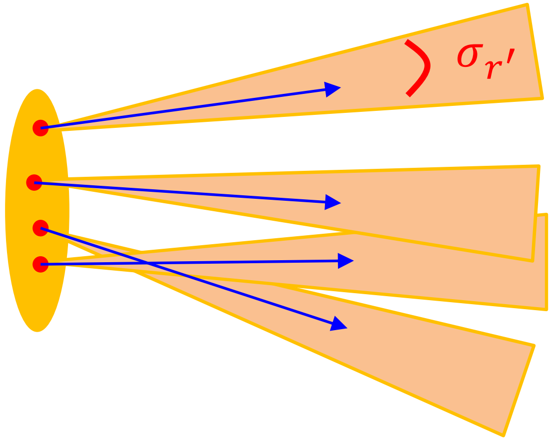
Phase space

Phase space

Real space



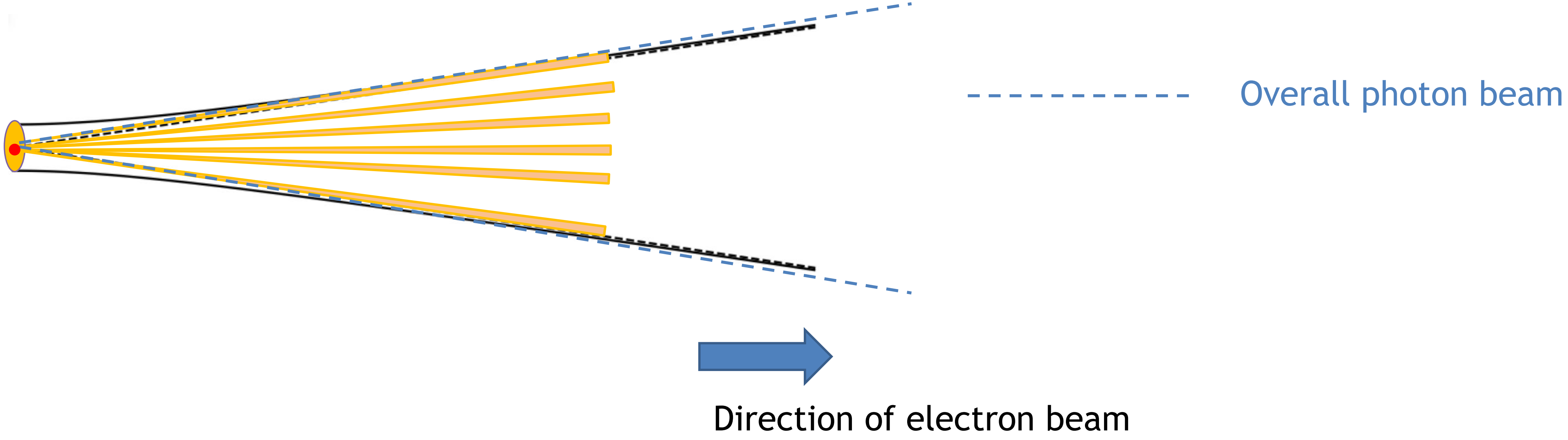
Phase space



Direction of electron beam

Electron beam dominate the divergence

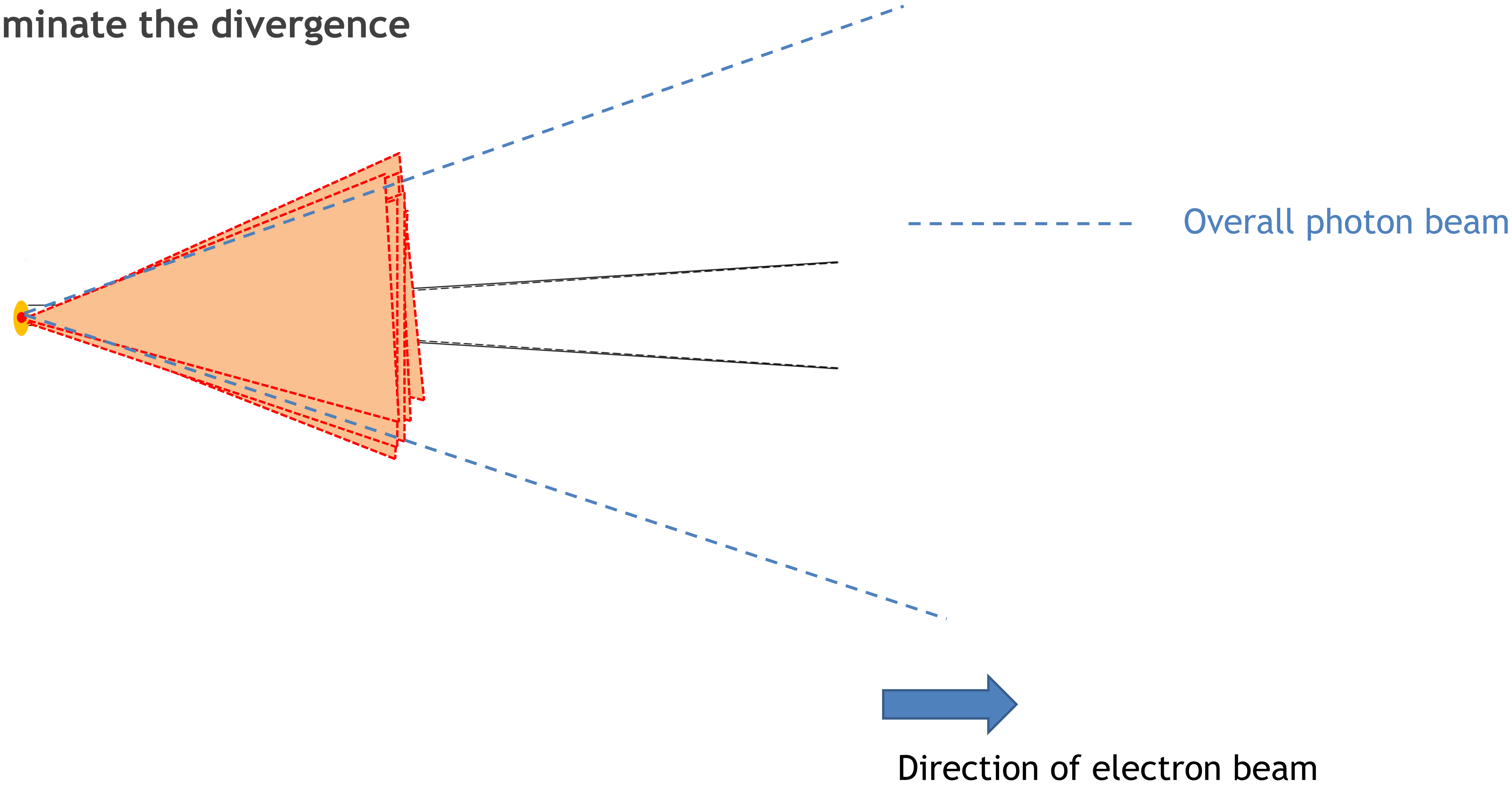
$\sigma_{x'} \gg \sigma_{r'}$



Phase space

Photon beam dominate the divergence

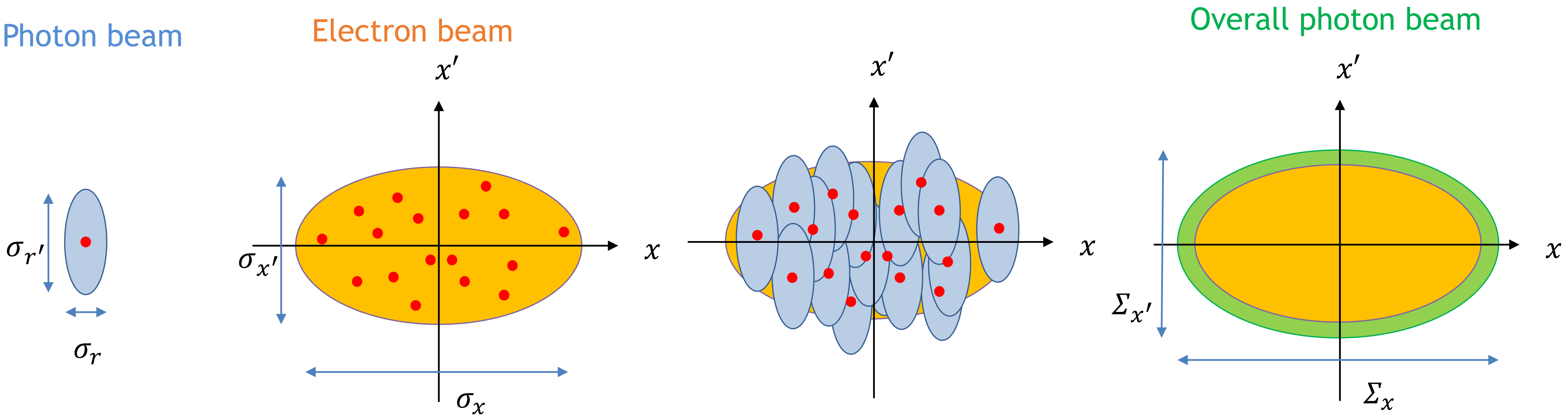
$\sigma_{x'} \ll \sigma_{r'}$



Brilliance: Phase space matching

- Overall photon beam: Convolution of electron beam distribution and photon beam distribution of single electron (or filament electron beam)
- Convolution of two Gaussian distribution is still a Gaussian distribution

$$\Sigma_x = \sqrt{\sigma_x^2 + \sigma_r^2}, \qquad \Sigma_{x'} = \sqrt{\sigma_{x'}^2 + \sigma_{r'}^2}$$



Brilliance: photon beam density in phase space

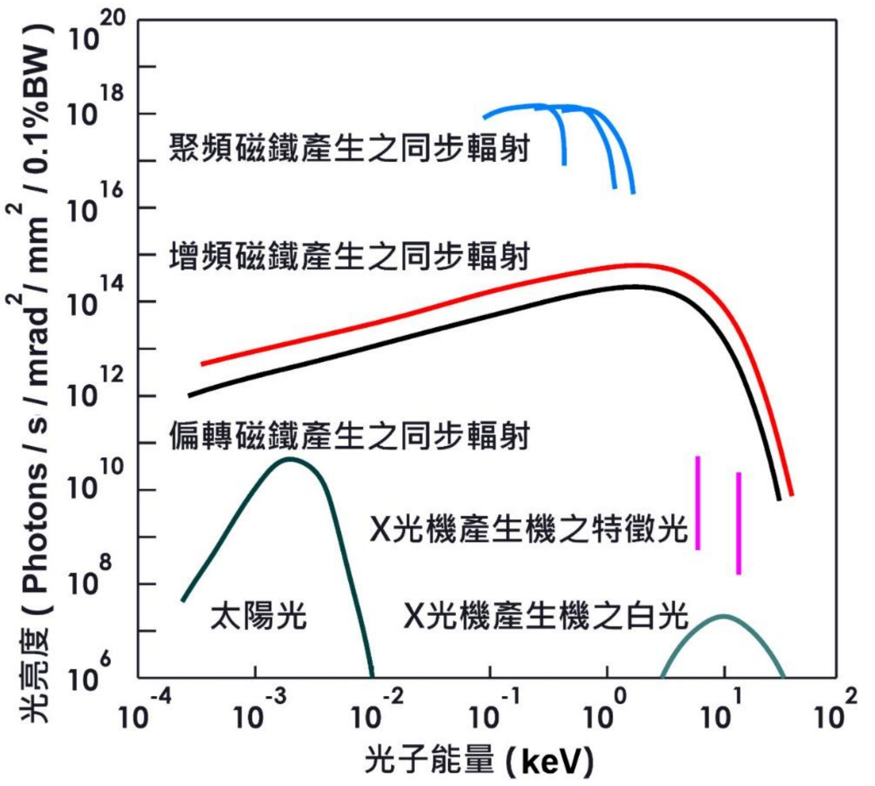
- **Brilliance (of undulator radiation): max photon beam density in phase space**

$$B_0 = \frac{F_{\mathrm{tot}}}{4\pi^2 \Sigma_x \Sigma_y \Sigma_{x'} \Sigma_{y'}}$$

Photon flux at a certain photon energy with 0.1% bandwidth:
→photon/s/0.1%BW

Effective photon beam size at source point
→mm²

Effective photon beam divergence
→mrad²



https://www.nsrrc.org.tw/chinese/lightsource.aspx

Brilliance: photon beam density in phase space

- **Electron beam dominate:** $\sigma_{x,y} \gg \sigma_r,\ \sigma_{x',y'} \gg \sigma_{r'}$

$$B_0 = \frac{F_{\text{tot}}}{4\pi^2 \sigma_x \sigma_y \sigma_{x'} \sigma_{y'}}$$

- **Photon beam dominates:** $\sigma_{x,y} \ll \sigma_r,\ \sigma_{x',y'} \ll \sigma_{r'}$

$$B_0 = \frac{F_{\text{tot}}}{4\pi^2 \sigma_r^2 \sigma_{r'}^2}$$

$$F_{\text{tot}} \propto I$$

TPS
Coupling 0.5%
ϵ_x : 1600 pm-rad
ϵ_y : 8 pm-rad
10 keV → 10 pm-rad ($\sigma_r \sigma_{r'} = \frac{\lambda}{4\pi}$)
100 eV → 1000 pm-rad

- **Diffraction limited source**

Design of synchrotron radiation source: keep reducing the emittance of the electron beam until diffraction limited condition is satisfied

Temporal (longitudinal) coherence and microbunch

Bunch length is longer than the wavelength:

$\sigma_l > \lambda \Rightarrow P \propto N_e$ → TPS: 500 mA in 600 bunches, each bunch:~1.4 nC $\Rightarrow N_e \approx 8.8 \times 10^9$

Bunch length is much shorter than the wavelength:

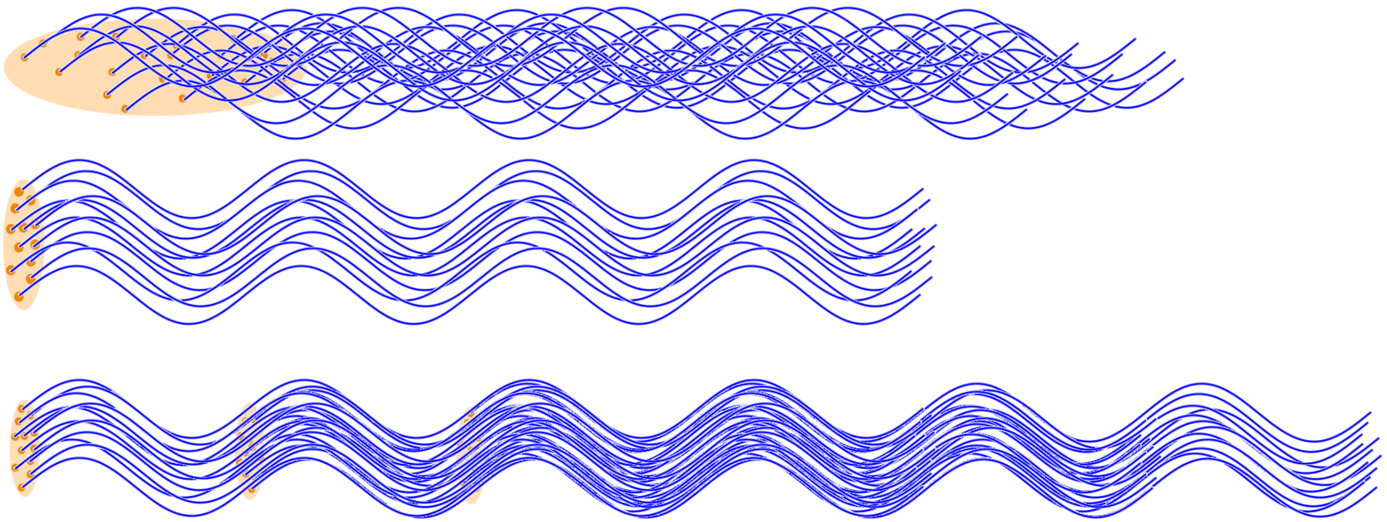
$\sigma_l \ll \lambda \Rightarrow P \propto N_e^2$

Typical bunch in SR source

e.g. TPS: bunch length ~ 3 mm

Microbunch

Microbunch train



Summary 2

Answer and New Goals

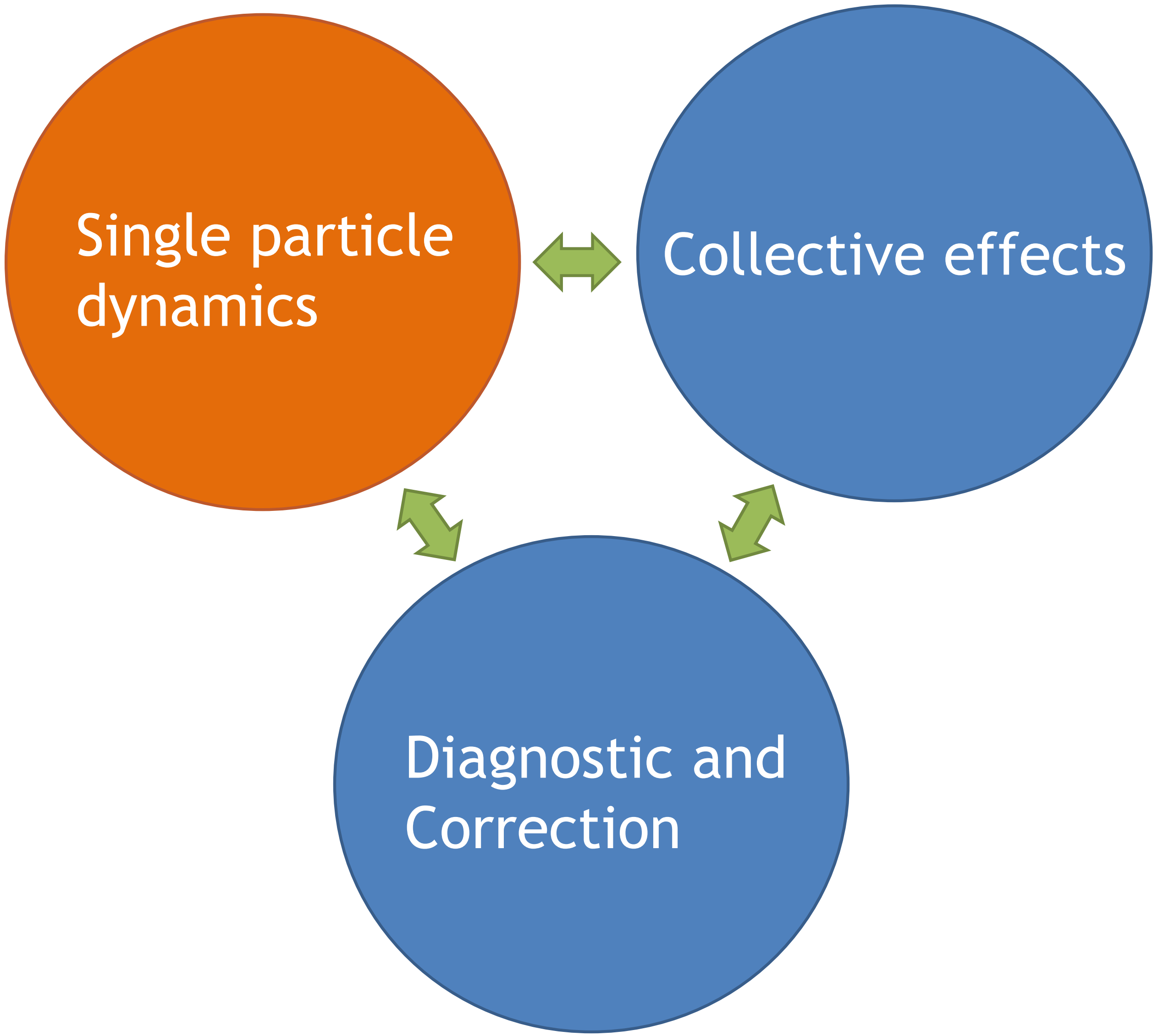
1. Particle: Electron
2. Magnetic field (~1 T, typical electromagnet or permanent magnet) is use to produce normal acceleration
3. γ: order of 10^3 ~ 10^4
4. Emittance: 10 pm-rad for hard x-ray
5. Bunch length: < wavelength length

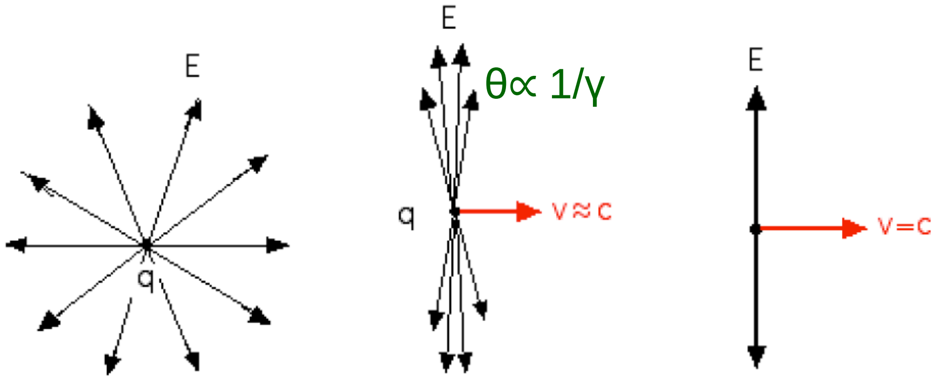
New Questions

1. How to generate electrons
2. How to design and arrange magnets to control electron beam (with introduction to transverse beam dynamics)
3. How to accelerate electron beam (with introduction to longitudinal beam dynamics)
4. How to control the emittance
5. How to reduce bunch length

Basic Accelerator Physics

Accelerator Physics





(a) stationary charge (b) a charge moving relativistically (c) when v=c

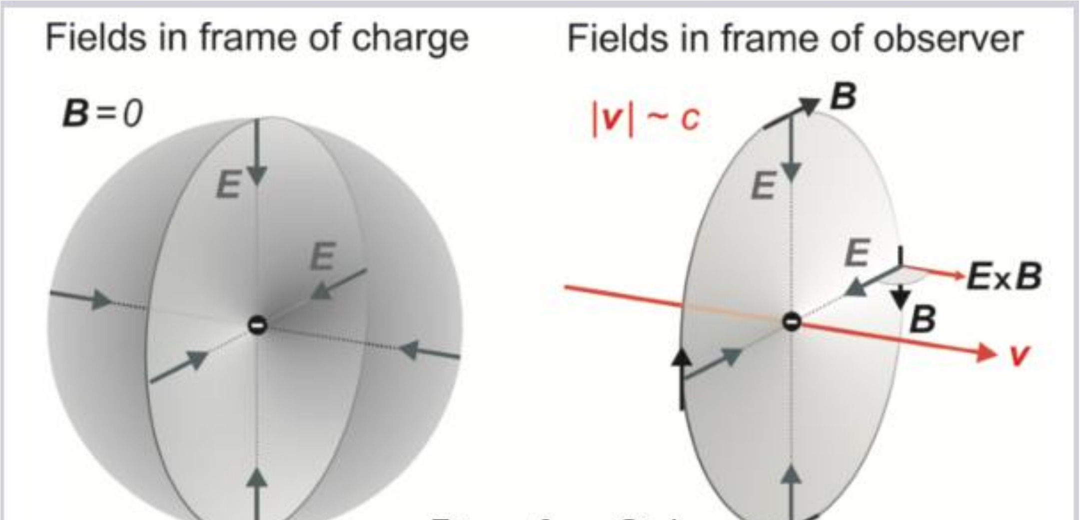
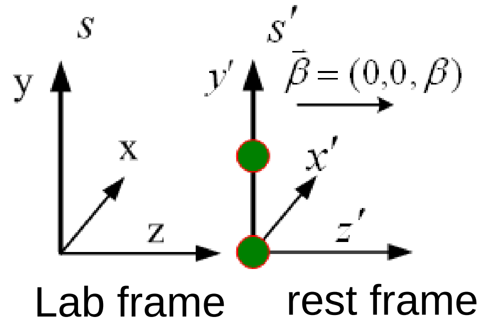


Figure from Stohr, Siegmann, "Magnetism"

The electric and magnetic field under Lorentz transformation between *S* and *S'* frame takes the following form

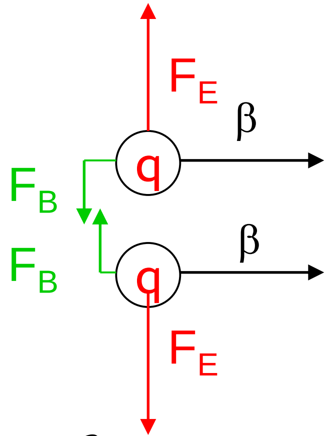


$$E_z = E'_z \qquad B_z = B'_z$$

$$E_x = \gamma(E'_x + c\beta B'_y) \qquad B_x = \gamma(B'_x - \frac{1}{c}\beta E'_y)$$

$$E_y = \gamma(E'_y - c\beta B'_x) \qquad B_y = \gamma(B'_y + \frac{1}{c}\beta E'_x)$$

The Lorentz force between two charged particles parallel moving at a distance of d is

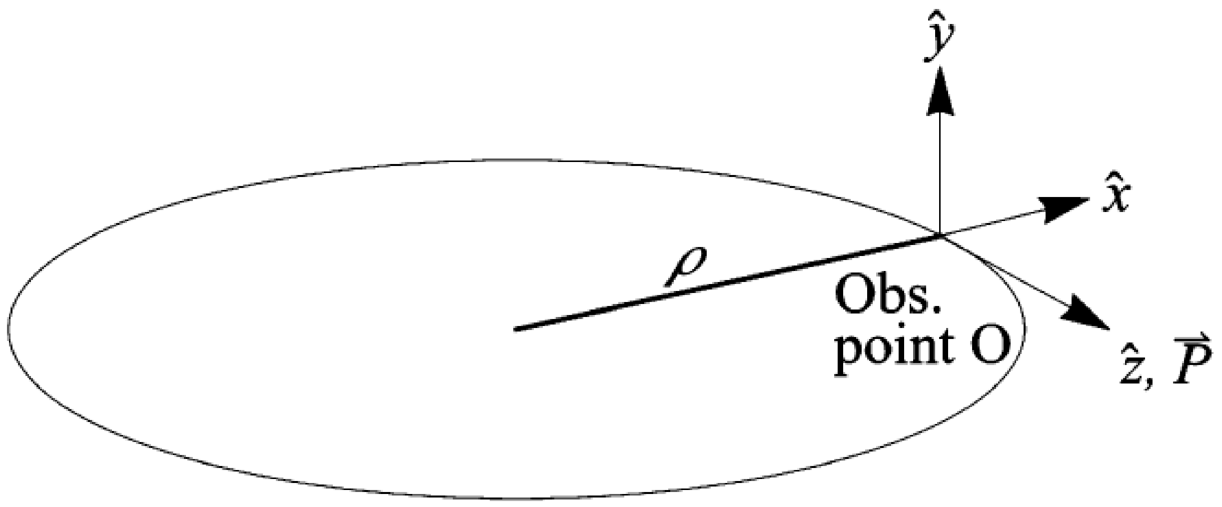


$$F_E = qE_y = \gamma q E'_y \qquad \vec{E}' = (0, E'_y, 0) \qquad \vec{B}' = (0,0,0)$$

$$F_B = qc(\vec{\beta} \times \vec{B})_y = qc(\beta_z B_x - \beta_x B_z) = qc\beta B_x = -\gamma\beta^2 q E'_y$$

$$F = F_E + F_B = \gamma(1-\beta^2) q E'_y = \frac{1}{\gamma} q E'_y = \frac{qE_y}{\gamma^2} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

- *The larger the beam energy, the weaker the space charge force*



Frenet-Serret Coordinate System

[ref] Lectures on Accelerator Physics, Alexander Wu Chao, World Scientific, 2020

- For an uniform B field, $\vec{B} = B_0\,\hat{y}$,
  the **ideal** particle trajectory is a circle.

- $(\hat{x}, \hat{y}, \hat{z})$ right-handed

- $\hat{x}$ horizontal transverse coordinate, points outward
- $\hat{y}$ vertical transverse coordinate, points upward
- $\hat{z}$ longitudinal coordinate, moving in clock-wise direction
  $(\vec{P} \parallel \hat{z})$

- Lorentz force points “inward” to balance the centrifugal force
- $\vec{B} \parallel \hat{y}$ for charge q > 0
- $\vec{B} \parallel (-\hat{y})$ for charge q < 0

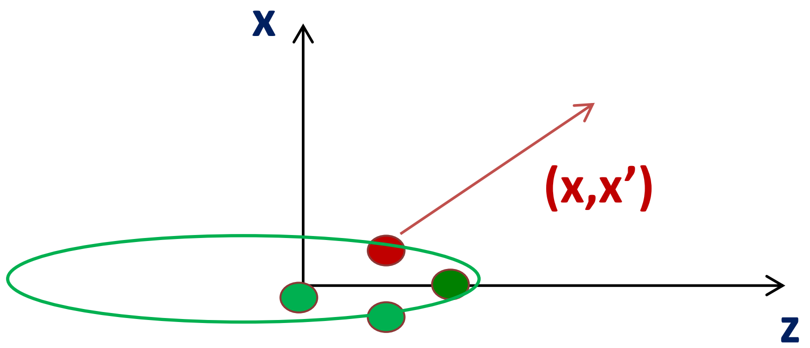
- For a planar accelerator,
  $\hat{y}$ is a constant,
  $\hat{x}$ and $\hat{z}$ rotate around the accelerator.

- 6-D phase space of beam

$$(x, x', y, y', z, \delta) \quad or \quad (x, x', y, y', t, \delta)$$

- Transverse phase space (horizontal and vertical)

$$(x, x');\ (y, y')$$
$$x' = v_x/v_z; \quad y' = v_y/v_z$$

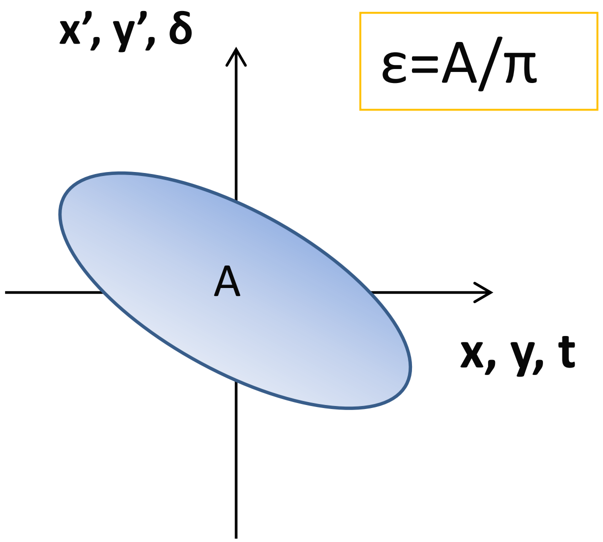


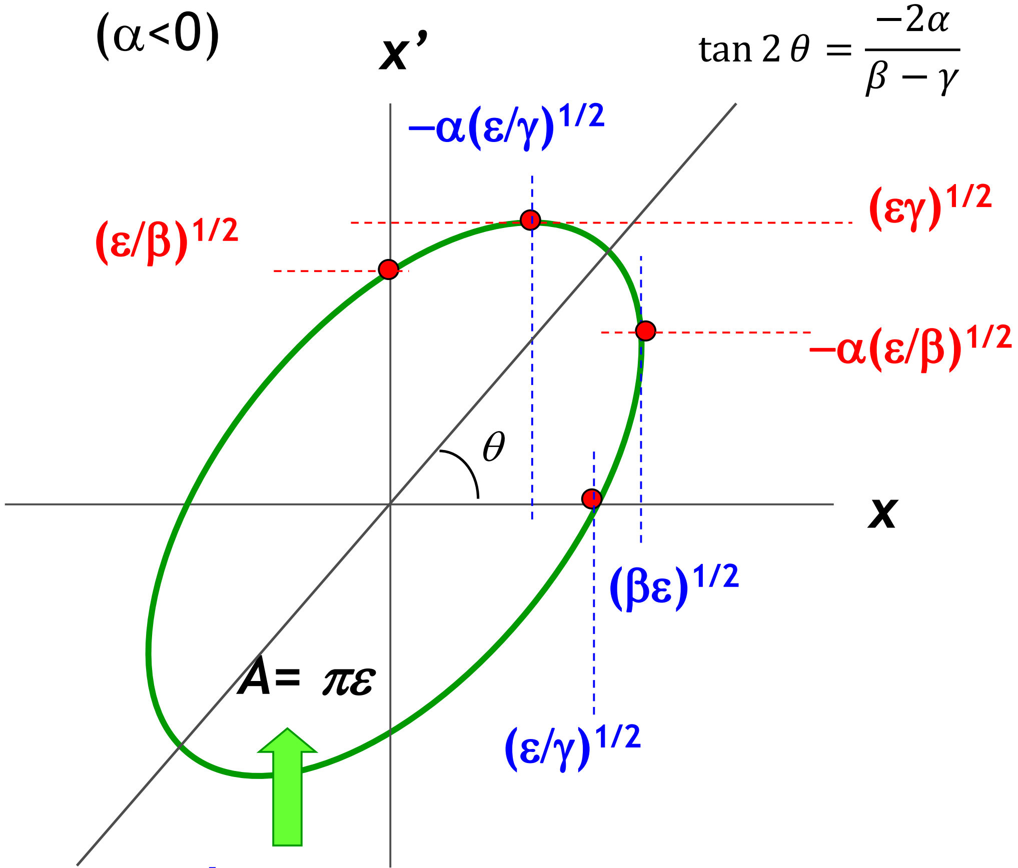
- Longitudinal phase space

$$(t, \delta); \quad \delta = \frac{\Delta p}{p} = \left(\frac{1}{\beta^2}\right)\frac{\Delta E}{E}$$

- Beam emittance

$$\epsilon = \frac{A}{\pi}, \ \varepsilon_x = \sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2}.$$





Emittance *(an invariant of betatron motion)*

- **Liouville's theorem**: the population density of particles in the phase space keeps the same constant under the conservative force of system (no energy difference)

$$x''(s) + k(s)x(s) = 0,$$

$$x(s) = \sqrt{\varepsilon}\sqrt{\beta}\cos(\varphi(s) - \varphi_0)\,,$$

$$\frac{1}{2}\left(\beta\beta'' - \frac{1}{2}\beta'^2\right) - \beta^2\varphi'^2 + \beta^2 k = 0,$$

$$\beta\varphi' = \text{constant}.$$

$$\varphi(s) - \varphi_0 = \int_0^s \frac{dS}{\beta(s)}.$$

$$\beta'' + 2k\beta - 2\gamma = 0, \qquad \gamma x^2 + 2\alpha x x' + \beta x'^2 = \varepsilon.$$

Courant-Snyder parameters are defined as,

$$\beta(s)$$

$$\alpha(s) = -\frac{\beta'(s)}{2}$$

$$\gamma(s) = \frac{1 + \alpha^2(s)}{\beta(s)}$$

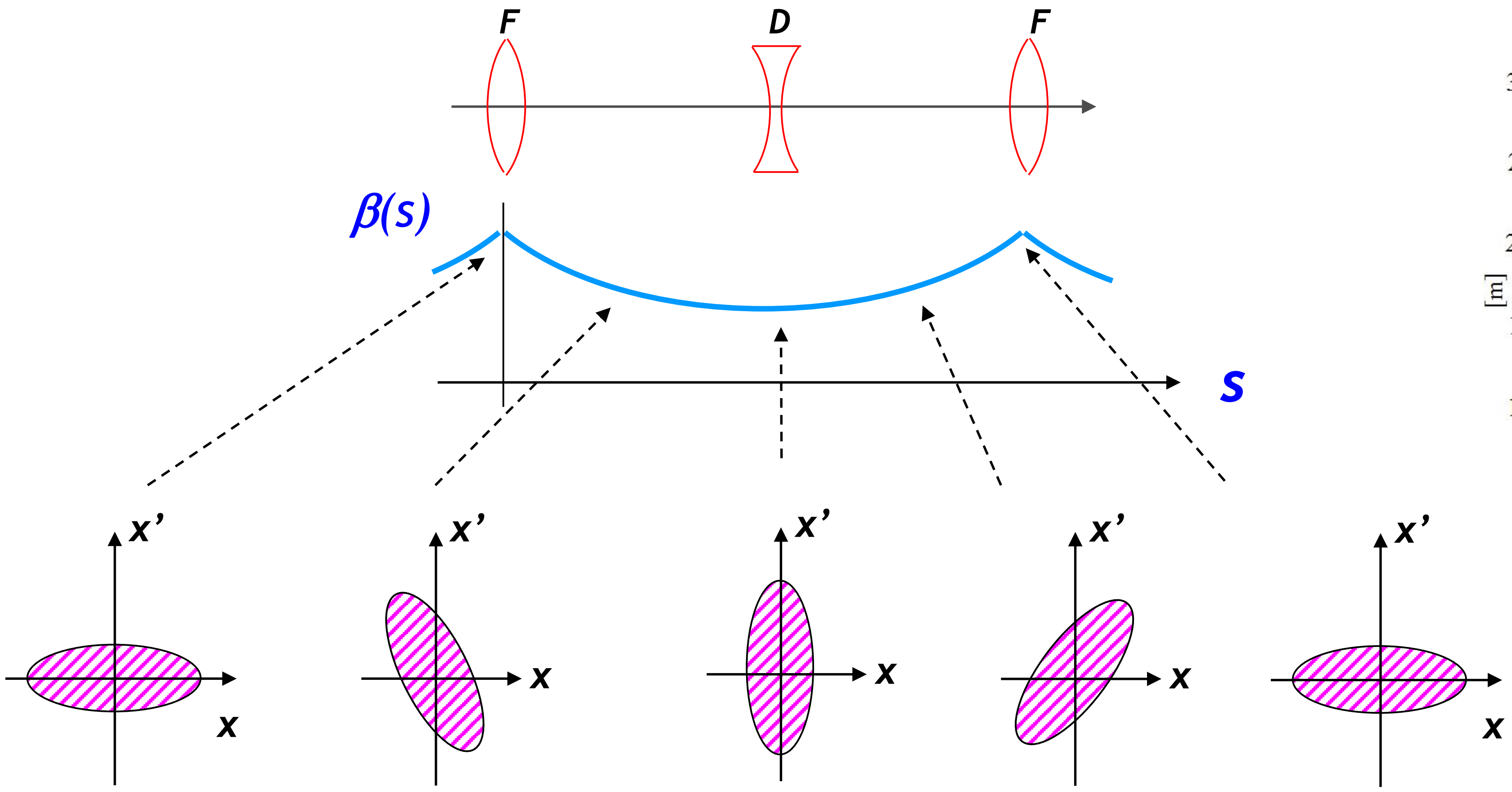
the beam size

$$\sigma(s) = \sqrt{\varepsilon \cdot \beta(s)}$$

the slope of beam envelope evolution

the beam divergence

$$\sigma'(s) = \sqrt{\varepsilon \cdot \gamma(s)}$$



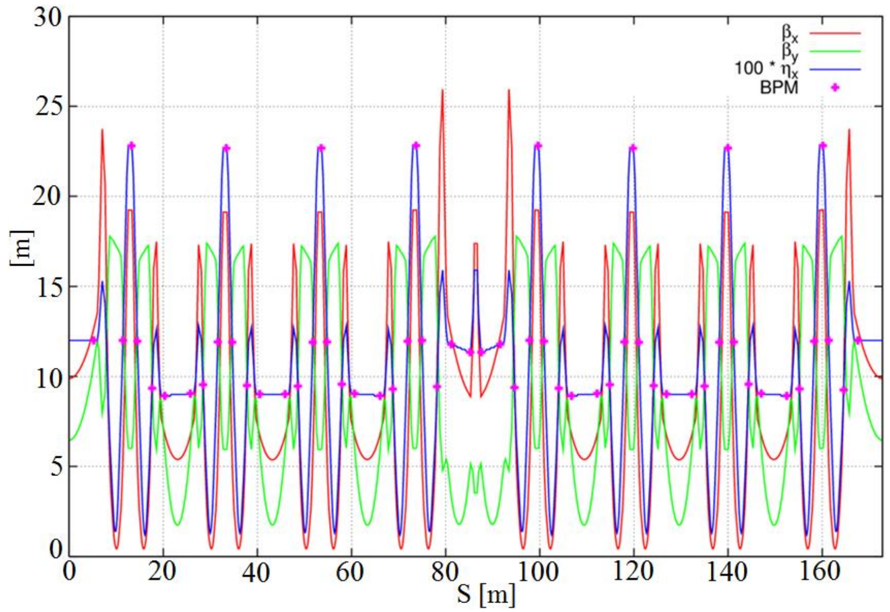
σ_{xmax}
$\alpha = 0$

Converging beam,
$\alpha > 0$

Beam waist
σ_{xmin}, $\alpha = 0$

Diverging beam,
$\alpha < 0$

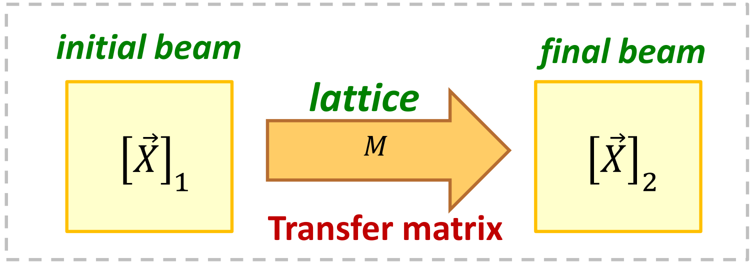
σ_{xmax}
$\alpha = 0$



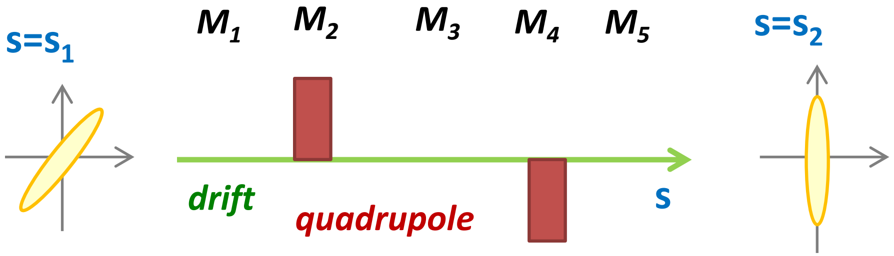
Lattice of TPS

the particle motion in a 6-D phase space can be expressed by the transfer matrix

$$[\vec{X}]_2 = \mathrm{M}[\vec{X}]_1, \qquad \vec{X} = (x, x', y, y', z, \delta), \qquad \delta = \Delta E/E = \Delta\gamma/\gamma$$



(*Example of beam transport line)*



$M=M_5M_4M_3M_2M_1$

$$\begin{bmatrix} x(s_2) \\ x'(s_2) \\ y(s_2) \\ y'(s_2) \\ z(s_2) \\ \delta(s_2) \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} & R_{16} \\ R_{21} & R_{22} & R_{23} & R_{24} & R_{25} & R_{26} \\ R_{31} & R_{32} & R_{33} & R_{34} & R_{35} & R_{36} \\ R_{41} & R_{42} & R_{43} & R_{44} & R_{45} & R_{46} \\ R_{51} & R_{52} & R_{53} & R_{54} & R_{55} & R_{56} \\ R_{61} & R_{62} & R_{63} & R_{64} & R_{65} & R_{66} \end{bmatrix} \begin{bmatrix} x(s_1) \\ x'(s_1) \\ y(s_1) \\ y'(s_1) \\ z(s_1) \\ \delta(s_1) \end{bmatrix},$$

where

$R_{11}(x|x_0)$,

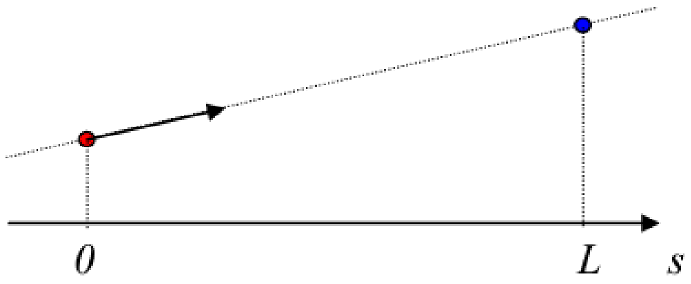
$R_{12}(x|x'_0),\ldots$

$R_{26}(x'|\delta_0)$,

$$R_{56}(z|\delta_0), T_{566}\left(z\,\middle|\,\delta_0^2\right), U_{5666}\left(z\,\middle|\,\delta_0^3\right),\ldots$$

[Ref]: Handbook of Accelerator Physics and Engineering, Alexander Wu Chao et al., World Scientific.

(1) Drift space



$$x_2 = x_1 + x_{p1} \cdot L$$

$$x_{p2} = x_{p1}$$

$$R_{drift} = \begin{bmatrix} 1 & L & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & L & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

- The slope remains constant
- Position varies linearly as distance

Symplecticity. M: transfer matrix for linear system, Jacobian for nonlinear system

$$M^{\mathrm{T}} S M = S \qquad S = \begin{bmatrix} S_2 & 0 & \cdots & 0 \\ 0 & S_2 & 0 & \vdots \\ \vdots & 0 & \ddots & \vdots \\ 0 & \cdots & \cdots & S_2 \end{bmatrix}, S_2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Symplectic → det $M = 1$ → Density of electron beam in phase space is conserved (emittance is conserved)

Lagrangian: $L = -mc^2\sqrt{1 - v^2/c^2} - e\Phi + e\vec{v}\cdot\vec{A}$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \vec{v}}\right) - \frac{\partial L}{\partial \vec{r}} = 0$$

$$\vec{P} = \frac{\partial L}{\partial \vec{v}} = \vec{p} + e\vec{A},$$

$$H = \vec{P}\cdot\vec{v} - L = c[m^2c^2 + (\vec{P} - e\vec{A})^2]^{1/2} + e\Phi,$$

$$\dot{x} = \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\partial H}{\partial P_x}, \qquad \dot{P}_x = \frac{\partial H}{\partial x}$$

$$\tilde{H} = -\left(1 + \frac{x}{\rho}\right)\left[\frac{(H - e\Phi)^2}{c^2} - m^2c^2 - (p_x - eA_x)^2 - (p_z - eA_z)^2\right]^{1/2} - eA$$

$$x' = \frac{\partial \tilde{H}}{\partial p_x}, \quad p_x' = -\frac{\partial \tilde{H}}{\partial x}$$

Canonical transformations

$$\tilde{H} = -\left(1 + \frac{x}{\rho}\right)\left[\frac{(H - e\Phi)^2}{c^2} - m^2c^2 - (p_x - eA_x)^2 - \left(p_y - eA_y\right)^2\right]^{1/2} - eA_z$$

$$x' = \frac{\mathrm{d}x}{\mathrm{d}z} = \frac{\partial \tilde{H}}{\partial p_x}, \qquad p'_x = \frac{\partial \tilde{H}}{\partial x}$$

S. Y. Lee Accelerator Physics 4th ed.

2D: uniform in longitudinal direction $A_x = A_y = 0$ $A_z = B_0 \,\mathrm{Re}\left[\sum_{n=0}^{\infty} \frac{b_n + ia_n}{n+1}(x+iy)^{n+1}\right]$

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}, \qquad B_x = \frac{1}{h_z}\frac{\partial A_z}{\partial y}, \qquad B_y = -\frac{1}{h_z}\frac{\partial A_z}{\partial x}, \qquad h_z = 1 + \frac{x}{\rho}$$

$$B(x,y) = B_y(x,y) + iB_x(x,y) = \sum_{n=0}^{\infty}(b_n + ia_n)(x+iy)^n = [b_0 + b_1 x - a_1 y + \cdots] + i[a_0 + b_1 y + a_1 x + \cdots]$$

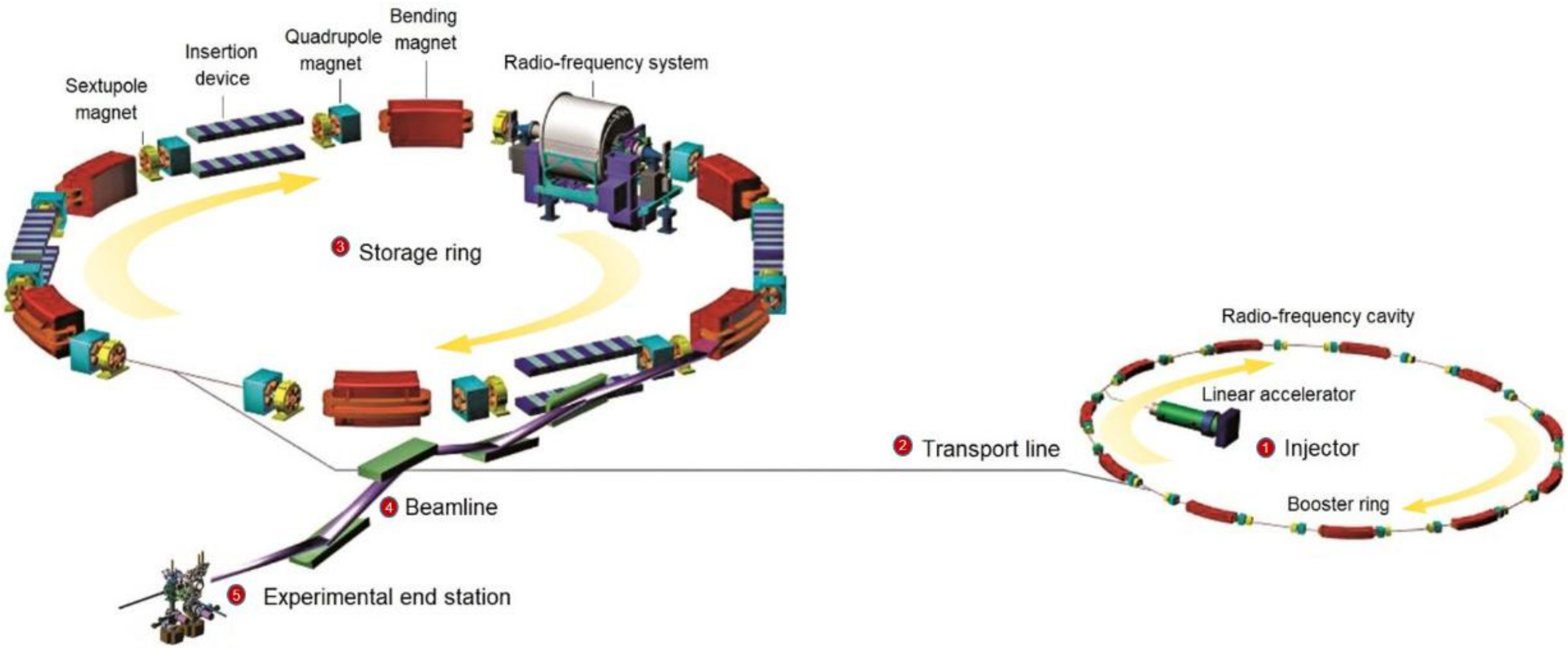
$$b_n = \frac{1}{B_0 n!}\frac{\partial^n B_y}{\partial x^n}, \qquad a_n = \frac{1}{B_0 n!}\frac{\partial^n B_x}{\partial x^n}$$

$n = 0$: dipole
$n = 1$: quadrupole
$n = 2$: sextupole
$n = 3$: octupole

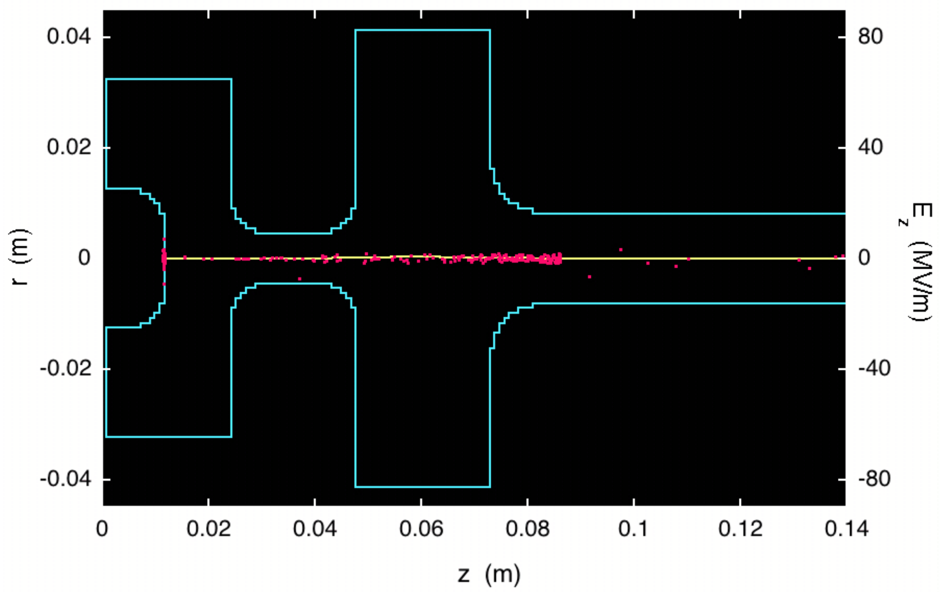
S. Y. Lee Accelerator Physics 4th ed.

New Questions

1. How to generate electrons
2. How to design and arrange magnets to control electron beam (with introduction to transverse beam dynamics)
3. How to accelerate electron beam (with introduction to longitudinal beam dynamics)
4. How to control the emittance
5. How to reduce bunch length


Bending
magnet
Quadrupole
magnet
Insertion
device
Sextupole
magnet
Radio-frequency system
3 Storage ring
Radio-frequency cavity
Linear accelerator
2 Transport line
1 Injector
Booster ring
4 Beamline
5 Experimental end station

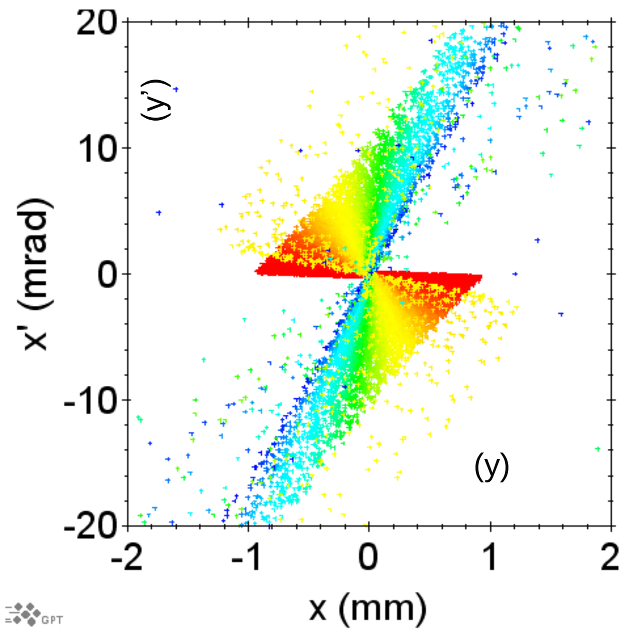
Simulated electron dynamics in a thermionic RF gun



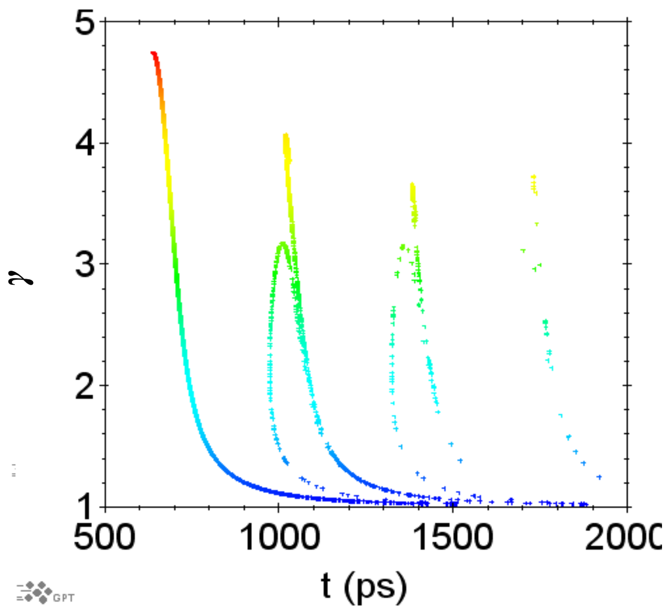
[ref] Hiroyuki Hama,
Animation of Charged Particle in Thermionic ITC RF Gun.

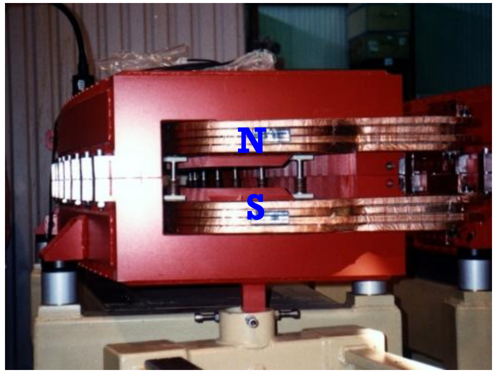
Particle distribution of generated electrons at the thermionic rf gun exit

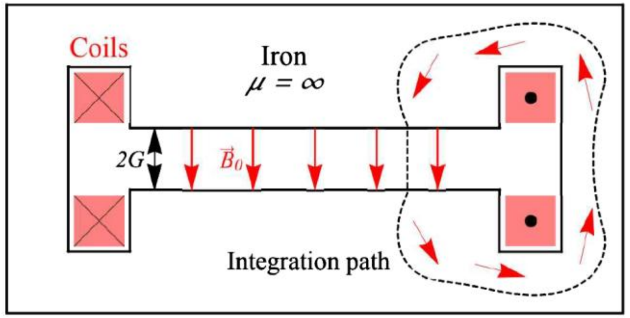
- transverse phase space -



- longitudinal phase space -

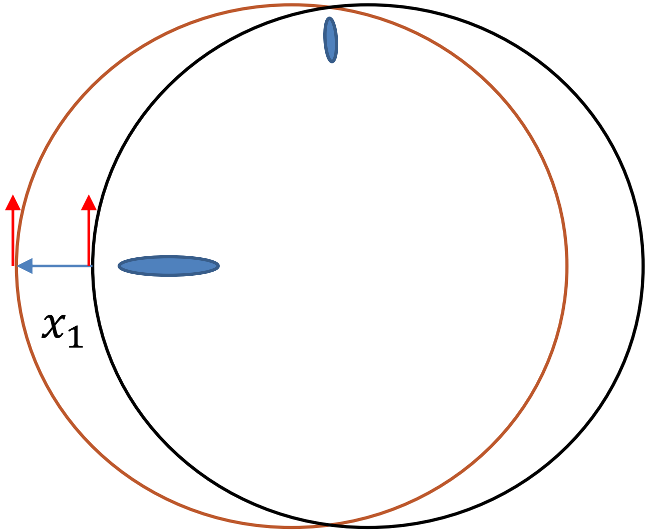






Magnetic fields for deflection (bending and focusing).

Geometric focusing effects



Magnetic rigidity

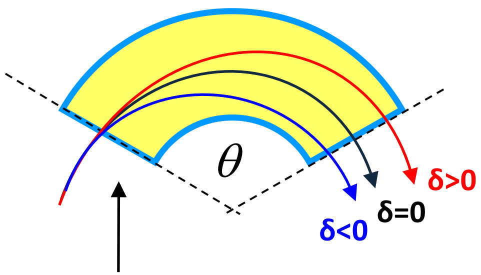
$$B\rho = \frac{p}{q} \rightarrow B\rho\ (\mathrm{T\ m}) = \frac{10}{2.998}\beta E\ (\mathrm{GeV})$$

TPS

$E = 3\ \mathrm{GeV}, \beta \approx 1, B = 1.19\ \mathrm{T} \rightarrow B \approx 8.4\ \mathrm{m}$

(3) Sector dipole magnet (hard-edge)

$\ell = \rho\theta$ arc length



Particle enters and exits normal to the magnet pole faces

$$R_{sector} = \begin{bmatrix} \cos\theta & \rho\sin\theta & 0 & 0 & 0 & \rho(1-\cos\theta) \\ -\frac{1}{\rho}\sin\theta & \cos\theta & 0 & 0 & 0 & \sin\theta \\ 0 & 0 & 1 & \ell & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \sin\theta & \rho(1-\cos\theta) & 0 & 0 & 1 & \rho(\theta-\sin\theta) \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

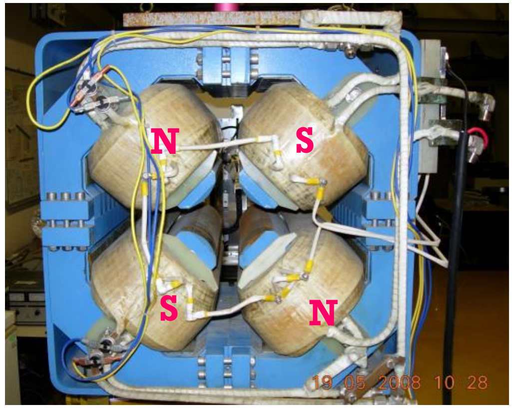
- a natural focusing effects in the horizontal deflection plane ; Vertical direction acts as a pure drift space
- **R16**: momentum dependent horizontal displacement term (larger momentum particle gets less bending)

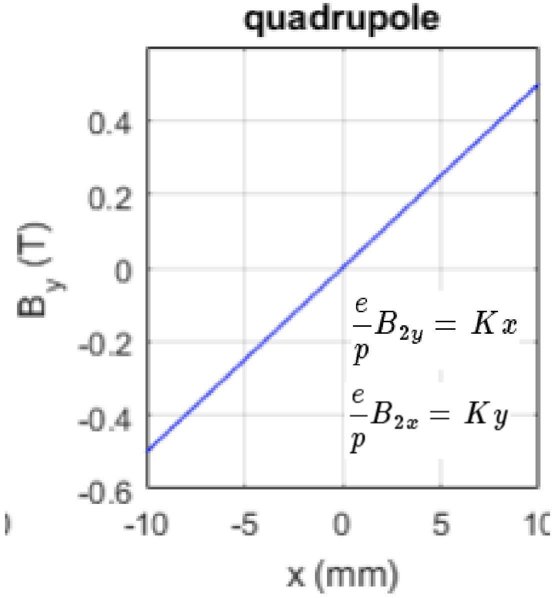
  → dispersion η → cause beam size broadening $\sigma_x = \sqrt{\varepsilon\beta} + \eta\delta$

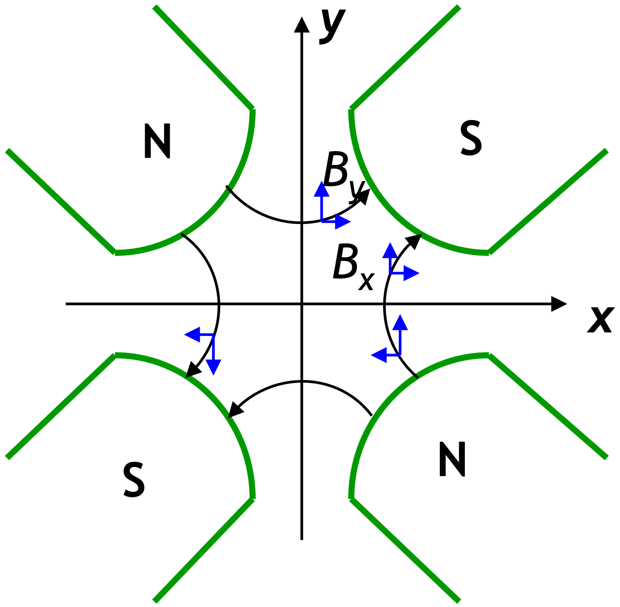
  → **control of R16 for dispersion compensation**
- **R56**: momentum dependent longitudinal displacement term (larger momentum particle has shorter traveling path)

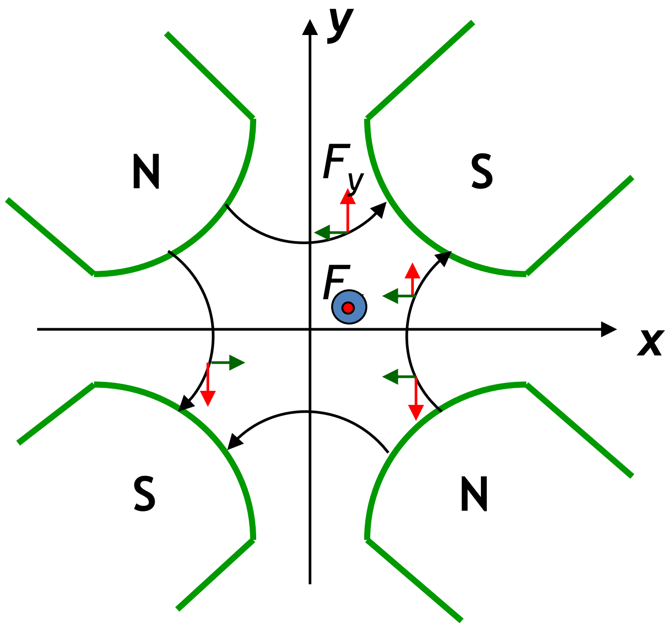
  → will lead to bunch lengthening → **control of R56 for the design of a bunch compressor**

2. Transverse beam dynamics



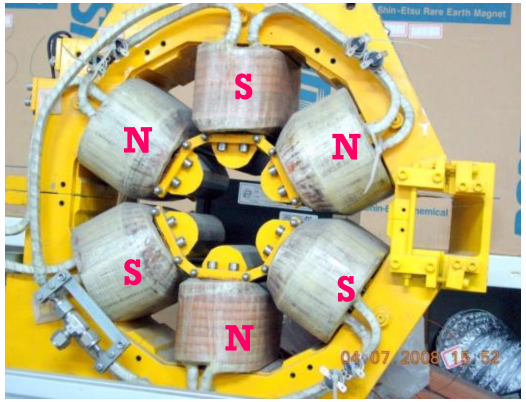


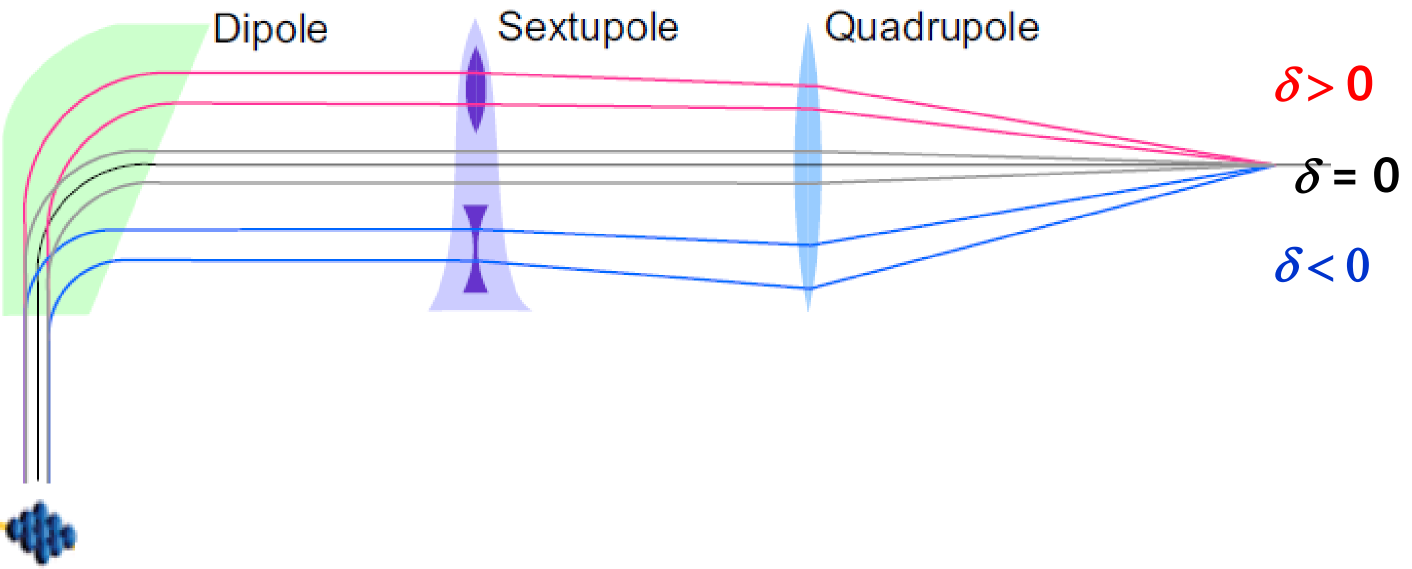


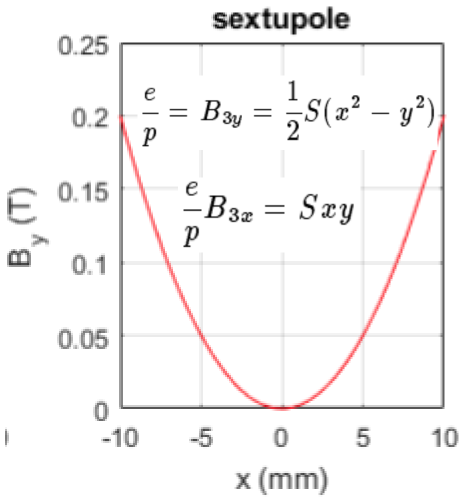


$$B_y(x, y) = b_1 x$$
$$B_x(x, y) = b_1 y$$

- Horizontal focusing, while vertical defocusing
- Needs at least two quadrupole magnets for beam focusing







- Sextupole magnet

+x particle, increase focusing strength

-x particle, decrease focusing strength

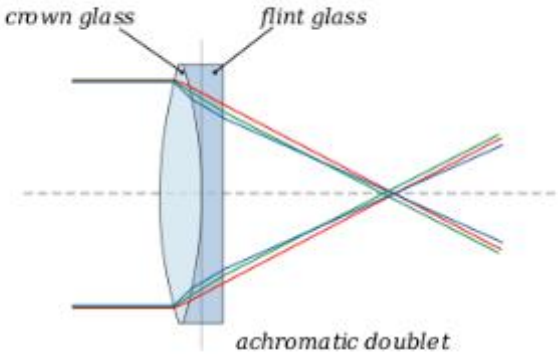
- Chromatic aberration originated from the quadrupole

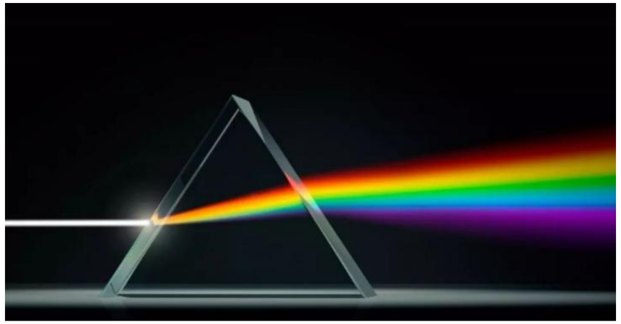
could be corrected by the inclusion of dispersive dipole + sextupole element

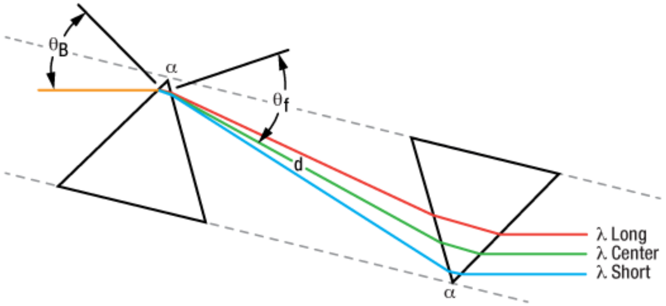
p.s. however, the nonlinear sextupole magnet will introduce the geometric aberration

2. Transverse beam dynamics

	Charged particle	Photon beam
Deflection	Bending magnet	Mirror、prism
Focusing	Quadruple magnet-s	Lens
Chromatic correction	quadrupole 、 Dipole+ sextupole	Lens complex
Dispersion compensation (or bunch length control)	Dipole pairs	Prism pairs

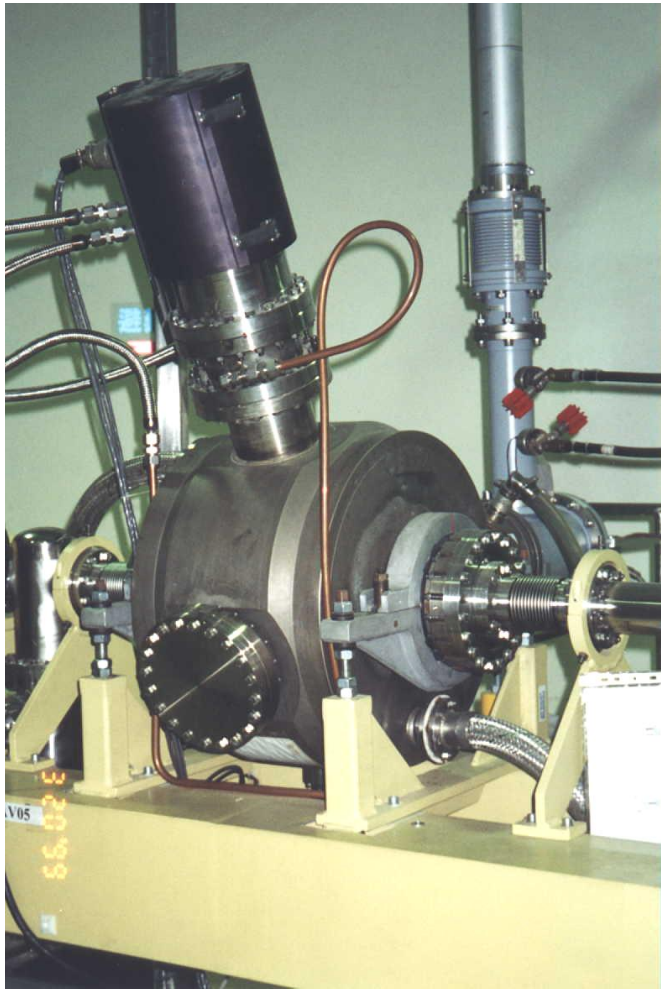




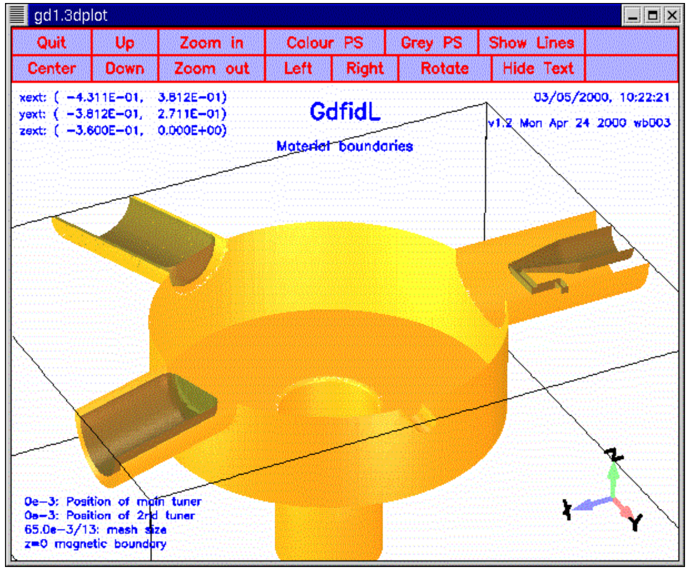


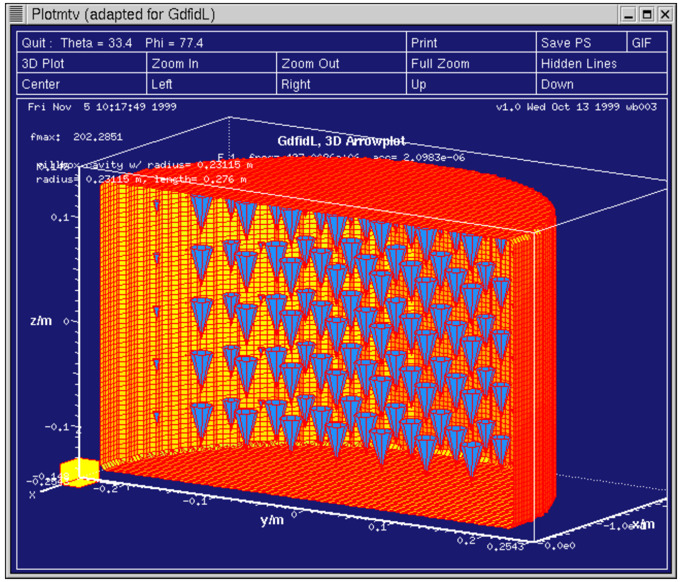
3. How to accelerate electron beam

- **Example of RF cavities:**

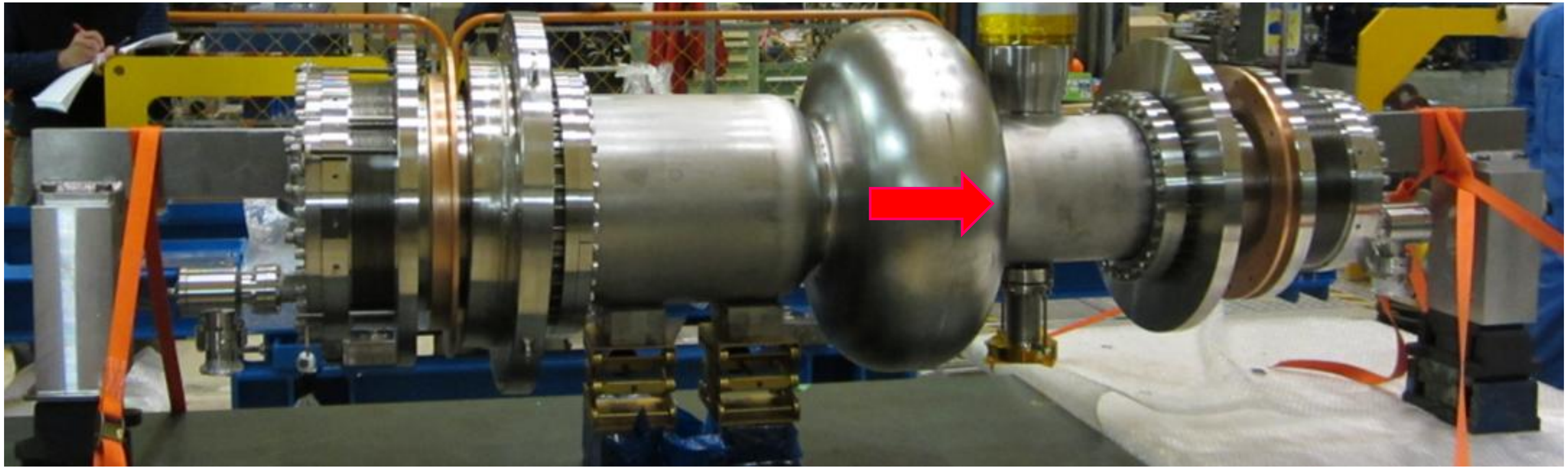


A pillbox cavity (NSRRC Booster)





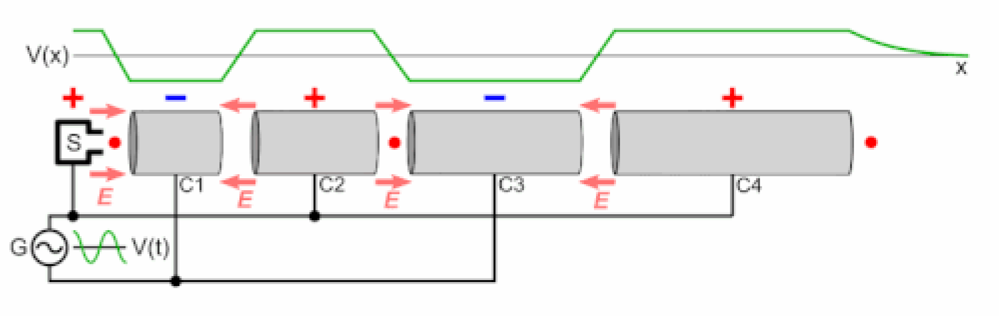
KEK-B type SRF cavity used at TPS storage ring



Ref: Accelerator Physics by Ping J. Chou

3. How to accelerate electron beam

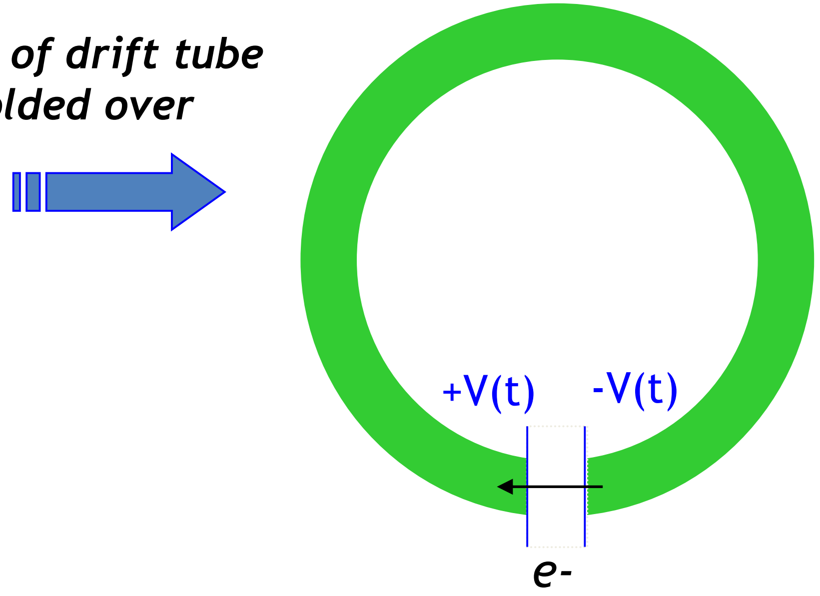
Linear



https://en.wikipedia.org/wiki/Particle_accelerator

Circular

A chain of drift tube been folded over



Electrostatic field is conservative
→ time varying field

Ref: Accelerator Physics by Ping J. Chou

3. Longitudinal beam dynamics

Speed and Path length

$\delta = \frac{\Delta p}{p_0}$: fractional momentum deviation

$$\frac{\Delta t}{t_0} = \eta_0 \frac{\Delta p}{p_0} = \left(\alpha_0 - \frac{1}{\gamma_0^2}\right)\frac{\Delta p}{p_0}$$

The effect due to the increase in speed with ***p***.
Before the e- reaches the relativistic speed, the larger the speed, the shorter time it takes to traverse one turn in the accelerator.

The change in the path length with ***p***.
When e- approaches the relativistic speed, the acceleration can only increase the momentum but not the velocity. Hence, the orbit radius increases.

Ref: Accelerator Physics by Ping J. Chou

γ_0: Lorentz factor of on-momentum particle
α: momentum compaction factor
L: path length
η: phase slip factor

$$L = L_0[1 + \alpha_0\delta(1 + \alpha_1\delta + \alpha_2\delta^2 + \cdots)]$$

$$\eta = \eta_0 + \eta_1\delta + \eta_2\delta^2 + \cdots$$

$\Delta t \rightarrow$ phase difference $\Delta\varphi$

$$\delta_{n+1} = \delta_n + \frac{eV}{\beta^2 E}(\sin\varphi_n - \sin\varphi_s)$$

$$\varphi_{n+1} = \varphi_n + 2\pi h \eta_0 \delta_{n+1}$$

Magnetic Bunching

- Step 1. RF chirp
- Step 2. dispersive section

- space for two-stage process
- wide range of adopted beam energy

< Issue >

- nonlineartity
- CSR emittance degradation,
- wake field …

Velocity Bunching

1 step in accelerating structure:

acceleration + compression

compact and simple operation

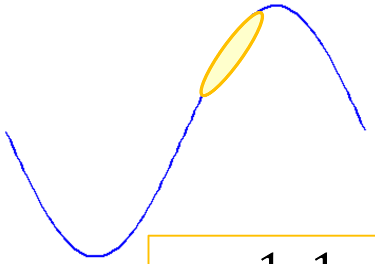
suitable for low energy beam

< Issue >

nonlinearilty

Space charge effects

Step 1. energy modulation - rf section



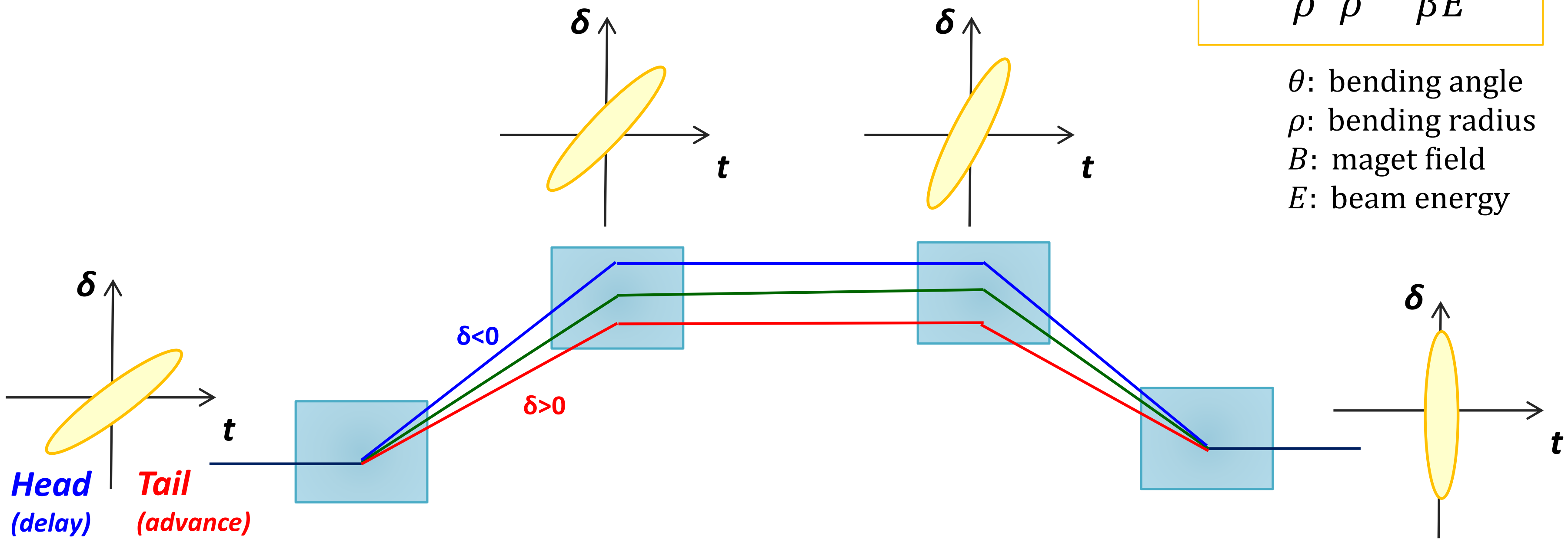
Step 2. dispersive region

– bending magnet, chicane (four-dipole system

→ Path difference of energy correlated beam

$$\theta \propto \frac{1}{\rho}, \frac{1}{\rho} \propto \frac{B}{\beta E}$$

θ: bending angle
ρ: bending radius
B: maget field
E: beam energy



trajectory of lower energy (larger bending angle, longer path, delay)

trajectory of central energy

trajectory of higher energy (smaller bending angle, shorter path, advance)

4. How to control the emittance

Equilibrium emittance of ring

$$\varepsilon_0 \propto E^2\theta^3 \qquad \varepsilon_0 \propto \frac{E^2}{(\text{Circumference})^3}$$

- the smaller bending angle
  (the concept of MBA lattice)

$$\sigma_x(s) = \sqrt{\varepsilon_x \cdot \beta_x(s)} \propto E$$

$$\sigma_x{}'(s) = \sqrt{\varepsilon_x \cdot \gamma_x(s)} \propto E$$

$$\sigma_\gamma/\gamma(s) \propto E$$

- *Lower beam energy is beneficial to the beam emittance*
- *Beam energy is mainly decided by the desired radiation photon beam energy*
- *As a result of advancements in magnet technology, the beam energy of recently designed machines is decreasing in order to reduce energy consumption while simultaneously improving beam emittance. (Spring8 → Spring8II: 8GeV→6GeV)*

Normalized emittance of linac

- The minimum normalized emittance of the whole injector is limited at the stage of beam generation.

$$\varepsilon_n = \sqrt{(\varepsilon^{th})^2 + (\varepsilon^{rf})^2 + (\varepsilon^{sc})^2 + \cdots}$$

- higher beam energy helps to reduce the geometric emittance, hence also the transverse electron beam size and divergence

$$\sigma_x(s) = \sqrt{\frac{\varepsilon_n}{\beta\gamma} \cdot \beta(s)} \propto \sqrt{\frac{1}{E}}$$

$$\sigma_x{}'(s) = \sqrt{\frac{\varepsilon_n}{\beta\gamma} \cdot \gamma(s)} \propto \sqrt{\frac{1}{E}}$$

$$\sigma_\gamma/\gamma(s) \propto \frac{1}{E}$$

- *damping of ε with acceleration with conserved ε_n improves beam quality*

4. How to control the emittance

SR

FACILITY NAME	Size and Location	Energy	Pulse length	Energy spread	Equilibrium emittance	Rep. rate
TPS 2016	C=518.4 m TAIWAN	3 GeV	22 ps	8.86×10^{-4}	1.6 nm-rad	~ 500 MHz (M-mode) ~580 kHz (S-mode)
MAX-IV 2017	C=528 m Sweden	3 GeV	400 ps	7.7×10^{-4}	320 pm-rad	~ 100 MHz (M-mode)
EBS 2020	C=844 m France	6 GeV	23 ps	9.3×10^{-4}	132 pm-rad	~ 350 MHz (M-mode)

LINAC FEL

FACILITY NAME	Size and Location	Energy	Pulse length	Energy spread	ε_{nx}	Geometric emittance	Rep. rate
SACLA, 2011	0.72 km RIKEN, JAPAN	8.5 GeV	10 fs	10^{-4}	0.7 mm-mrad	42 pm-rad	30-60 Hz
PAL-XFEL 2017	1.1 km Pohang, Korea	10 GeV	10 – 100 fs	----	0.5 mm-mrad	26 pm-rad	60 Hz
LCLS-II, 2022	3.5 km SLAC, USA	4 GeV	50 fs	$< 10^{-3}$	0.45 mm-mrad	57 pm-rad	0.62 MHz

4. How to control the emittance

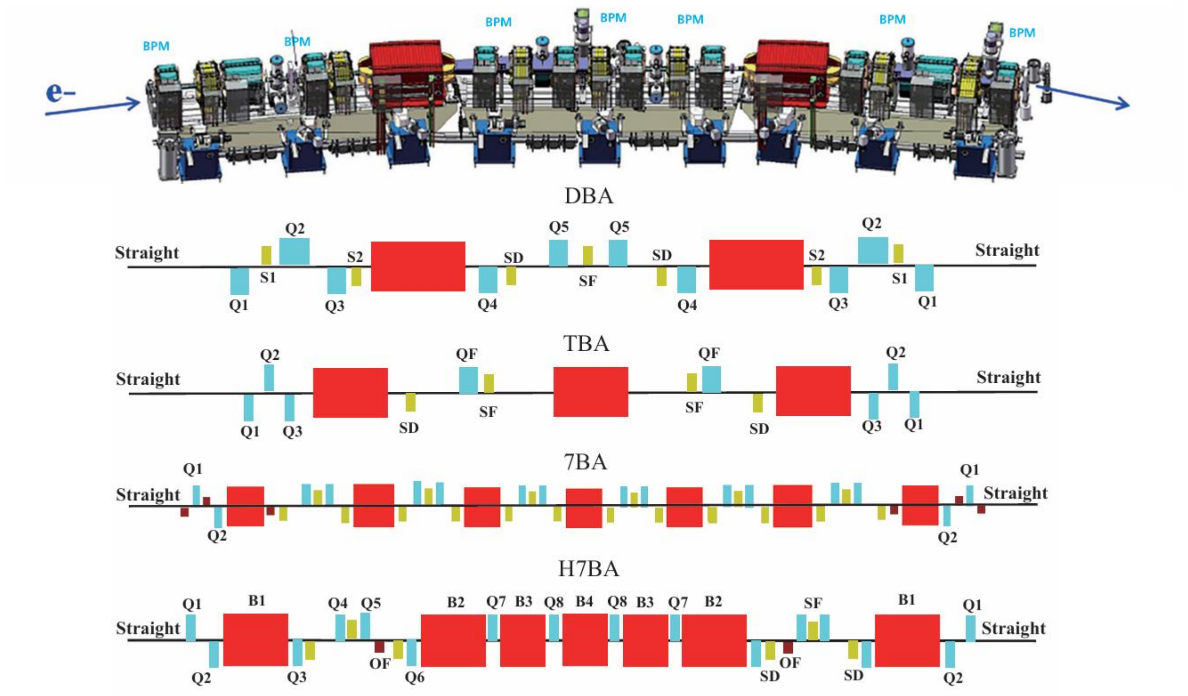
- Quantum excitation and radiation damping

  The emittance, energy spread and bunch length of electron beam are determined by the equilibrium of excitation and damping.

- Lattice design

Deflection angle per bending magnet

1. Reduce deflection angle: excitation↓ $\epsilon_n \approx\propto \gamma^2\theta^3$
2. Transverse gradient dipole: damping↑
3. Longitudinal gradient dipole: excitation↓
4. Reverse dipole: excitation↓
5. Damping wiggler (or Robinson wiggler): damping↑
6. Coupling



圖二　儲存環磁格單元，依序為 DBA、TBA、7BA、H7BA，其中二極磁鐵為紅色、四極磁鐵為水藍色、六極磁鐵為綠色、八極磁鐵為咖啡色。

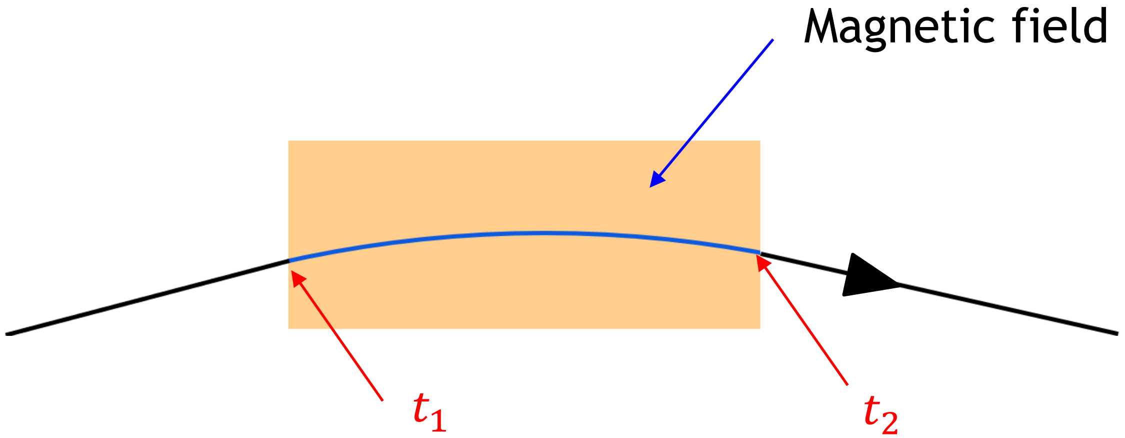
同步輻射光源之加速器磁格簡介 by 邱茂森
https://www.nsrrc.org.tw/NsrrcWebSystem/UPLOADS/CHINESE/PUBLISH_BRIEF/125/5a32ffdcc9.pdf

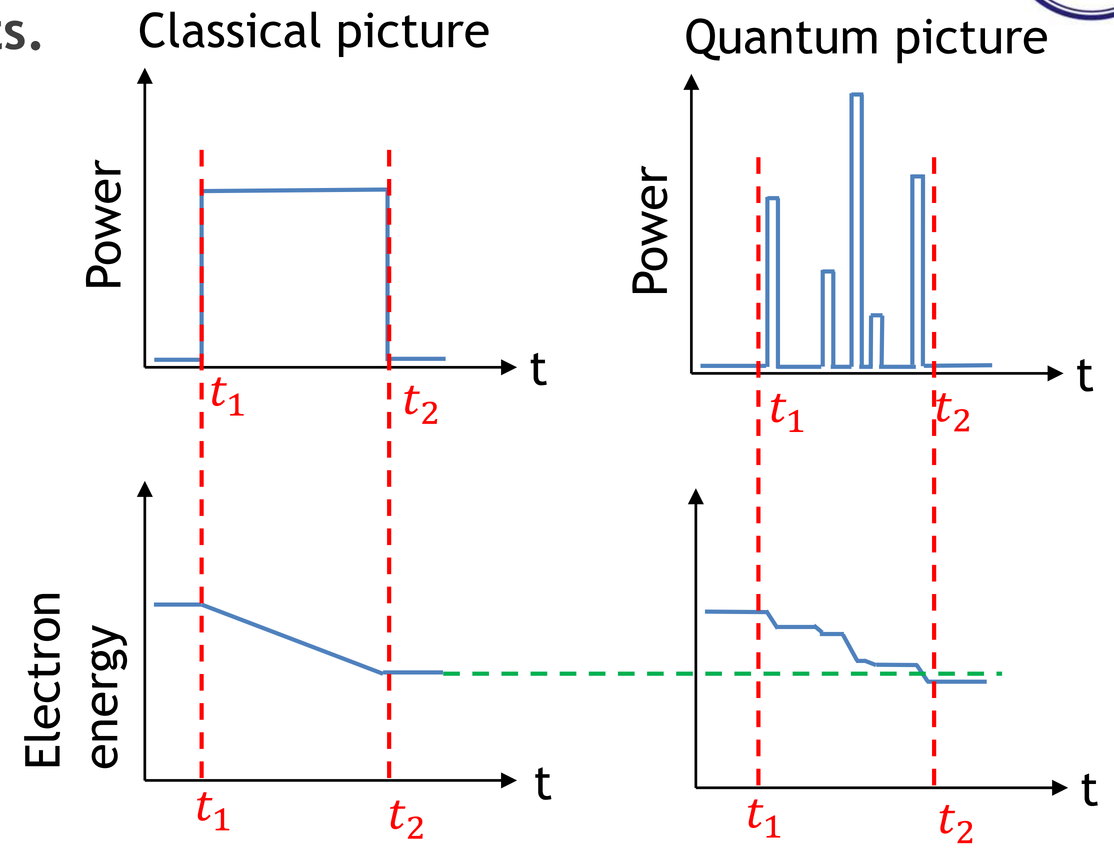
4. How to control the emittance

- Longitudinal dynamics is used to describe the effects.
- Radiation damping

$$P = \frac{\beta^4 e^2 c^3}{2\pi} C_\gamma \frac{E^4}{\rho^2}$$

- Quantum excitation





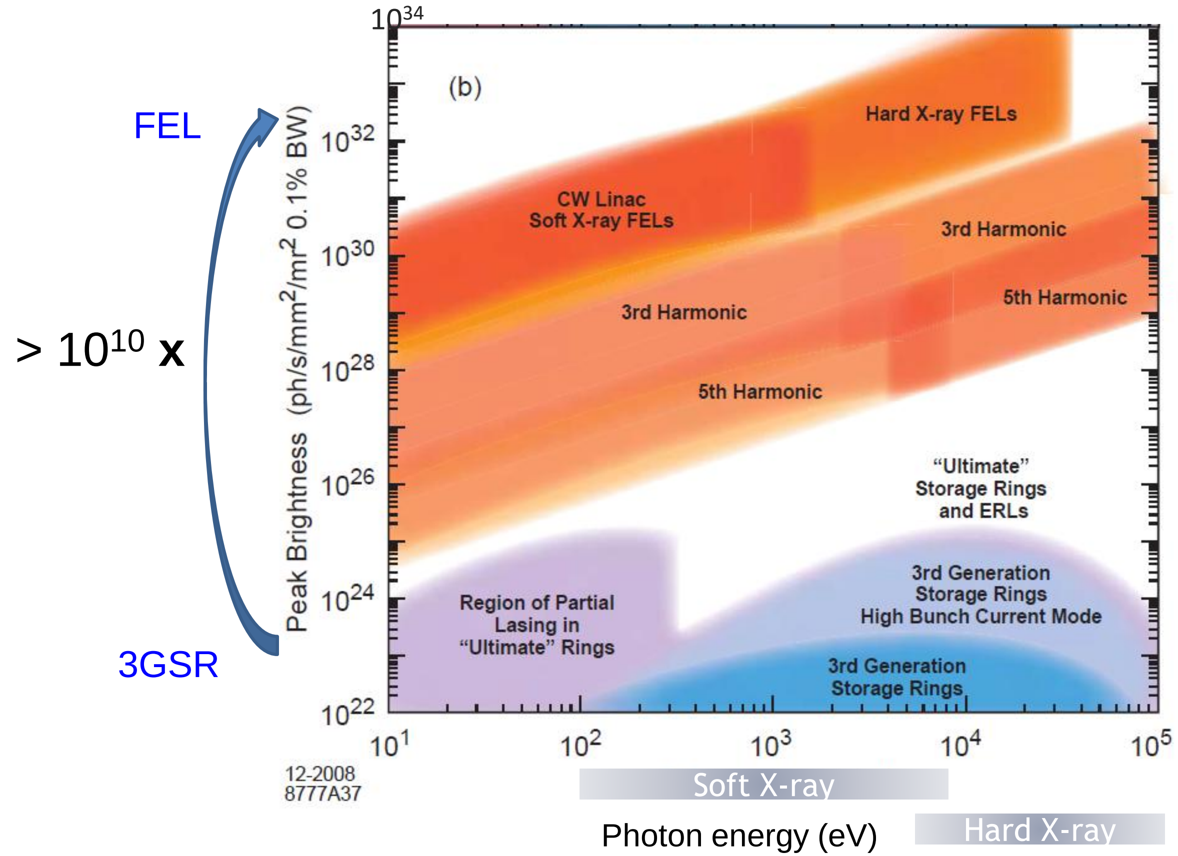
- Averaged number of photon emitted per revolution: $N_\gamma = \frac{5\pi}{\sqrt{3}}\alpha\gamma, \alpha \approx \frac{1}{137}$

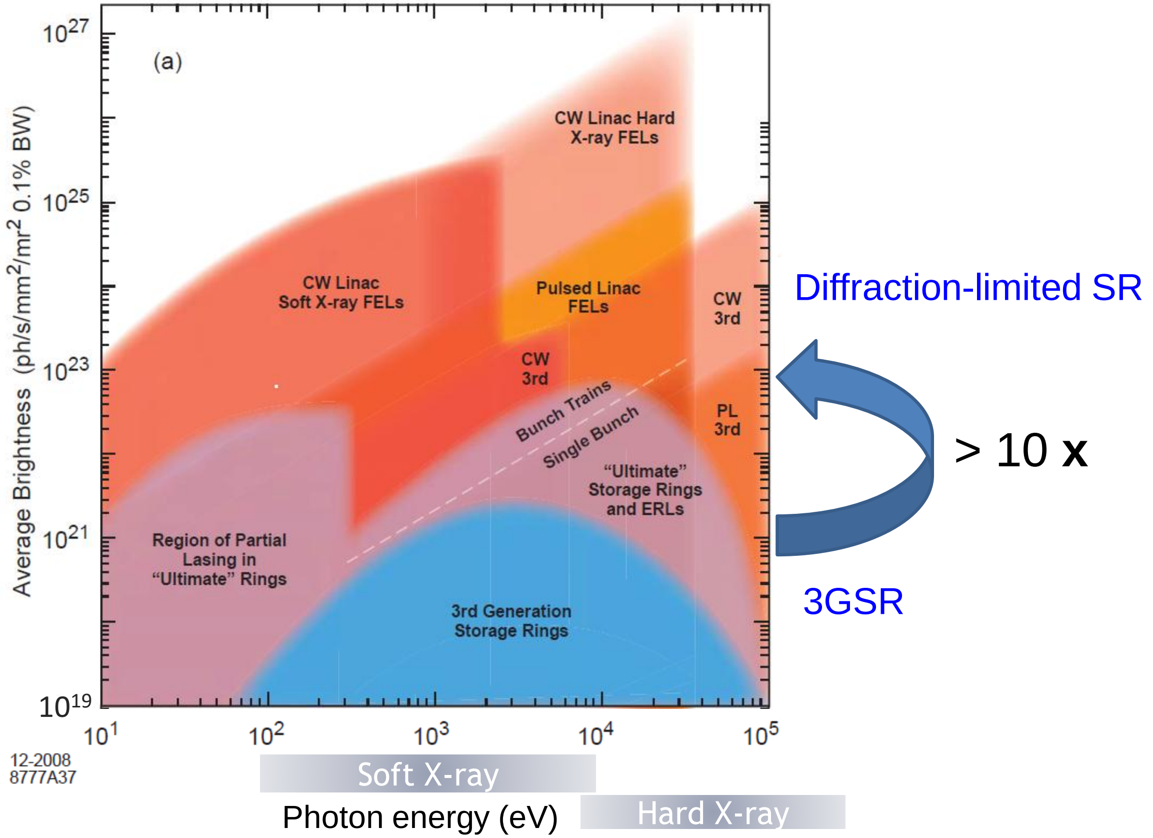
TPS: $N_\gamma \approx 389$ (~8 per bending)

- Both phenomena not only happens in longitudinal motion, but also in transverse motion.

Science and Technology of Future Light Sources — A White Paper (SLAC-R-917), 2008

[Ref.] https://www.slac.stanford.edu/pubs/slacreports/reports17/slac-r-917.pdf
Science and Technology of Future Light Sources — A White Paper (SLAC-R-917)



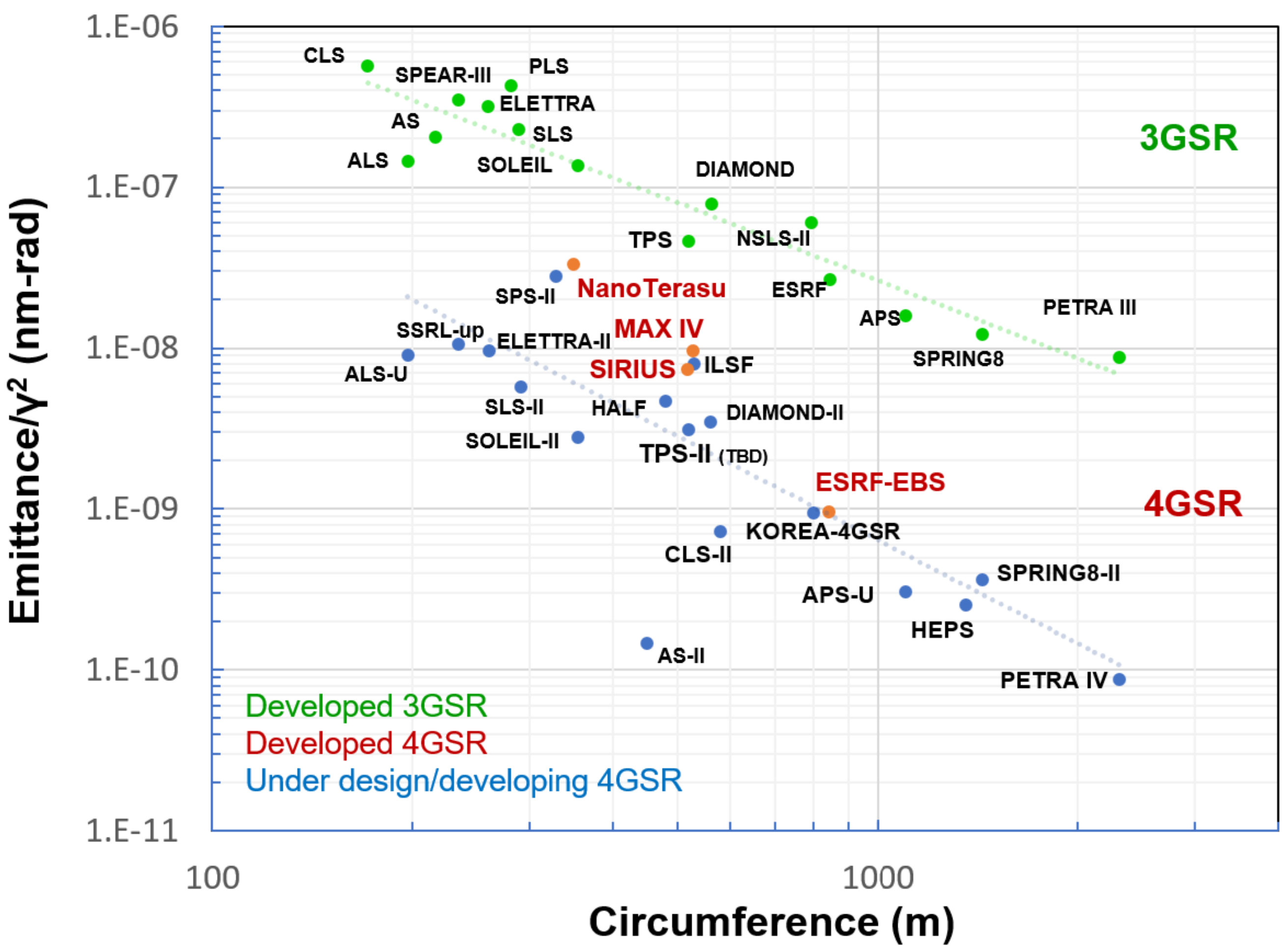


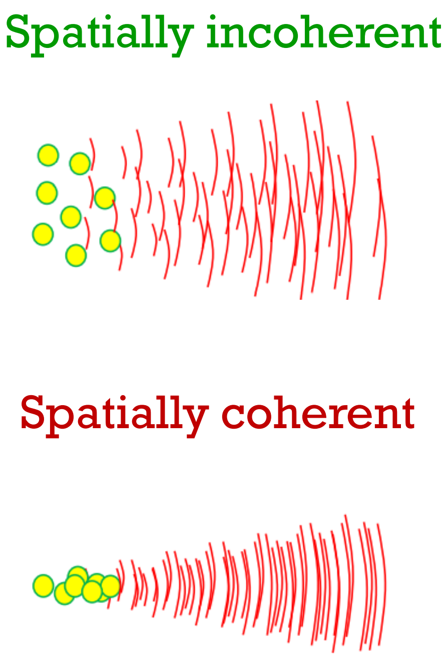
$$B = \frac{flux}{(2\pi)^2 \Sigma_x \Sigma_{x'} \Sigma_y \Sigma_{y'}} \left[\frac{photons}{sec-mm^2-mrad^2-0.1\%B.W.}\right]$$

$$B_{avg} = B \times \Delta T \times R_p$$

4. How to control the emittance

- Toward smaller electron beam emittance ring to increase the light brightness, spatial coherent flux, and coherent fraction.

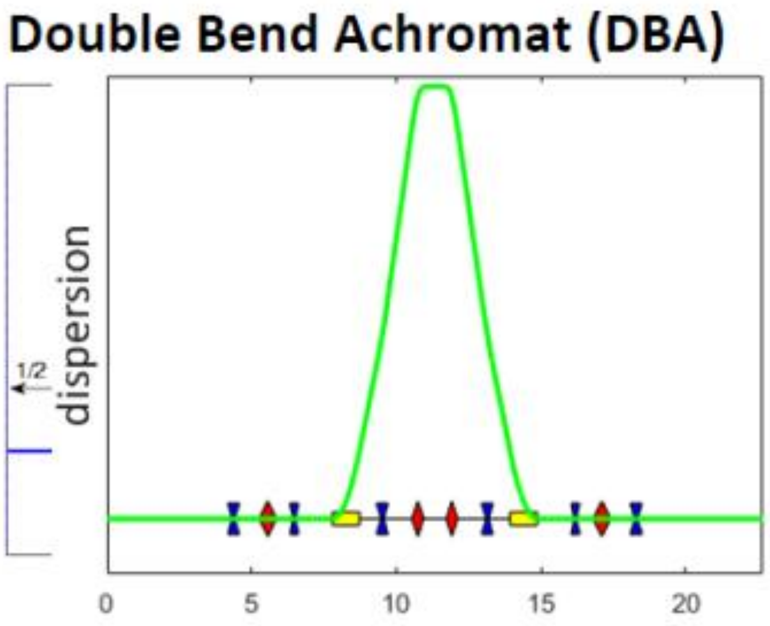


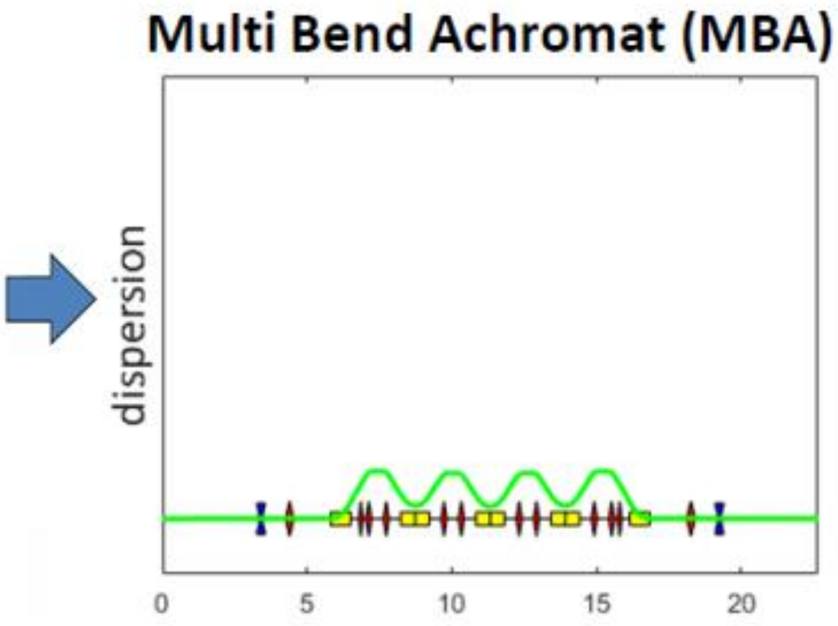


4. How to control the emittance

$$\varepsilon_0 = C_q \Upsilon^2 \frac{<H/|\rho|^3>}{j_x <1/|\rho|^2>}$$

$$\varepsilon_0 \propto E^2\theta^3$$

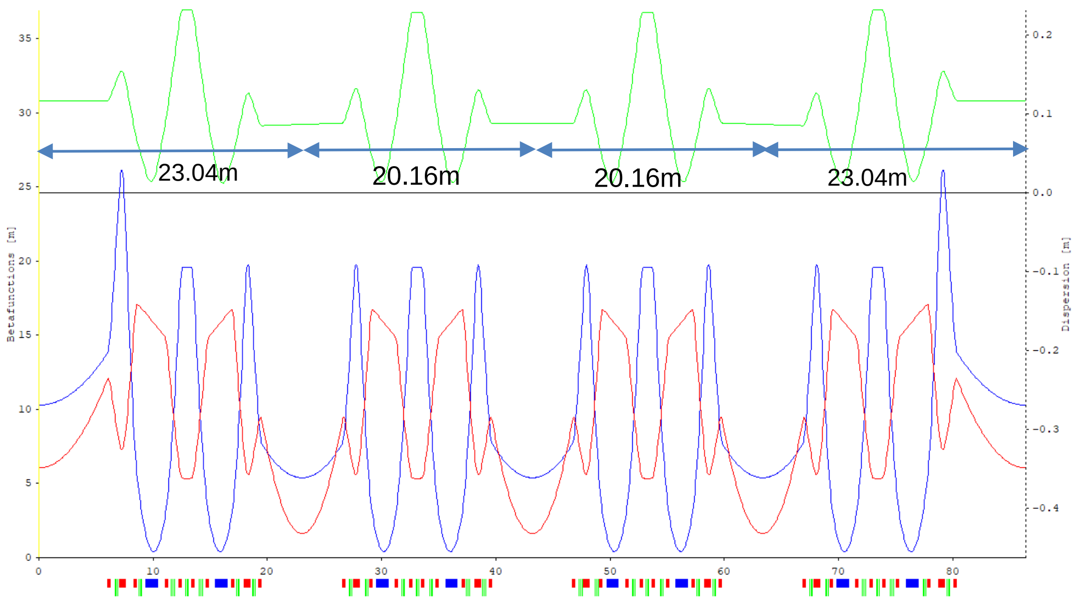




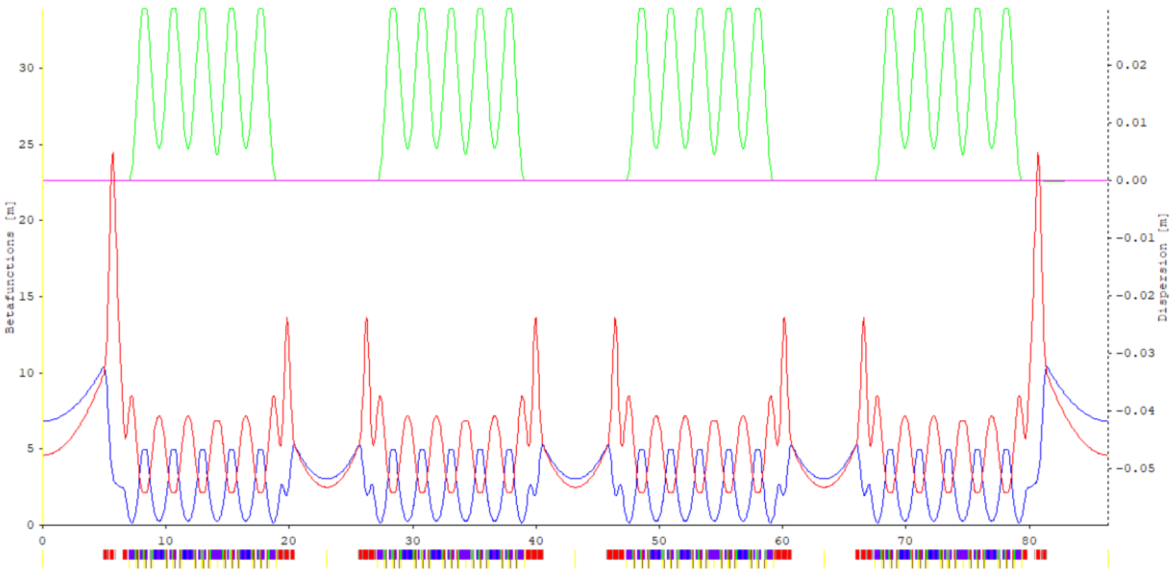
[Ref] Courtesy of R. Bartolini

4. How to control the emittance

TPS DBA lattice



TPS-II 6BA lattice



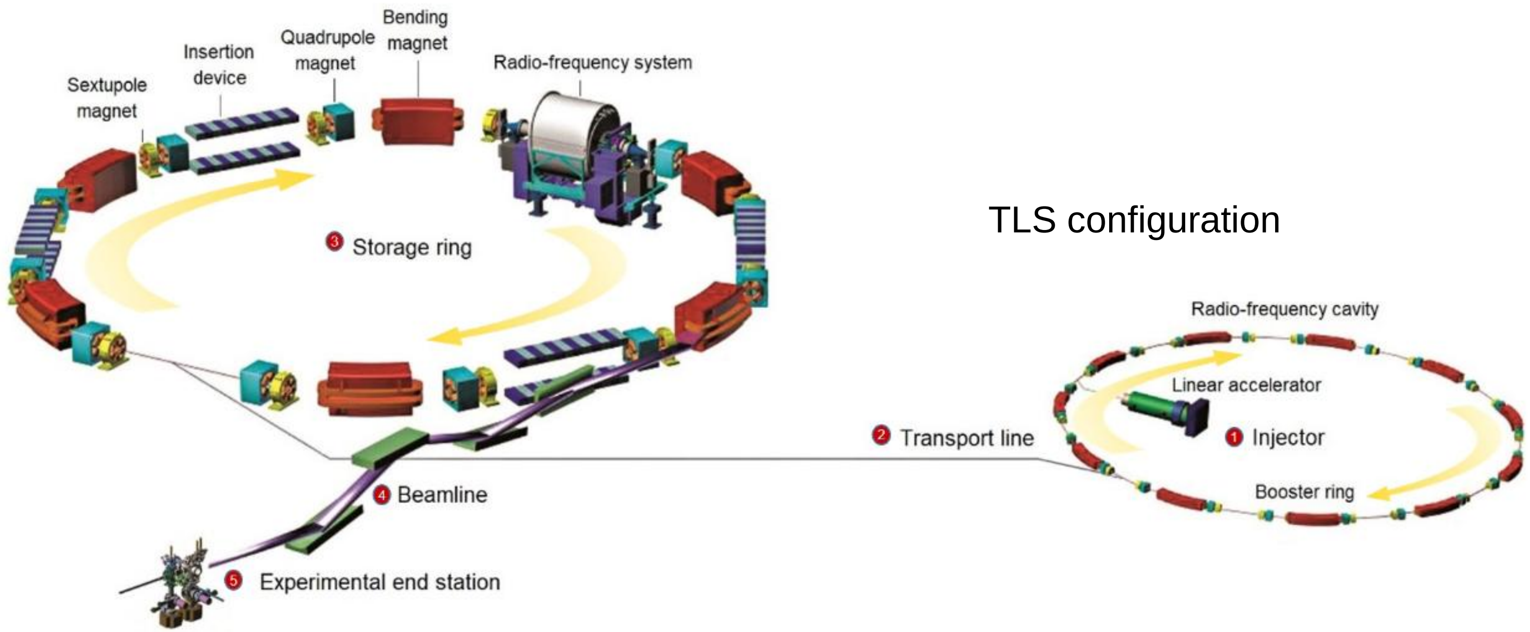
DBA lattice, 518.4 m, 3 GeV
ε = 1600 pm-rad
νx = 26.19, νy = 13.25
$\alpha = 2.4 \times 10^{-4}$
ξx = -75, ξy = -27

Dipole numbers
48 → 144
Ɛ / 10 times improve

6BA lattice
ε = 67 pm-rad
νx = 66.815, νy = 19.695
$\alpha = 0.58 \times 10^{-4}$

SR
(STORAGE RING)

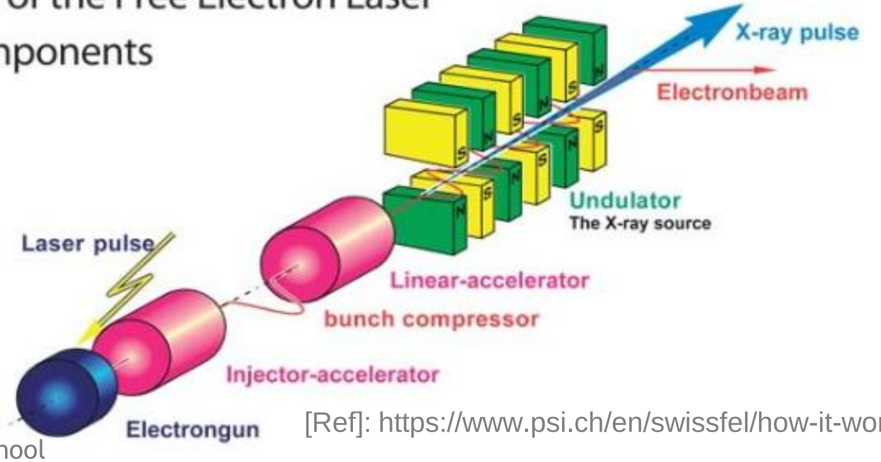
- # of users
- Wider spectrum
- Higher rep. rate
- Stability and flexibility



TLS configuration

Schematic design of the Free Electron Laser with different components

1) Electrongun
2) Injector
3) Accelerator
4) Undulator

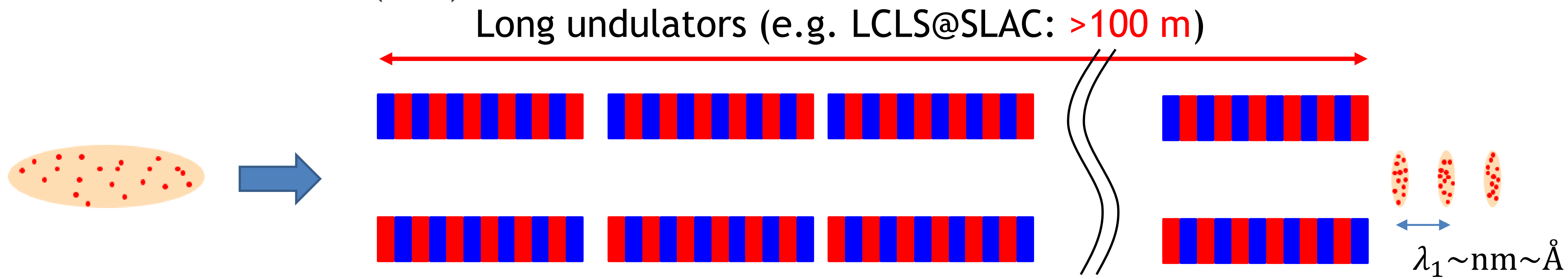


[Ref]: https://www.psi.ch/en/swissfel/how-it-works

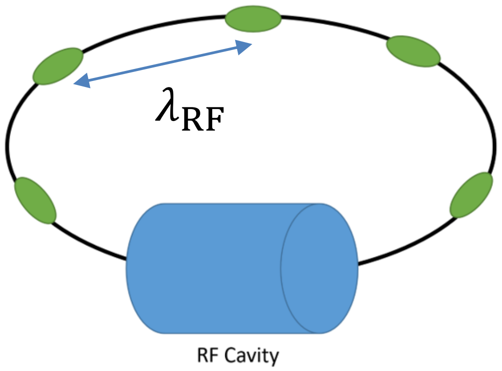
FEL
(FREE ELECTRON LASER)

- Higher peak brightness (> 10^{10} x than SR)
- ~ fully spatial coherence
- better temporal coherence (~ 1% for SASE， ~100 % for seeded FEL)
- Ultrashort time-resolved related discoveries

(High gain) Free electron laser (FEL)

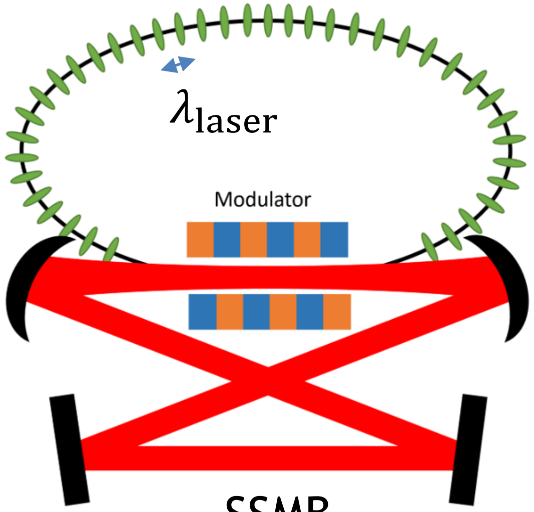


Steady-state microbunch (SSMB):



Conventional storage ring

λ_{RF}~m



SSMB

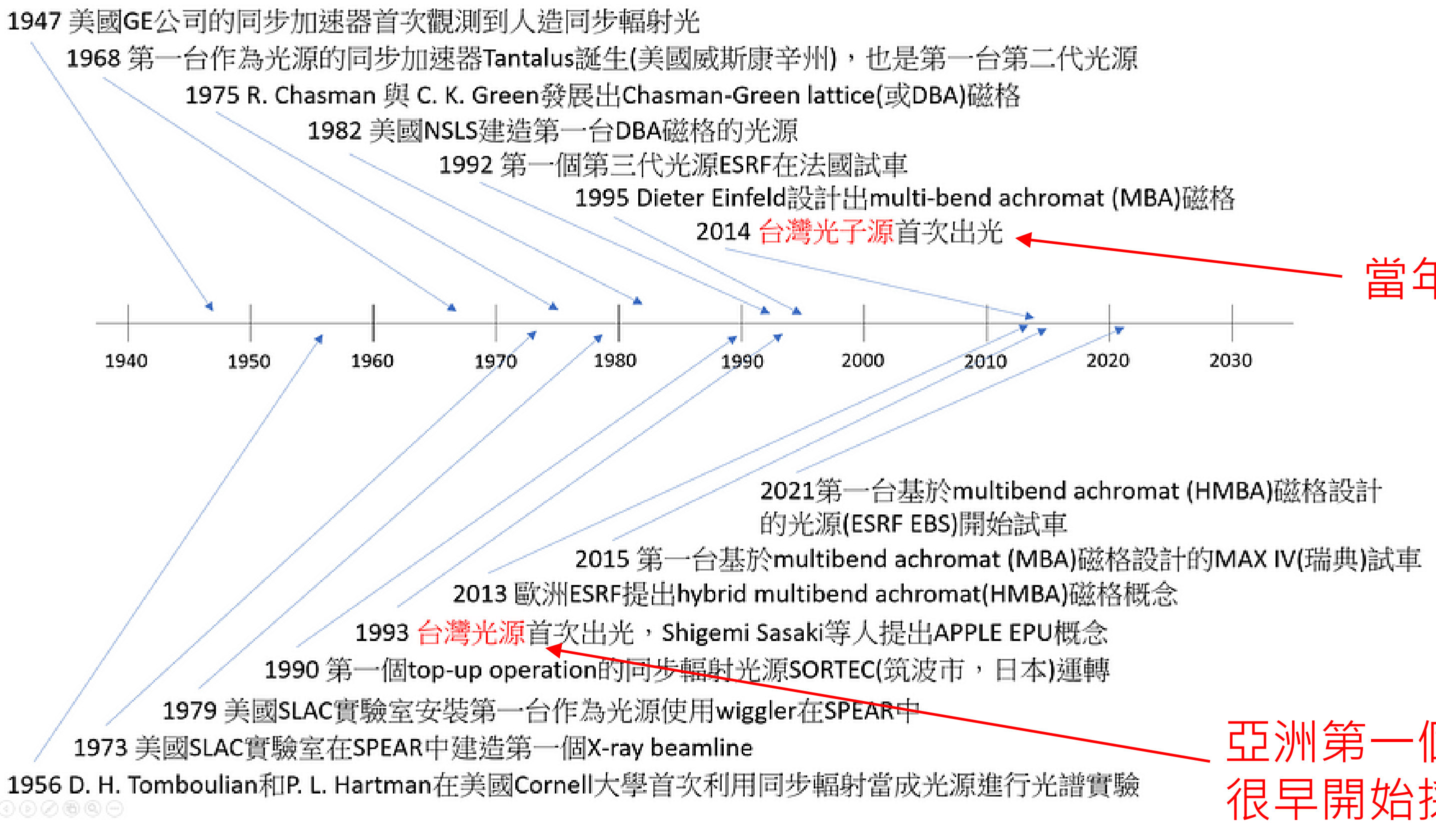
λ_{laser}~μm

Brief history of synchrotron radiation source

英國建造第一台X-ray波段的同步輻射光源@Daresbury Laboratory(FODO lattice) emittamce=1500 nm-rad

美國APS-upgraded(H7BA) emittamce<40 pm-rad

Brilliance increase>10^{11} times (including enhancement of undulator)



當年世界最亮

亞洲第一個第三代光源
很早開始採用Top-up operation

同步輻射光源簡史

https://medium.com/@cheongchoon/%E5%90%8C%E6%AD%A5%E8%BC%BB%E5%B0%84%E5%85%89%E6%BA%90%E7%B0%A1%E5%8F%B2-32f5064ee0f3

1975:

R. Chasman 與 C. K. Green發展出Chasman-Green lattice，又稱double-bend achromat (DBA)磁格

Preliminary design of a dedicated synchrotron radiation facility

https://ieeexplore.ieee.org/document/4327987

並為National Synchrotron Light Source設計了兩台同步輻射光源(能分別為700 MeV和2 GeV)

Design of a national dedicated synchrotron radiation facility

https://www.osti.gov/servlets/purl/7334216

1976:

Brian M. Kincaid提出helical undulator的概念

A short-period helical wiggler as an improved source of synchrotron radiation

https://pubs.aip.org/aip/jap/article-abstract/48/7/2684/506076/A-short-period-helical-wiggler-as-an-improved

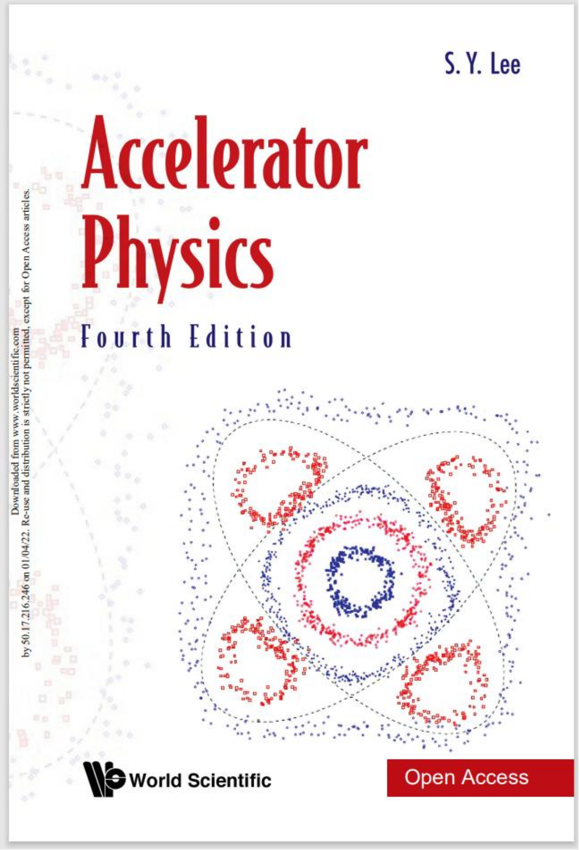
1979:

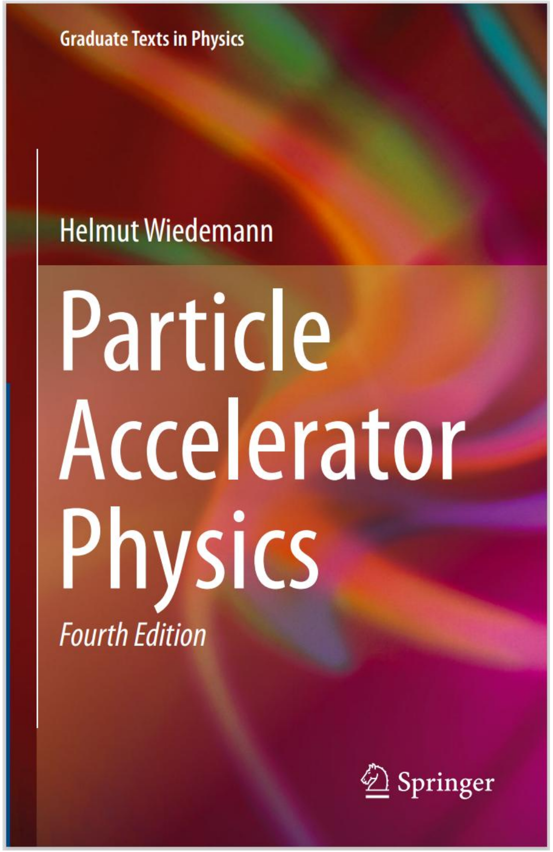
美國SLAC實驗室安裝第一台作為光源使用的電磁鐵wiggler至Stanford Synchrotron Radiation Light source 的SPEAR中
Initial Operation of SSRL Wiggler in SPEAR

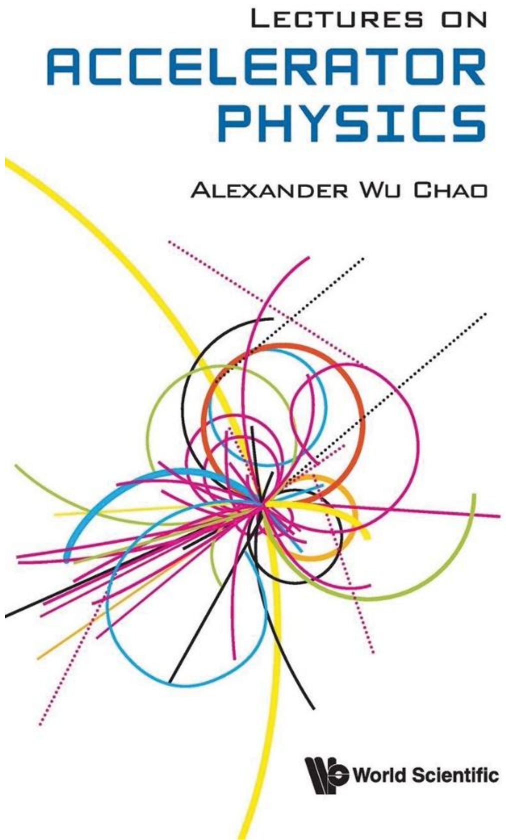
https://ieeexplore.ieee.org/document/4330617

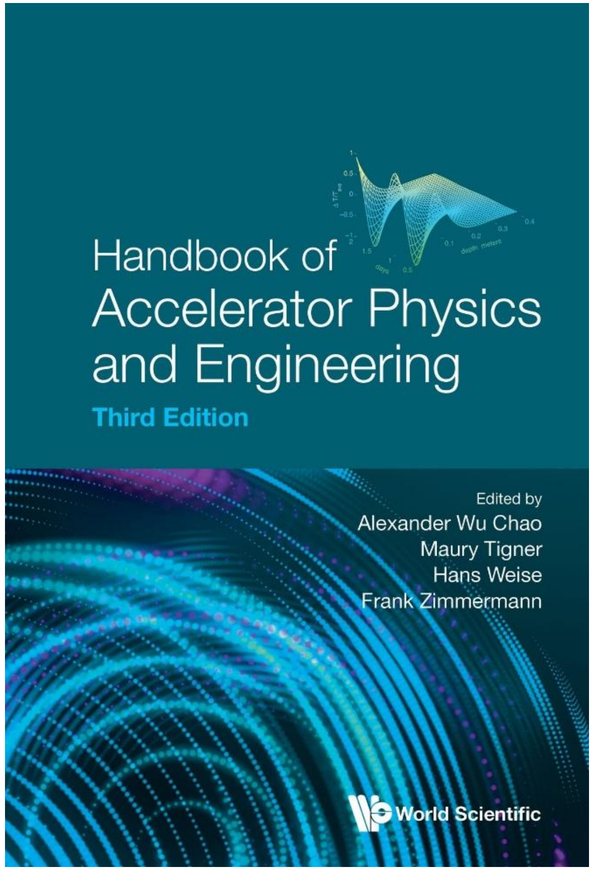
Wiggler and Undulator Magnets

https://web.archive.org/web/20150923175408/http://www.askmar.com/Magne


S. Y. Lee
Accelerator Physics
Fourth Edition
World Scientific
Open Access


Graduate Texts in Physics
Helmut Wiedemann
Particle Accelerator Physics
Fourth Edition
Springer


Lectures on
Accelerator Physics
Alexander Wu Chao
World Scientific


Handbook of
Accelerator Physics
and Engineering
Third Edition
Edited by
Alexander Wu Chao
Maury Tigner
Hans Weise
Frank Zimmermann
World Scientific

Open access

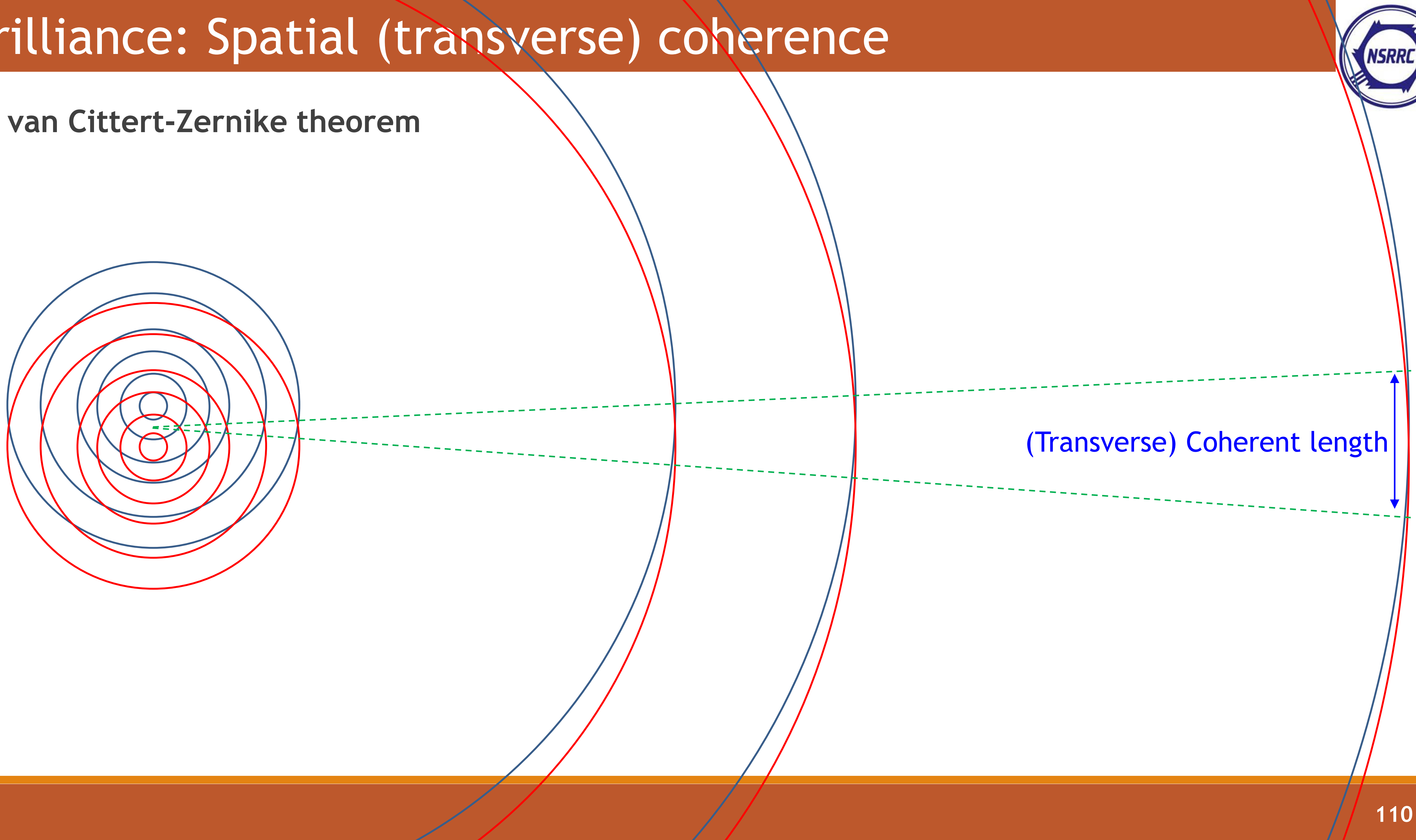
Welcome to Join NSRRC

@Nuan-Ya Huang, 2024 FEL Summer School

Thanks for your attention!!

Brilliance: Spatial (transverse) coherence

- van Cittert-Zernike theorem



Brilliance: Spatial (transverse) coherence

- van Cittert-Zernike theorem (For incoherent source)

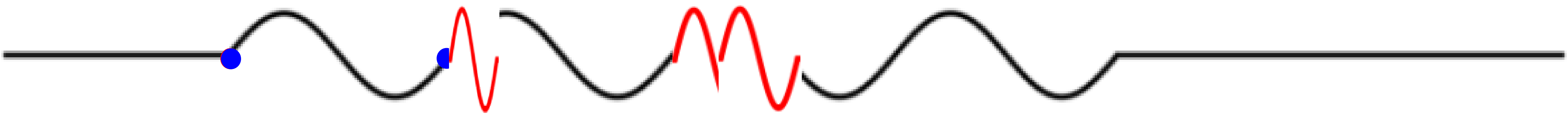
  TPS is not only partial coherent but maybe near fully coherent in vertical direction

TPS
Coupling 0.5%
ϵ_x : 1600 pm-rad
ϵ_y : 8 pm-rad

10 keV → 10 pm-rad ($\sigma_r \sigma_{r'} = \frac{\lambda}{4\pi}$)
100 eV → 1000 pm-rad

Thirteen Rouen Ducks Create Coherent Waves
https://www.youtube.com/watch?v=4o48J4streE

Resonance wavelength and frequency



Time-shifting property of Fourier transformation

$$\mathcal{F}(\omega) = \int f(t)e^{-i\omega t}\mathrm{d}t$$

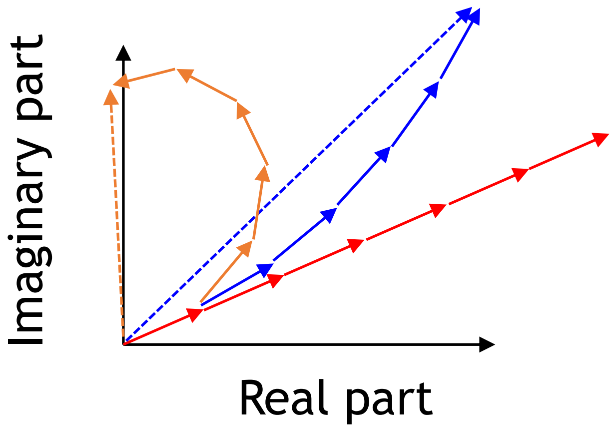
$$f_1(t) \to \mathcal{F}_1(\omega)$$

$$f_2(t) = f_1(t - \Delta t)$$

$$f_2(t) \to \mathcal{F}_2(\omega) = e^{-i\omega\Delta t}\mathcal{F}_1(\omega)$$

$\omega\Delta t = 2n\pi$ →coherently summation

Phase diagram



Undulator radiation

Fundamental harmonic

$$\lambda_1 = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

On-axis odd harmonics shows up since longitudinal component of velocity is not constant

$$\lambda_h = h\lambda_1, \qquad h = 1,3,5,\cdots$$

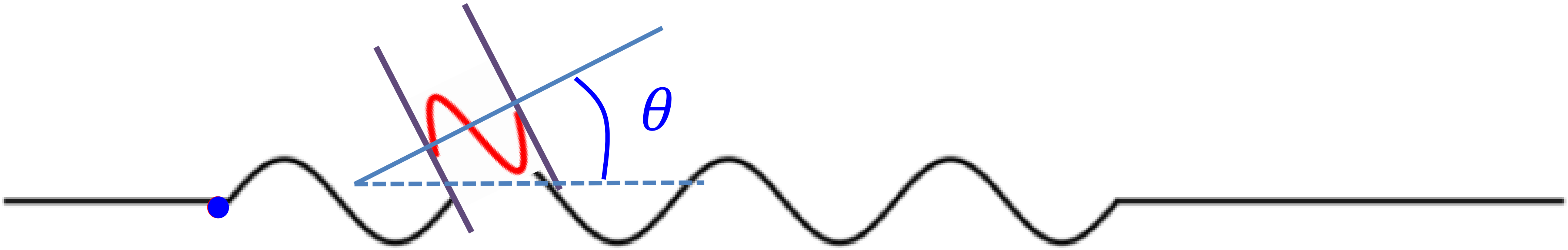
Averaged longitudinal velocity

$$\beta^2 = \beta_\perp^2 + \beta_{||}^2 = 1 - \frac{1}{\gamma^2}, \qquad \beta_{||} \approx 1 - \frac{1+K^2}{2\gamma^2}$$

Resonance wavelength and angular frequency of an off-axis observer

$$\lambda_1 = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right)$$

$$\omega_1 = \omega_u \frac{2\gamma^2}{1 + \frac{K^2}{2} + \gamma^2\theta^2}$$



Undulator radiation

For the on-axis spectrum:

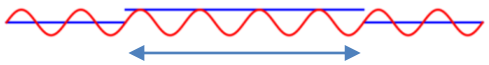
resonance photon energy has max photon flux

However, considering the overall spectrum

Resonance photon energy is not the peak value

Estimation

Radiation pulse



Red: sinusoidal

Blue: box function

$T_u = N_u/\omega_1$

Convolution theorem

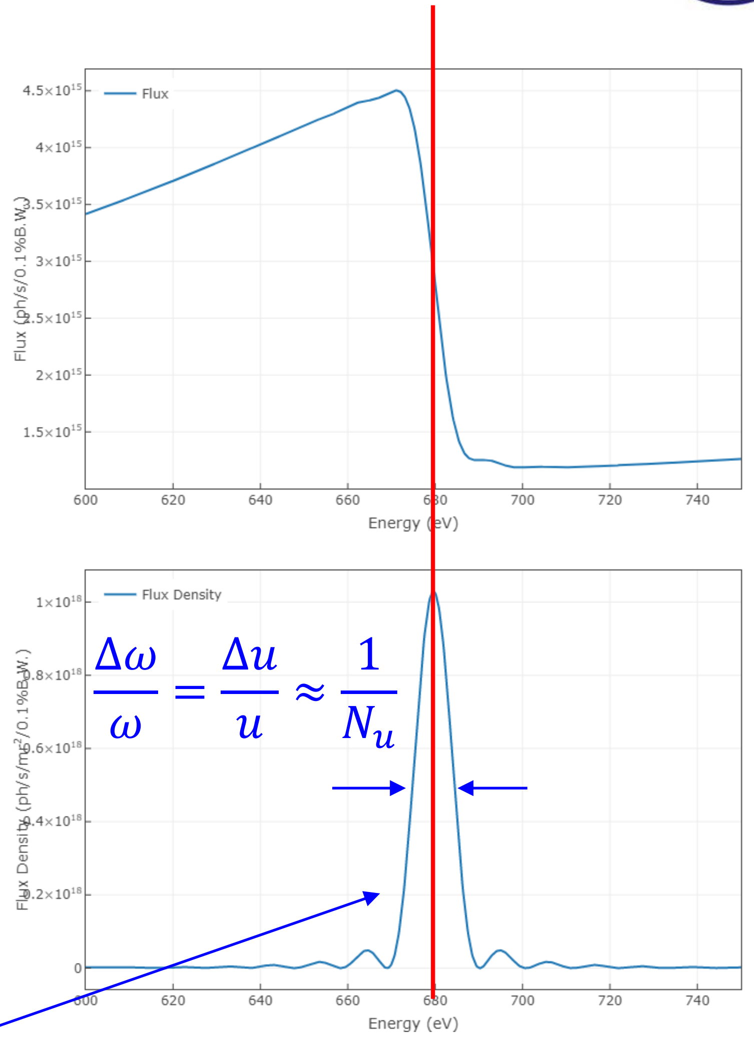
$$f(t) \rightarrow \mathcal{F}(\omega), g(t) \rightarrow \mathcal{G}(\omega)$$

$$h(t) = f(t)g(t) \rightarrow \mathcal{F}(\omega) * \mathcal{G}(\omega)$$

$$f(t) = \mathrm{rect}\left(\frac{t}{T_u}\right) \rightarrow \mathcal{F}(\omega) = \mathrm{sinc}(T_u\omega/2)$$

Form factor of undulator radiation spectrum

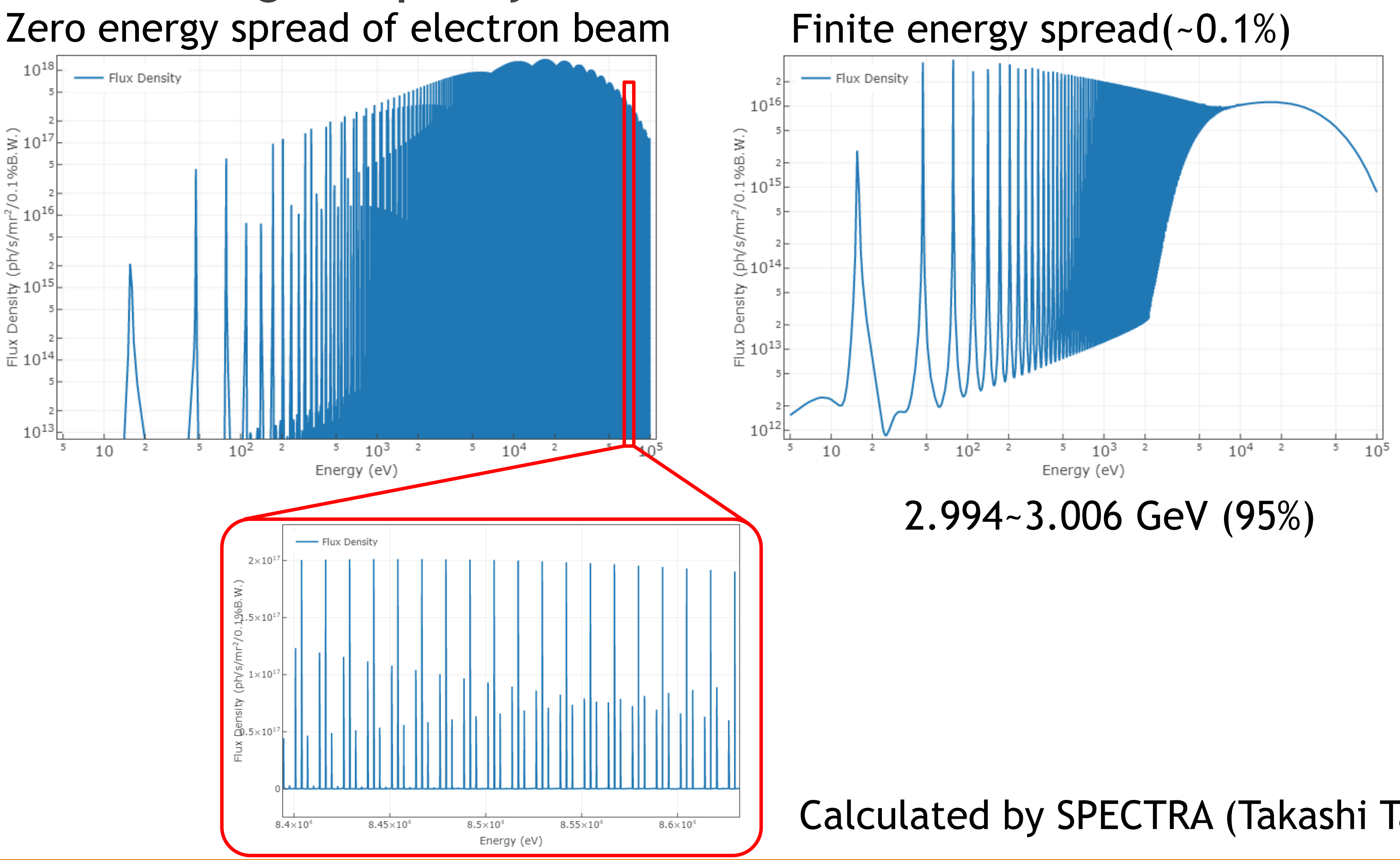
$$F(\omega) \propto \mathrm{sinc}^2\left(\pi N_u \frac{\Delta\omega}{\omega}\right)$$



$$\frac{\Delta\omega}{\omega} = \frac{\Delta u}{u} \approx \frac{1}{N_u}$$

Undulator radiation

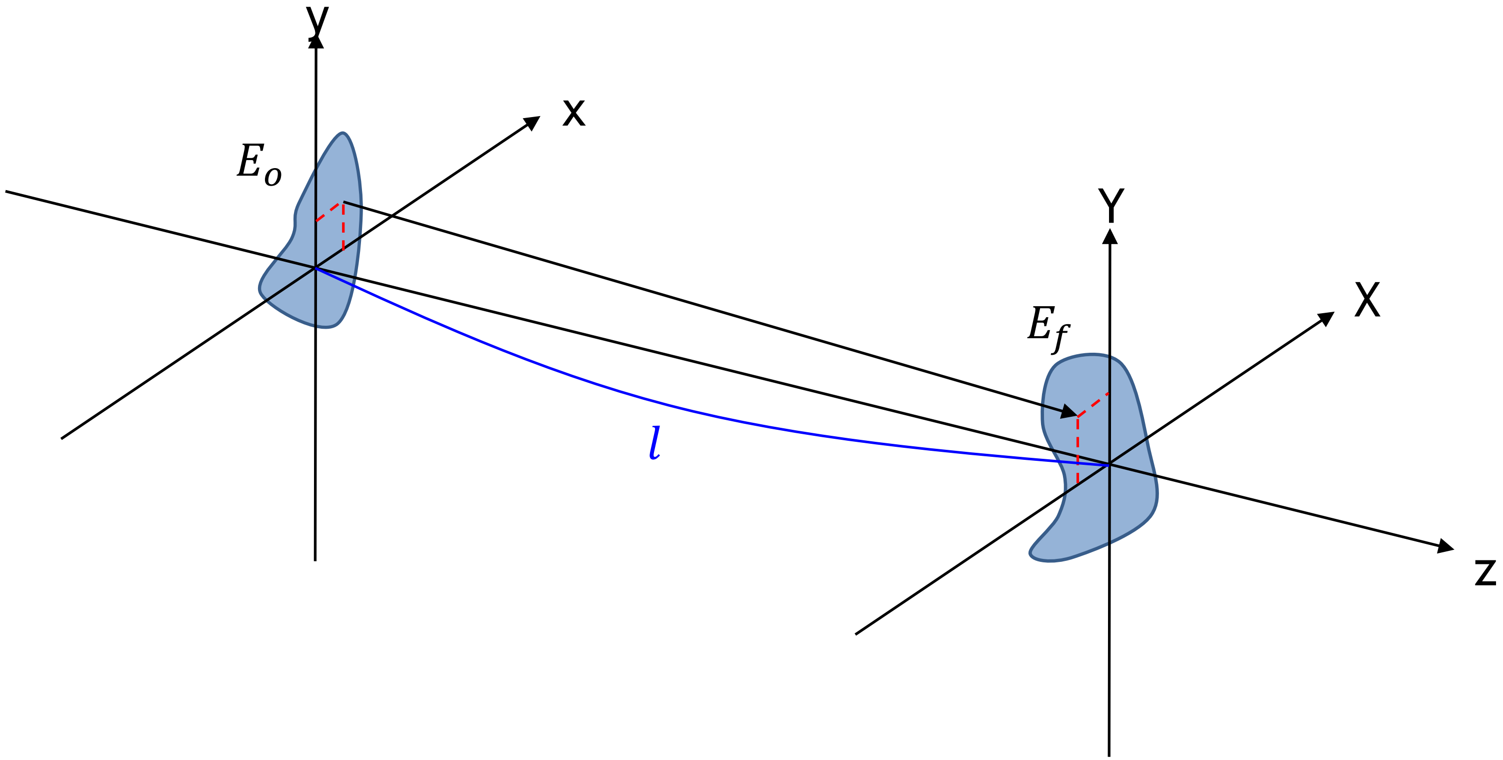
Energy spread of electron beam cause the high harmonics merge together and results in a synchrotron radiation spectrum at high frequency



Calculated by SPECTRA (Takashi Tanaka)

Propagation of wave: Huygens-Fresnel principle

Every points is a source of spherical wave



$$k = \frac{2\pi}{\lambda}$$

$$E_f(X,Y) = \frac{-i}{\lambda l} \int E_o(x,y) \exp\left\{ik\left[\frac{(X-x)^2 + (Y-y)^2}{2l}\right] + l\right\}$$

Undulator radiation

Spatial and angular distribution of radiation field are Fourier pair

$$\mathbf{r} = (x, y), \boldsymbol{\phi} = (\phi_x, \phi_y)$$

$$E(\mathbf{r}) = \int \mathcal{E}(\boldsymbol{\phi}) \exp(ik\boldsymbol{\phi} \cdot \mathbf{r})\, \mathrm{d}^2\, \boldsymbol{\phi} \quad \Leftrightarrow \quad \mathcal{E}(\boldsymbol{\phi}) = \frac{1}{\lambda^2} \int E(\mathbf{r}) \exp(-ik\boldsymbol{\phi} \cdot \mathbf{r})\, \mathrm{d}^2\, \mathbf{r}$$

Decomposition into plane waves in different direction

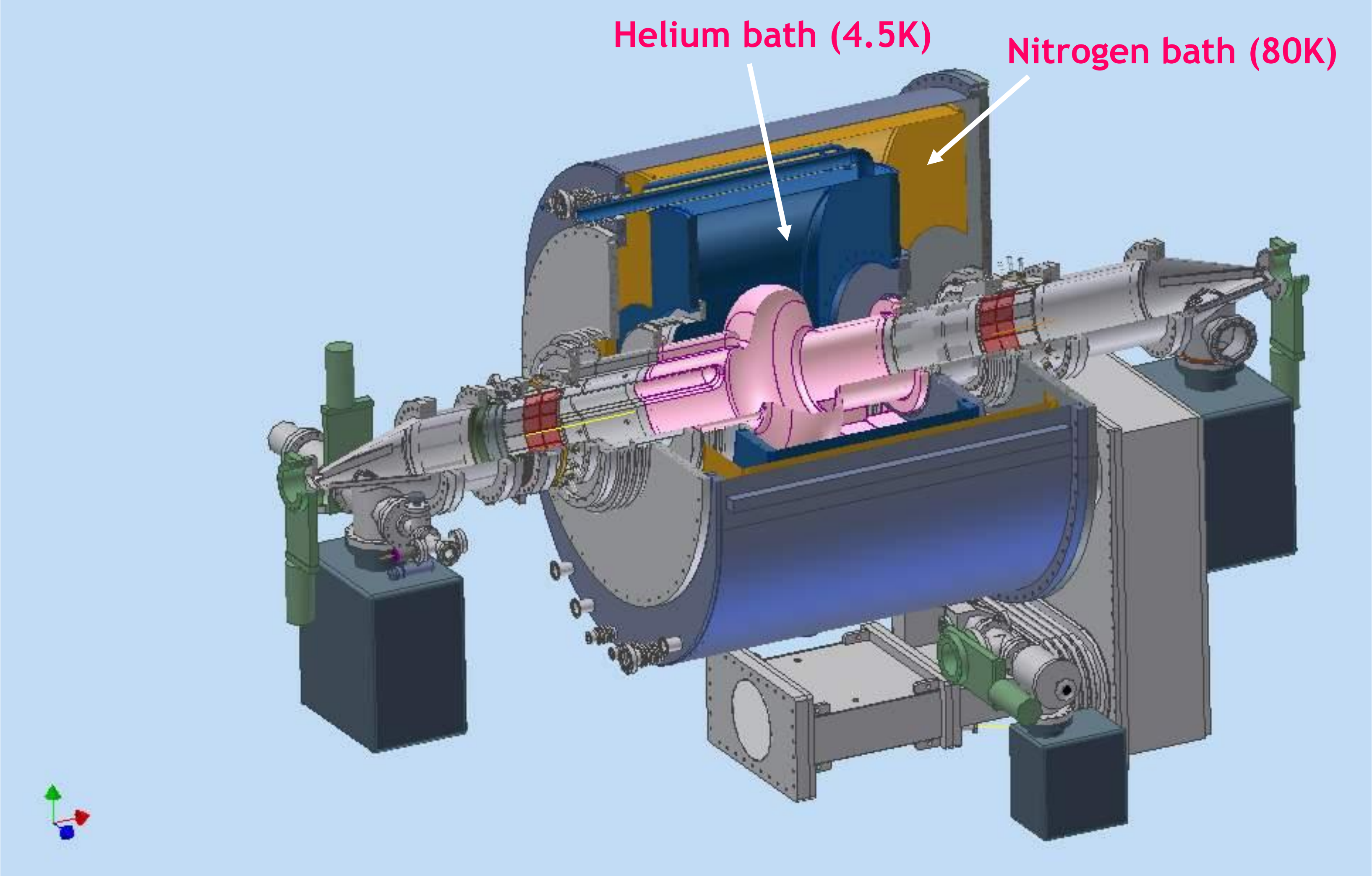
Small angle approximation

$$\boldsymbol{\phi}^2 \ll 1 \rightarrow \sqrt{1 - \boldsymbol{\phi}^2} \approx 1 - \frac{\boldsymbol{\phi}^2}{2}$$

$$E_f(X, Y) = \frac{-i}{\lambda l} \int E_o(x, y) \exp\left\{ik\left[\frac{(X - x)^2 + Y^2}{2l}\right] + l\right\} \quad \Leftrightarrow \quad \mathcal{E}_f(\boldsymbol{\phi}) = \mathcal{E}_o(\boldsymbol{\phi}) \exp\left[ikl\left(1 - \frac{\boldsymbol{\phi}^2}{2}\right)\right]$$

3. How to accelerate electron beam

The cut-away view of SRF module at Taiwan Light Source



Ref: Accelerator Physics by Ping J. Chou

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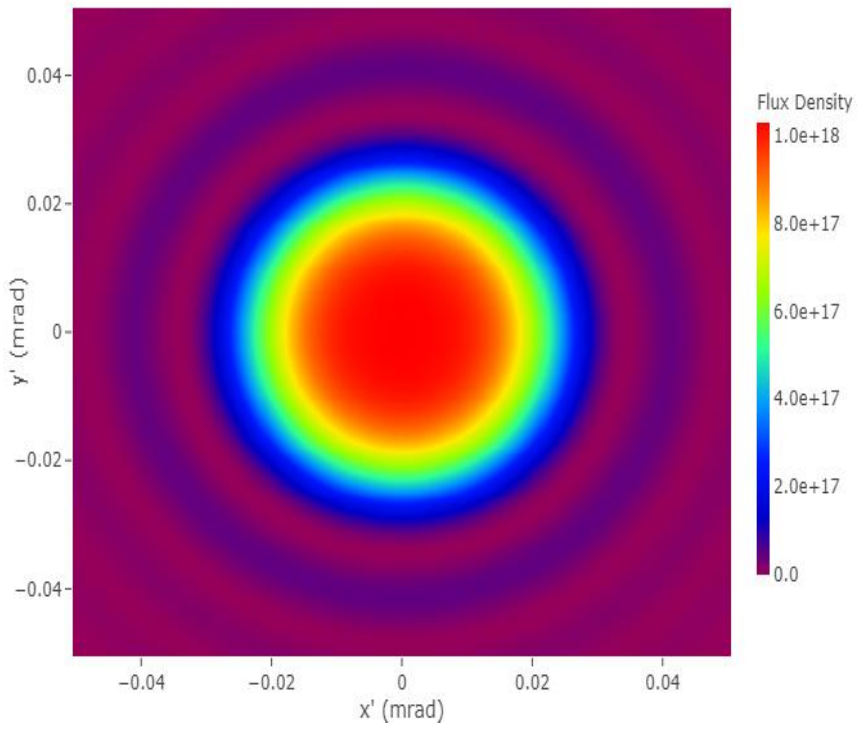
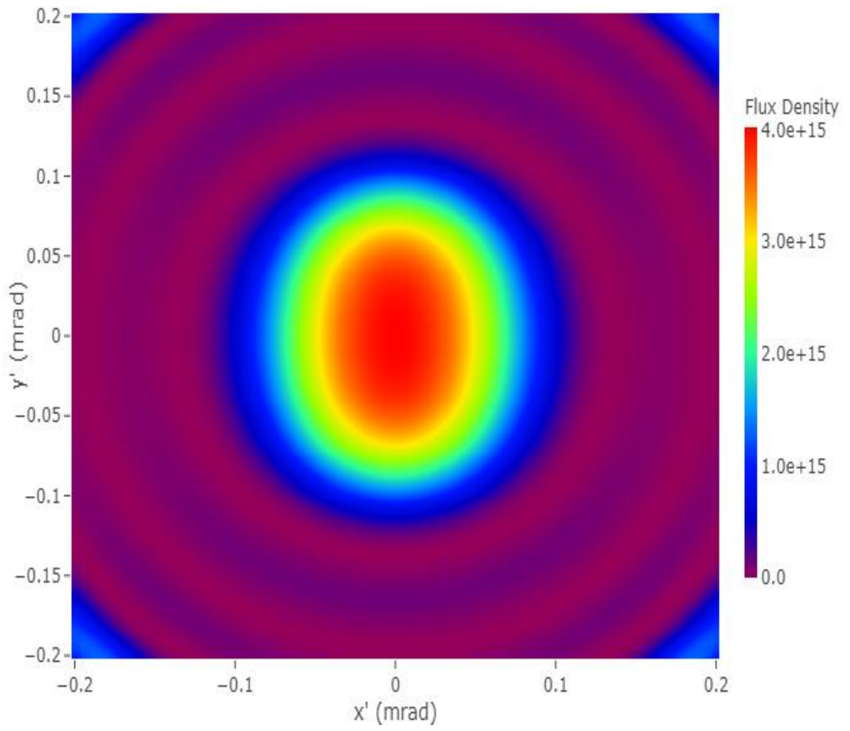
Undulator radiation

Typical angular distribution of 1st harmonic undulator radiation

$N_u \gg 1$



~60 periods



4 main periods

Both x and y direction: $\propto \ \mathrm{sinc}^2 \left[\frac{\pi L}{2\lambda}\theta^2\right]$

Distribution also determined by Bessel functions

Brilliance: Spatial (transverse) coherence

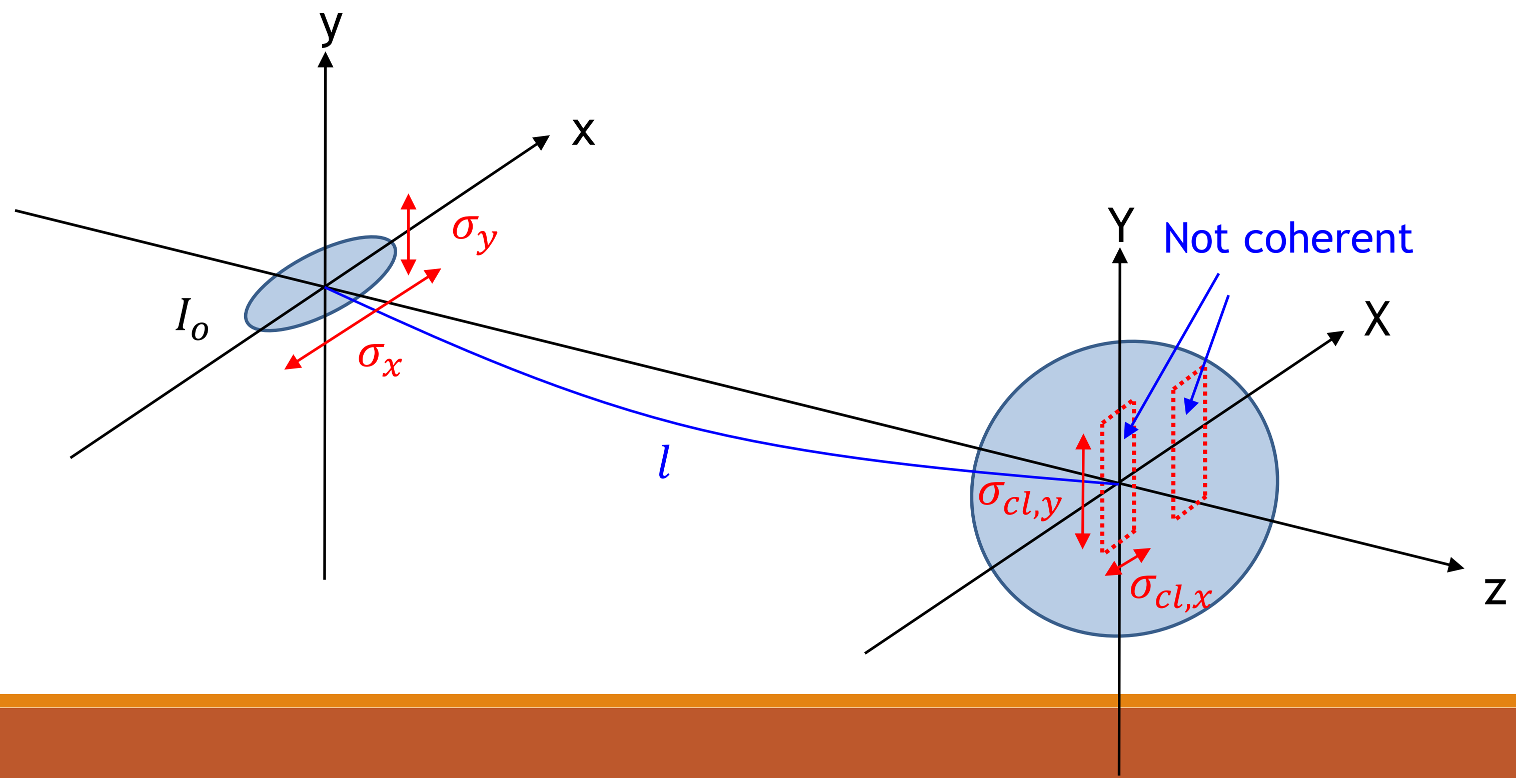
- **van Cittert-Zernike theorem:**

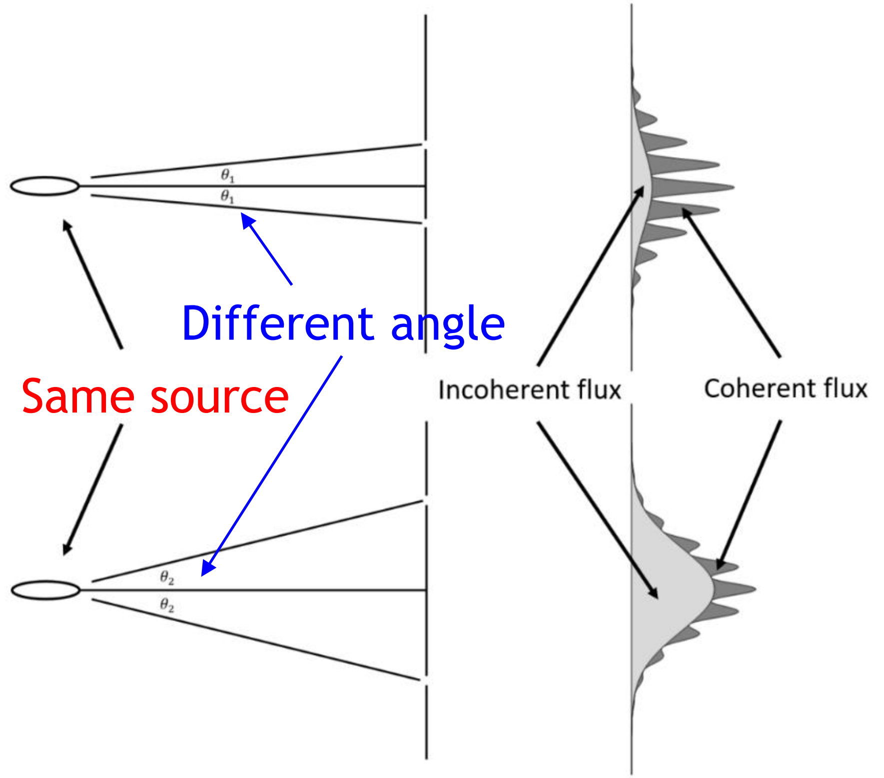
For incoherent source: degree of coherence far away from the source is proportional

to the Fourier transform of the intensity distribution $I_o = \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right)$ of the source.

- **Coherent length**

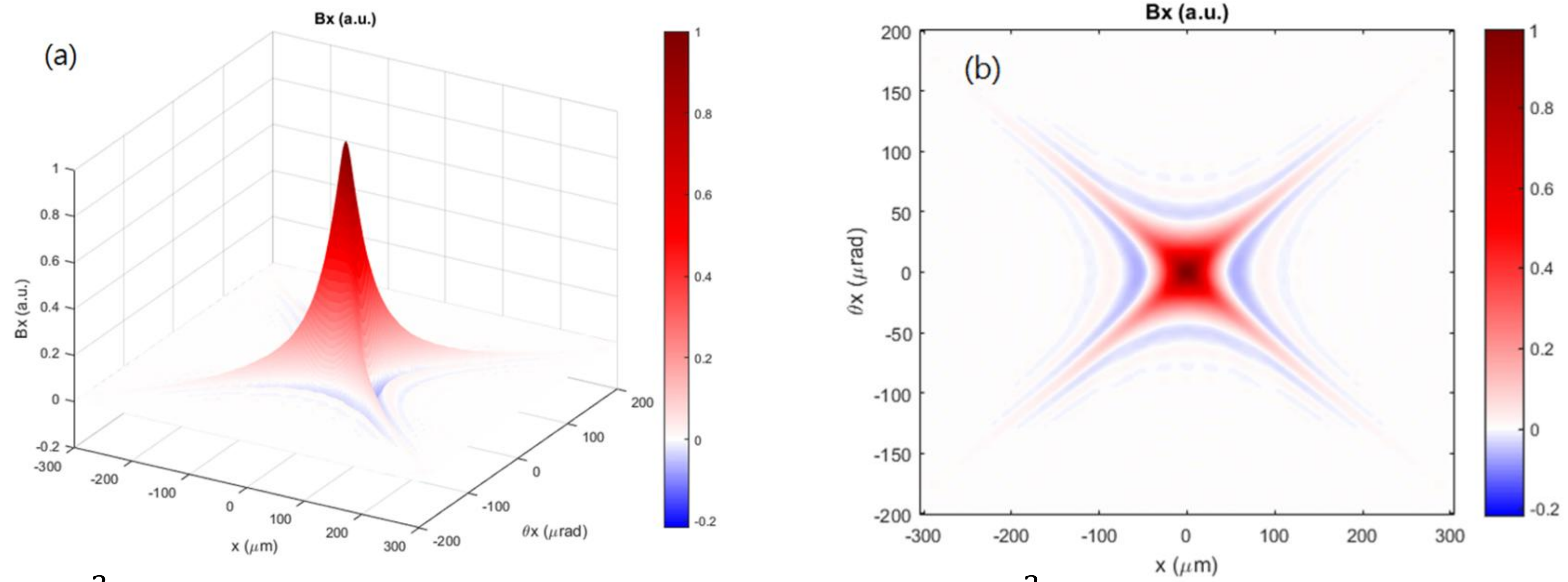
$$|\gamma(\boldsymbol{r}_1, \boldsymbol{r}_2)| \propto \exp\left[-\frac{(\Delta X)^2}{2\sigma_{cl,x}^2} - \frac{(\Delta Y)^2}{2\sigma_{cl,y}^2}\right], \sigma_{cl,x,y} = \frac{\lambda l}{2\pi\sigma_{x,y}}$$





Brilliance: Phase space

- Overall photon beam: Convolution of electron beam distribution and photon beam distribution of single electron (or filament electron beam)
- Transverse electron beam distribution: Gaussian distribution (explained later)
- Photon beam distribution: Wigner distribution function (approximated by Gaussian distribution)



$$W(\boldsymbol{r},\boldsymbol{\theta};\omega)=\left(\frac{1}{\lambda}\right)^2\int\langle E\left(\boldsymbol{r}-\frac{\boldsymbol{r}'}{2};\omega\right)E^*\left(\boldsymbol{r}+\frac{\boldsymbol{r}'}{\boldsymbol{2}};\omega\right)\rangle\, e^{ik\boldsymbol{r}'\cdot\boldsymbol{\theta}}\mathrm{d}^2\boldsymbol{r}'=\left(\frac{1}{\lambda}\right)^2\int\langle\mathcal{E}\left(\boldsymbol{\theta}-\frac{\boldsymbol{\theta}'}{2};\omega\right)\mathcal{E}^*\left(\boldsymbol{\theta}+\frac{\boldsymbol{\theta}'}{\boldsymbol{2}};\omega\right)\rangle\, e^{-ik\boldsymbol{r}\cdot\boldsymbol{\theta}'}\mathrm{d}^2\boldsymbol{\theta}'$$

Brilliance: Phase space

- **Electron beam distribution** $\rho_e(\boldsymbol{r},\boldsymbol{\theta}) = \frac{1}{4\pi^2\sigma_x\sigma_y\sigma_{x'}\sigma_{y'}}\exp\left(-\frac{x^2}{2\sigma_x^2}-\frac{y^2}{2\sigma_y^2}-\frac{\theta_x^2}{2\sigma_{x'}^2}-\frac{\theta_y^2}{2\sigma_{y'}^2}\right)$

- **Photon beam distribution** $B_{fi}(\boldsymbol{r},\boldsymbol{\theta}) = B_0\exp\left(-\frac{x^2}{2\sigma_r^2}-\frac{y^2}{2\sigma_r^2}-\frac{\theta_x^2}{2\sigma_{r'}^2}-\frac{\theta_y^2}{2\sigma_{r'}^2}\right)$

- **Overall photon beam distribution**

$$B(\boldsymbol{r},\boldsymbol{\theta}) = \rho_e(\boldsymbol{r},\boldsymbol{\theta}) * B_{fi}(\boldsymbol{r},\boldsymbol{\theta}) = \frac{F_{\text{tot}}}{4\pi^2\Sigma_x\Sigma_y\Sigma_{x'}\Sigma_{y'}}\exp\left(-\frac{x^2}{2\Sigma_x^2}-\frac{y^2}{2\Sigma_y^2}-\frac{\theta_x^2}{2\Sigma_{x'}^2}-\frac{\theta_y^2}{2\Sigma_{y'}^2}\right)$$

$$\Sigma_x = \sqrt{\sigma_x^2+\sigma_r^2},\quad \Sigma_y = \sqrt{\sigma_y^2+\sigma_r^2},\quad \Sigma_{x'} = \sqrt{\sigma_{x'}^2+\sigma_{r'}^2},\quad \Sigma_y = \sqrt{\sigma_{y'}^2+\sigma_{r'}^2}$$

- **Brilliance (max value of the overall photon beam distribution in phase space)**

$$B_0 = B(\boldsymbol{0},\boldsymbol{0}) = \frac{F_{\text{tot}}}{4\pi^2\Sigma_x\Sigma_y\Sigma_{x'}\Sigma_{y'}} \qquad F_{\text{tot}} = \int B(\boldsymbol{r},\boldsymbol{\theta})\mathrm{d}^2\boldsymbol{r}\mathrm{d}^2\boldsymbol{\theta}$$

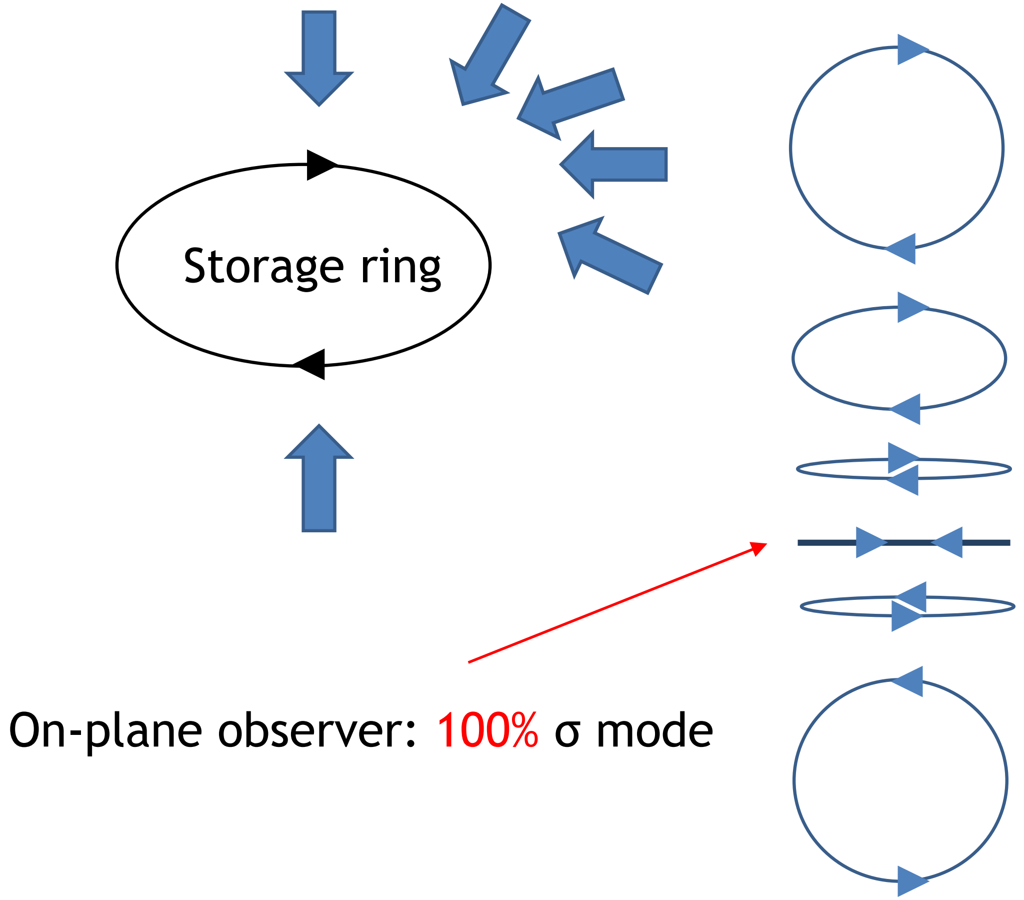
Polarization

σ mode: in bending plane

π mode: perpendicular to bending plane

Power of σ and π modes 7:1

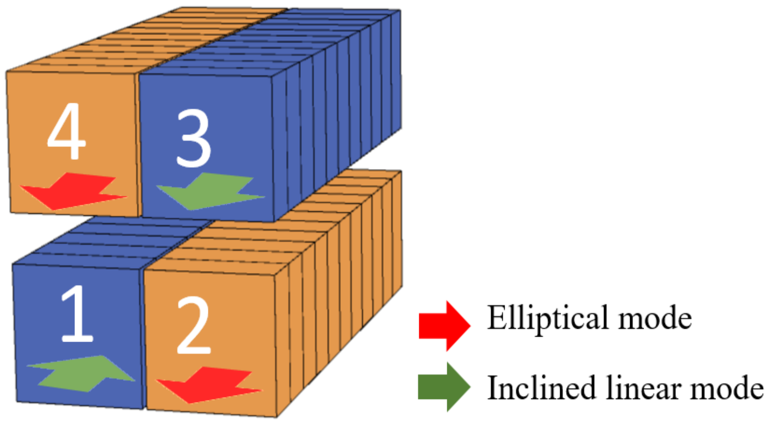
To obtain circular polarized radiation: off-plane



Polarization

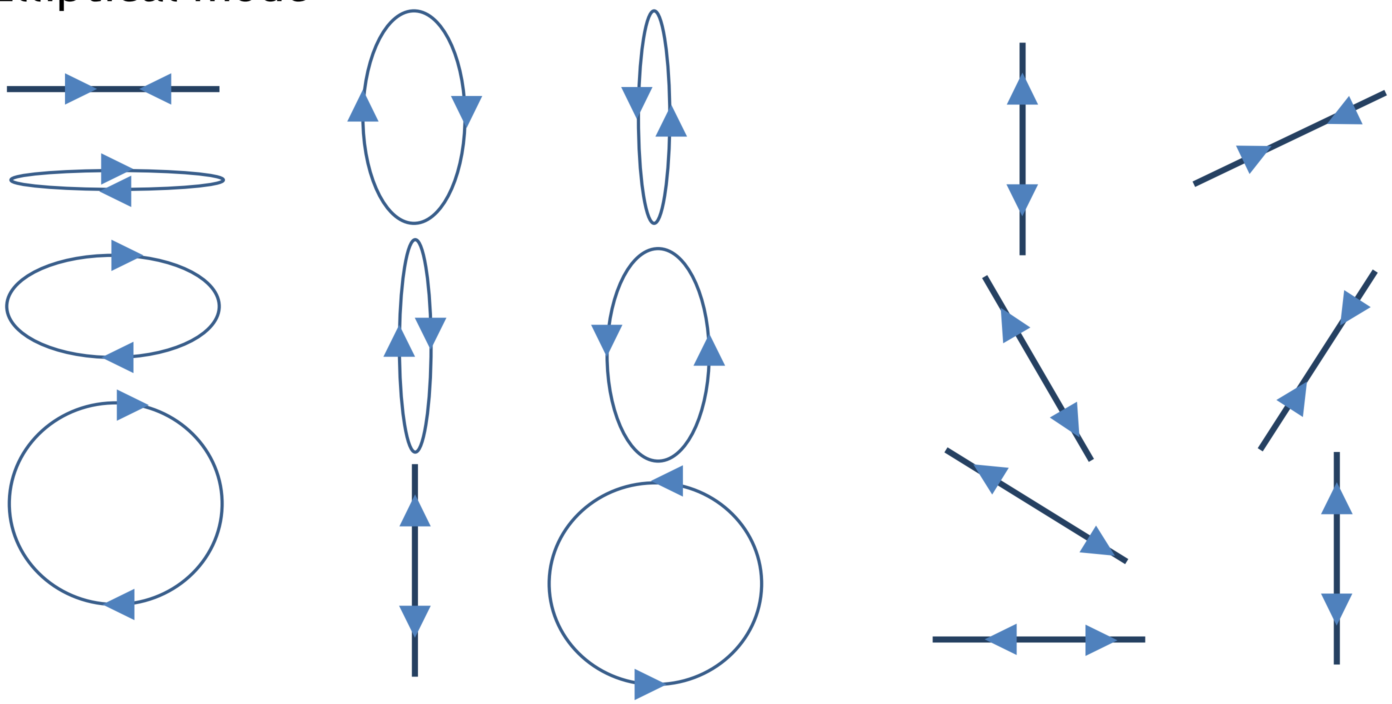
Polarized radiation by undulator:

- Elliptical polarized undulator

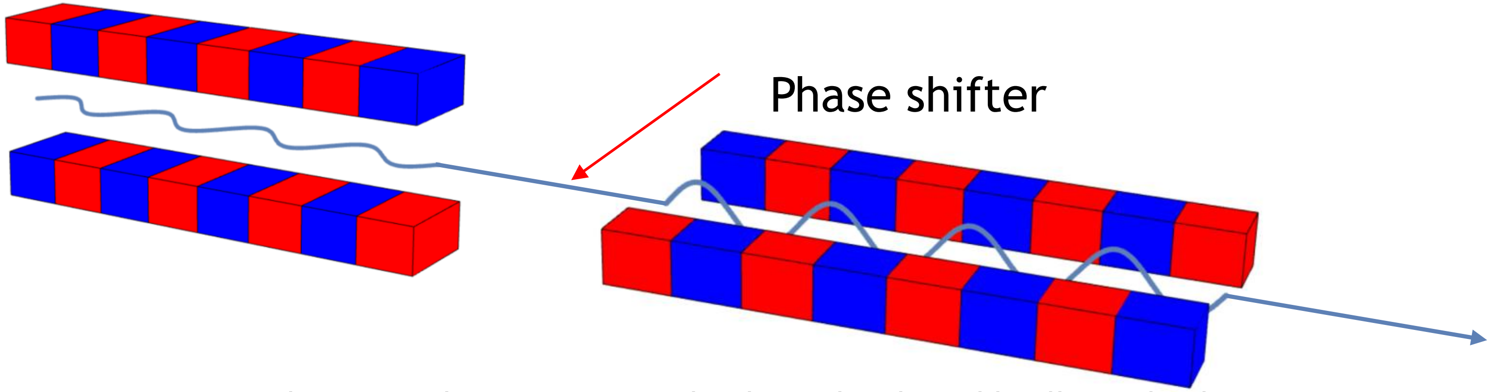


Elliptical mode

Inclined linear mode



- Cross undulator

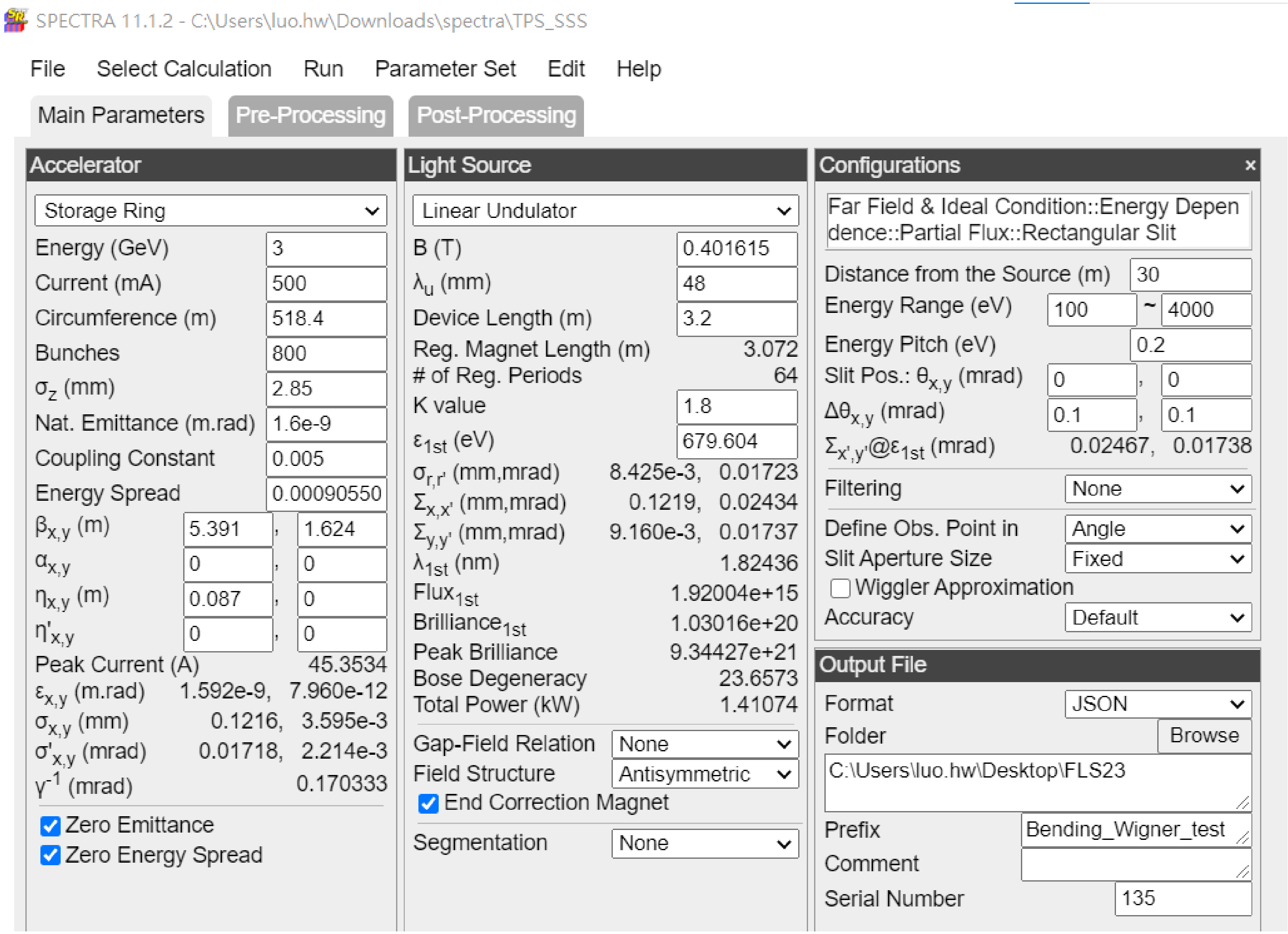


Kwang Je Kim, A synchrotron radiation source with arbitrarily adjustable elliptical polarization
https://www.sciencedirect.com/science/article/abs/pii/0167508784903545

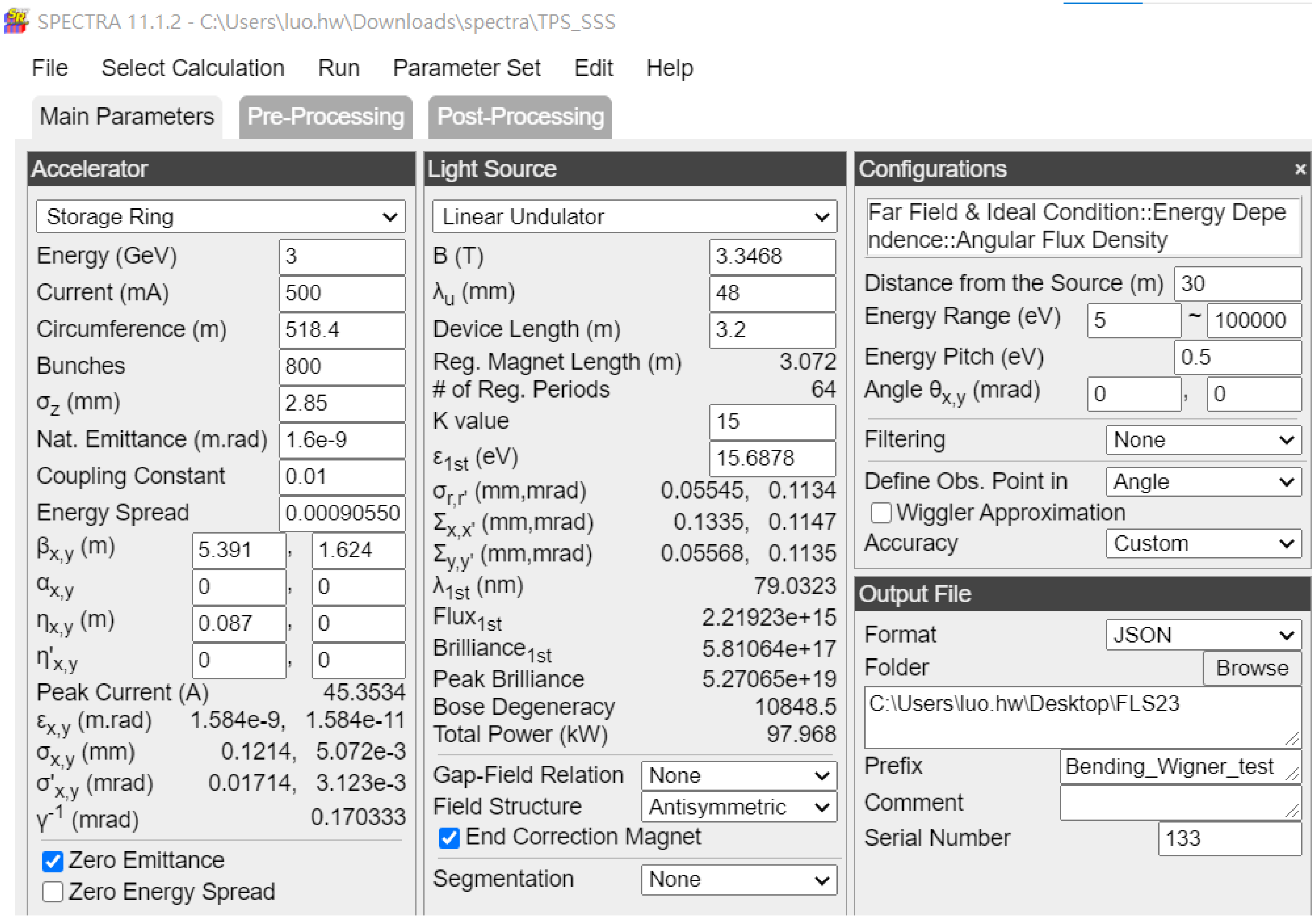
Acknowledgement: Grok3 helps me makes some Manim animations.

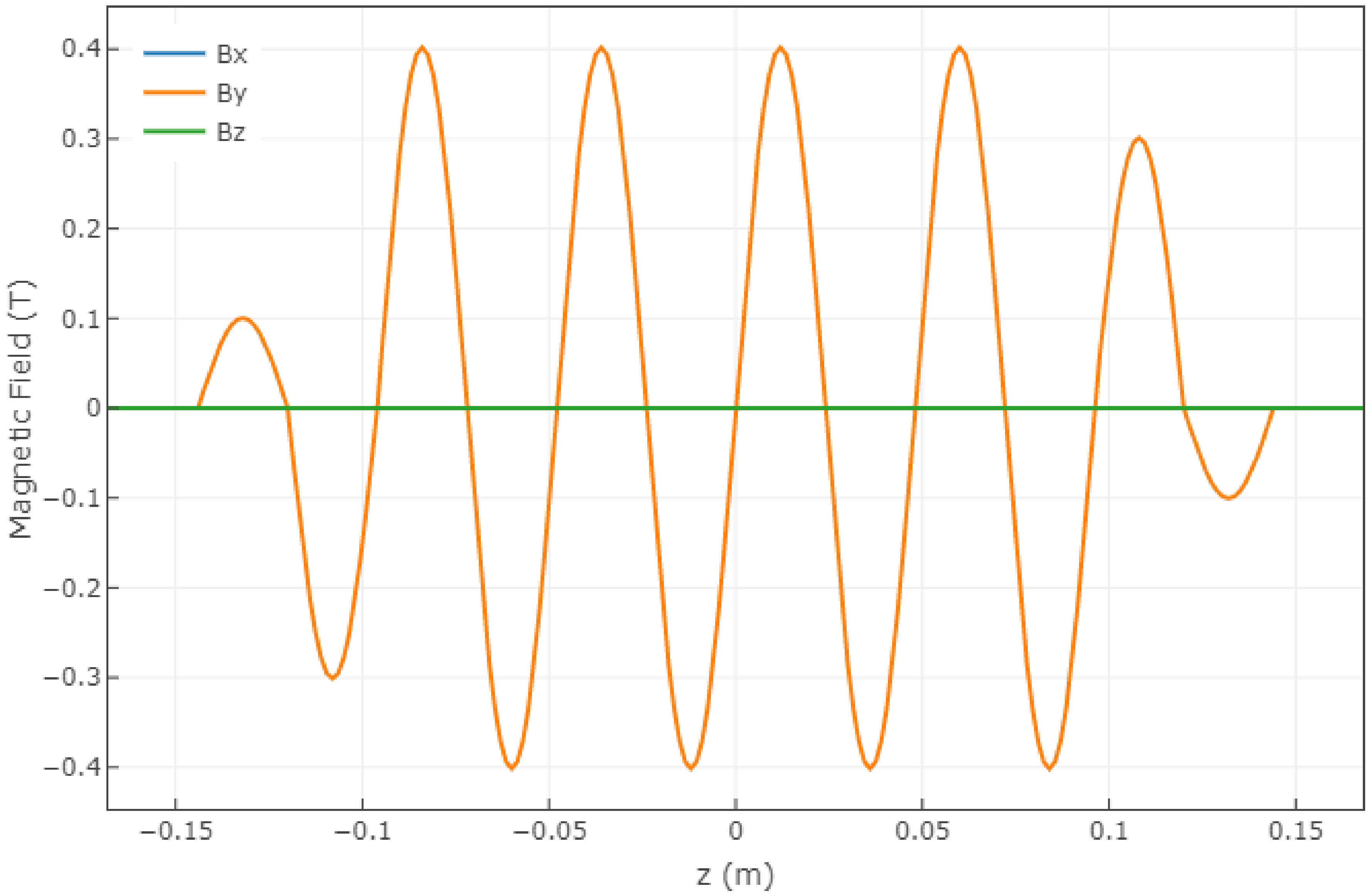
Appendix

# of synchrotron radiation



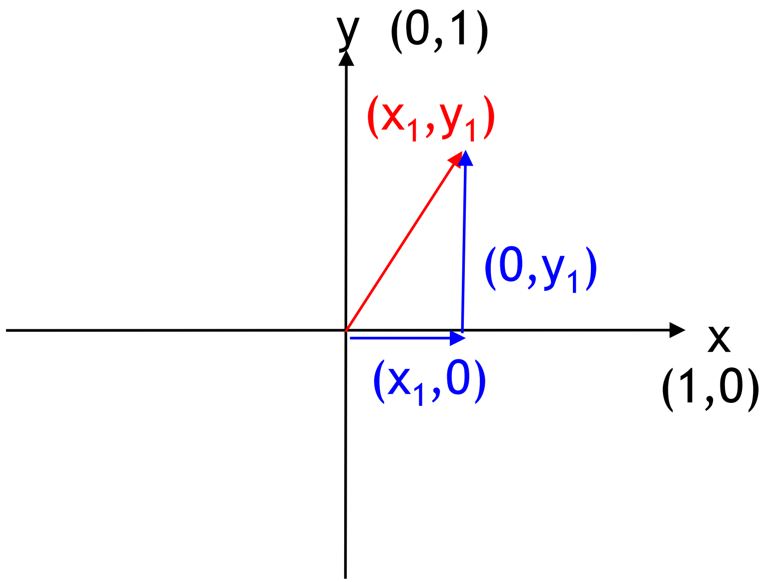
# of synchrotron radiation

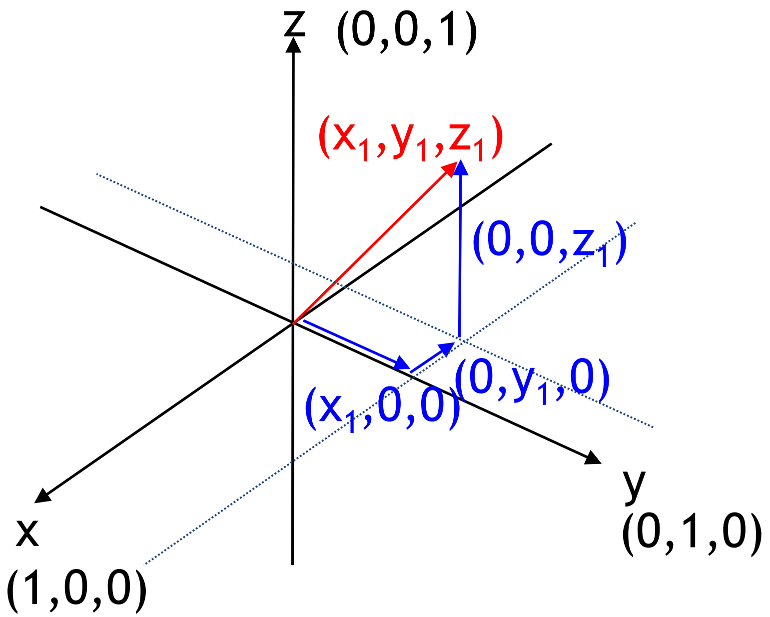




Bx
By
Bz
0.4
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
−0.4
Magnetic Field (T)
−0.15
−0.1
−0.05
0
0.05
0.1
0.15
z (m)

Projection of vector by dot product



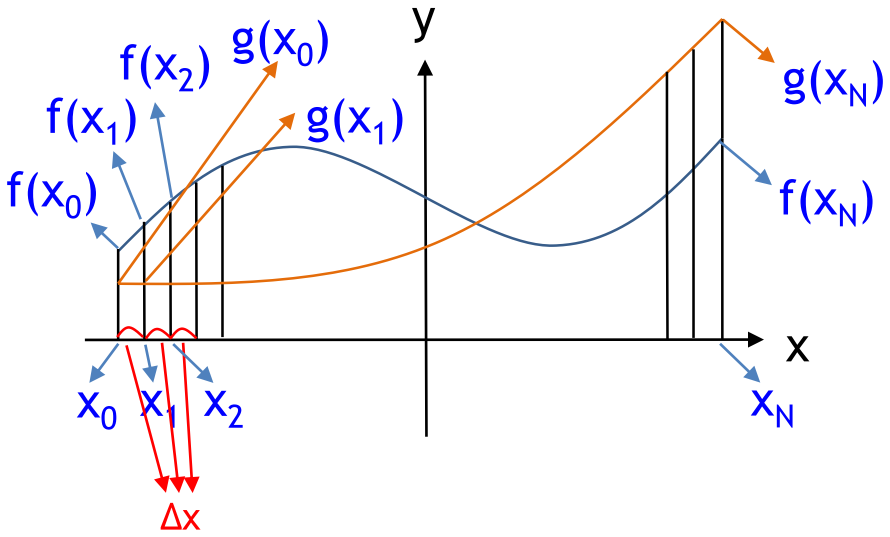


$(x_1,y_1).(1,0)=(x_1,0)$
$(x_1,y_1).\ (0,1)=(0,y_1)$

$(x_1,y_1,z_1).(1,0,0)=(x_1,0,0)$
$(x_1,y_1,z_1).(0,1,0)=(0,y_1,0)$
$(x_1,y_1,z_1).(0,0,1)=(0,0,z_1)$

Introduction to Fourier analysis

Integration of multiplication of functions in terms of vector dot product



$$\int_{x_0}^{x_N} f(x_N)dx \approx [f(x_0)+f(x_1)+f(x_2)+\ldots+f(x_{N-1})]\Delta x$$

$$\int_{x_0}^{x_N} f(x_N)g(x_N)dx \approx [f(x_0)*g(x_0)+f(x_1)*g(x_1)+f(x_2)*g(x_2)+\ldots+f(x_{N-1})*g(x_{N-1})]\Delta x$$

$$f(x_0)*g(x_0)+f(x_1)*g(x_1)+f(x_2)*g(x_2)+\ldots+f(x_{N-1})*g(x_{N-1}) = [f(x_0), f(x_1), f(x_2),\ldots,f(x_{N-1})].[g(x_0), g(x_1), g(x_2),\ldots,g(x_{N-1})]$$

$\mathbf{f}=[f(x_0), f(x_1), f(x_2),\ldots,f(x_{N-1})]$

$\mathbf{g}=[g(x_0), g(x_1), g(x_2),\ldots,g(x_{N-1})]$

$$\Rightarrow \int f(x)g(x)dx \approx (\mathbf{f}\cdot\mathbf{g})\,\Delta x$$

Fourier transformation: projection of a function on sin and cos functions

$$\mathcal{F}(\omega) = \int f(t)e^{-i\omega t}\mathrm{d}t = \int f(t)(\cos\omega t - i\sin\omega t)\mathrm{d}t = \int f(t)\cos\omega t\,\mathrm{d}t - i\int f(t)\sin\omega t\,\mathrm{d}t$$

Real part: cos (even)

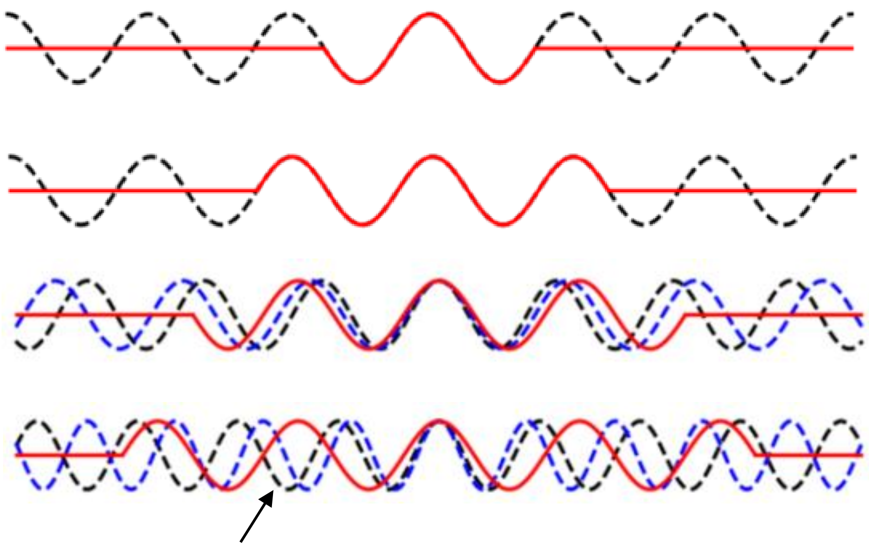
Imaginary part: sin (odd)

Orthogonality of sinusoidal functions (they **perpendicular** to each other)

$$\int \sin\omega t\cos\omega t\,\mathrm{d}t = 0$$

$$\int \sin\omega_1 t\sin\omega_2 t\,\mathrm{d}t = \int \cos\omega_1 t\cos\omega_2 t\,\mathrm{d}t = \delta(\omega_1 - \omega_2)$$

Red: reference period length



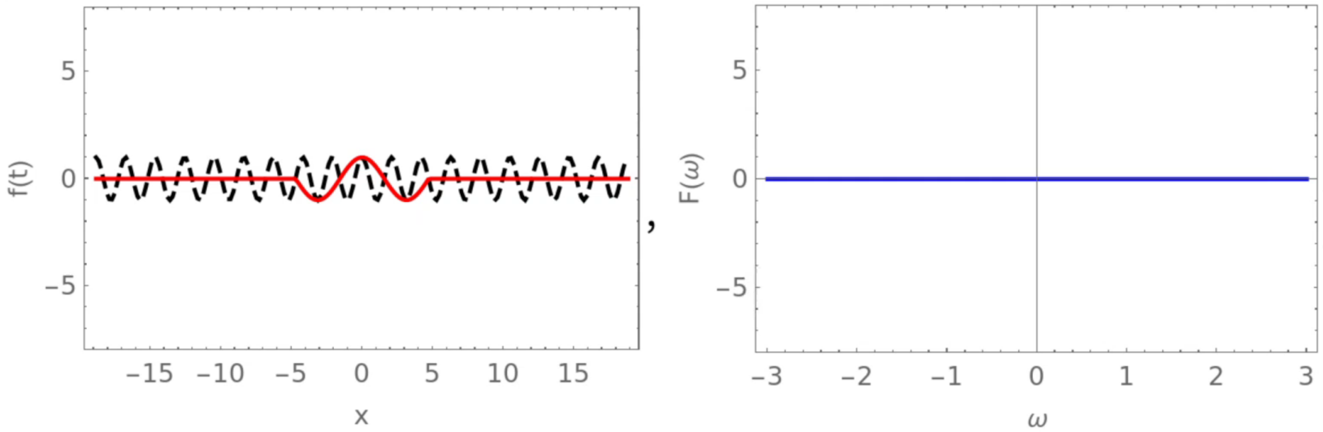
Black: 1.1 period length
Blue: 1.2 period length

Black: 1.4 period length
Blue: 1.6 period length

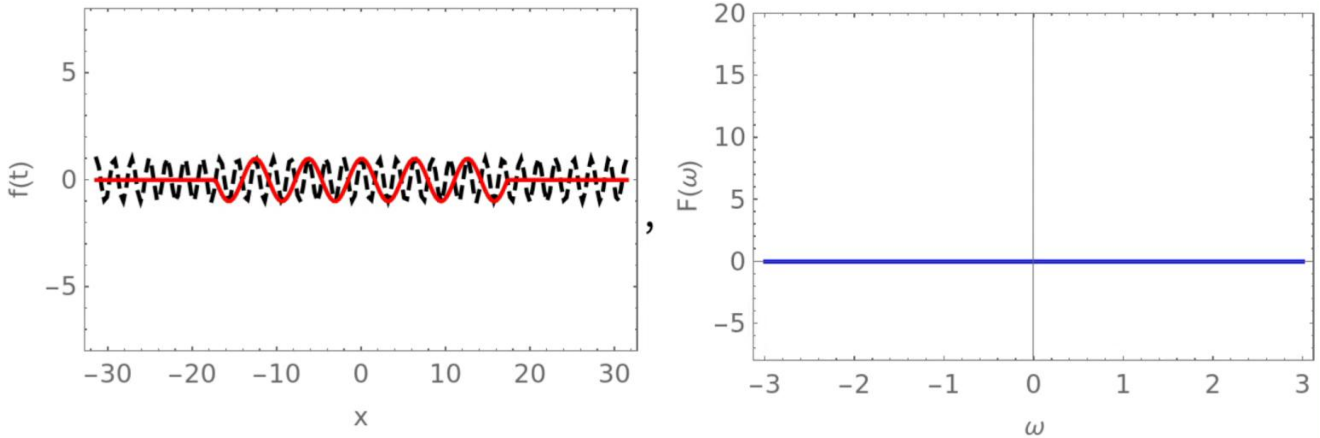
Cancellation due to difference period length

Fourier transformation of finite sinusoidal functions (Only real part for even functions)

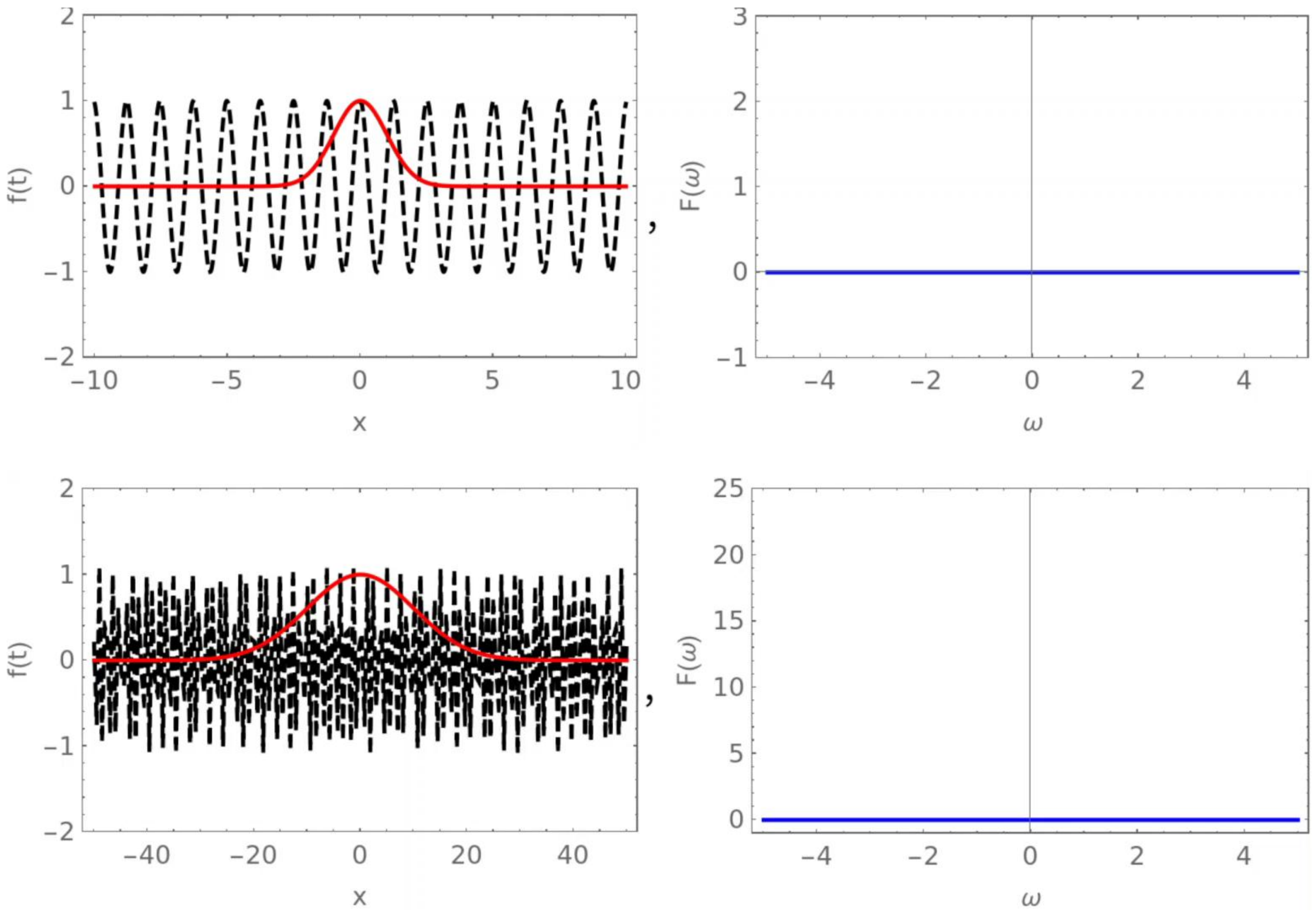
1.5 period



5.5 period



Fourier transformations of Gaussian distributions are still Gaussian distributions

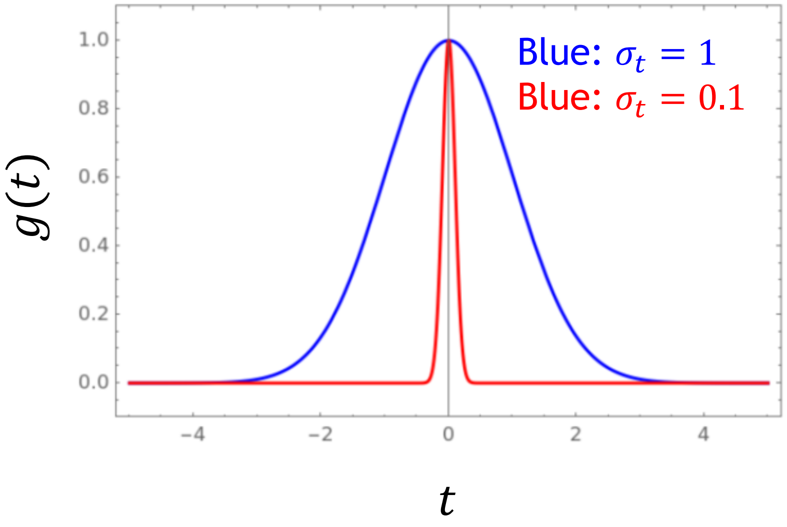


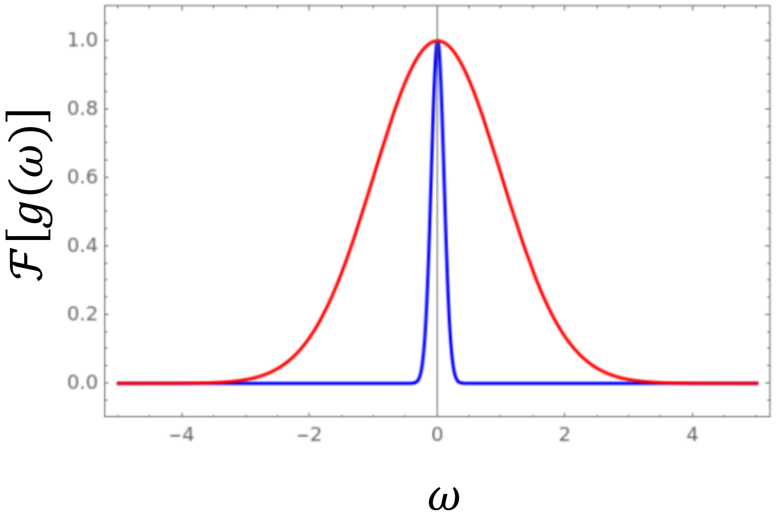
Fourier transformation of a Gaussian distributions is still a Gaussian distributions

$$\mathcal{F}[\omega] = \mathcal{F}\left[e^{-\frac{t^2}{2\sigma_t^2}}\right] = \int e^{-\frac{t^2}{2\sigma_t^2}} e^{-i\omega t} dt = \sqrt{2\pi}\sigma_t e^{-\frac{\sigma_t^2\omega^2}{2}} = \sqrt{2\pi}\sigma_t e^{-\frac{\omega^2}{2\sigma_\omega^2}} \propto e^{-\frac{\omega^2}{2\sigma_\omega^2}} \quad \rightarrow \sigma_\omega = \frac{1}{\sigma_t}$$

The multiplication of the conjugate pair of standard values is a constant $\rightarrow \sigma_t\sigma_\omega = 1$

$$\mathcal{F}[f] = \mathcal{F}\left[e^{-\frac{t^2}{2\sigma_t^2}}\right] = \int e^{-\frac{t^2}{2\sigma_t^2}} e^{-i2\pi f t} dt = \sqrt{2\pi}\sigma_t e^{-\frac{\sigma_t^2(2\pi f)^2}{2}} \propto e^{-\frac{\omega^2}{2\sigma_f^2}} \quad \rightarrow \sigma_t\sigma_f = \frac{1}{2\pi}$$





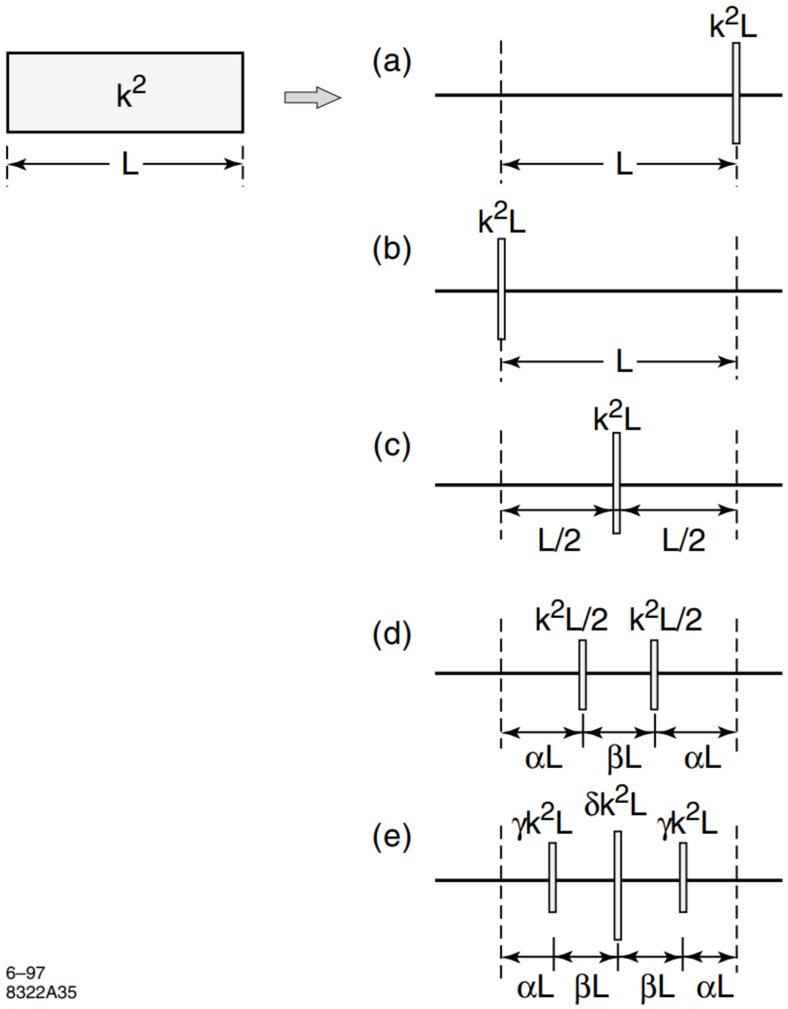


Figure 7.1: Symplectification Models.

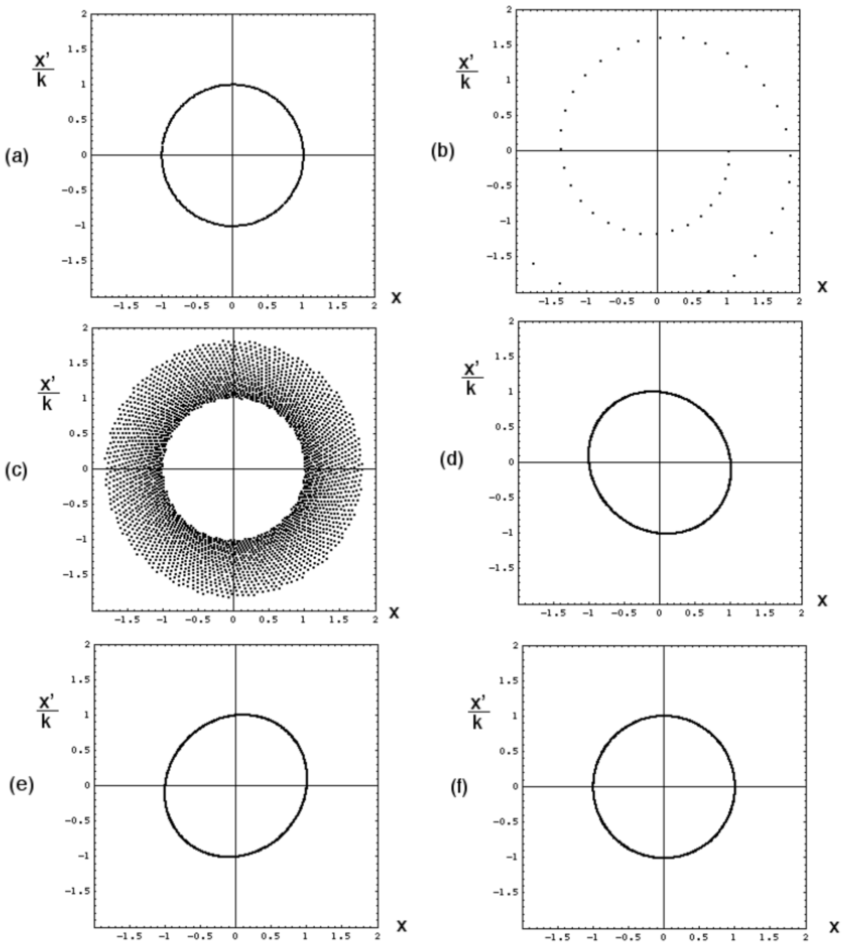


Figure 7.2: Phase space trajectories of a particle traversing a quadrupole as predicted using various tracking algorithms. (a) the exact map (7.4), (b) non-symplectic map (7.6), (c) nonsymplectic map (7.8), (d) symplectic map (7.11), (e) symplectic map (7.12), (f) symplectic thin-lens map (7.13).

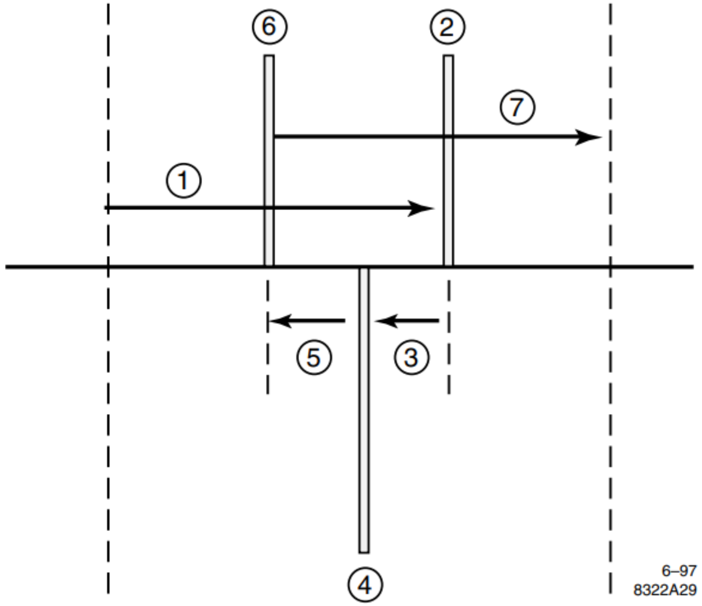


Figure 7.3: Seven steps in the 4-th order symplectic integration.

Table 1: Various techniques of explicit canonical integration for an accelerator element of strength S and length L. The symbol (L) means a drift length L. The symbol (SL) means a kick of integrated strength (SL). Values of $\alpha, \beta, \gamma, \delta$ are given by Eq.(7.19). See also Eq.(7.57) for a 6-th order integration.

Integrator	Model	Error
1st order	$(L)(SL)$	$\mathcal{O}(L^2)$
1st order	$(SL)(L)$	$\mathcal{O}(L^2)$
Ray tracing	$(\frac{L}{n})(\frac{SL}{n})\cdots$ repeat n times	$\mathcal{O}(\frac{L^2}{n})$
2nd order(thin-lens)	$(\frac{L}{2})(SL)(\frac{L}{2})$	$\mathcal{O}(L^3)$
Ray tracing	$(\frac{L}{2n})(\frac{SL}{n})(\frac{L}{2n})\cdots$ repeat n times	$\mathcal{O}(\frac{L^3}{n^2})$
4th order	$(\alpha L)(\gamma SL)(\beta L)(\delta SL)(\beta L)(\gamma SL)(\alpha L)$	$\mathcal{O}(L^5)$
Ray tracing	$(\frac{\alpha L}{n})(\frac{\gamma SL}{n})(\frac{\beta L}{n})(\frac{\delta SL}{n})(\frac{\beta L}{n})(\frac{\gamma SL}{n})(\frac{\alpha L}{n})\cdots$ repeat n times	$\mathcal{O}(\frac{L^5}{n^4})$