

Laser-driven plasma accelerators and applications

黃振崗

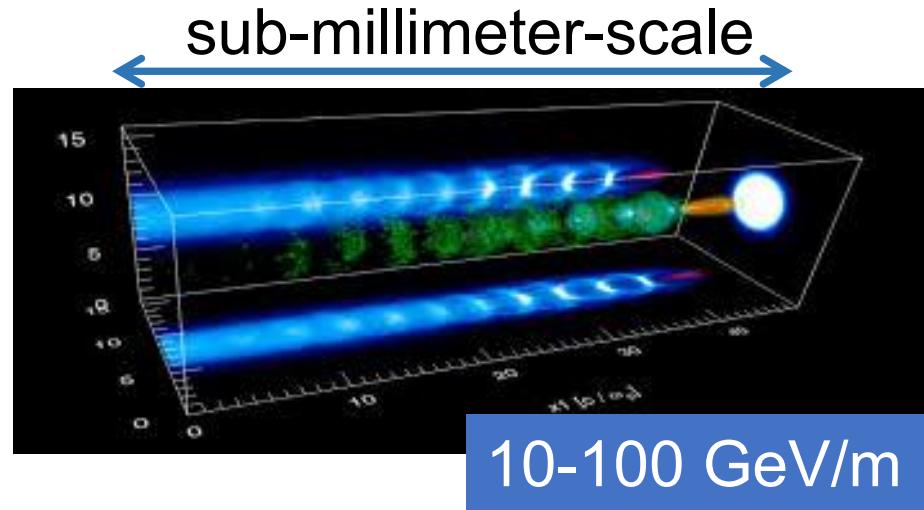
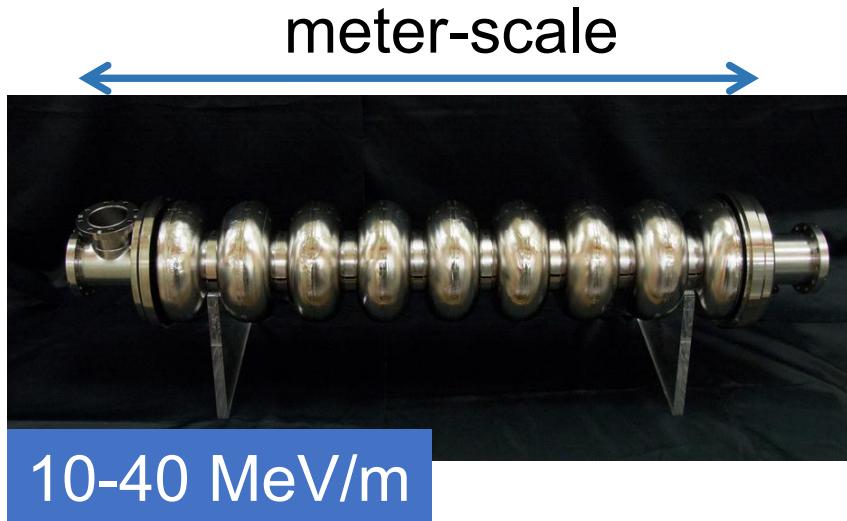
Department of Physics, National Central University



Winter School on Free Electron Lasers 2026



Plasma as an accelerator



- Conventional accelerators are large and expensive
 - Ex. CERN: \$8 billion to construct and \$1 billion pro year to maintain
- Conventional accelerators cannot achieve better than a few 10 MV/m or material breakdown will happen
- Laser plasma acceleration enables development of “compact” accelerators

Plasma as an accelerator



Length of plasma:
4 centimeters

Astra Gemini Laser
2 GeV electron beam ~ \$4 M

Diamond light source
3 GeV electron beam ~ \$400 M

**Storage ring
circumference:
561.6 meters**

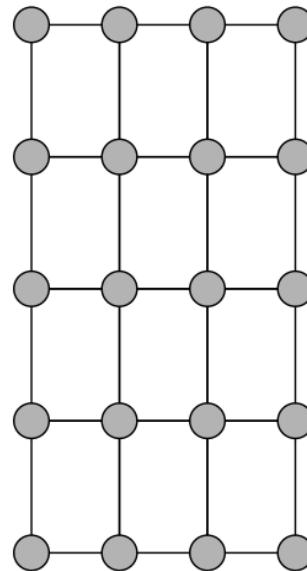
Contents

- Introduction of laser and plasma physics
- Laser wakefield accelerator (LWFA)
- Applications of laser-driven plasma accelerators

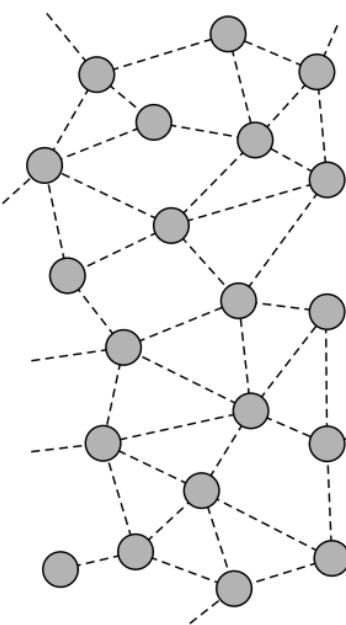
What is a plasma

■ The “fourth state of matter”

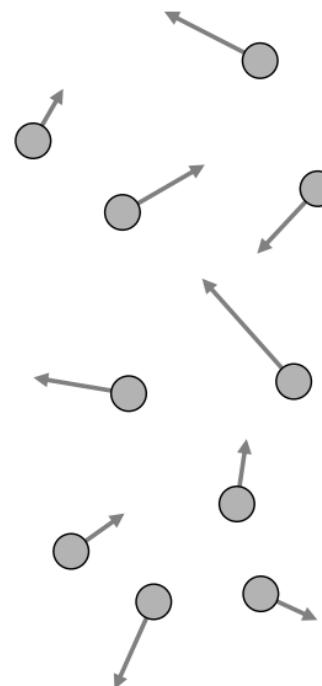
solid



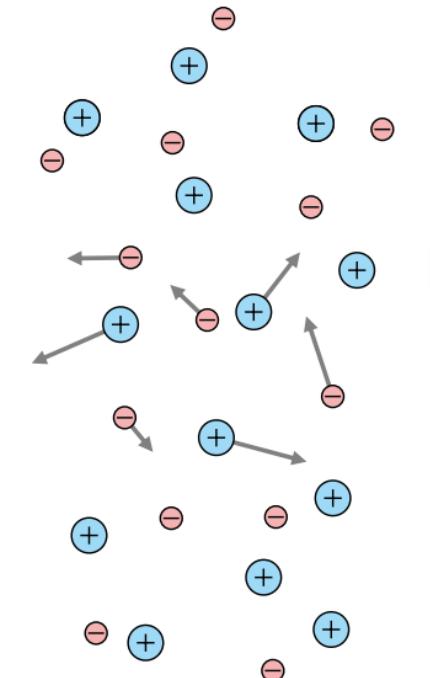
liquid



gas



plasma

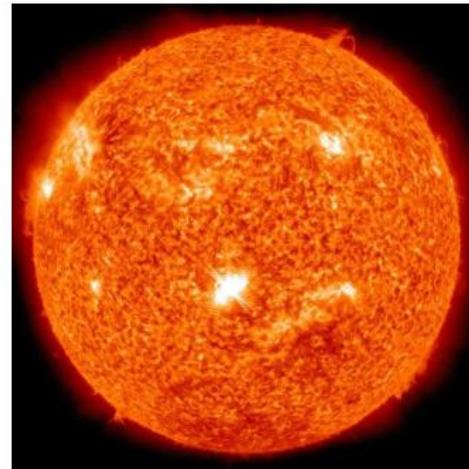


● atom/molecule

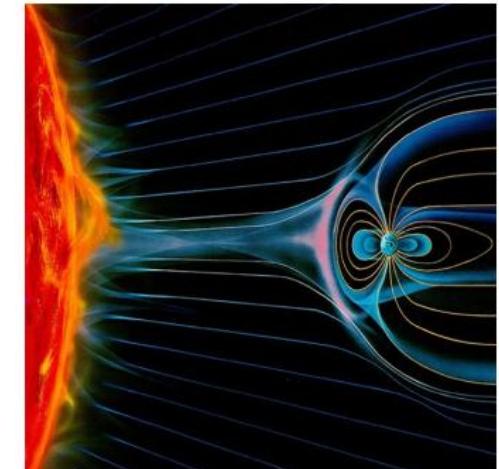
⊕ ion ⊖ electron

Plasma is ubiquitous in nature

- The “fourth state of matter”
- More than 99% of the visible universe is plasma



star



solar wind



ionosphere



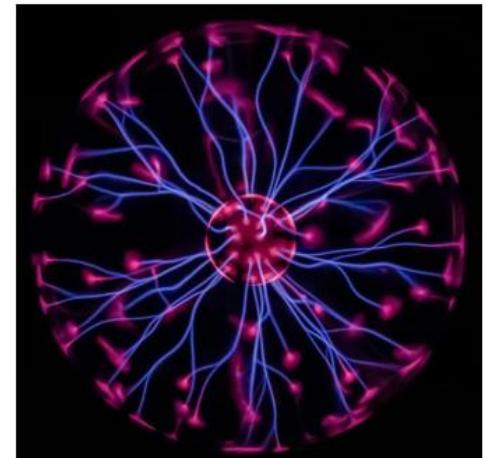
lightening

Plasma on earth

- The “fourth state of matter”
- More than 99% of the visible universe is plasma
- If you heat a gas enough, it becomes a plasma—a glowing, electrified 'soup' of particles.



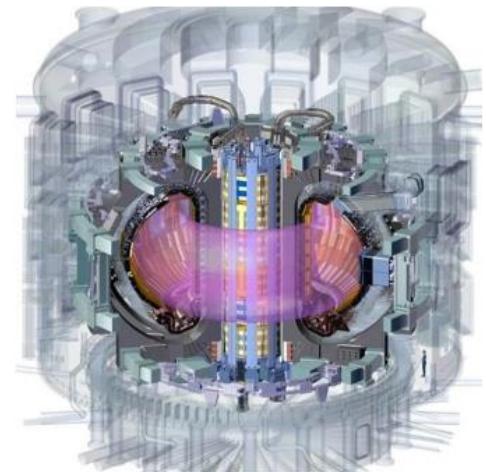
fluorescent tube



plasma ball



arc

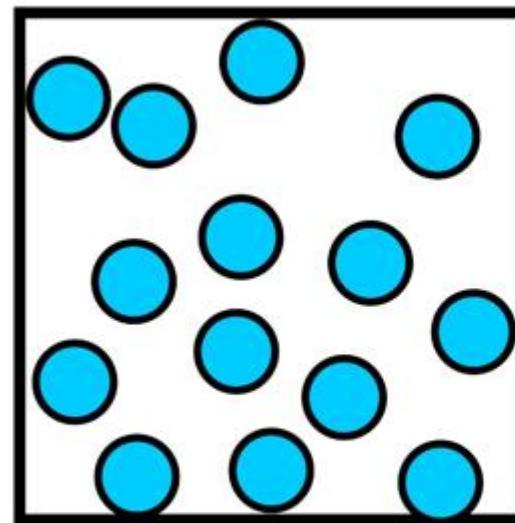


tokamak plasma

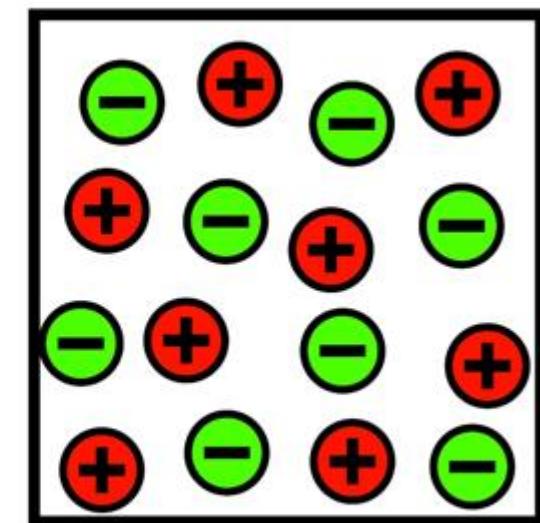
Plasma on earth

- The “fourth state of matter”
- More than 99% of the visible universe is plasma
- If you heat a gas enough, it becomes a plasma—a glowing, electrified 'soup' of particles.
- Definition:

A plasma is a **quasi-neutral** gas of charged and neutral particles which exhibits **collective** behavior.



Gas



Plasma



Adding **heat**

Quasi-neutrality of plasma

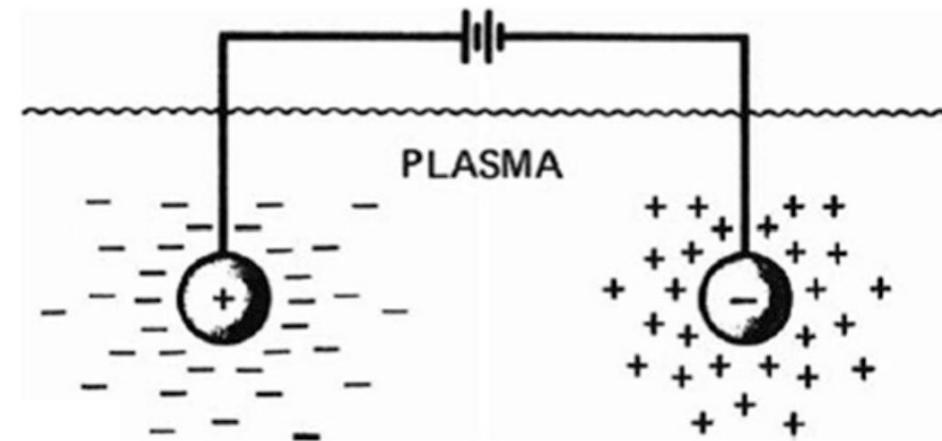
■ Debye shielding and Debye length

The plasma has ability to "screen" out electric fields. Charges rearrange themselves to cancel out external electrical field.

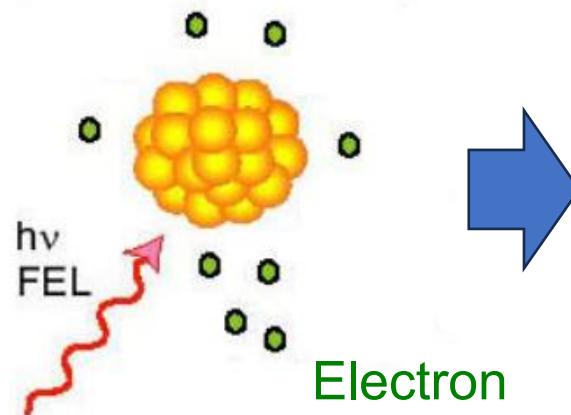
$$\lambda_D = \sqrt{\frac{\epsilon_0 k_B T_e}{n_e e^2}} \propto \sqrt{\frac{T_e}{n_e}}$$

scale length $\gg \lambda_D$:
quasi-neutral fluid

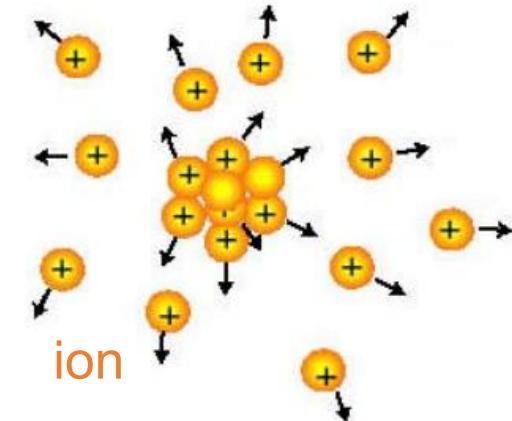
scale length $\ll \lambda_D$:
collection of charged particles



In extreme conditions,
quasi-neutrality can fail



Coulomb explosion



Collective behaviors of plasma

■ Fundamental criterion of plasma

$$N_D = n_e \lambda_D^3 \gg 1, \quad L \gg \lambda_D$$

1. Many particles within screening distance
2. Individual collisions less important than collective field

■ Oscillation at plasma frequency

$$\omega_p = \left(\frac{n_e e^2}{\epsilon_0 m_e} \right)^{1/2} \propto \sqrt{n_e}$$

where

n_e : electron density

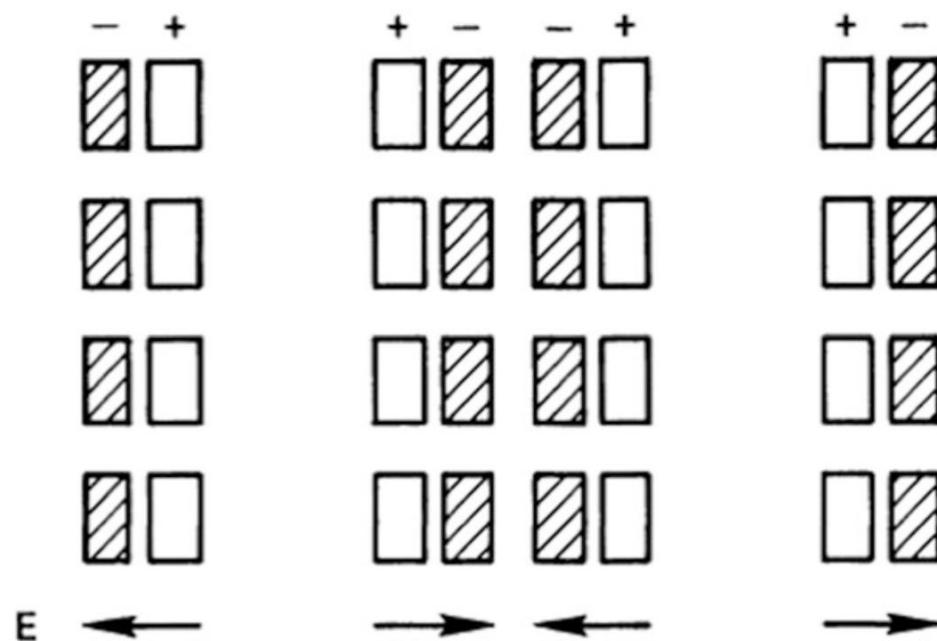
e : electron charge

m_e : mass of electron density

ϵ_0 : Vacuum permittivity

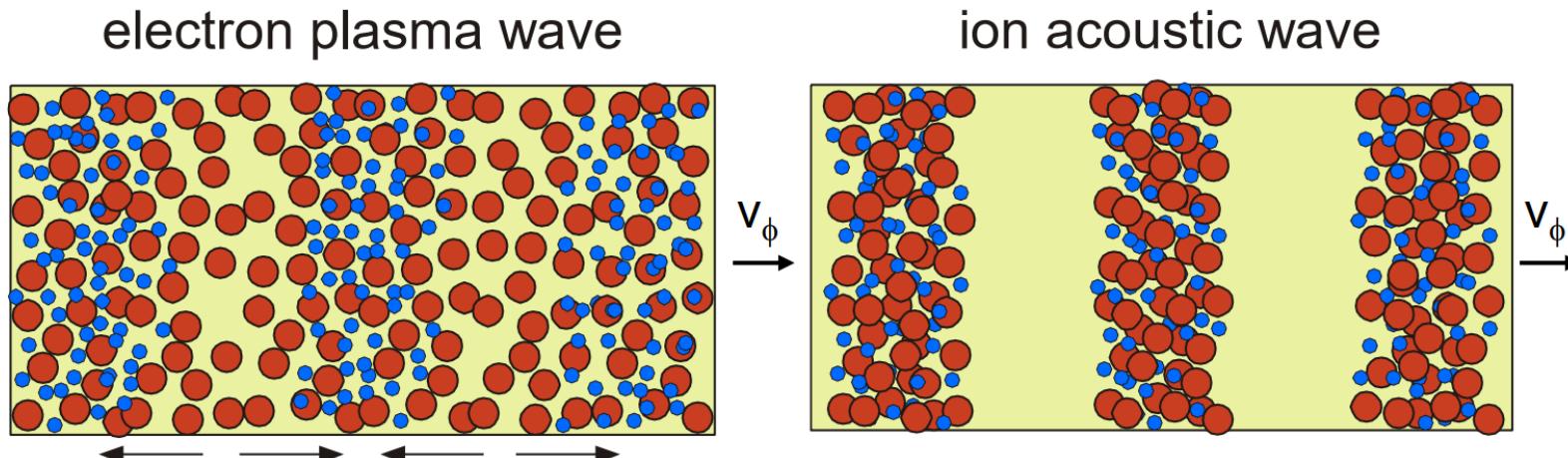
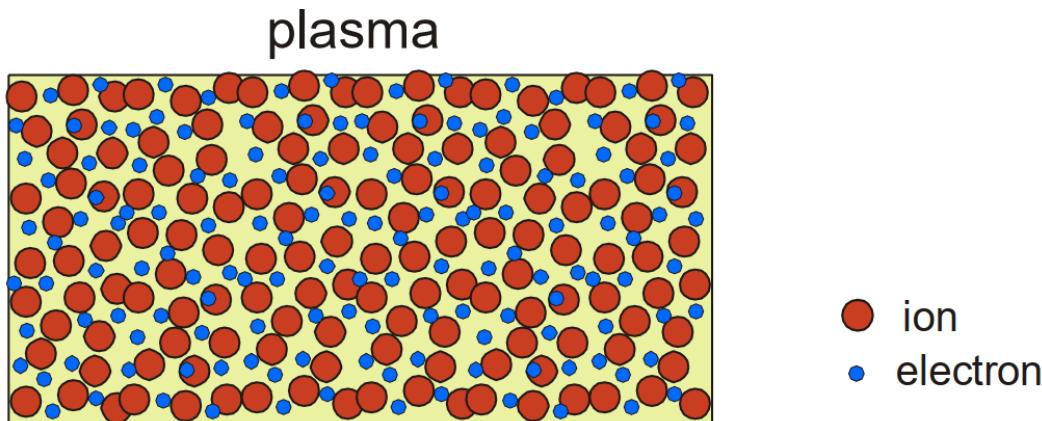
Plasma wavelength

$$\lambda_p = \frac{2\pi v_g}{\omega_p} \cong \frac{2\pi c}{\omega_p}$$



Treat plasma as a conducting fluid

Plasma waves



time scale: picosecond,
ion fixed

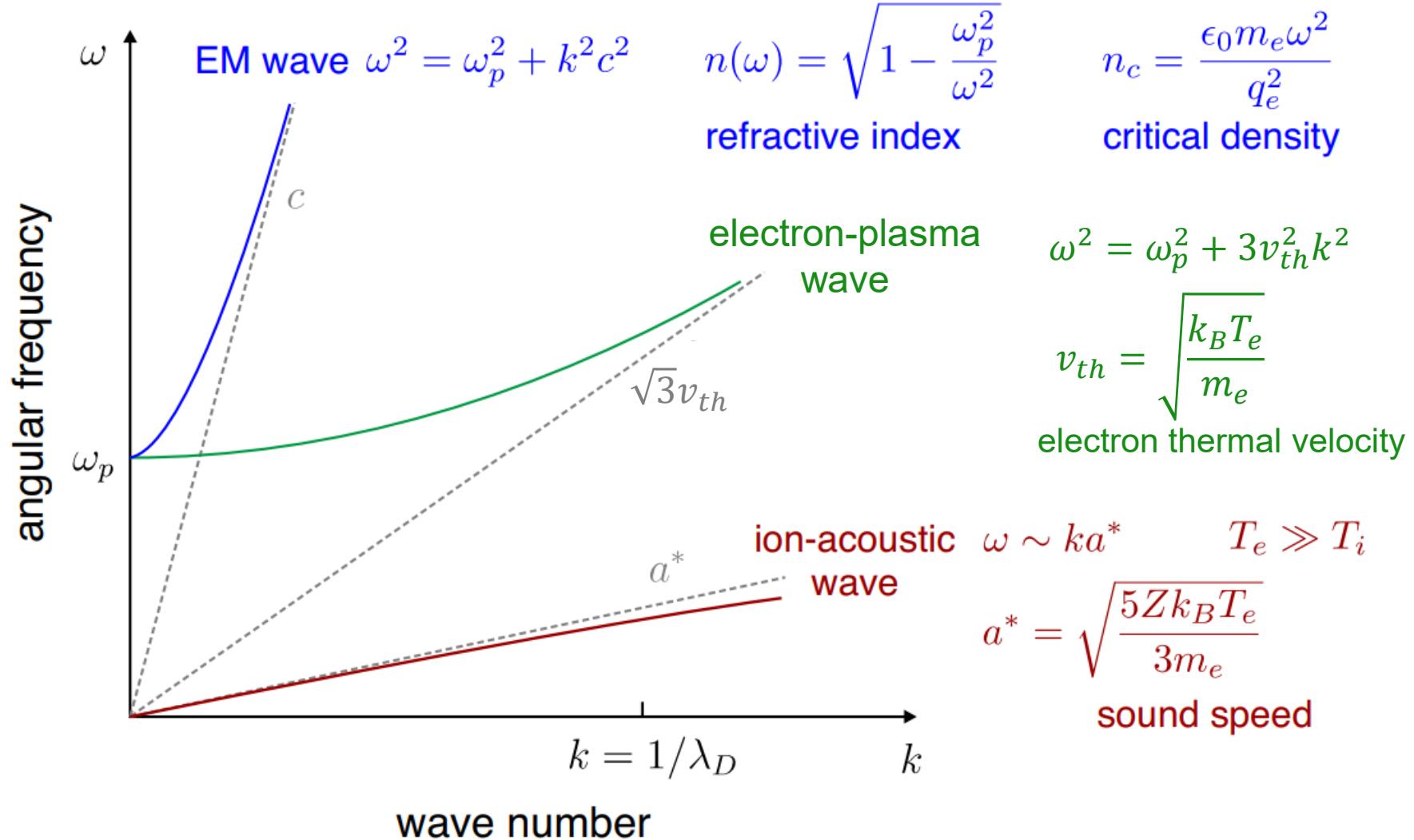
time scale: nanosecond

Waves in plasma

- Dispersion relations for 3 fundamental waves in uniform, unmagnetized plasmas:

Transverse:
electromagnetic wave

Longitudinal:
electron-plasma wave
ion-acoustic wave



Light waves in plasma

■ The dispersion relation of EM wave in a plasma

$$\omega^2 = \omega_p^2 + k^2 c^2$$

(with relativistic correction)

■ Index of refraction of plasma

$$n(\omega) = \sqrt{1 - \frac{\omega_p^2}{\omega^2}}$$

$$n = \sqrt{1 - \frac{\omega_p^2}{\gamma \omega^2}}$$

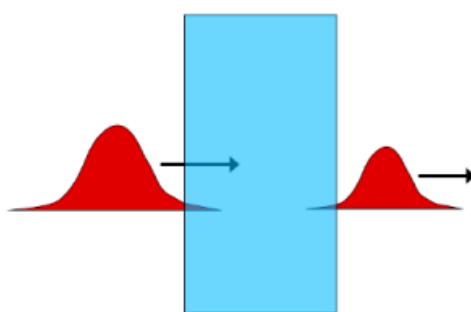
γ : Lorentz factor

EM waves cannot propagate in a plasma when $\omega < \omega_p$.

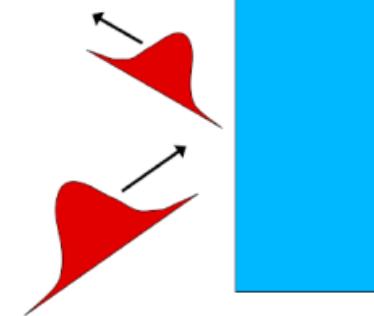
■ Critical density

$$n_c = \frac{\epsilon_0 m_e}{e^2} \omega^2$$

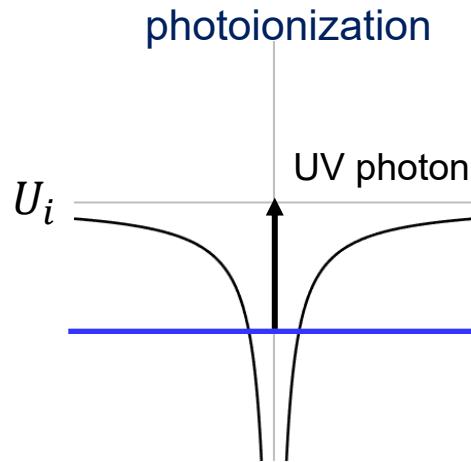
underdense



overdense



Laser-induced plasma



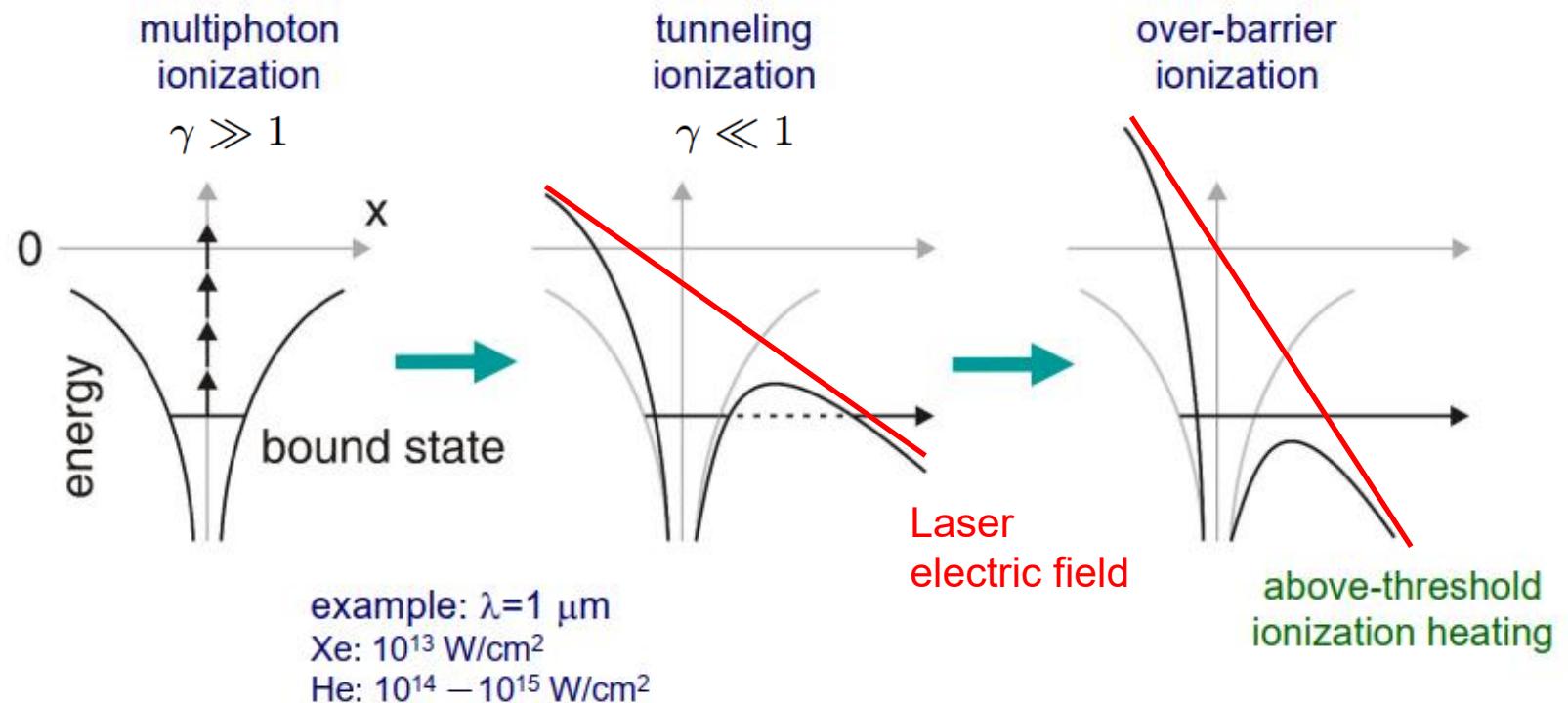
Keldysh parameter

$$\gamma = (U_i/2U_p)^{1/2}$$

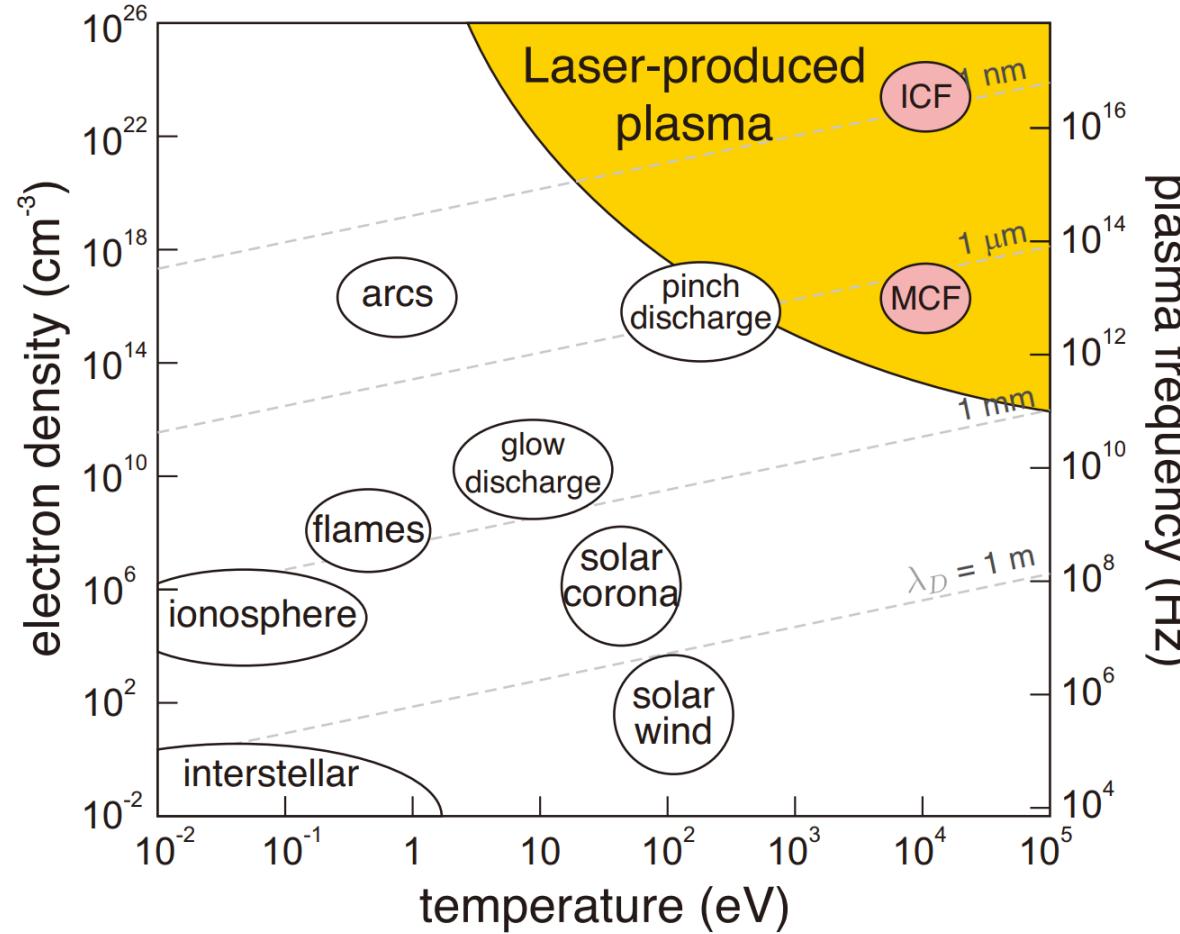
U_i : ionization potential

U_p : ponderomotive potential

Optical-field ionization



Laser-induced plasma



ICF: inertial confinement fusion (laser fusion) (LLNL, USA)
MCF: magnetic confinement fusion (ITER, France)

Fundamental of laser-plasma interaction

■ Normalized vector potential

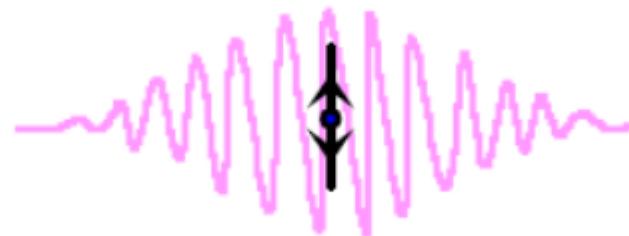
$$a_0 = \frac{eA_0}{m_e c} = \frac{eE_0}{\omega m_e c} \sim \sqrt{I}\lambda$$

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} \quad \mathbf{B} = \nabla \times \mathbf{A}$$

where
 A_0 : peak vector potential
 E_0 : peak electric field
 c : vacuum light speed

■ Interaction of electron with intense laser field

$a_0 \ll 1$ (non-relativistic)



$a_0 > 1$ (relativistic)

when $v \rightarrow c$
 $m_0 \rightarrow \gamma m_0$
 $v \times B \rightarrow E$



figure-8 motion (for linear polarization)

$$m_e \frac{\partial \mathbf{v}}{\partial t} \cong e \mathbf{E}$$

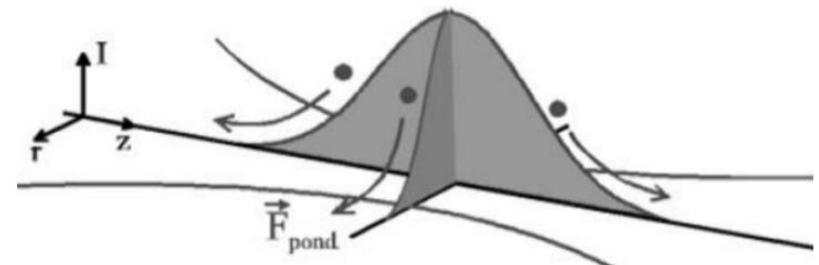
$$m_e \frac{\partial \mathbf{v}}{\partial t} = e \left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B} \right)$$

Ponderomotive force

- Intense light can push charged particles toward regions of lower intensity (optical pressure)

$$\mathbf{F}_{\text{pond}} = -\frac{q^2}{2m\omega^2} \nabla \langle E^2 \rangle = -\frac{1}{2} mc^2 \nabla \langle a^2 \rangle$$

It's a nonlinear, time-averaged force exerted by inhomogeneous oscillating EM field on charged particles.



- Mass difference between an electron and an ion
 - Ions are over 1800 heavier than electrons
 - An intense laser's ponderomotive force instantly expels electrons
 - The massive ions barely move, creating a stationary uniform background
- Ponderomotive force is the main driving force of laser plasma accelerators

The birth of laser plasma accelerator

VOLUME 43, NUMBER 4

PHYSICAL REVIEW LETTERS

23 JULY 1979

Laser Electron Accelerator

T. Tajima and J. M. Dawson

Department of Physics, University of California, Los Angeles, California 90024

(Received 9 March 1979)

An intense electromagnetic pulse can create a weak of plasma oscillations through the action of the nonlinear ponderomotive force. Electrons trapped in the wake can be accelerated to high energy. Existing glass lasers of power density 10^{18} W/cm^2 shone on plasmas of densities 10^{18} cm^{-3} can yield gigaelectronvolts of electron energy per centimeter of acceleration distance. This acceleration mechanism is demonstrated through computer simulation. Applications to accelerators and pulsers are examined.

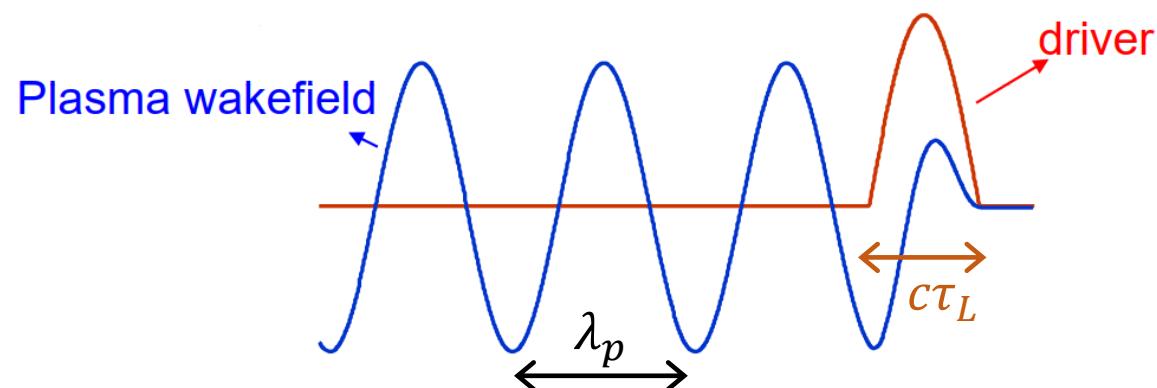
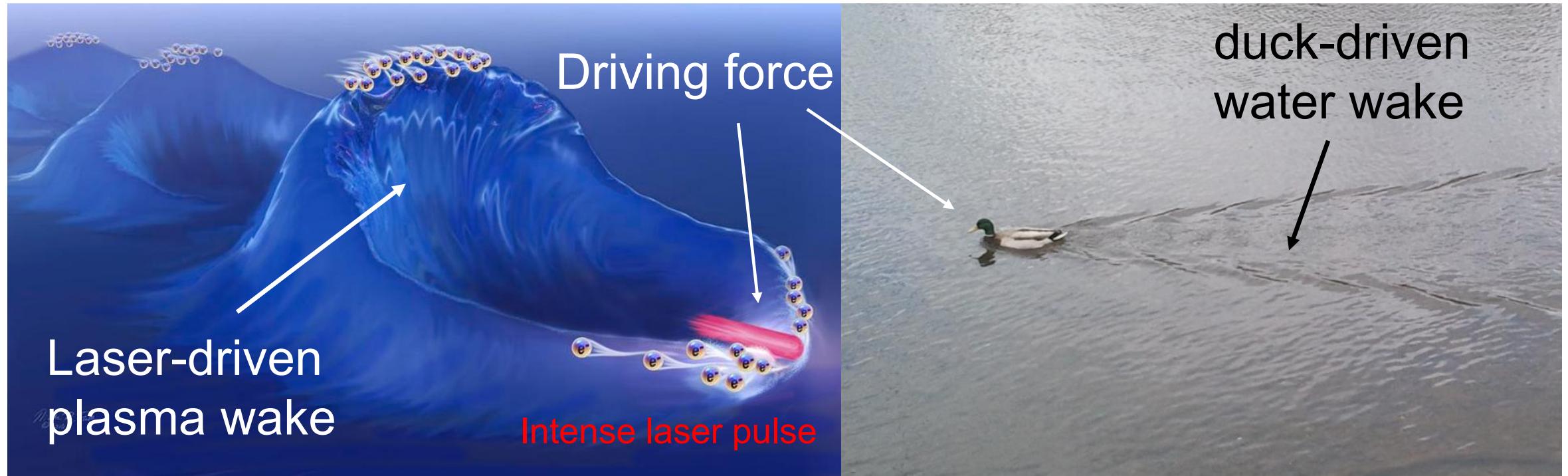


Toshiki Tajima



John Dawson

Plasma wake



Laser-wakefield accelerator (LWFA)

VOLUME 43, NUMBER 4

PHYSICAL REVIEW LETTERS

23 JULY 1979



Toshiki Tajima



John Dawson

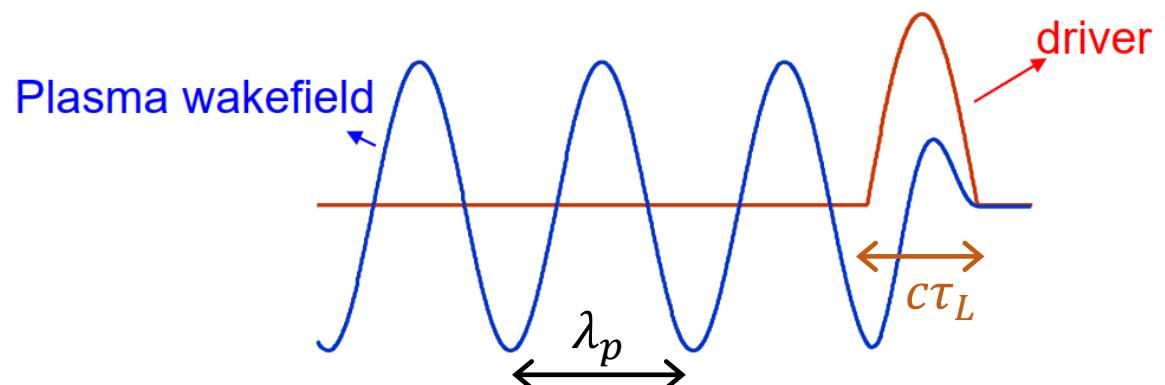
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The problem: In 1979, lasers were either high-energy (but long-pulsed) or short-pulsed (but very weak).

Similar concept using electron beams

VOLUME 54, NUMBER 7

PHYSICAL REVIEW LETTERS

18 FEBRUARY 1985

Acceleration of Electrons by the Interaction of a Bunched Electron Beam with a Plasma

Pisin Chen^(a)

Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

and

J. M. Dawson, Robert W. Huff, and T. Katsouleas

Department of Physics, University of California, Los Angeles, California 90024

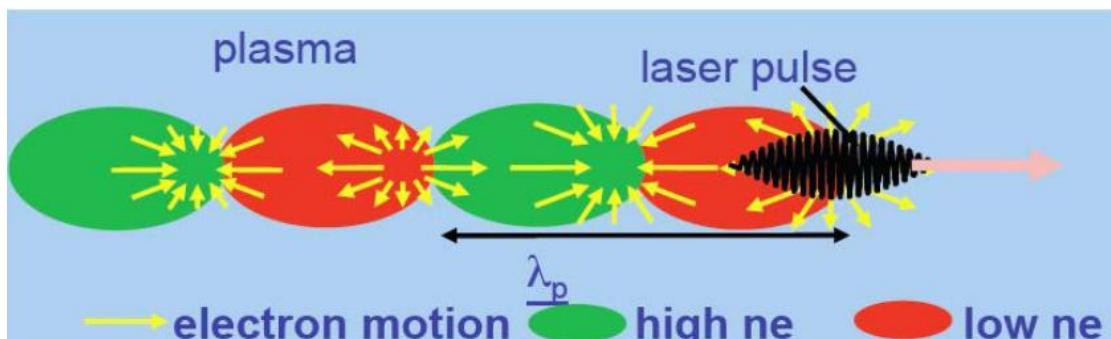
(Received 20 December 1984)

A new scheme for accelerating electrons, employing a bunched relativistic electron beam in a cold plasma, is analyzed. We show that energy gradients can exceed 1 GeV/m and that the driven electrons can be accelerated from $\gamma_0 mc^2$ to $3\gamma_0 mc^2$ before the driving beam slows down enough to degrade the plasma wave. If the driving electrons are removed before they cause the collapse of the plasma wave, energies up to $4\gamma_0 mc^2$ are possible. A noncollinear injection scheme is suggested in order that the driving electrons can be removed.

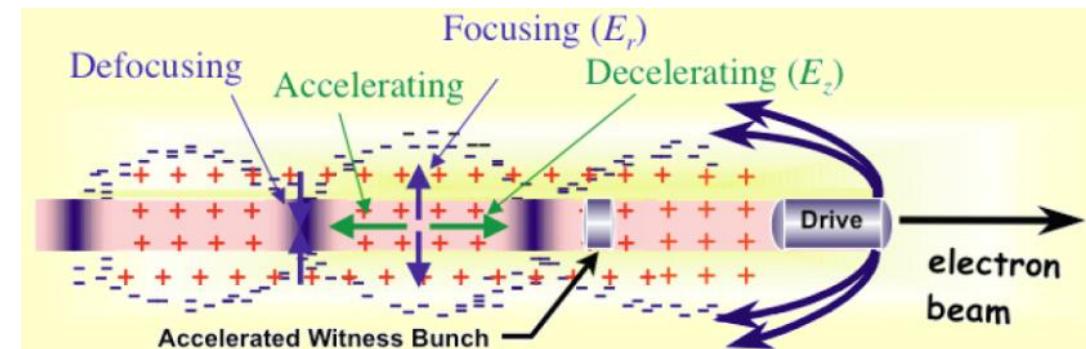


陳丕燊

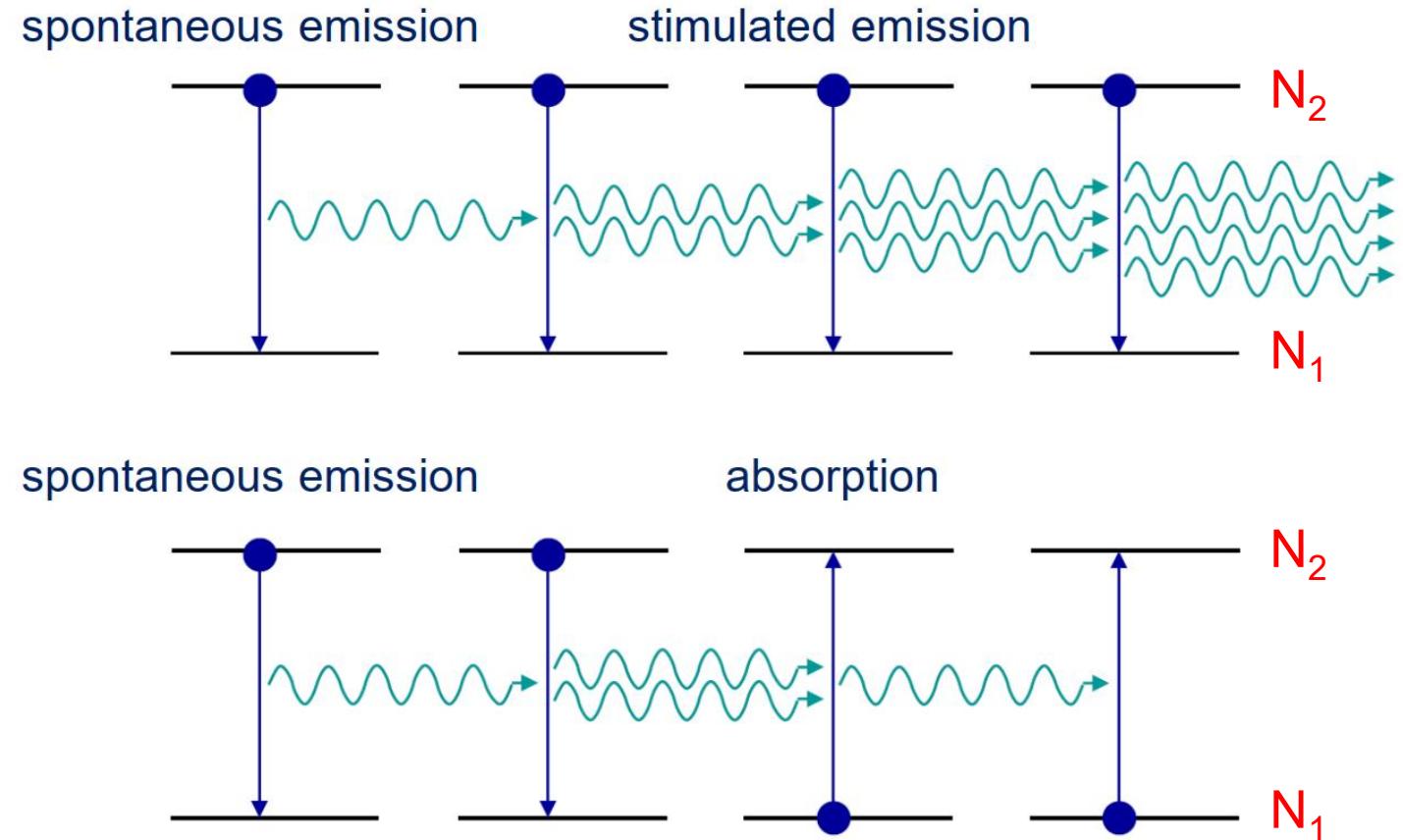
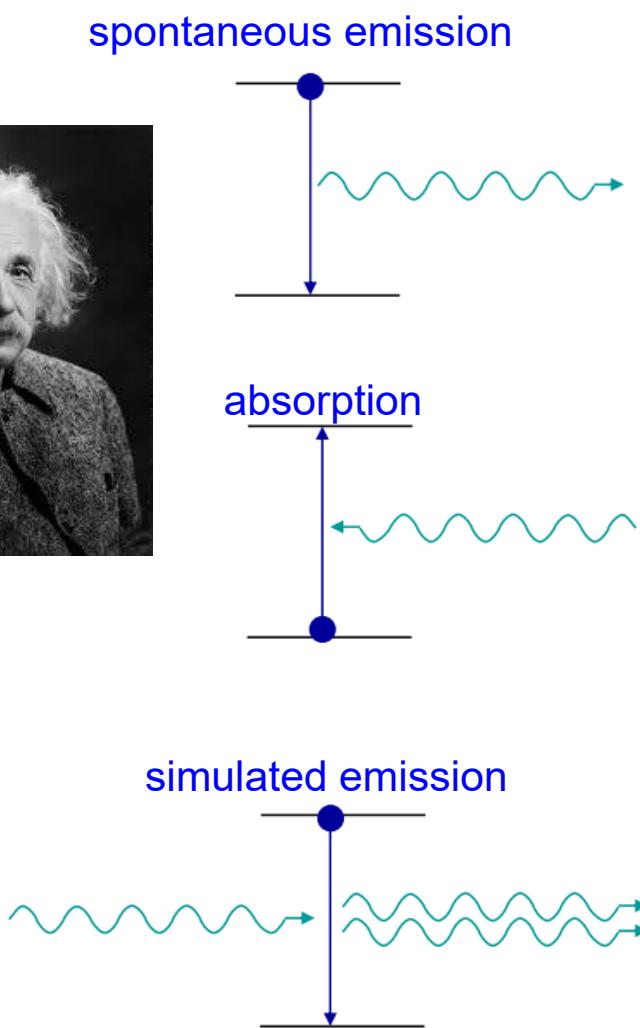
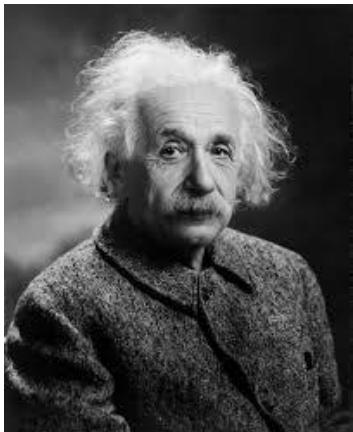
Laser-driven acceleration



Beam-driven acceleration



Optical amplification

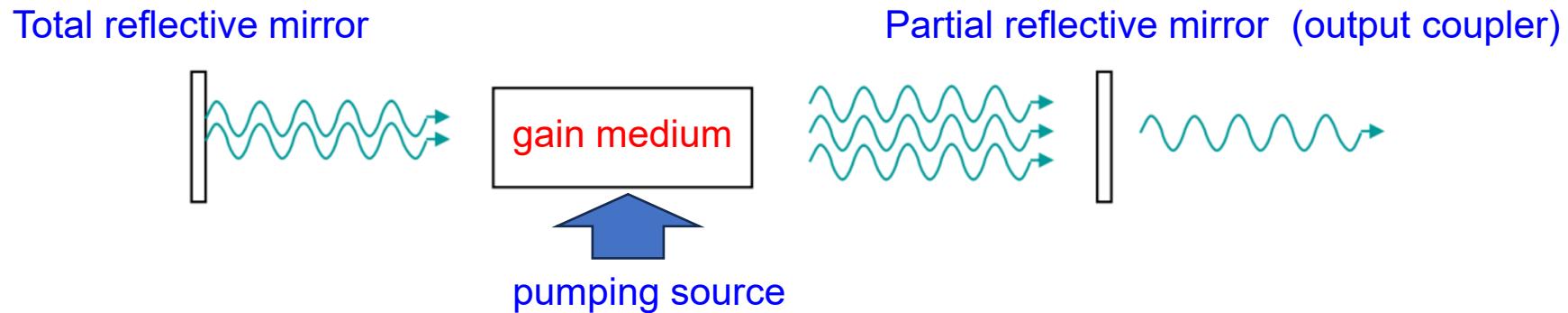


- Amplification conditions:
 $N_2 > N_1$ (population inversion)

LASER

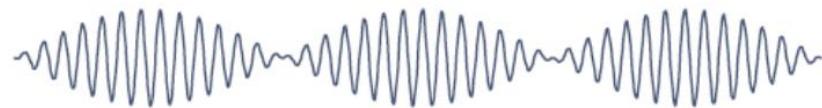
(Light Amplification by Stimulated Emission of Radiation)

- A laser is a device that emits light through a process of optical amplification based on the stimulated emission.
- Three principal parts of a laser:
 - (1) gain medium
 - (2) pumping source
 - (3) optical cavity

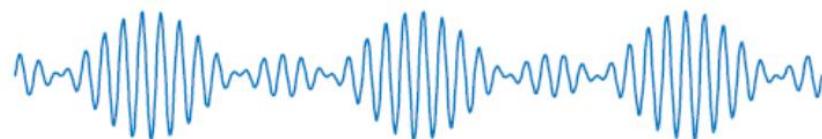


- Functions of the optical oscillator
 1. confine the laser propagation
 2. limit the laser frequency
 3. increase the effective length of the amplifier

How to generate an ultrafast laser pulse



2 frequency components in phase



3 frequency components in phase



4 frequency components in phase



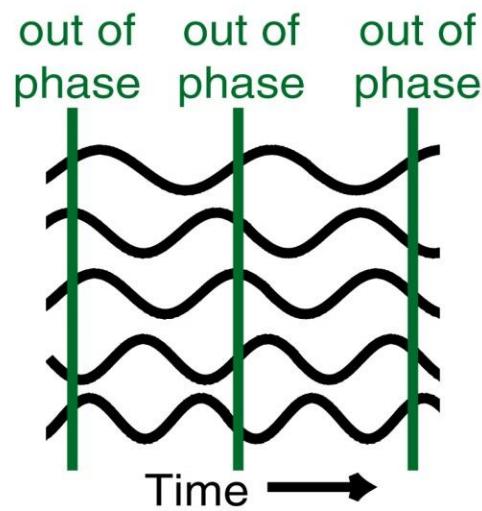
10 frequency components in phase



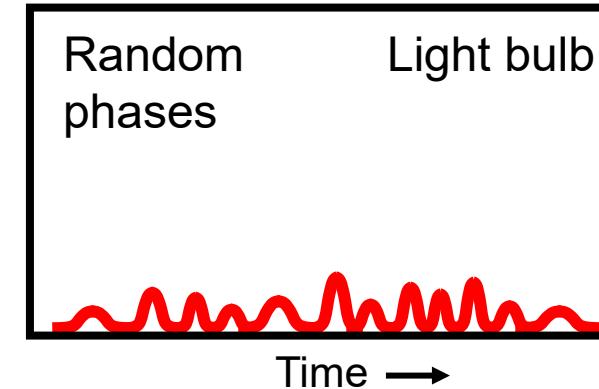
10 frequency components **in random phase**

Mode-locking: fs pulse

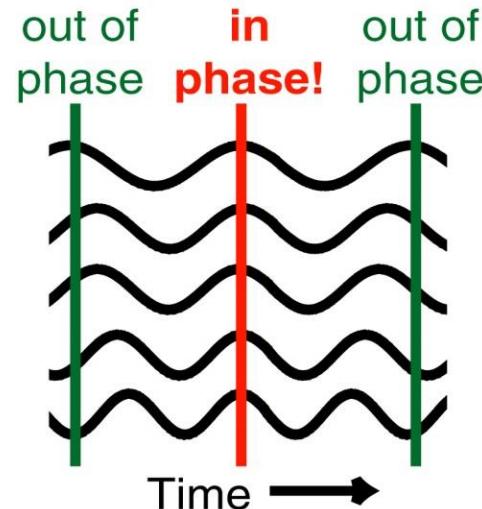
Random
phases
of all
laser
modes



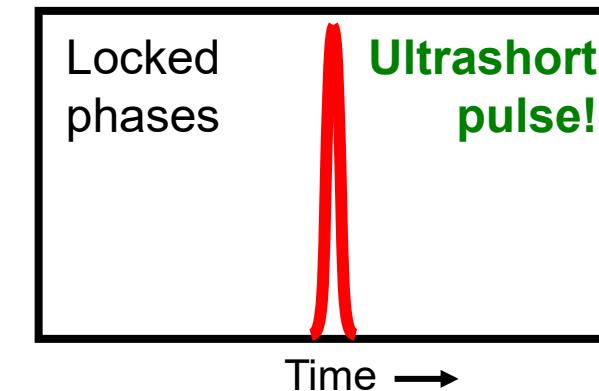
Intensity vs. time



Locked
phases
of all
laser
modes

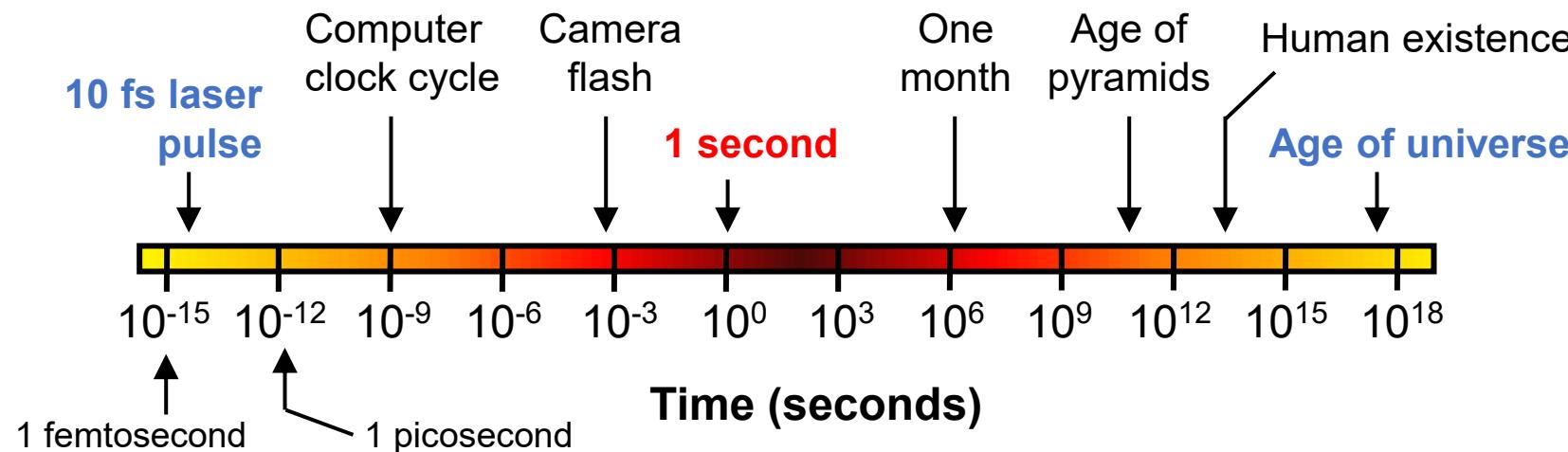


Intensity vs. time



Timescales of ultrafast lasers

It's now routine to generate femtosecond pulses in a university lab.



3000萬年:1秒 \approx 1秒:30 femtosecond

The generation of such pulses was motivated by the need to measure ps and fs events in chemistry, physics, and engineering that occur on such timescales.

Kilo (k)	10^{+3}
Mega (M)	10^{+6}
Giga (G)	10^{+9}
Tera (T)	10^{+12}
Peta (P)	10^{+15}
Exa (E)	10^{+18}

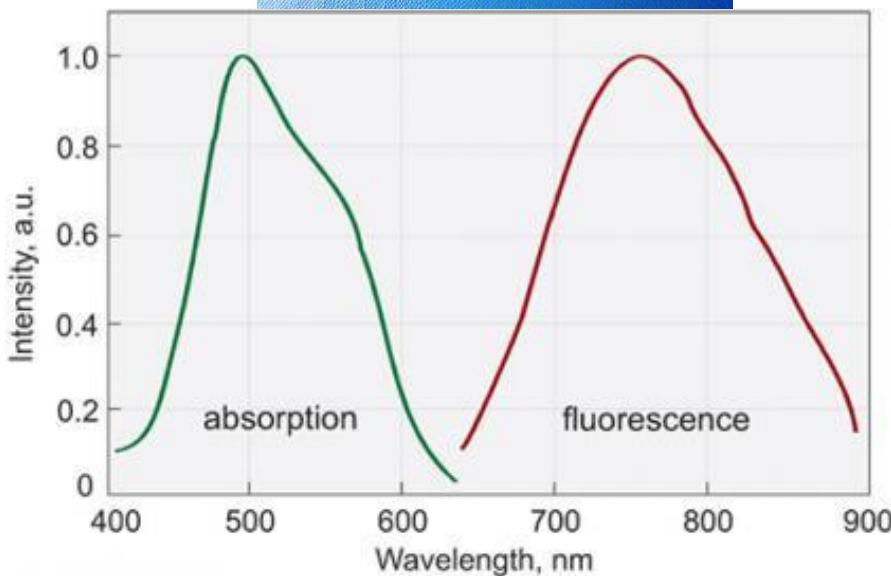
milli (m)	10^{-3}
micro (μ)	10^{-6}
nano (n)	10^{-9}
pico (p)	10^{-12}
femto (f)	10^{-15}
atto (a)	10^{-18}

We learned how generate ultra-intense pulses in 1985

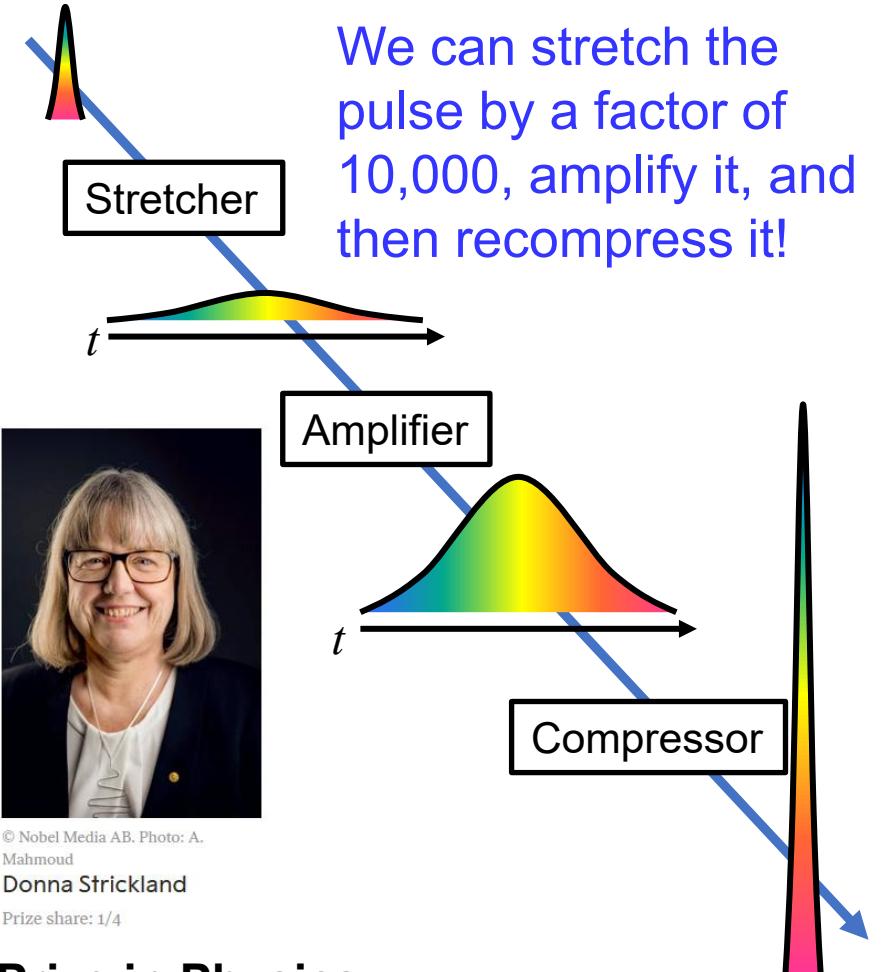
Discovery of Ti:sapphire crystal

(titanium-doped sapphire, $\text{Al}_2\text{O}_3:\text{Ti}^{3+}$)

➤ broad emission spectrum: ~650-1000 nm



Chirped-pulse amplification (CPA)



© Nobel Media AB. Photo: A. Mahmoud
Gérard Mourou

Prize share: 1/4

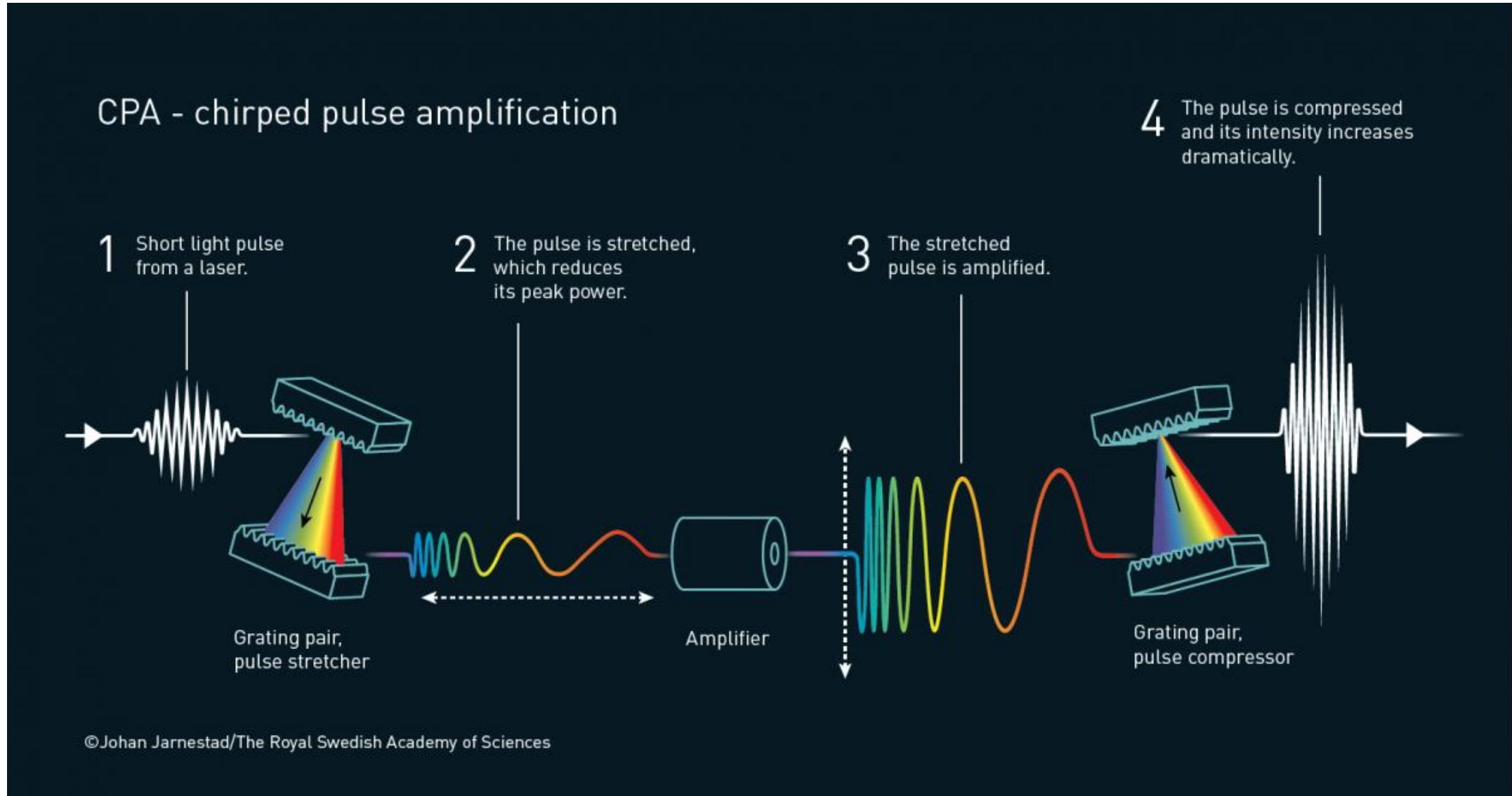


© Nobel Media AB. Photo: A. Mahmoud
Donna Strickland

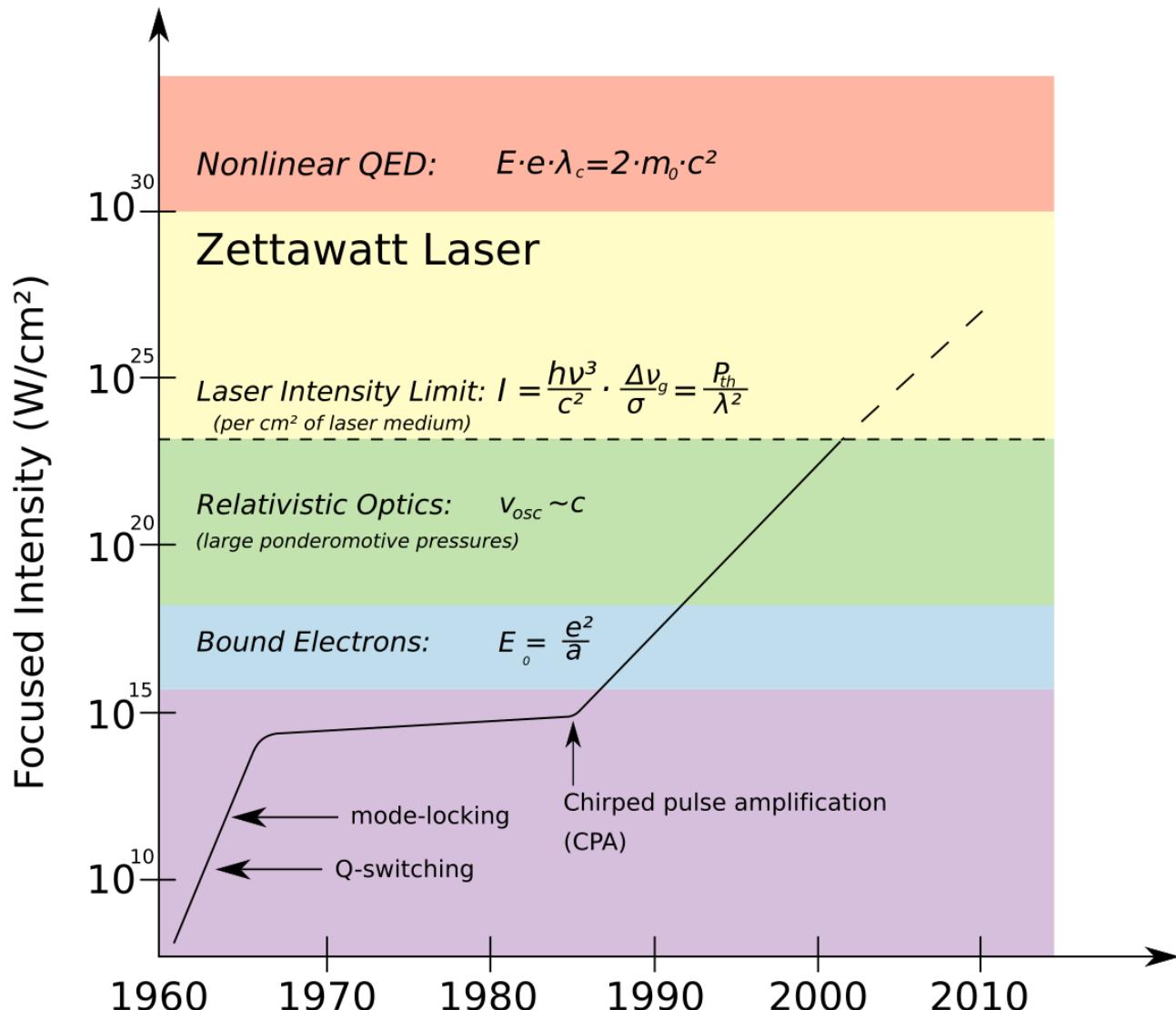
Prize share: 1/4

2018 Nobel Prize in Physics

How to generate an ultra-intense laser pulse



Laser intensity progress



1960 Invention of laser

1962 Q-switching technique

1963 Mode-locking technique

1985 Chirped pulse amplification (CPA)

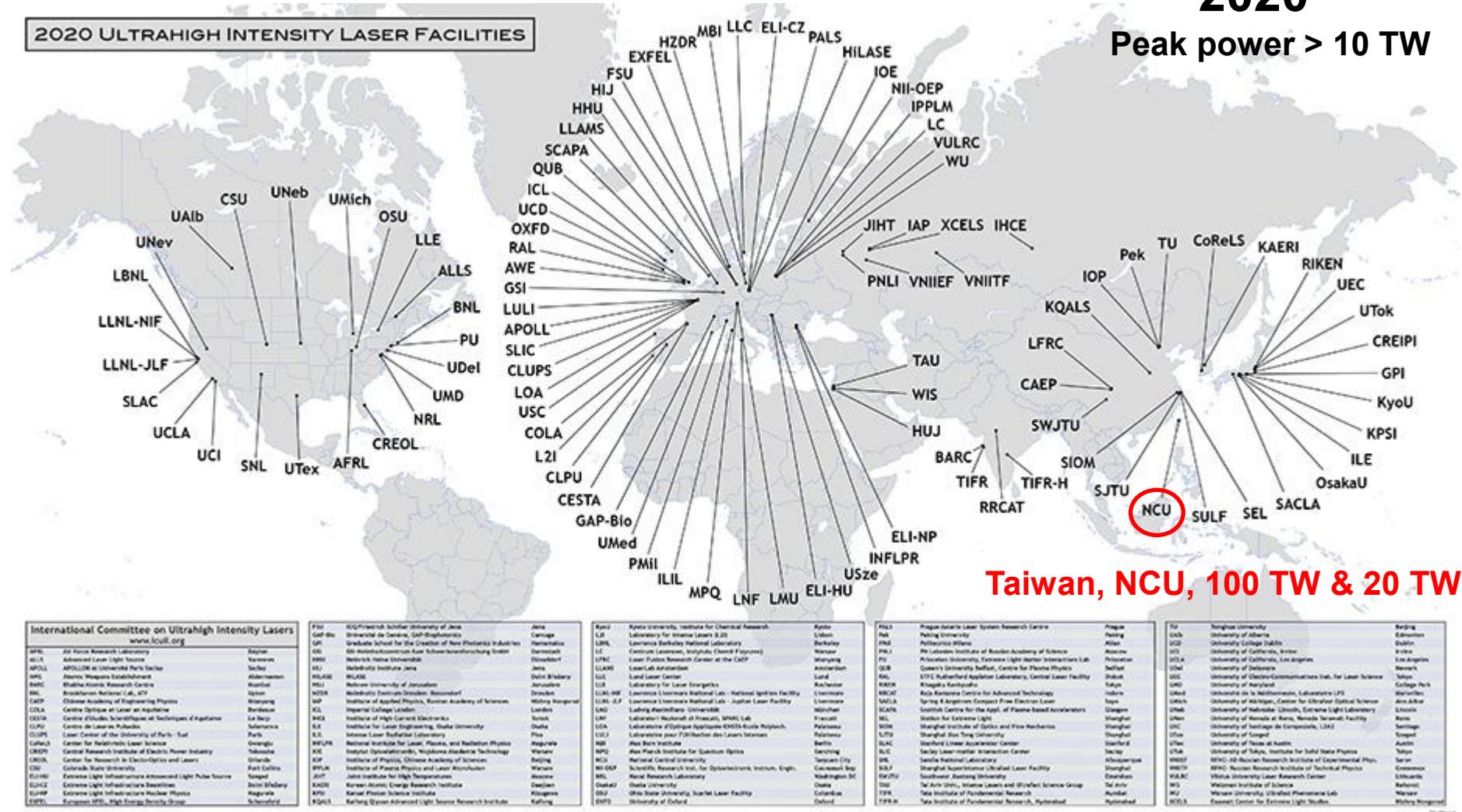
1986 Ti:sapphire used as gain medium

2021 Highest laser intensity achieved
 $I_0 = 1.1 \times 10^{23} \text{ W/cm}^2$
at CoReLS, South Korea

Ultrahigh Intensity Laser Facilities Worldwide

2020

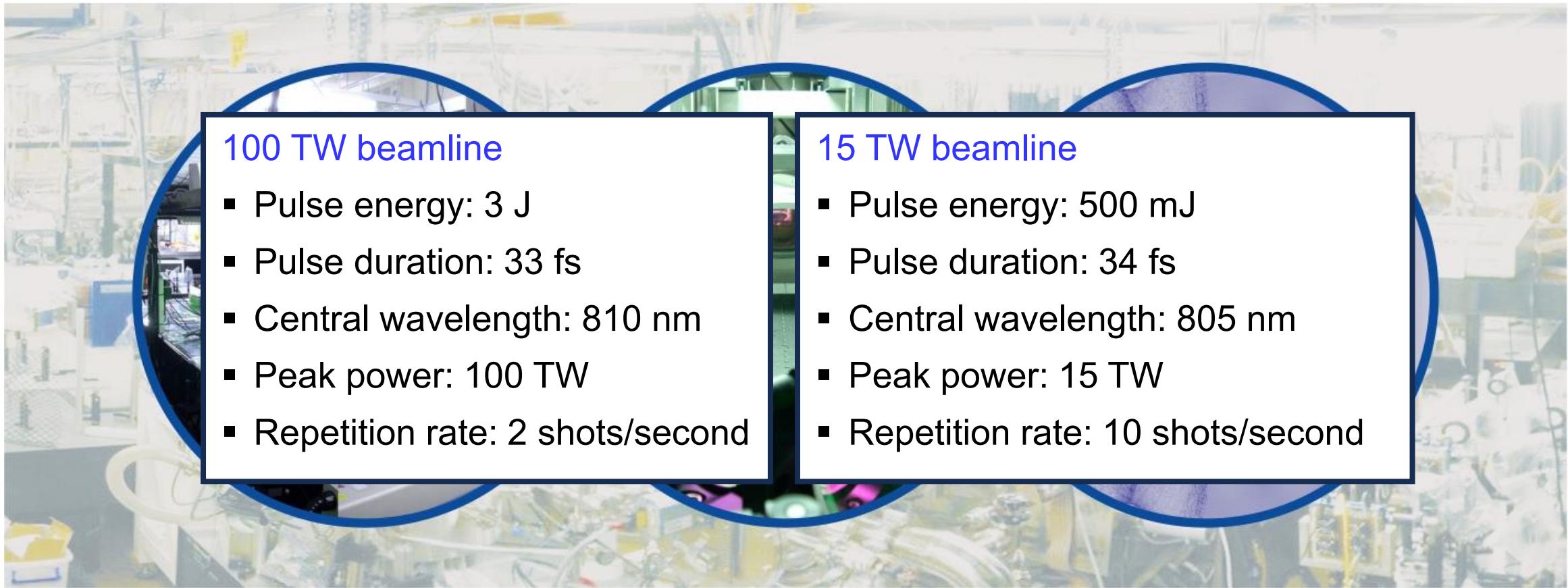
Peak power > 10 TW



100 TW Laser System at NCU

High-Field Physics and Ultrafast Technology Laboratory

Exploring the Interaction between Ultra-intense Electromagnetic Field and Matter



Laser and Experimental Areas

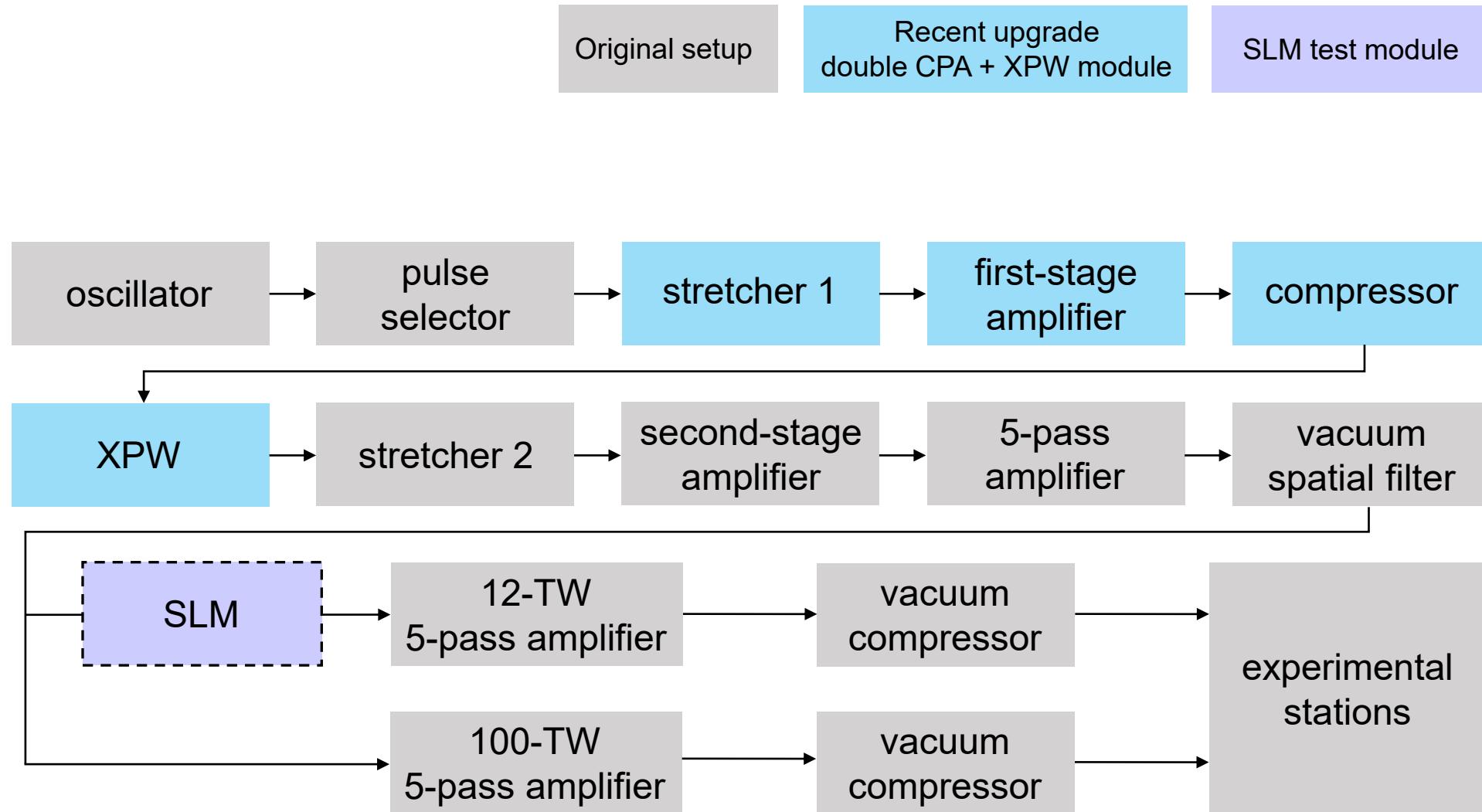
Laser area



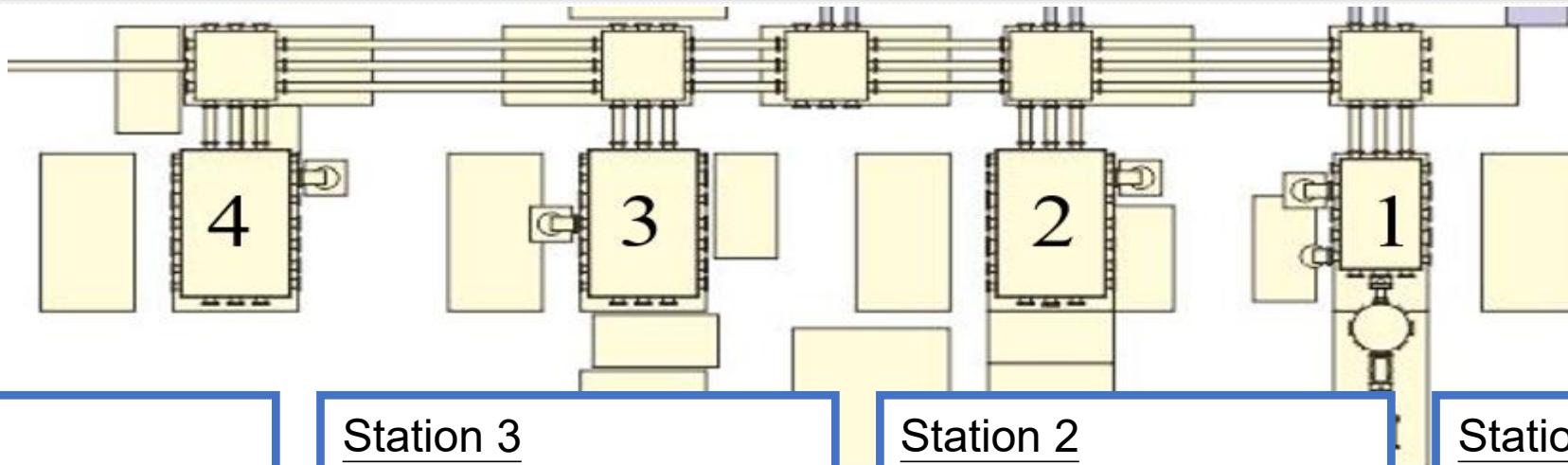
Experimental area



Block diagram of the system



Experimental stations at NCU L310



Station 4

Laboratory astrophysics,
High-repetition rate electron accelerator

PI



Chih-Hao Pai

Station 3

Proton accelerator and its applications,
Proton-boron interaction

PI



Chih-Hao Pai

Station 2

Electron accelerator and its applications,
Basic laser-plasma interactions

PI

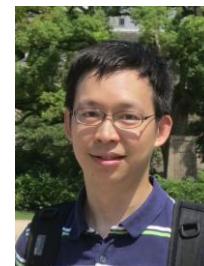


Chen-Kang Huang

Station 1

Advanced light source,
High-harmonic generation (HHG) and its applications

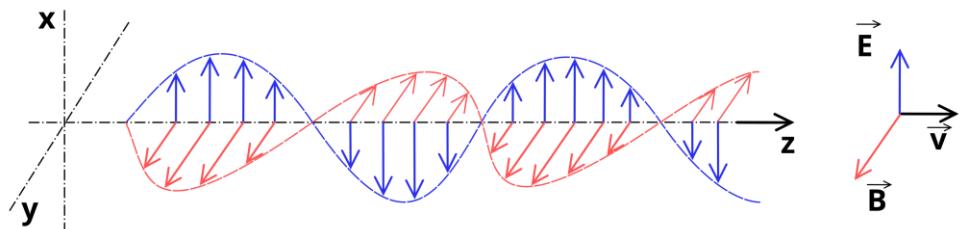
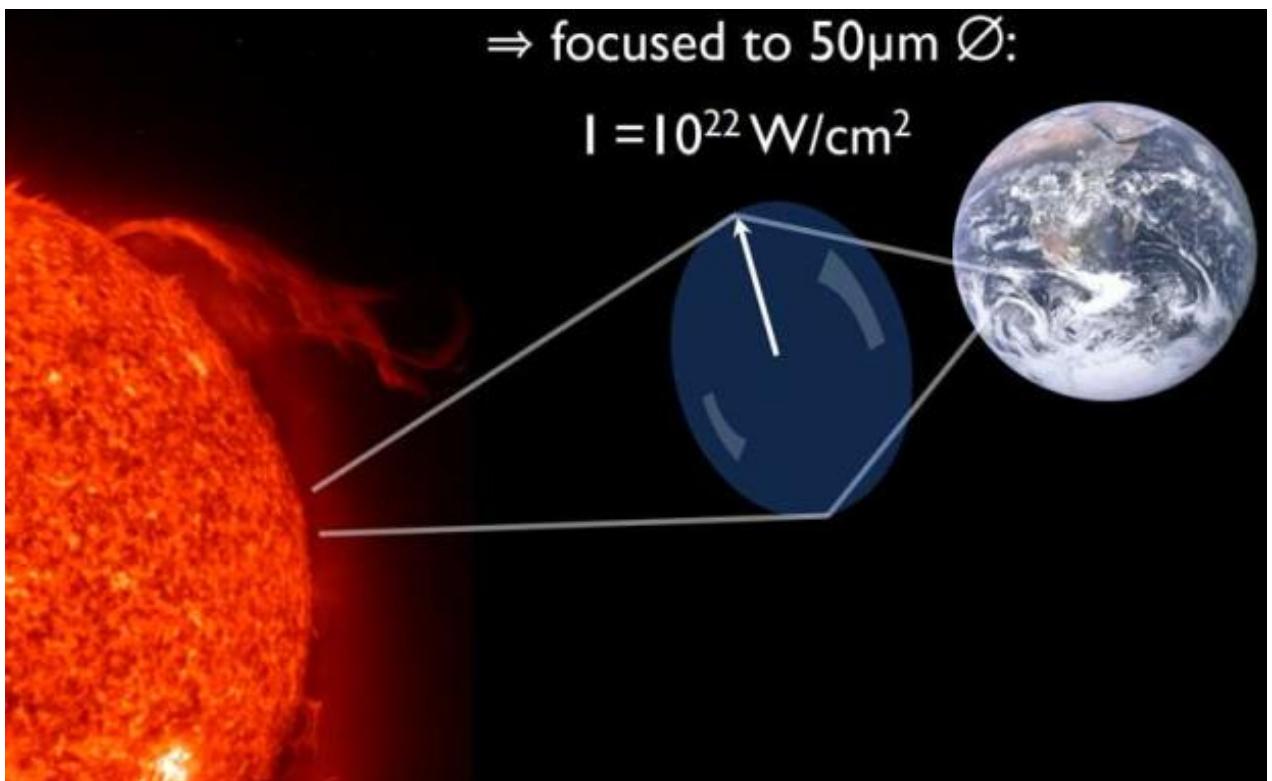
PI



Hsu-hsin Chu

How strong is a 100-TW laser

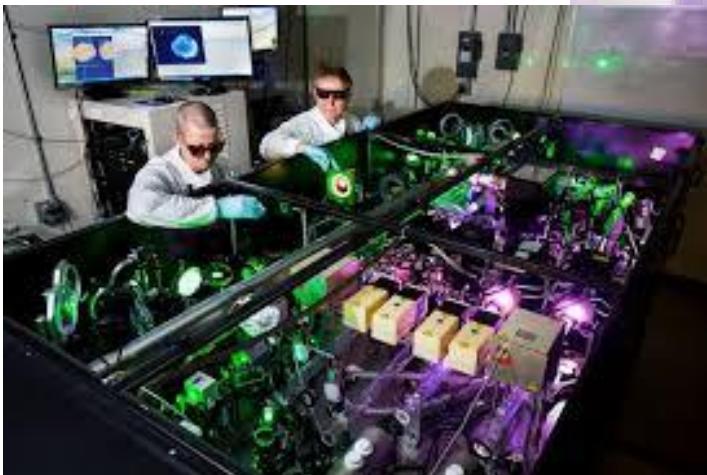
- Focusing a 100-TW of power into a few micrometer spot results in an **intensity** of order 10^{21} W/cm^2
- The **electric field strength** corresponding to this intensity is about 10^{13} V/m
- This is equivalent to focusing all of the sunlight incident on the earth onto a single human hair!



$$\text{power} = \frac{\text{energy}}{\text{time}}$$

$$\text{intensity} \propto \frac{\text{power}}{\text{area}}$$

How strong can a laser's field be



Laser pointer

$\sim 10^{-10}$ Tesla

Geomagnetic field

$\sim 10^{-5}$ T

MRI machine

~ 1.5 T

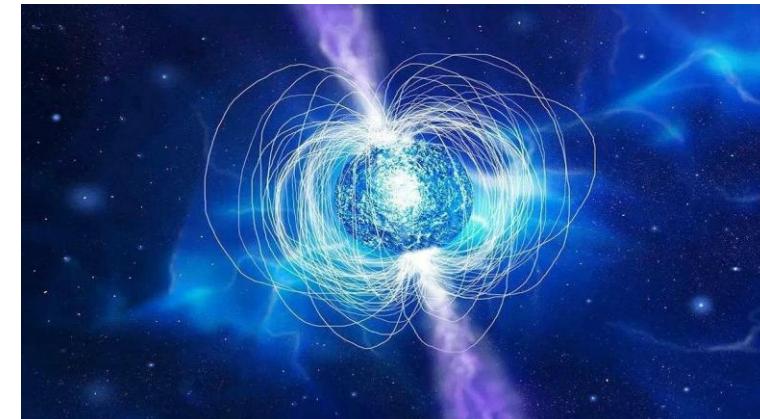
Superconducting magnet

~ 100 T

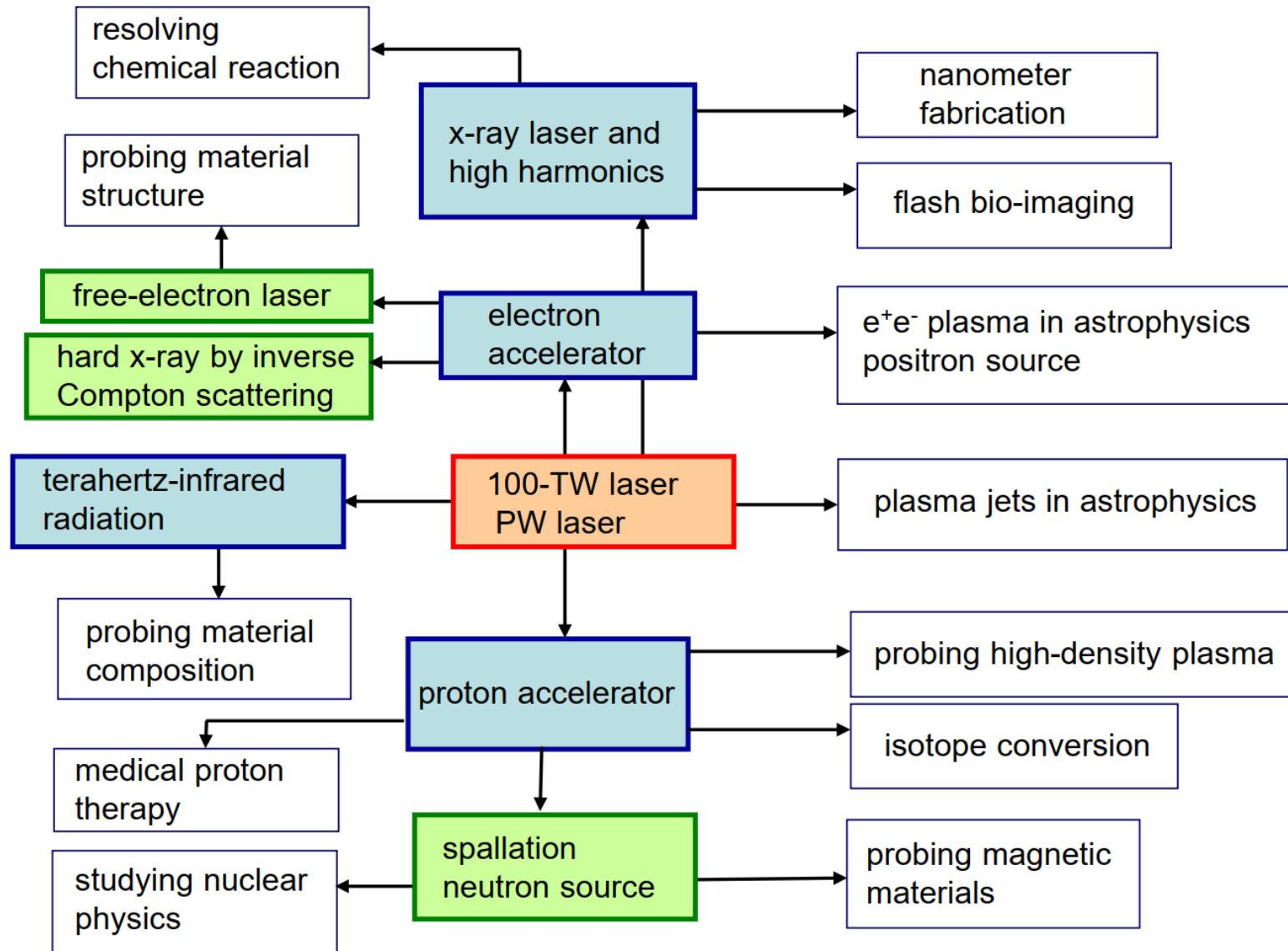
Focused intense laser $\sim 10^5 - 10^6$ T

Magnetar (磁星)

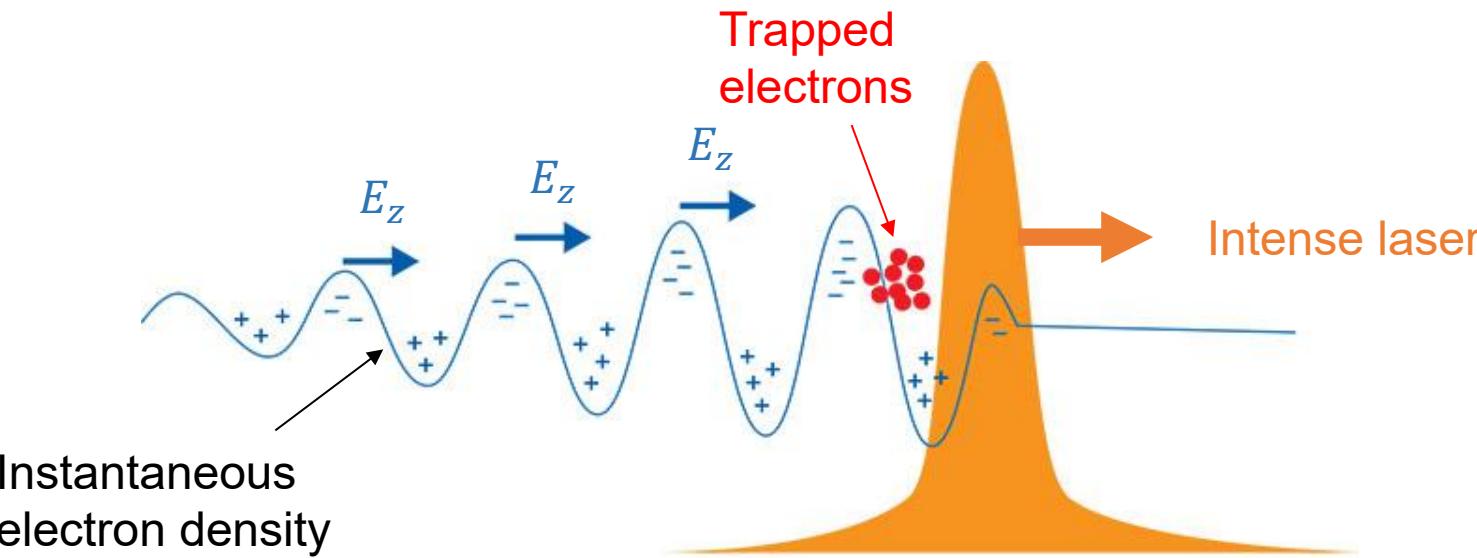
$\sim 10^9 - 10^{11}$ T



Ultrafast lasers for scientific research



The Laser Wakefield Accelerator (LWFA)



The **phase velocity** of the plasma wave (the wake) is equal to the **group velocity** of the driving laser

$$v_{ph} = v_g \sim c$$
$$v_g = c \sqrt{1 - \frac{\omega_p^2}{\omega_L^2}}$$

$$E_{\max} = \frac{m_e \omega_p c}{e} \approx 96 \sqrt{n_e [\text{cm}^{-3}]} \text{ V/m} \quad \text{for } n_e = 10^{18} \text{ cm}^{-3}, E_{\max} \approx 100 \text{ GV/m}$$

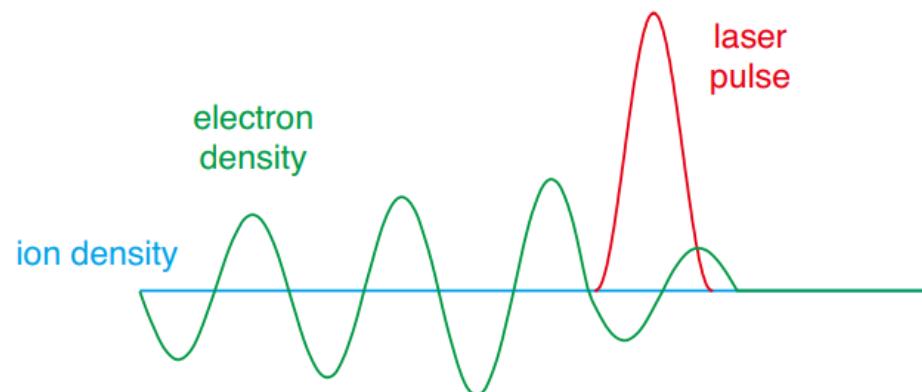
Wakefield excitation: 1D fluid model

■ Low intensity limit ($a_0 \ll 1$)

$$\left(\frac{\partial^2}{\partial \zeta^2} + k_p^2 \right) \phi = k_p^2 \frac{\hat{a}^2}{4}$$

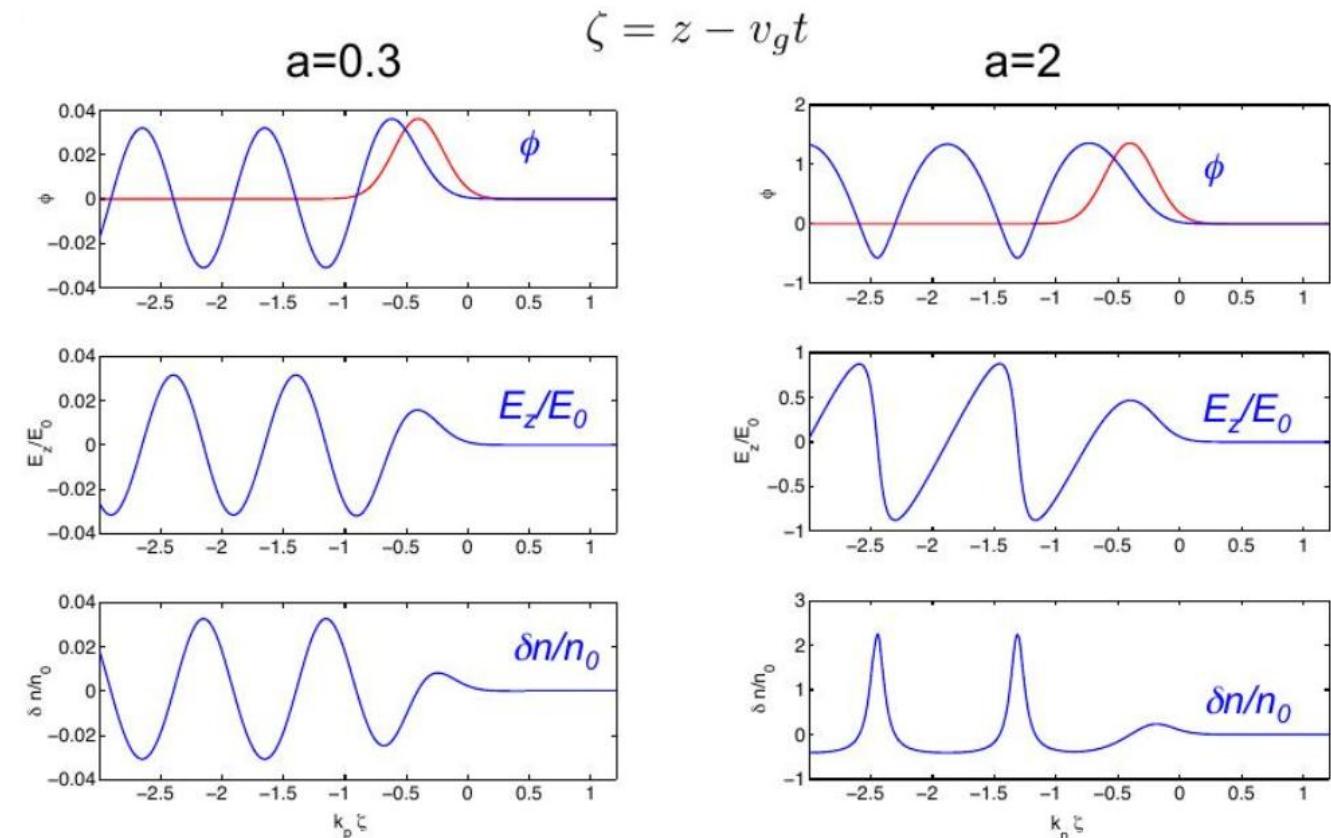
■ 1D nonlinear fluid theory which works for $a_0 > 1$

$$\frac{\partial^2 \phi}{\partial \zeta^2} = k_p^2 \gamma_p^2 \left[\beta_p \left(1 - \frac{1 + a^2}{\gamma_p^2 (1 + \phi)^2} \right)^{-1/2} - 1 \right]$$



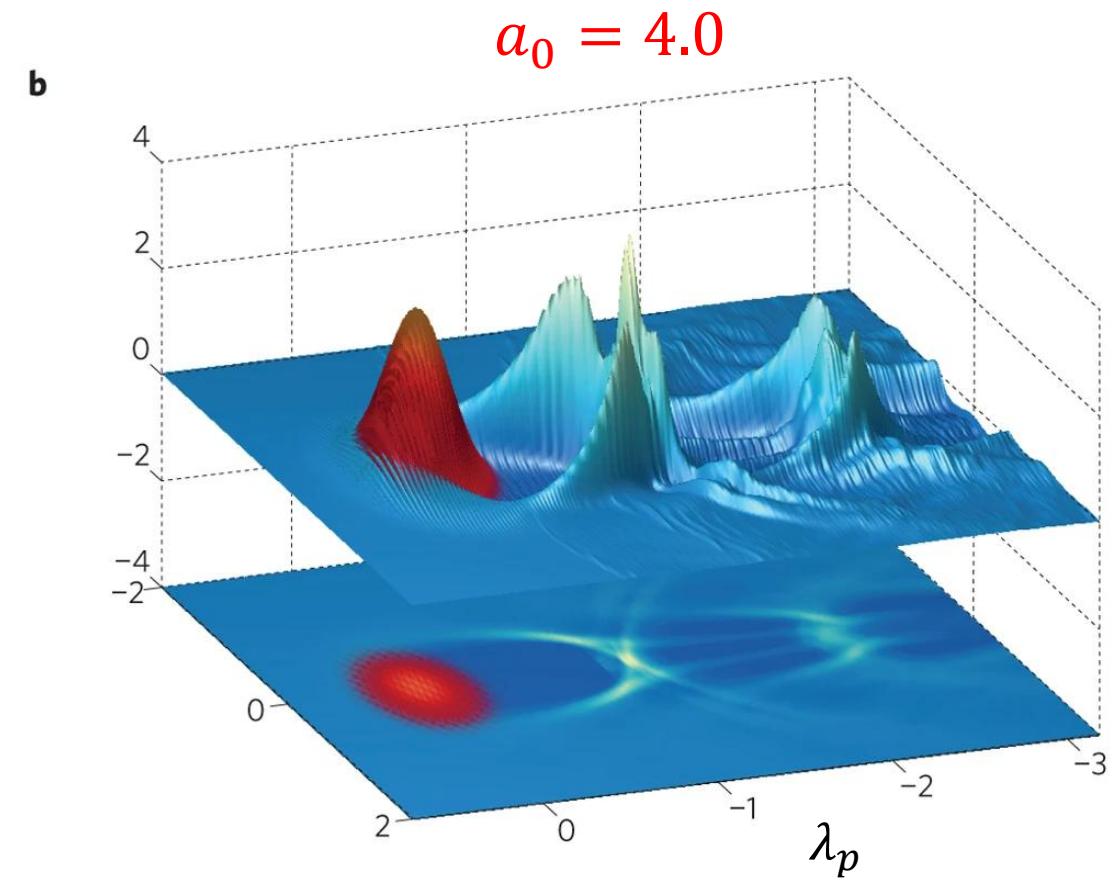
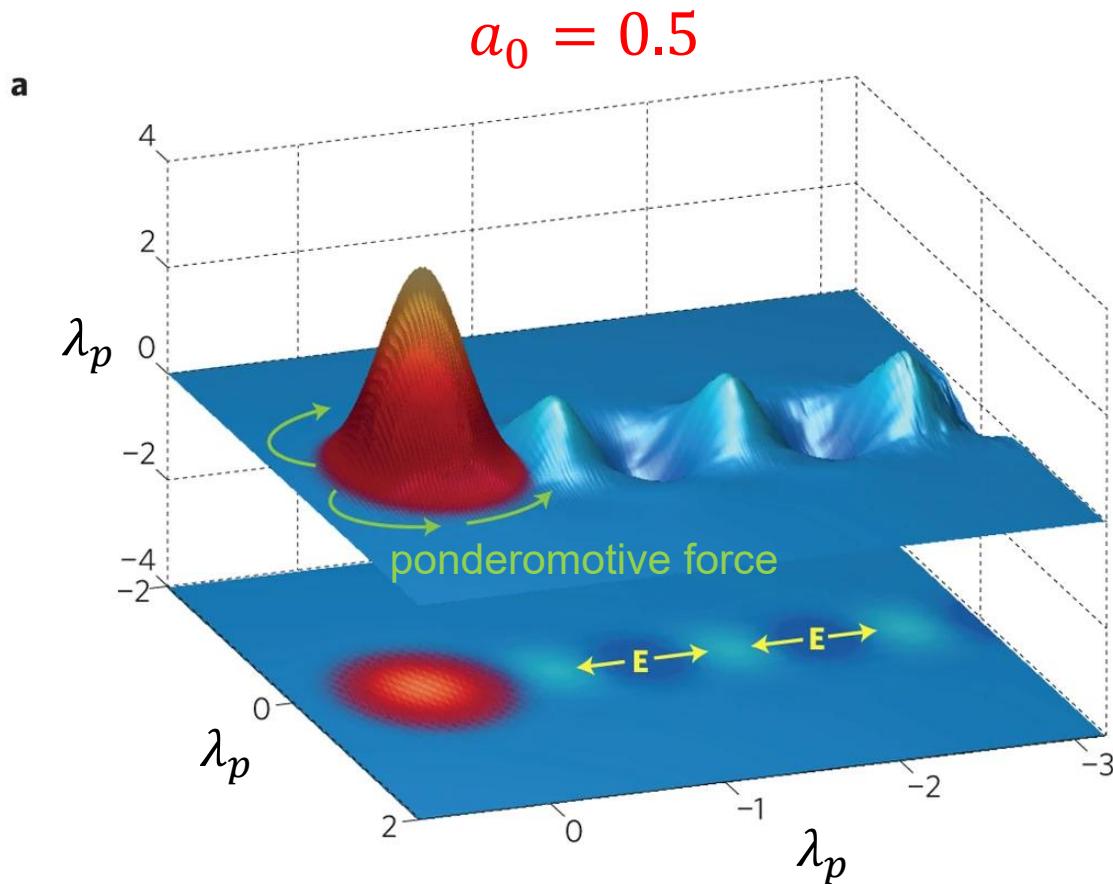
for a gaussian pulse:

$$a(\zeta) = a_0 \exp(-\zeta^2/2L_0^2) \cos(K_0 z - \omega_0 t)$$

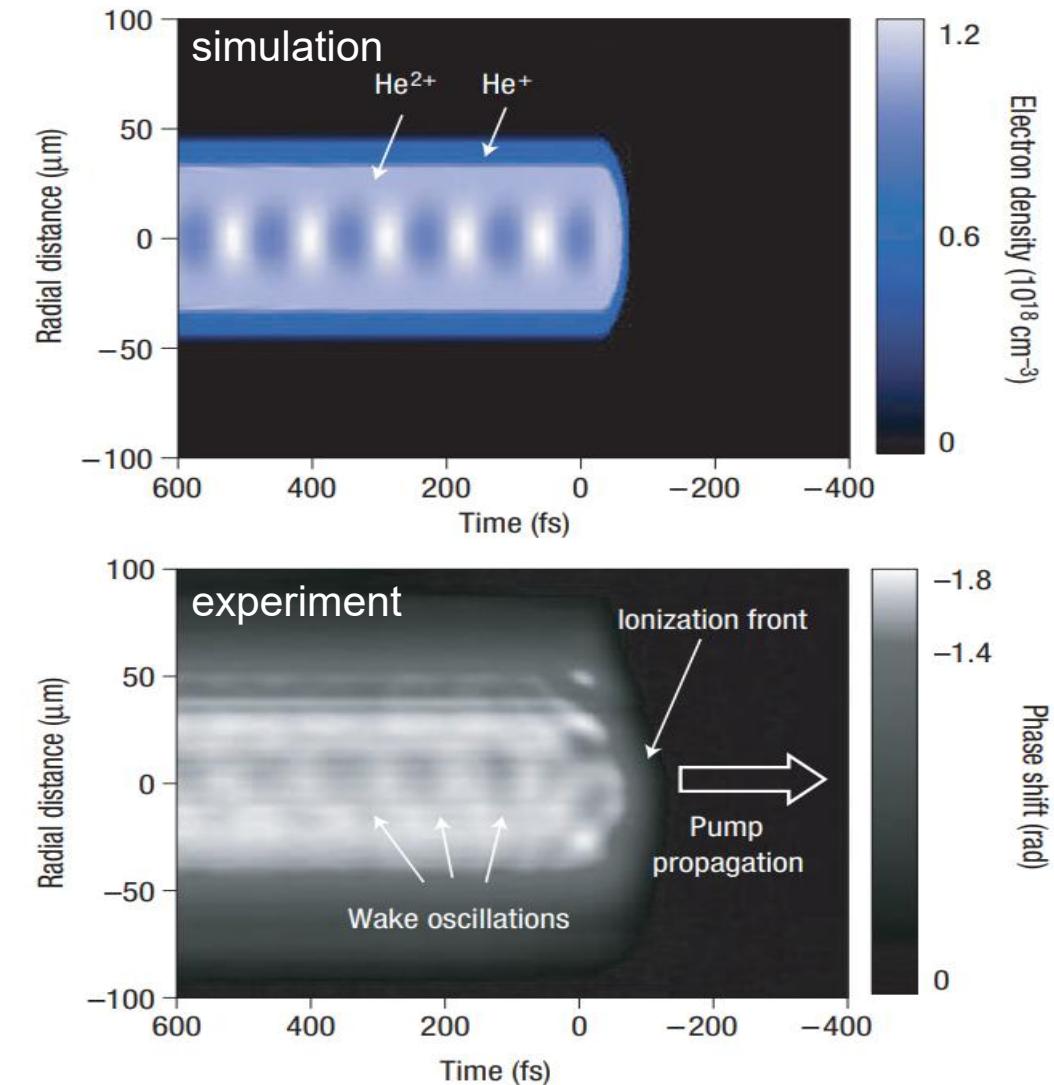
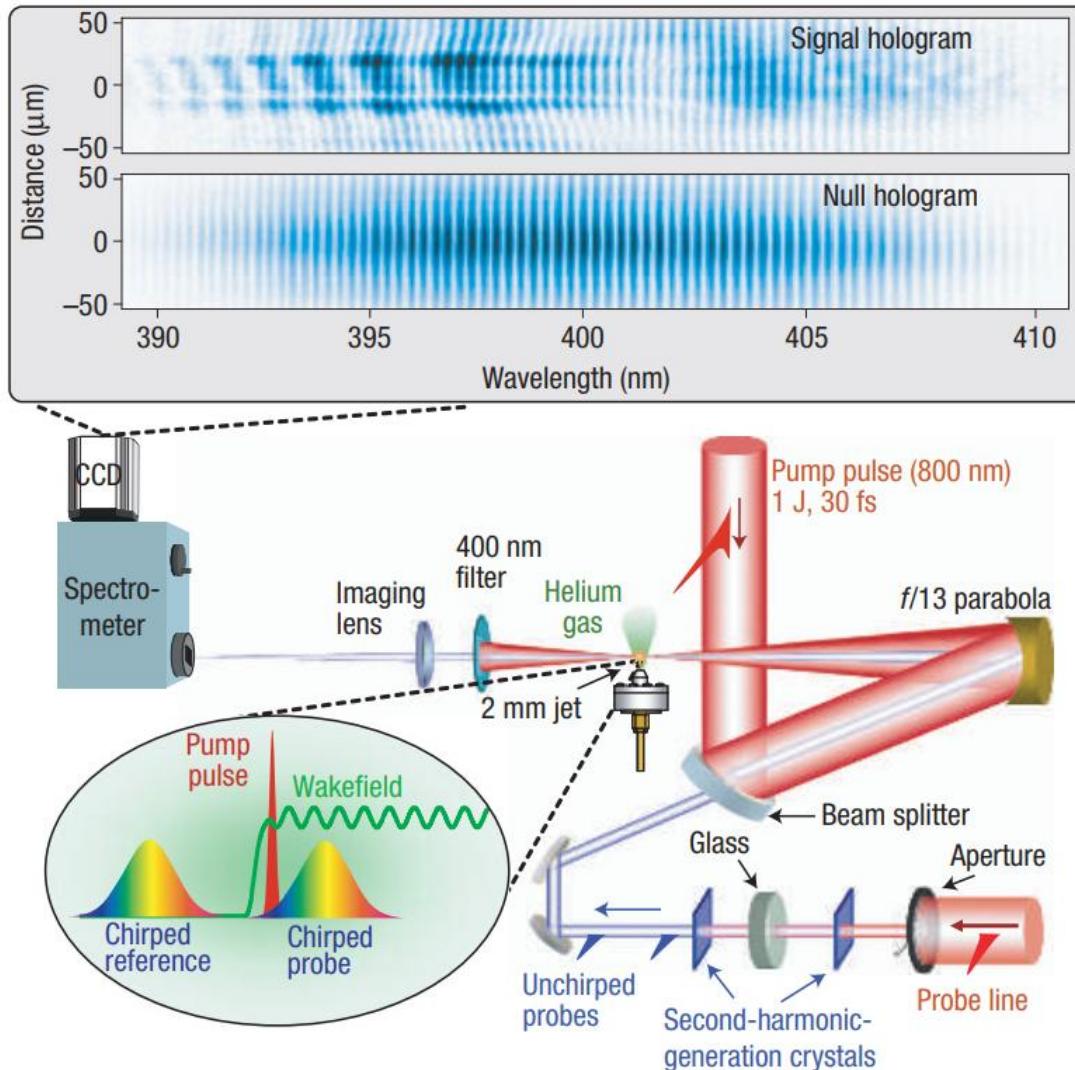


Wakefield excitation: 3D particle-in-cell simulation

- Simulation using 3D particle-in-cell (PIC) code OSIRIS

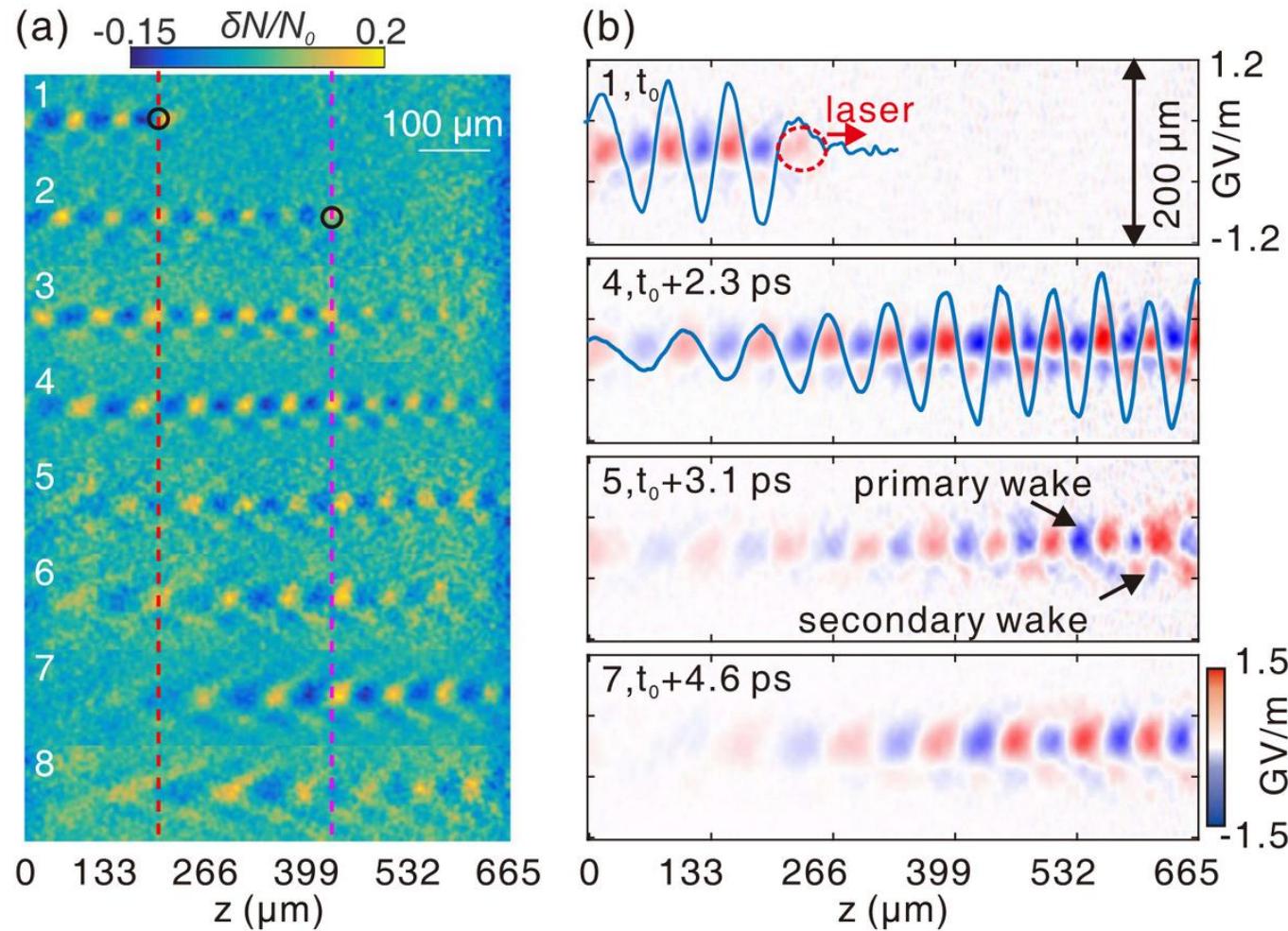
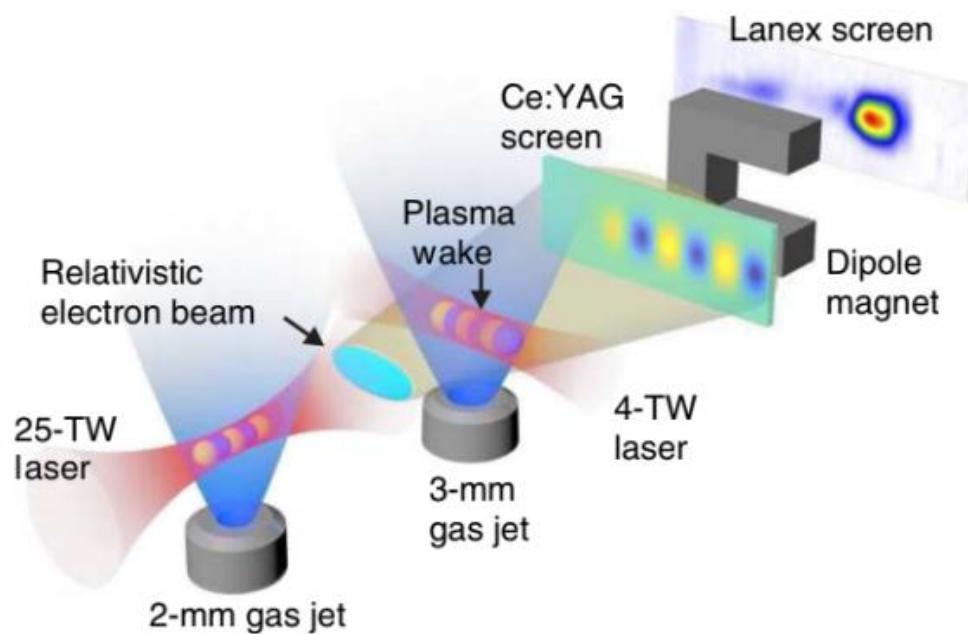


Probing plasma wakefield



Probing plasma wakefield

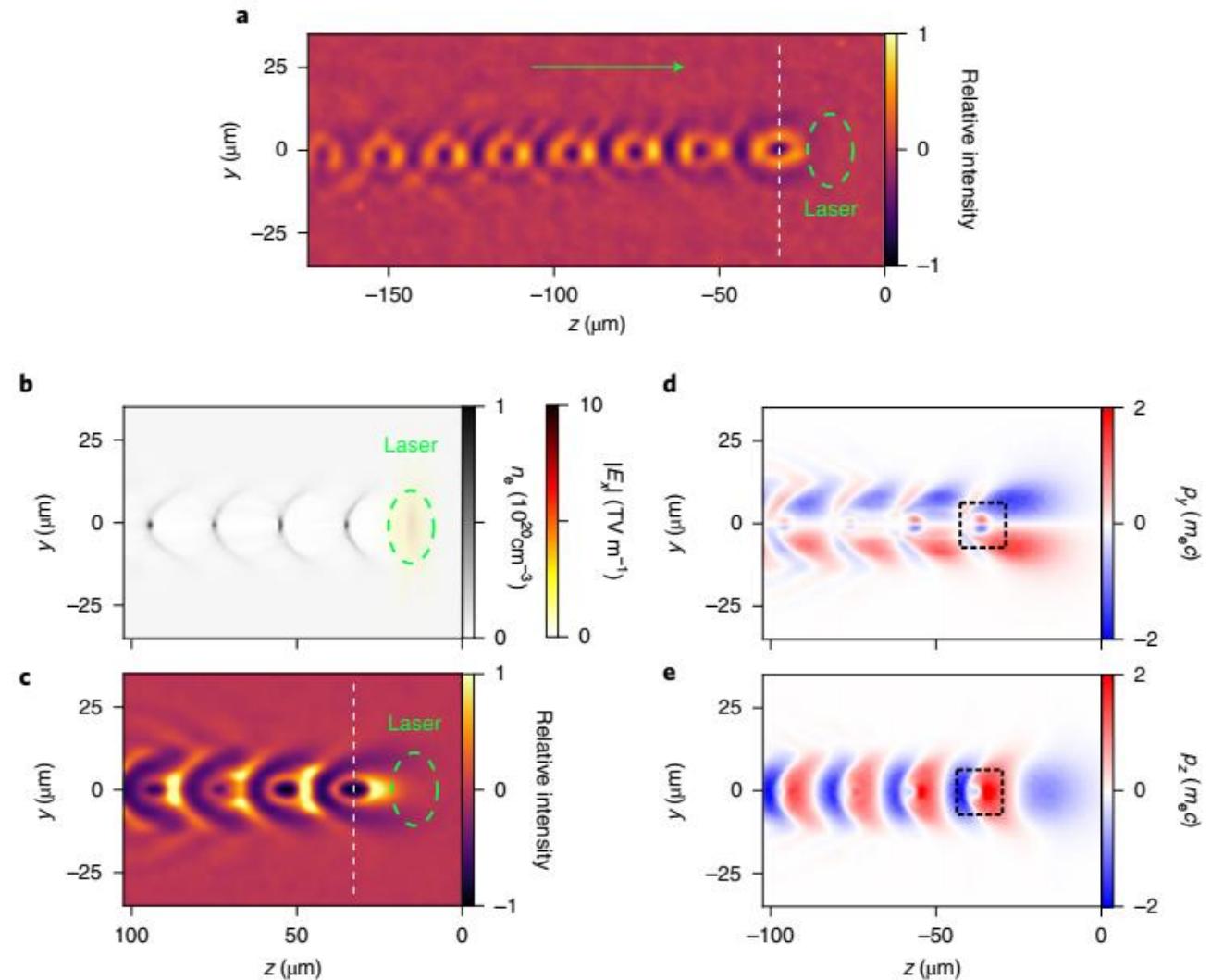
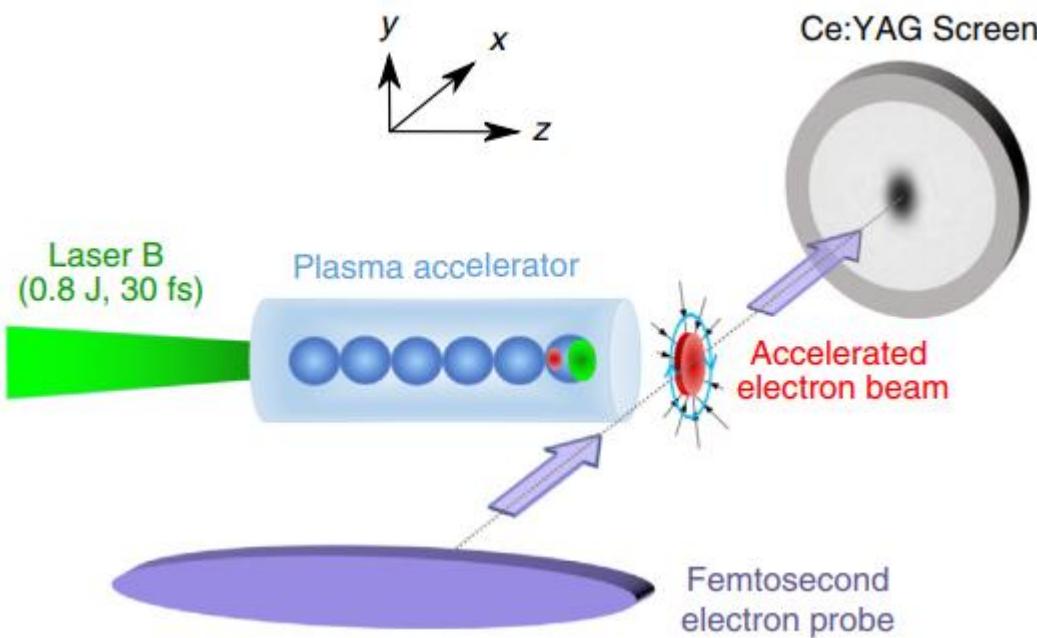
- Femtosecond electron beams from LWFA as the probe.
- Plasma wakefield can strongly bend the transversing electron beam.



Physical Review Letters **119**, 064801 (2017).

Plasma Physics and Controlled Fusion **60** (4), 044013 (2018).

Observing nonlinear plasma wakefield

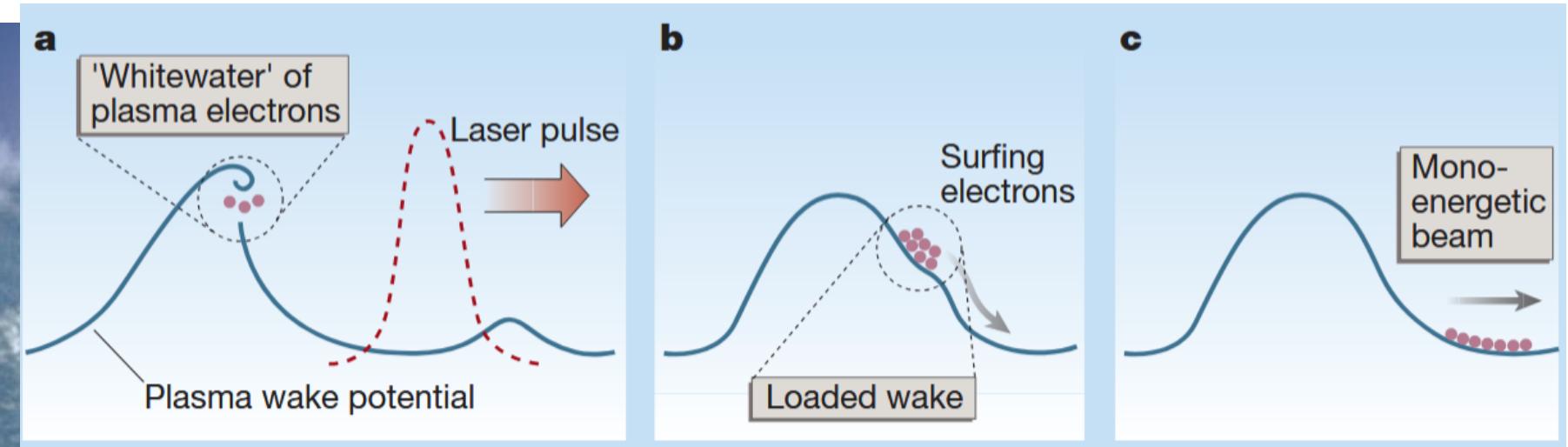


Surf on the plasma wave

Electrons hang ten on laser wake

Thomas Katsouleas

Electrons can be accelerated by making them surf a laser-driven plasma wave. High acceleration rates, and now the production of well-populated, high-quality beams, signal the potential of this table-top technology.



Trapped electrons gain energy by maintaining an optimal phase relationship with plasma wave's electric field.

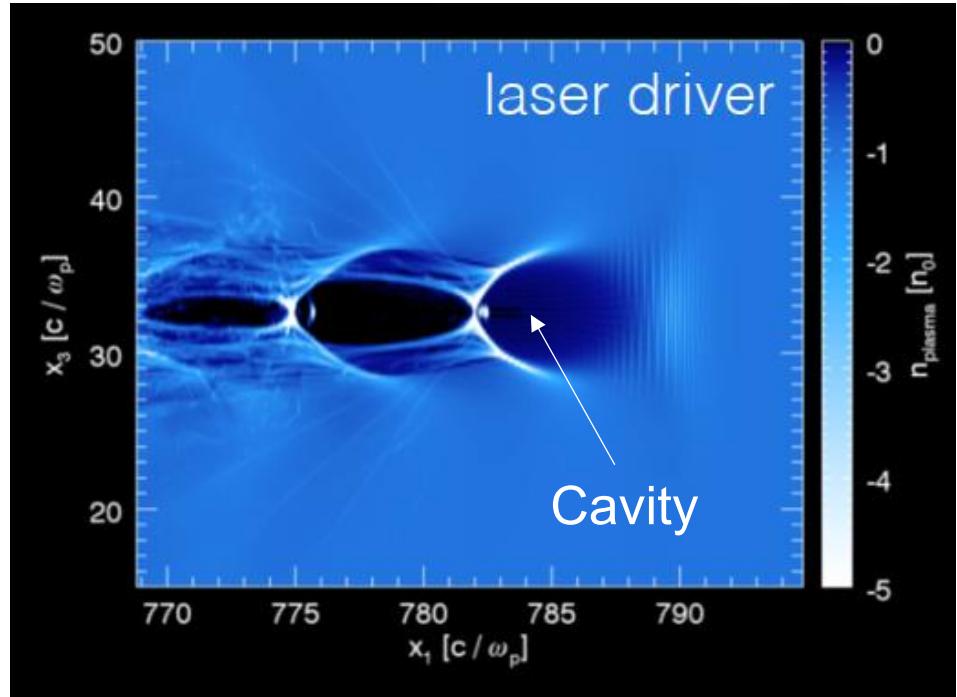


Injection at right phase!

The blow-out (bubble) regime

- When the drive beam is strong enough to completely expel all the electrons near the focal spot
- A preferred regime for high-quality electron acceleration:
 - Linear and large longitudinal field
 - Linear focusing field
 - High efficient ($\sim 20\%$)

$$a_0 > 2$$



- We can estimate the bubble size (r_b) by balancing the ponderomotive force and space charge force of the ionic bubble

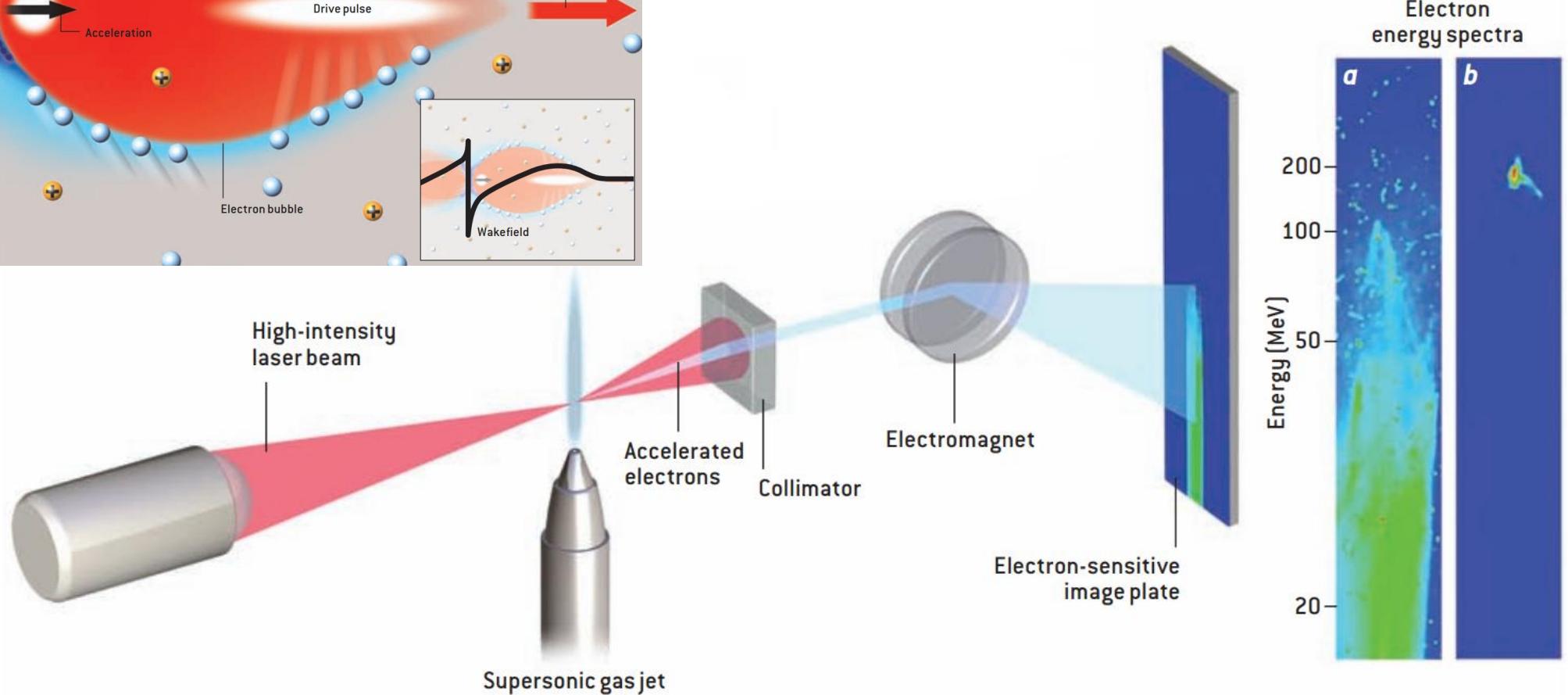
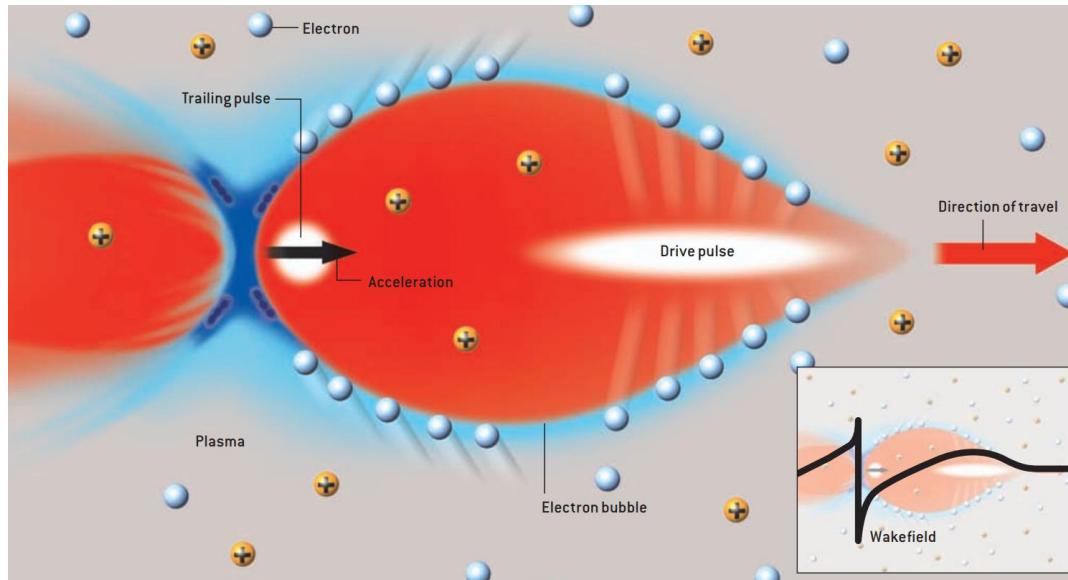
$$\mathbf{F}_p = -\frac{1}{2} m_e c^2 \nabla \frac{a^2}{\gamma} \approx m_e c^2 a_0 / w_0$$

$$\nabla \cdot \mathbf{E} = \frac{-e(n_e - n_i)}{\epsilon_0} = \frac{en_0}{\epsilon_0} \Rightarrow \mathbf{F}_{sc} = -e\mathbf{E} \approx -e^2 n_0 r_b / \epsilon_0$$

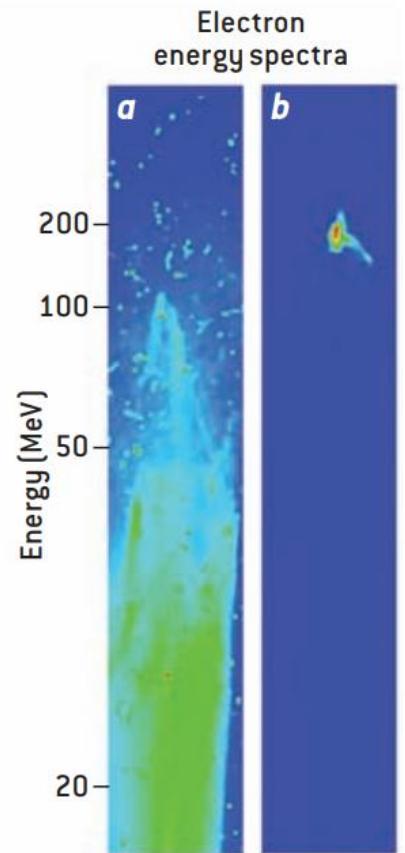
$$\mathbf{F}_p + \mathbf{F}_{sc} = 0 \Rightarrow m_e c^2 \frac{a_0}{w_0} = \frac{e^2 n_0 r_b}{\epsilon_0} \quad r_b \approx \frac{a_0}{k_p^2 w_0}$$

At matched spot size: $r_b \approx 2\sqrt{a_0} \frac{c}{\omega_p}$

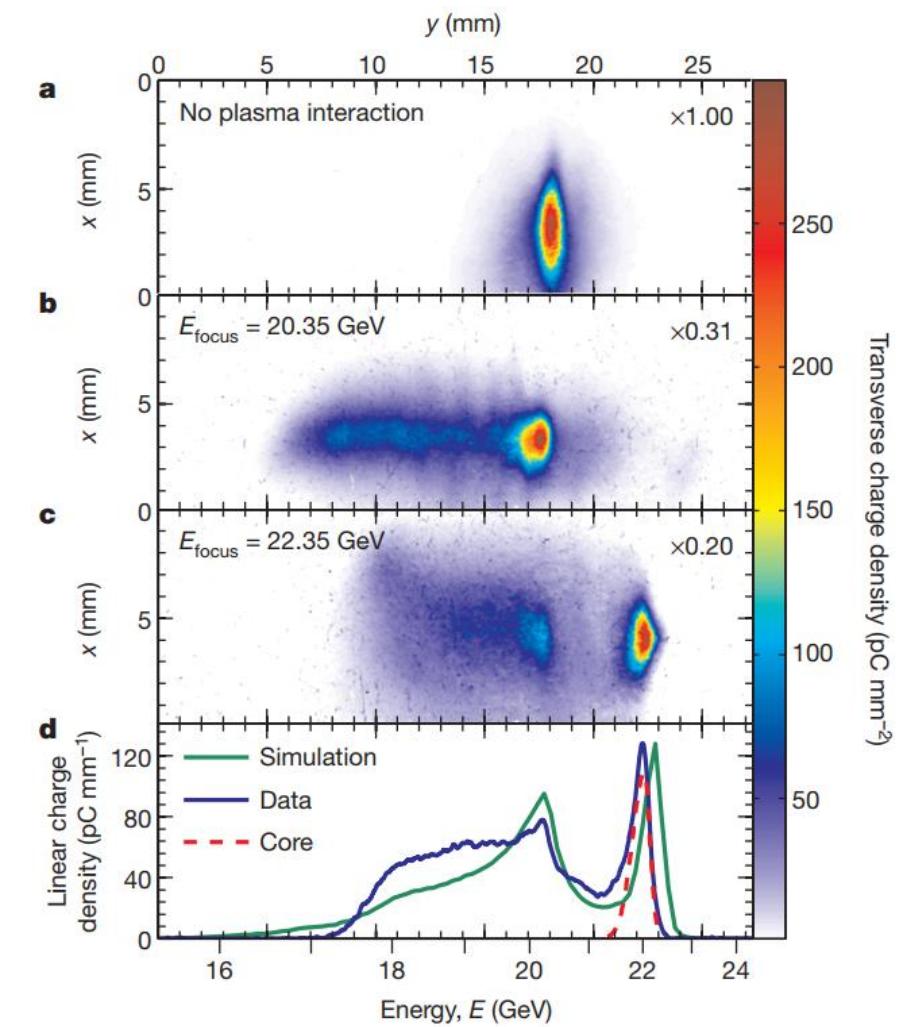
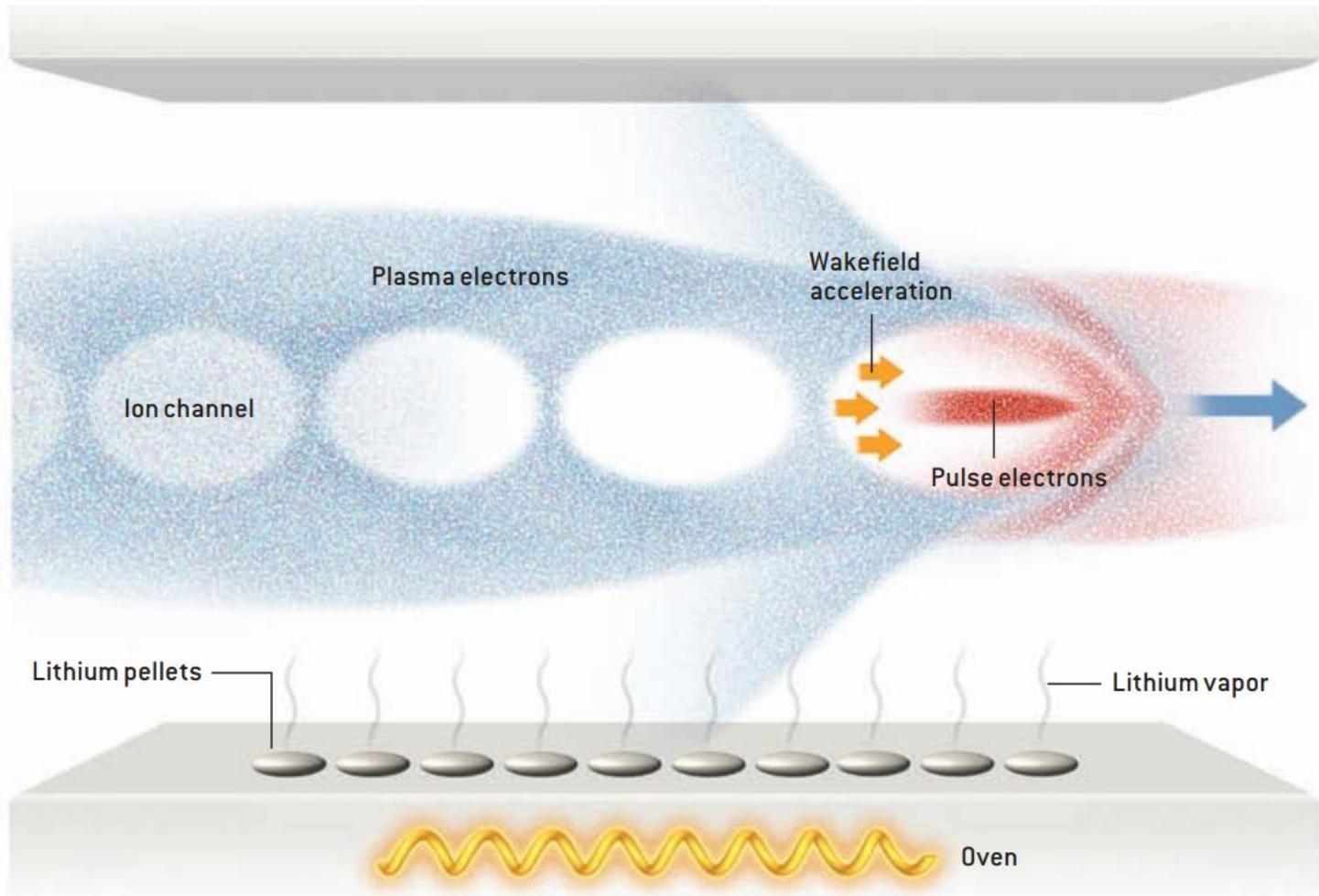
Experimental demonstration of LWFA



Mono-energetic electron bunch
observed in the blow-out regime



Experimental demonstration of PWFA



Injection mechanisms

- For a surfer to “catch a wave”, he must swim to get up to speed before the wave arrives
- we must find a way of accelerating electrons up to the correct speed for them to be trapped by the wave and accelerate



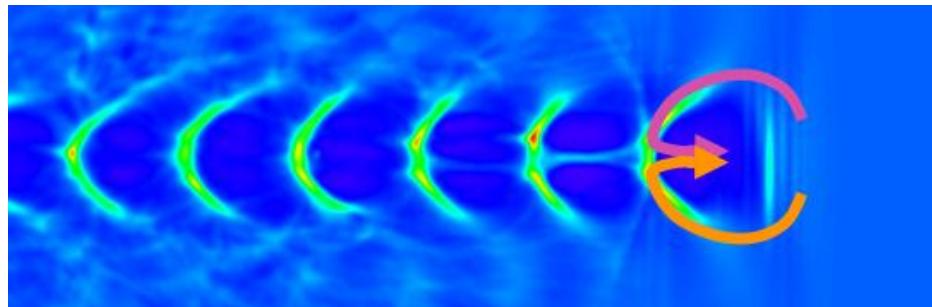
■ Why is injection so important?

- Injection strongly influences the performances of the accelerators:
repetition rate, charge, beam quality (emittance, energy spread)
- In a laser plasma accelerator, it is important to decouple the injection from the acceleration mechanism

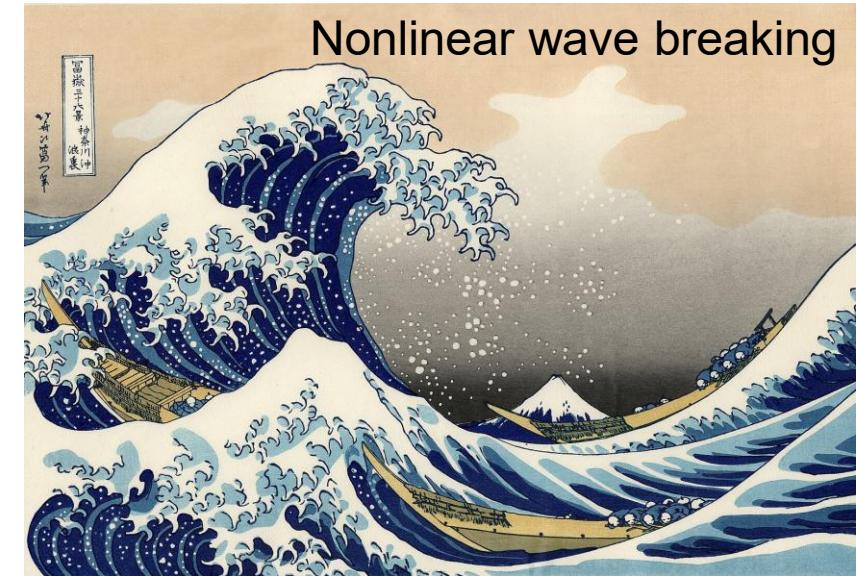
■ Difficulties

- Injection of a short bunch
- Synchronization between laser and injection beam

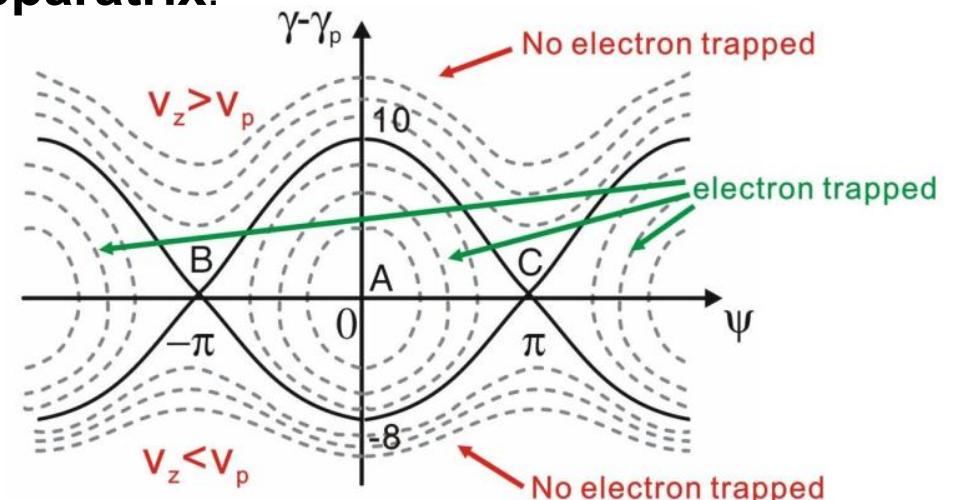
Self-injection



- When the plasma wakefield reaches the **wave-breaking limit**, its structure becomes highly nonlinear, allowing background electrons to be self-injected into the accelerating phase.
- In the blowout (bubble) regime, these electrons are captured at the rear of the ion cavity at nearly the same time and location
 - quasi-monoenergetic electron beams



Separatrix:



Self injection

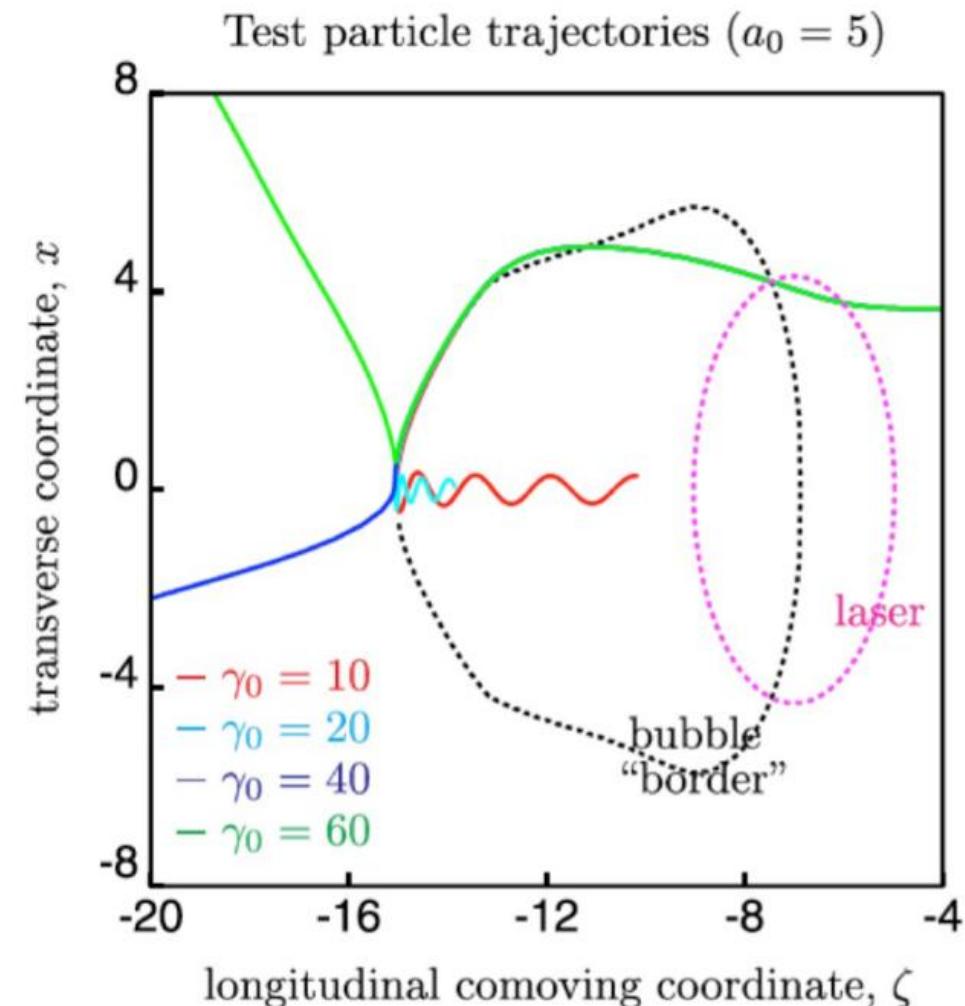
- Requires large amplitude wakefield — high laser intensity

$$a_0 = 0.85\lambda[\mu\text{m}] \left(I[10^{18}\text{W/cm}^2] \right)^{1/2} \gtrsim 4$$

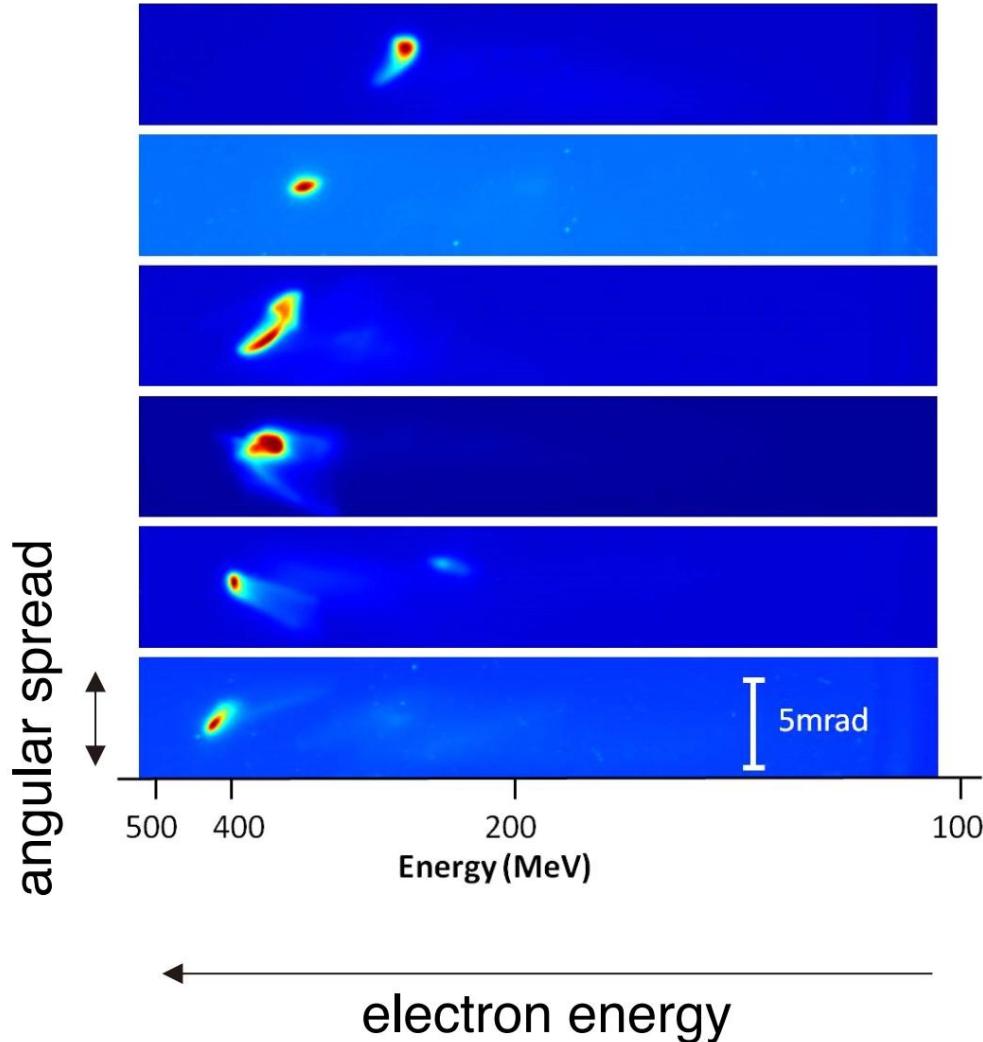
- Requires high plasma density — slow plasma wave phase velocity

$$\gamma_p \sim \frac{\omega_L}{\omega_p} \propto n^{-1/2}$$

- Sub-micron emittance observed from self-trapping



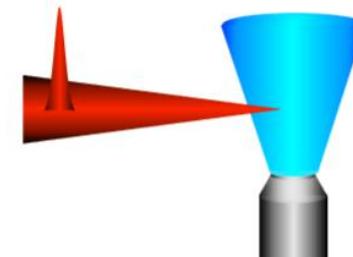
Mono-energetic electron beam at NCU



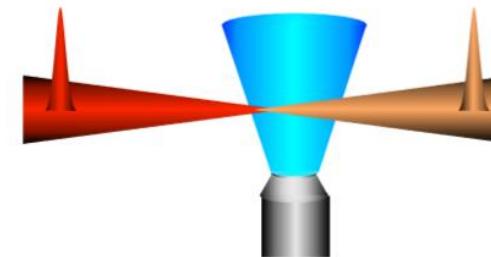
- condition:
 - driving pulse energy = 2 J
 - pulse duration = 36 fs
 - focal spot size w_0 = 27 μm
 - He jet length = 4 mm
 - atom density = $2.8 \times 10^{18} \text{ cm}^{-3}$
- accelerated electron:
 - maximum energy: 430 MeV
 - energy spread: 2~5%
 - beam divergence: ~ 1 mrad
 - charge: 1~5 pC

Advanced injection schemes

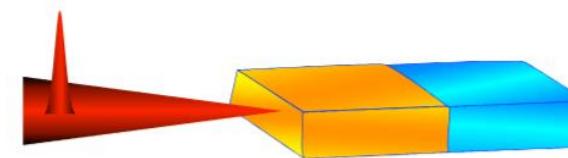
- External injection



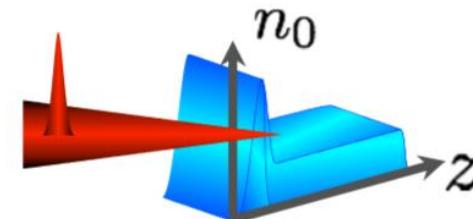
- Colliding pulse injection



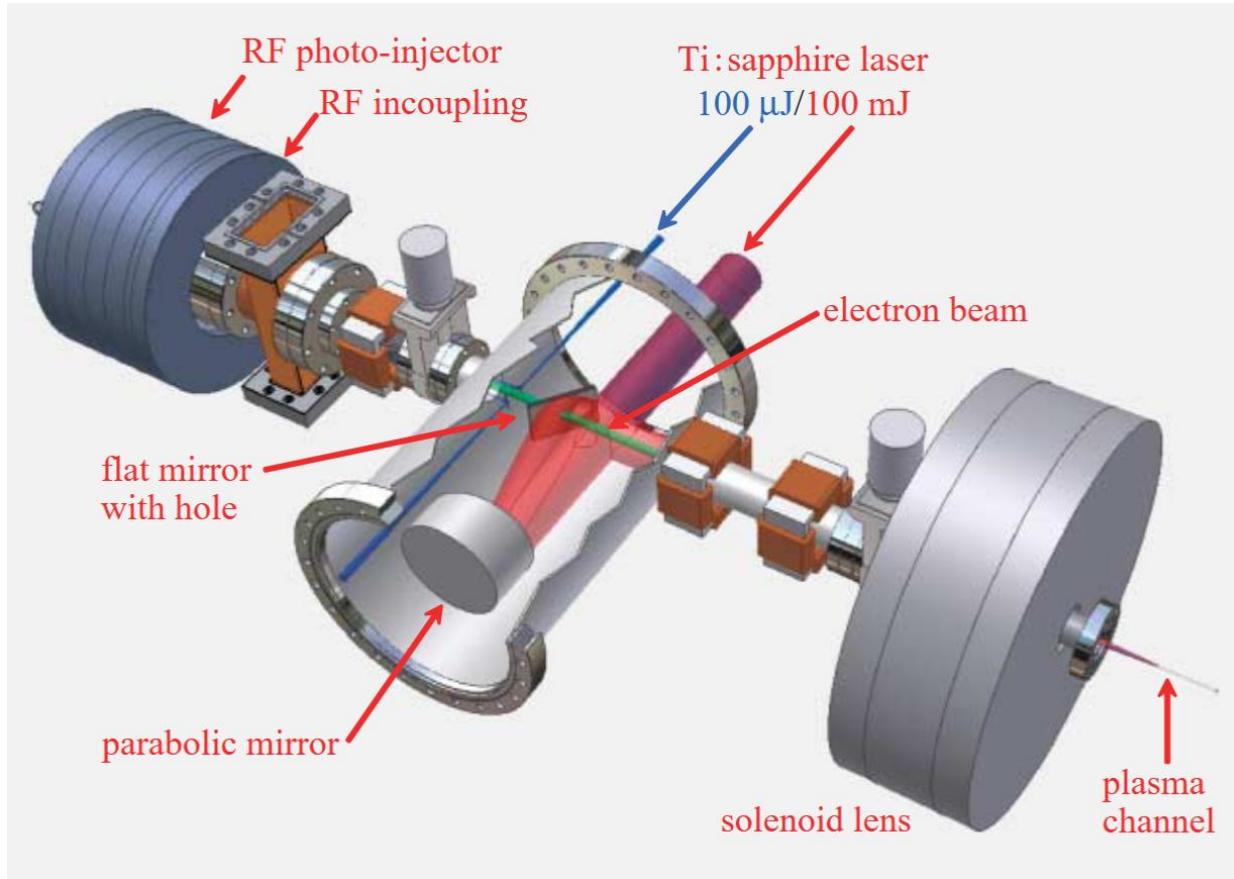
- Ionization injection



- Density gradient injection
(down-ramp injection)

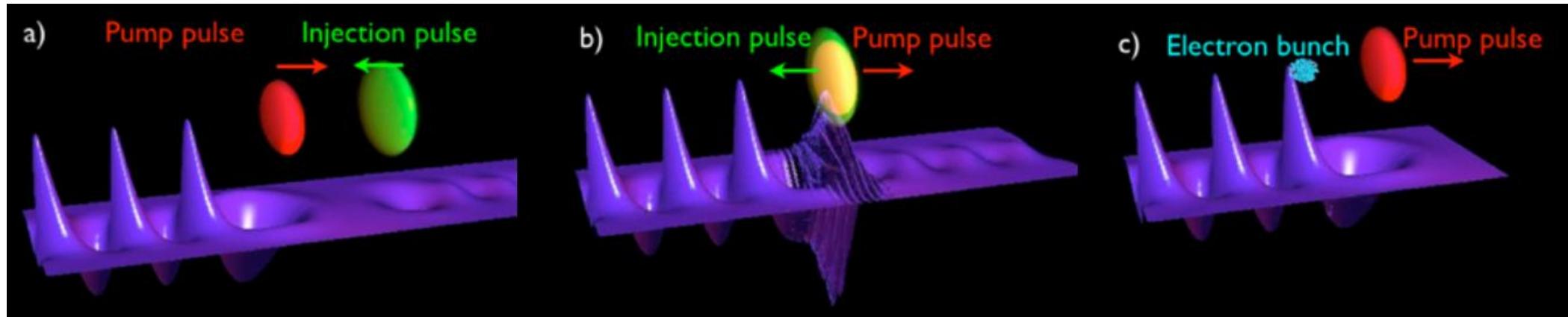


External injection

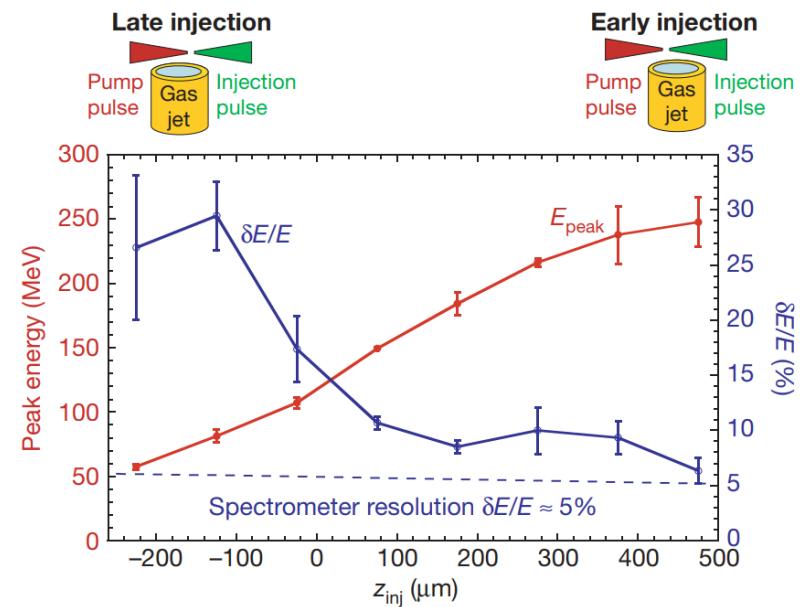


- uses a conventionally produced electron beam and tries to put it into the correct part of the plasma wave
- requires exquisite alignment and timing between the electron beam and the laser
- requires a very small, short electron bunch ($\sigma_z, \sigma_{x,y} < \lambda_p$)
- The idea is to operate in a linear or quasi-linear regime as this is thought to be more stable

Colliding pulse injection

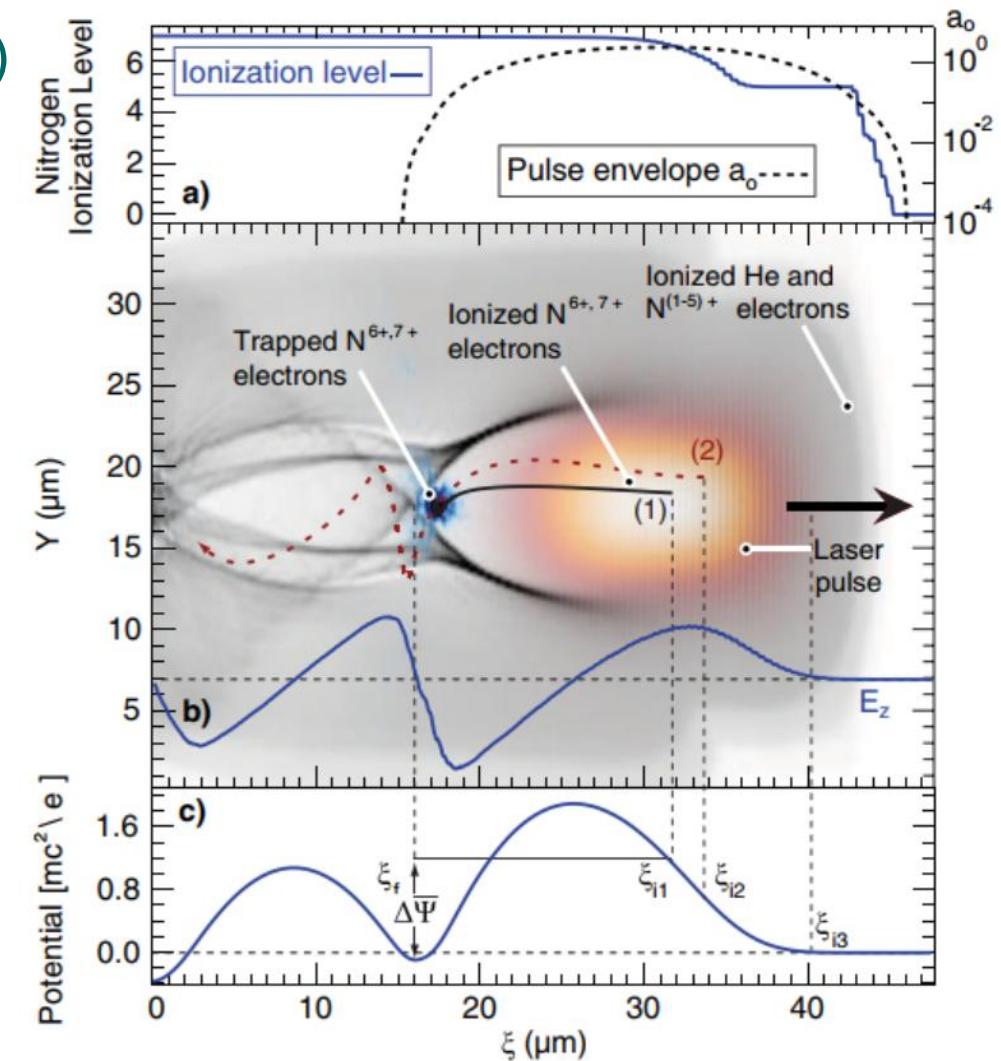
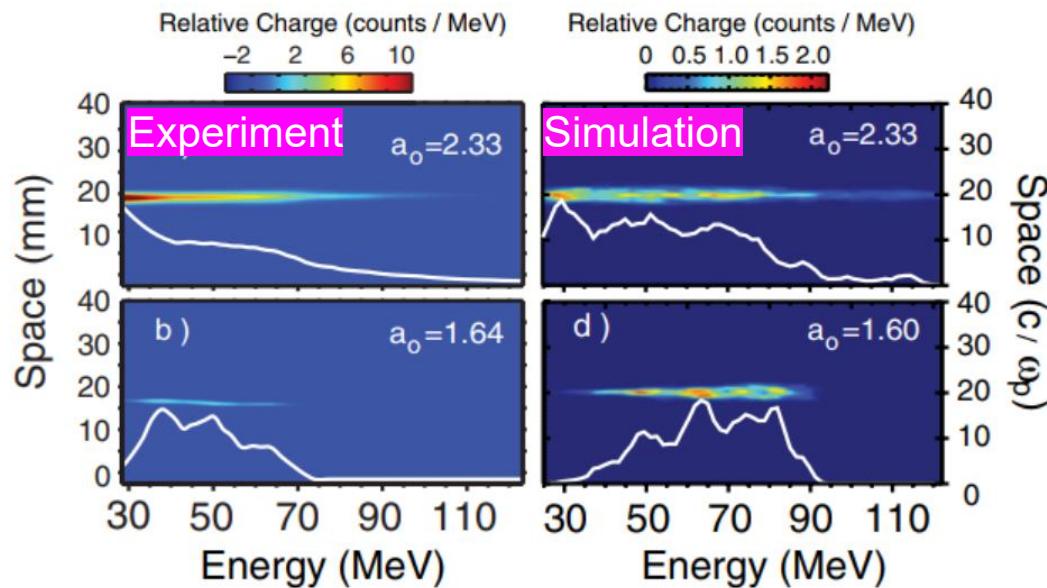


- A moderately intense laser drives a non-linear plasma wave (but below the self-injection threshold)
- A second laser pulse collides with the first, resulting “beatwave”.
- The beatwave pre-accelerates electrons locally and injects
- Injection is local and short (30 fs) → monoenergetic beams



Ionization injection

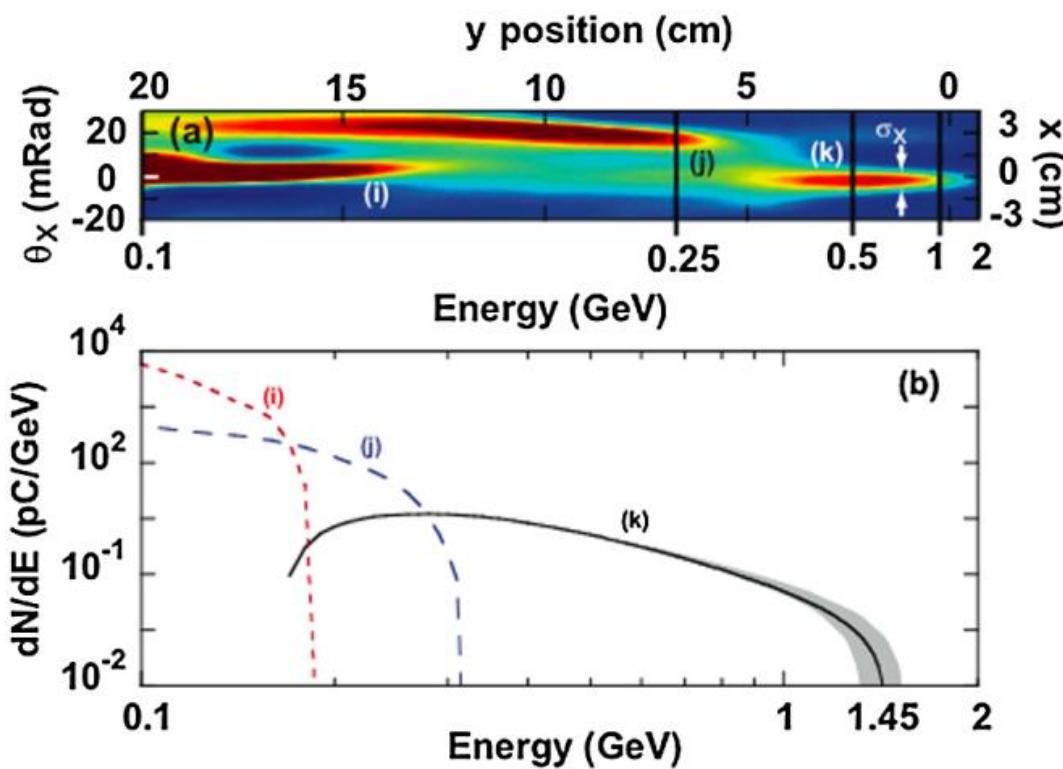
- Ionization of inner shells of high Z atoms (N, Kr, Ar) near the peak laser intensity
- These electrons that are “born” inside the bubble are much more easily trapped
- Ionization injection is continuous, so leading to **high charge (> 100 pC)** but **large energy spread**



Ionization injection

Continuous ionization injection:

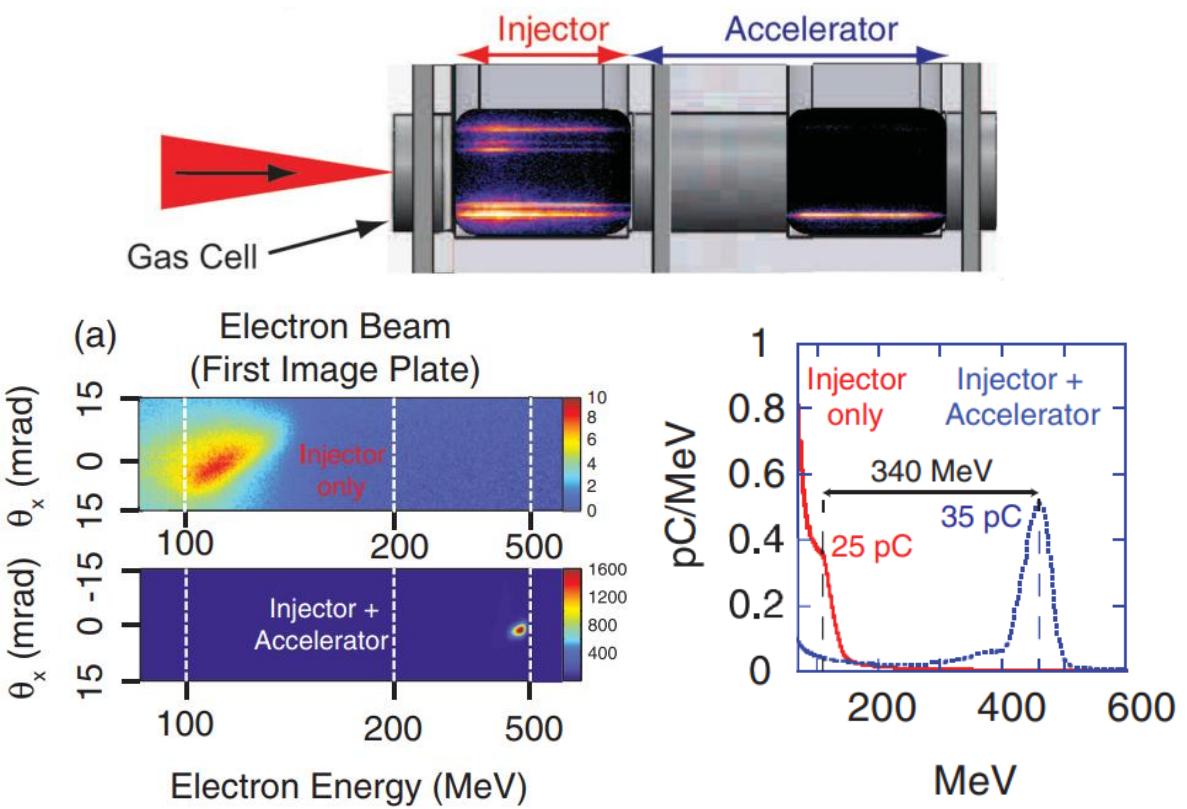
- beyond 1 GeV has been observed



C. E. Clayton et al., *PRL* **105**, 105003 (2010)
B. B. Pollock et al., *PRL* **107**, 045001 (2011)

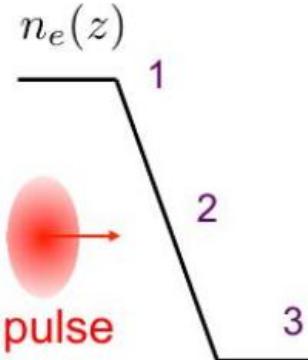
This problem can be overcome by using a two-compartment gas cell

- The first one contains high-Z gas (injector)
- The second one contains pure He (accelerator)

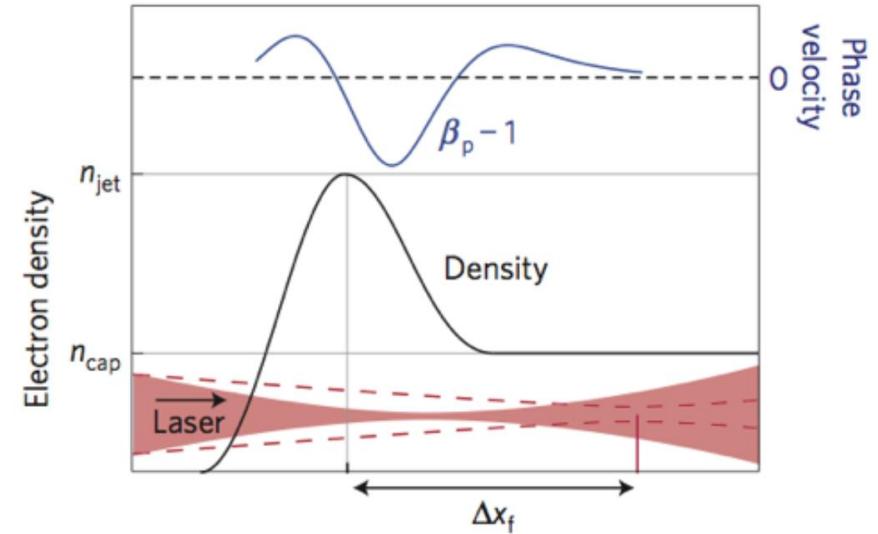
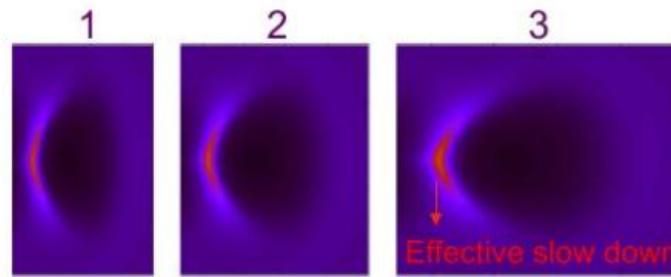


Density gradient injection

key idea: Slow down the plasma wave



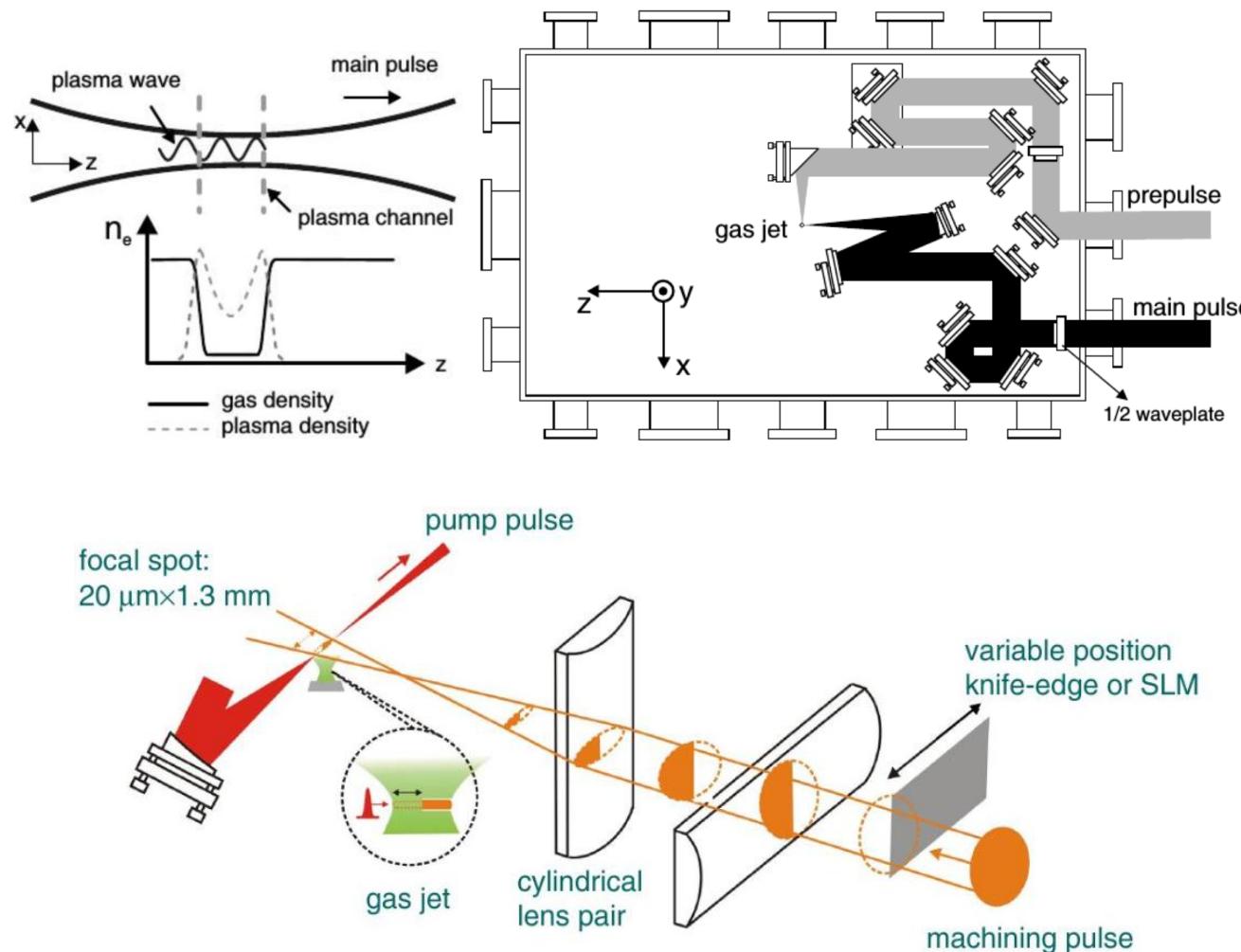
$$\lambda_p(z) \propto 1/\sqrt{n_e(z)}$$



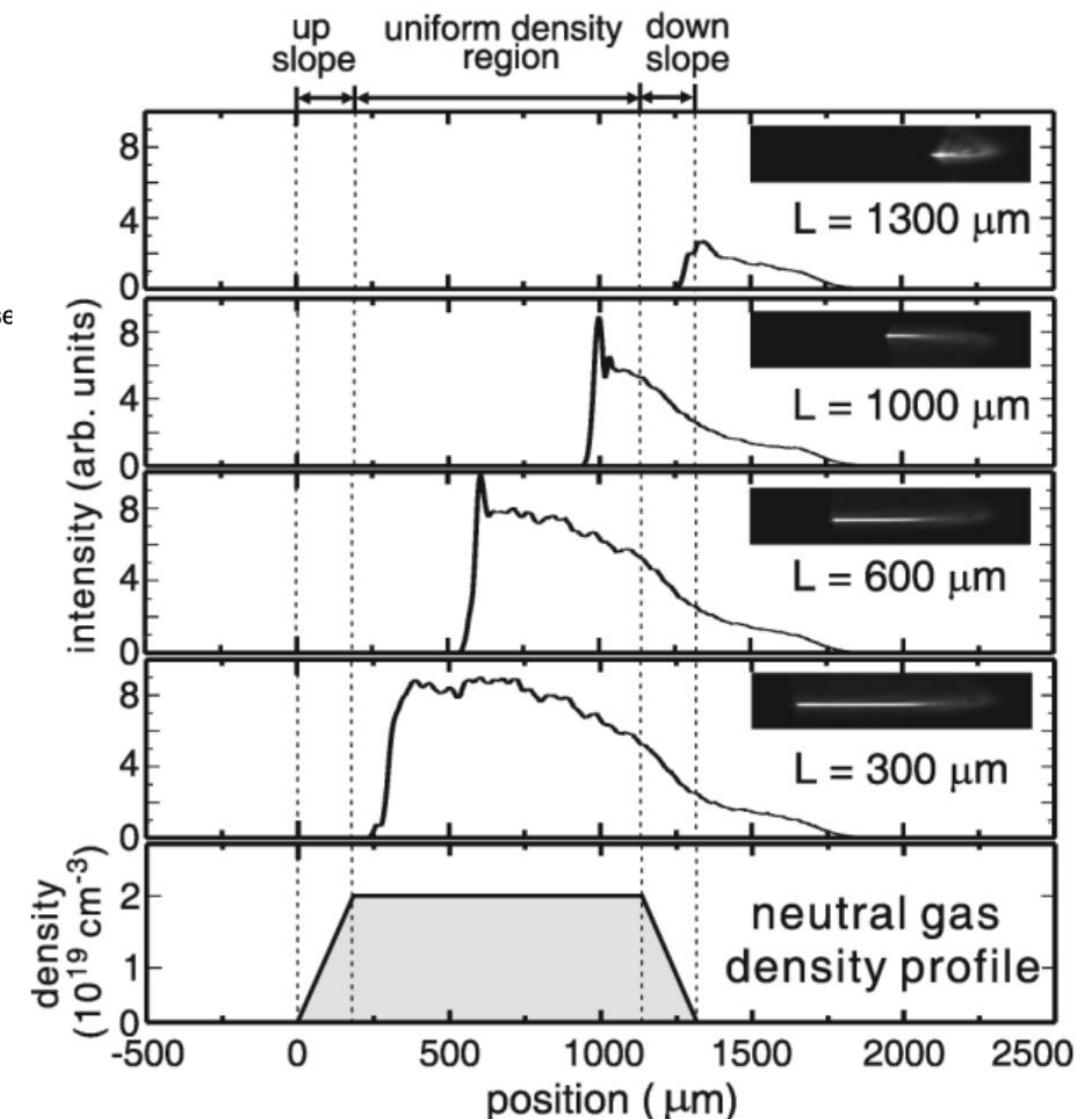
- In the density down-ramp, λ_p increases
 - Cause the plasma wave to elongate
 - Effective slow down of the phase velocity of the plasma wave
- Slowing phase velocity facilitates trapping and decrease the threshold for self-injection
- By tuning the position of the laser focus relative to the down-ramp, the position of injection can be control:

localized injection → stable and low energy spread

Optical-induced density gradient

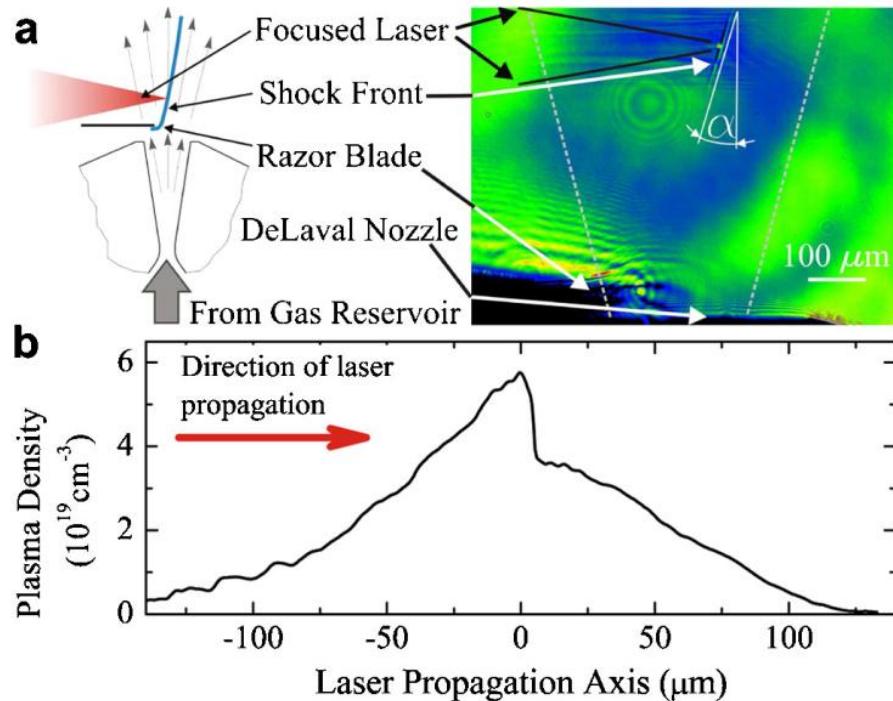


C. T. Hsieh et al., *PRL* **96**, 095001 (2006)



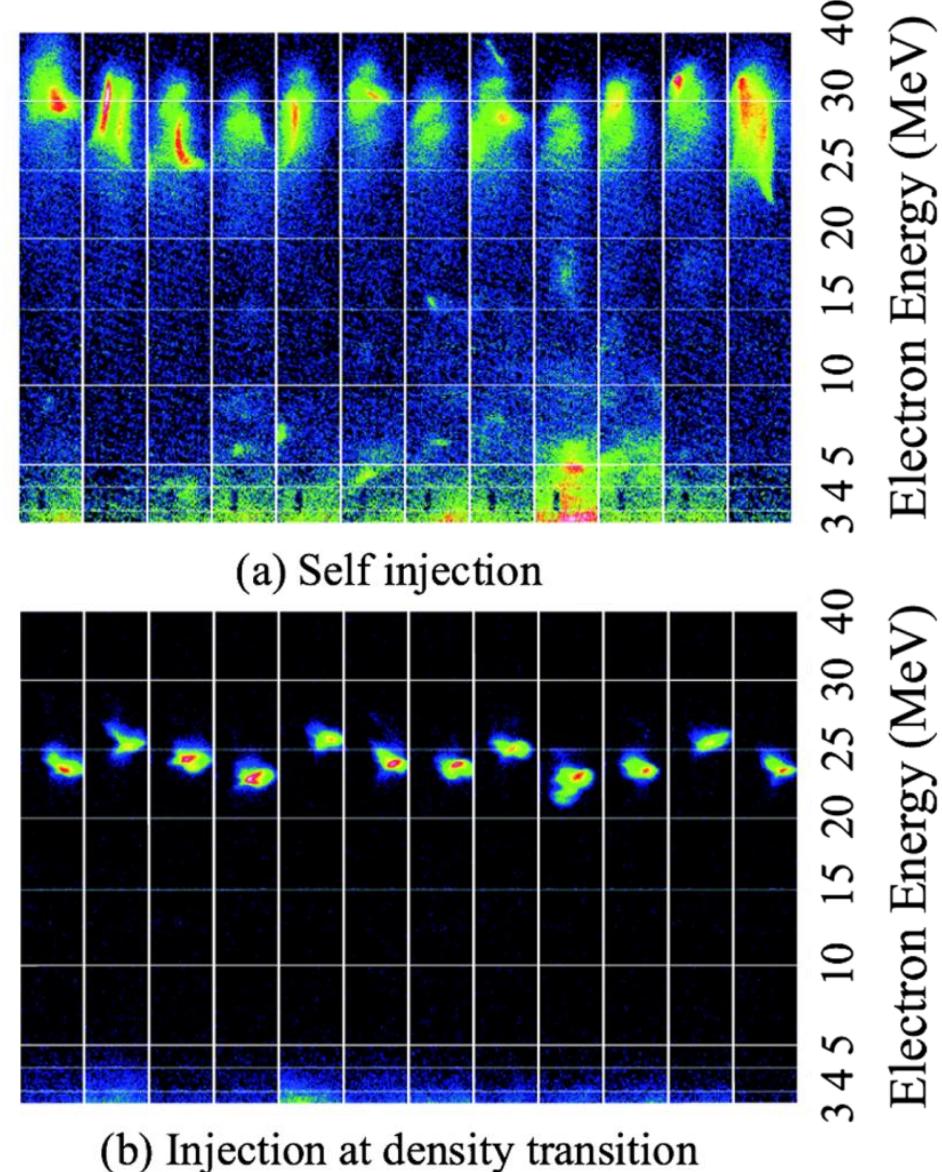
Mechanical-induced density gradient

Shock in the gas flow



stable, relatively narrow energy spread

Schmidt et al., *Phys. Rev. Acc. Beams* 3, 091301 (2010)



Energy gain in blow-out regime

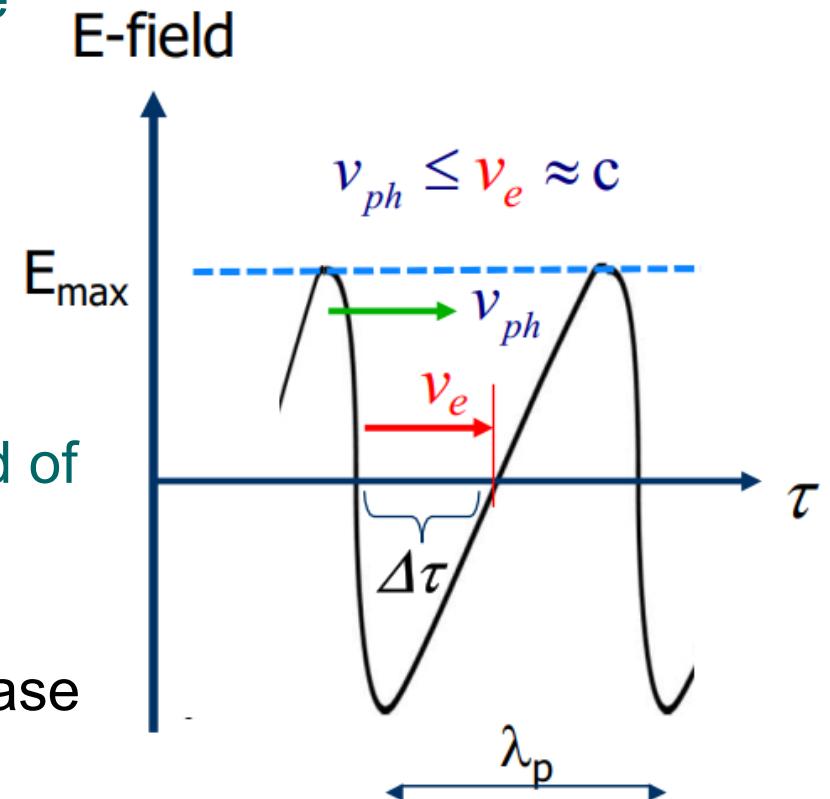
- Using the equation for the electric field and matched bubble radius, we can estimate the field strength of the bubble:

$$E(r) = \frac{en_0 r}{\epsilon_0} \quad r_b \approx 2\sqrt{a_0} \frac{c}{\omega_p} \quad E_{\max} \approx \sqrt{a_0} \frac{m_e c \omega_p}{e}$$

For $a_0 = 3$ and $n_0 = 4 \times 10^{18} \text{ cm}^{-3}$,
the maximum field is 330 GV/m

- Injected electrons are quickly accelerated to near the speed of light, which is faster than the phase velocity of the plasma wave ($v_{ph} < v_e \approx c$)

- The electrons eventually cross from the accelerating phase into the deaccelerating phase of the wave
- Dephasing length (L_d): the maximum distance an electron can be accelerated before it begins to lose energy
- Dephasing is the fundamental limit to energy gain in LWFA

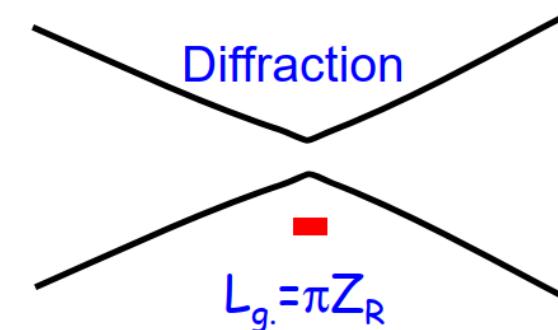


Limitations to acceleration

1. Laser **diffraction**

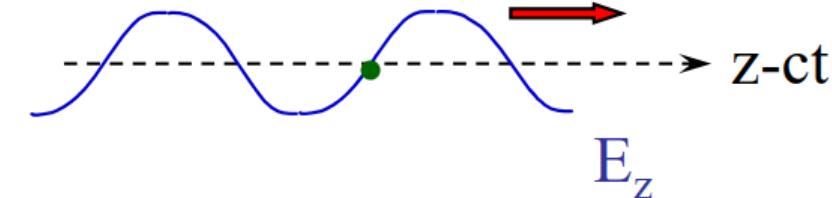
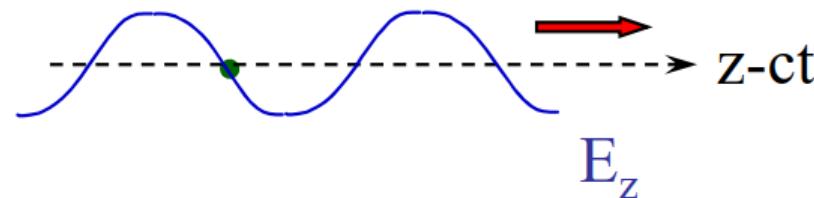
The laser intensity drops over the Rayleigh range

$$Z_R = \frac{\pi w_0^2}{\lambda_L}$$



2. Plasma wave **dephasing** (L_d : dephasing length)

The accelerated electrons outrun the plasma wave



3. Laser energy **depletion** (L_{pd} : pump depletion length)

Laser energy deposition into wave excitation

Dephasing length and depletion length

■ Dephasing length

- The time for trapped electrons to move half a plasma wave out of phase

$$t_d = \frac{\lambda_p}{2c(\beta_e - \beta_p)} \approx \frac{\lambda_p}{c} \frac{n_{cr}}{n_e}$$

where

$$\beta_e \approx 1$$

$$\beta_p = \frac{v_g}{c} = \left(1 - \frac{n_e}{n_{cr}}\right)^{1/2}$$

$$L_d \cong ct_d = \lambda_p \frac{n_{cr}}{n_e}$$

■ Pump depletion length

Electric field energy density of plasma wave:

$$U_{\text{plasma}} = \frac{1}{4} \epsilon_0 E_{z0}^2 \quad E_{z0} = \delta \frac{m_e c \omega_p}{e}$$

Energy in plasma wave cross section A, length L

$$W_{\text{plasma}} = U_{\text{plasma}} A L$$

EM field energy density of laser:

$$U_{\text{laser}} = \frac{1}{2} \epsilon_0 E_0^2 \quad E_0 = a_0 \frac{m_e c \omega_0}{e}$$

Energy in plasma wave cross section A, duration τ

$$W_{\text{laser}} = U_{\text{laser}} A c \tau$$

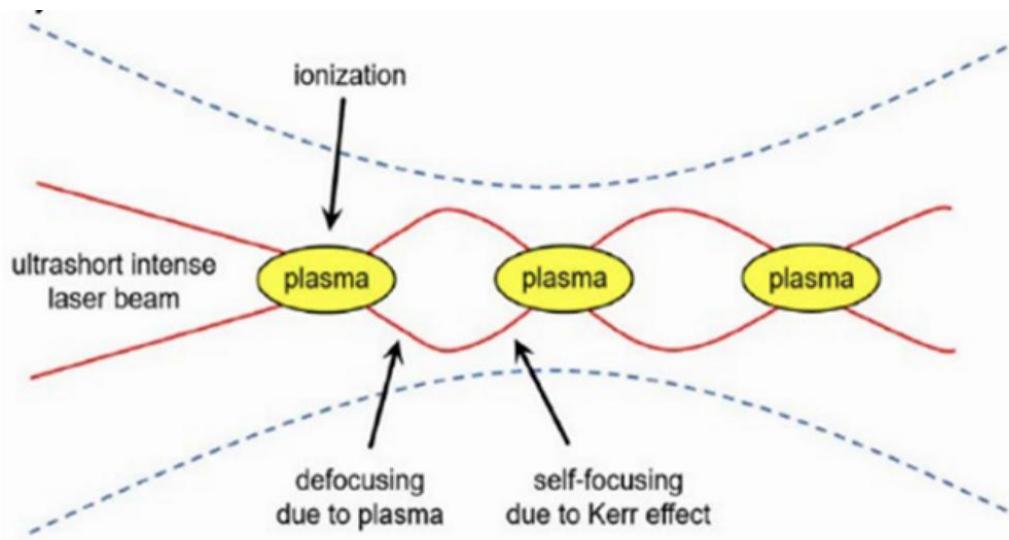
$$L_{pd} = \frac{c U_{\text{laser}}}{U_{\text{plasma}}} = 2 \epsilon \left(\frac{a_0}{\delta}\right)^2 \frac{n_{cr}}{n_e} \lambda_p$$

It is optimal to tailor parameters to match L_d and L_{pd}

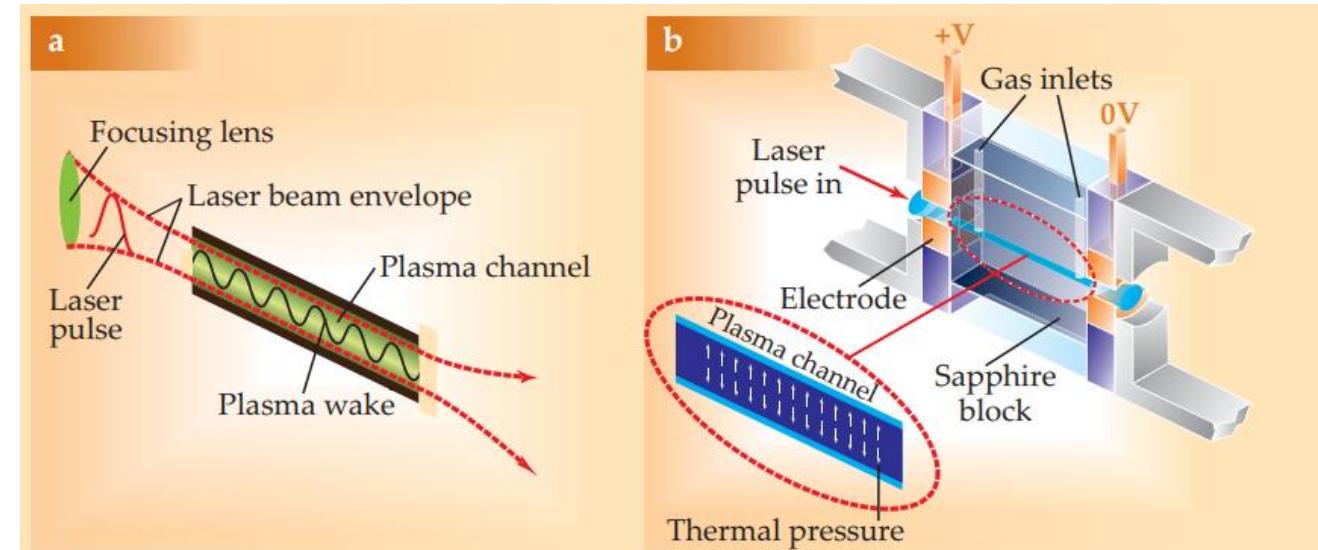
Increase the guiding distance

■ To overcome the diffraction

Self-guiding (relativistic self-focusing)



Through pre-formed plasma channel (plasma waveguide)



hydrodynamic
waveguide

capillary discharge
waveguide

Ponderomotive self-channeling

Expulsion of electrons

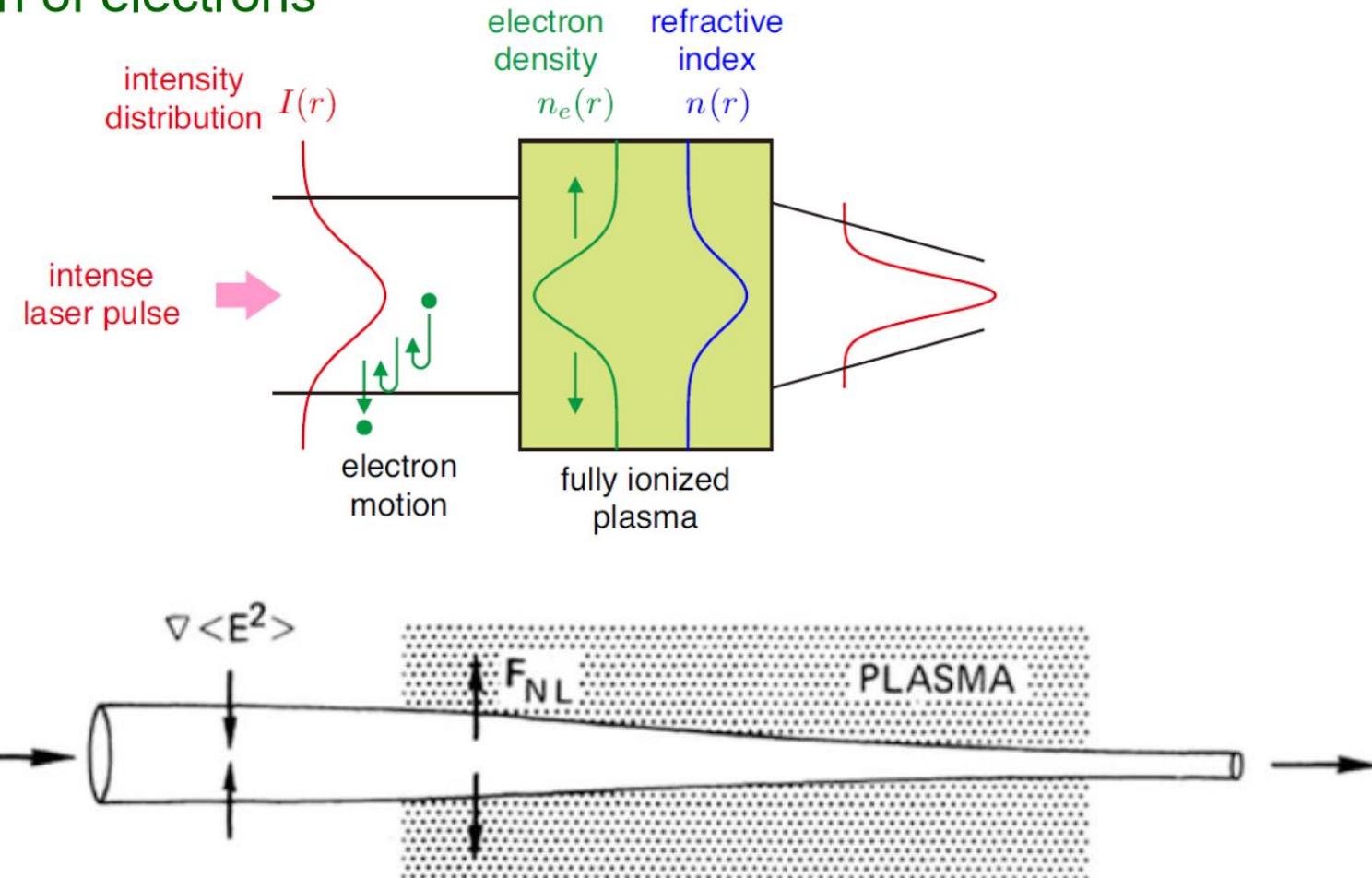
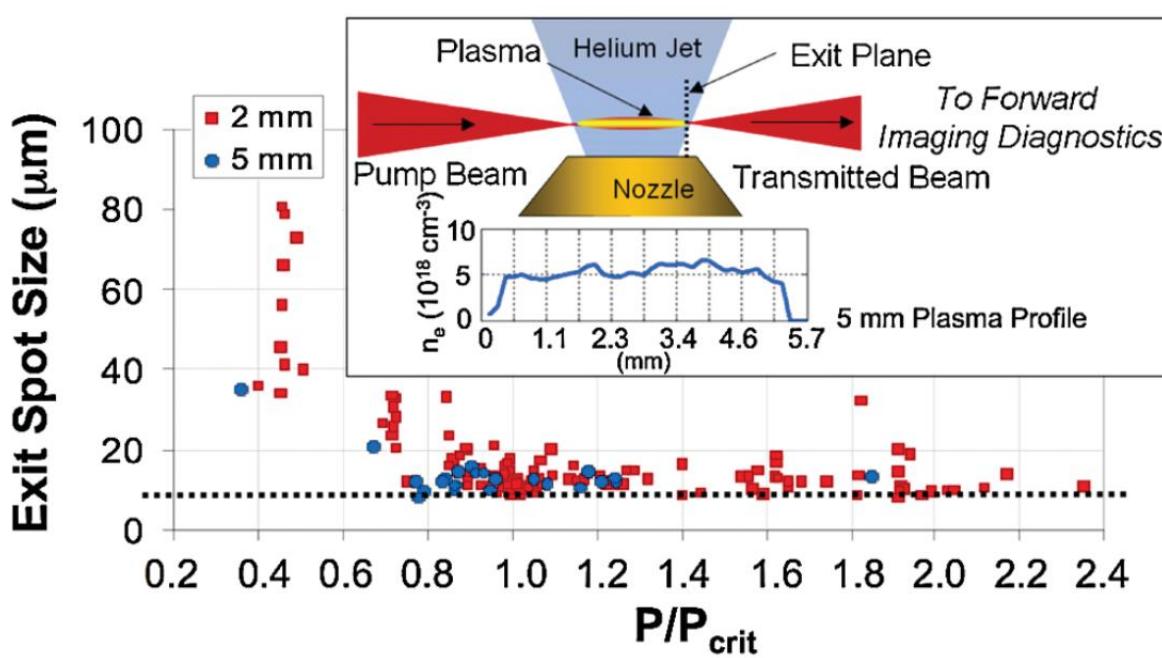
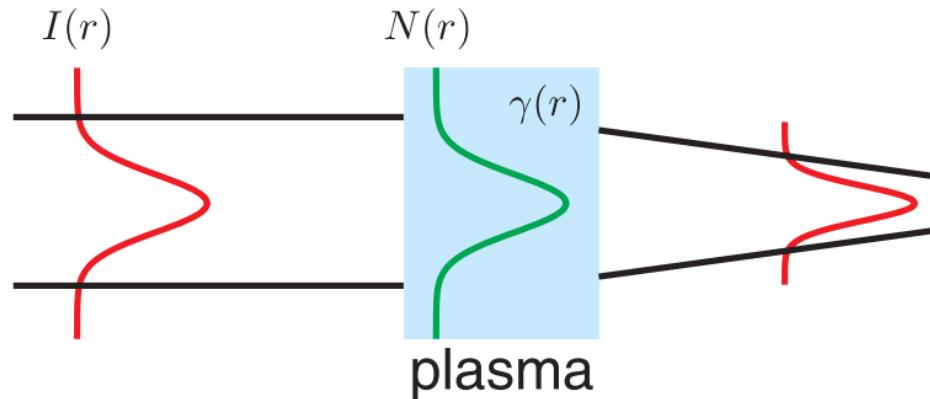


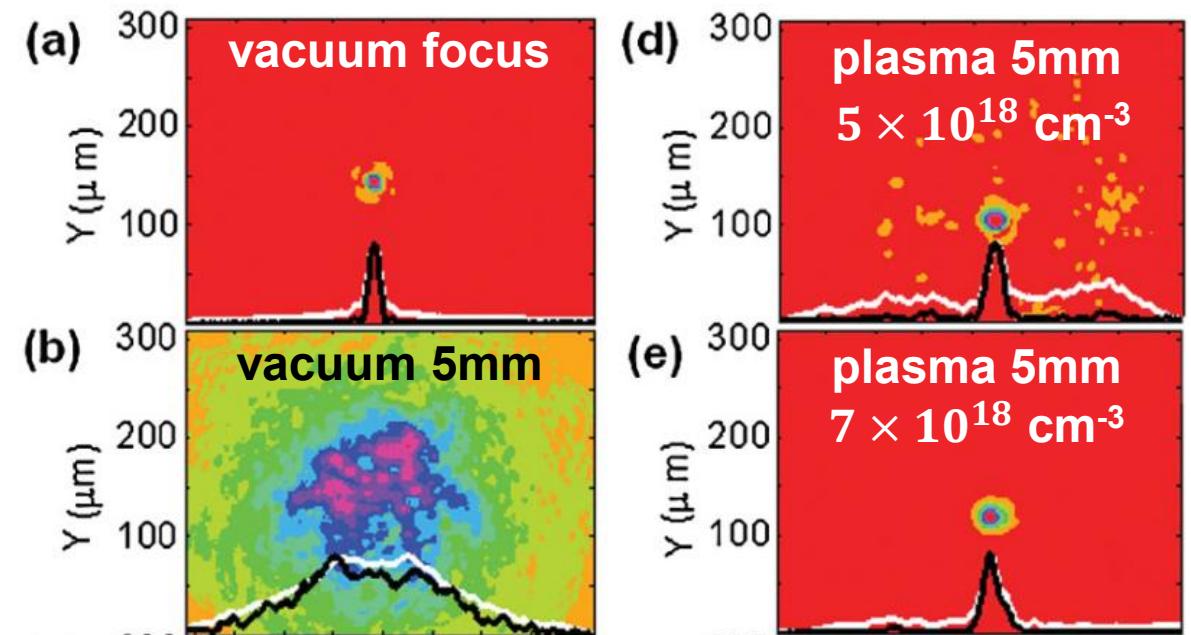
Fig. 8.12 Self-focusing of a laser beam is caused by the ponderomotive force

Relativistic self-guiding

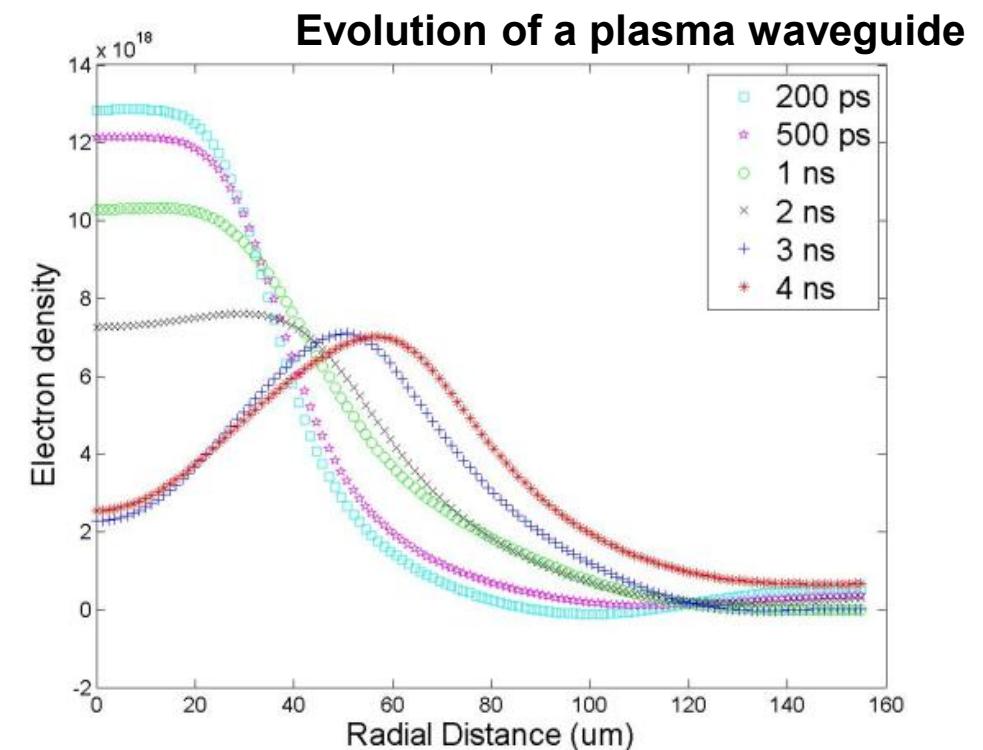
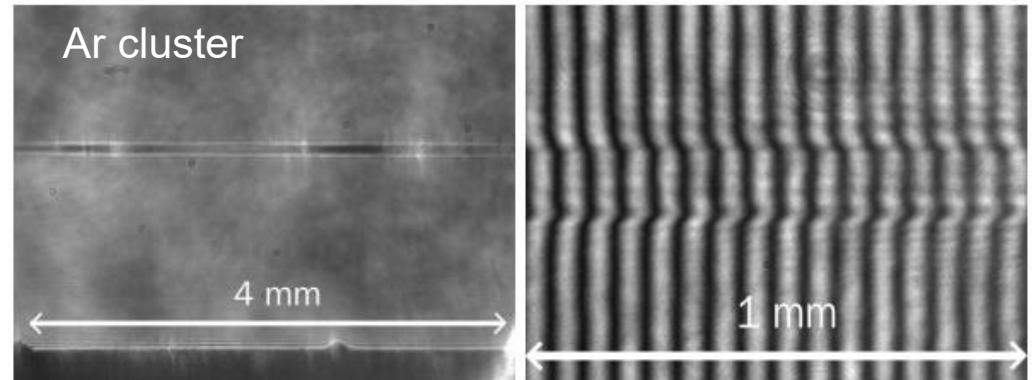
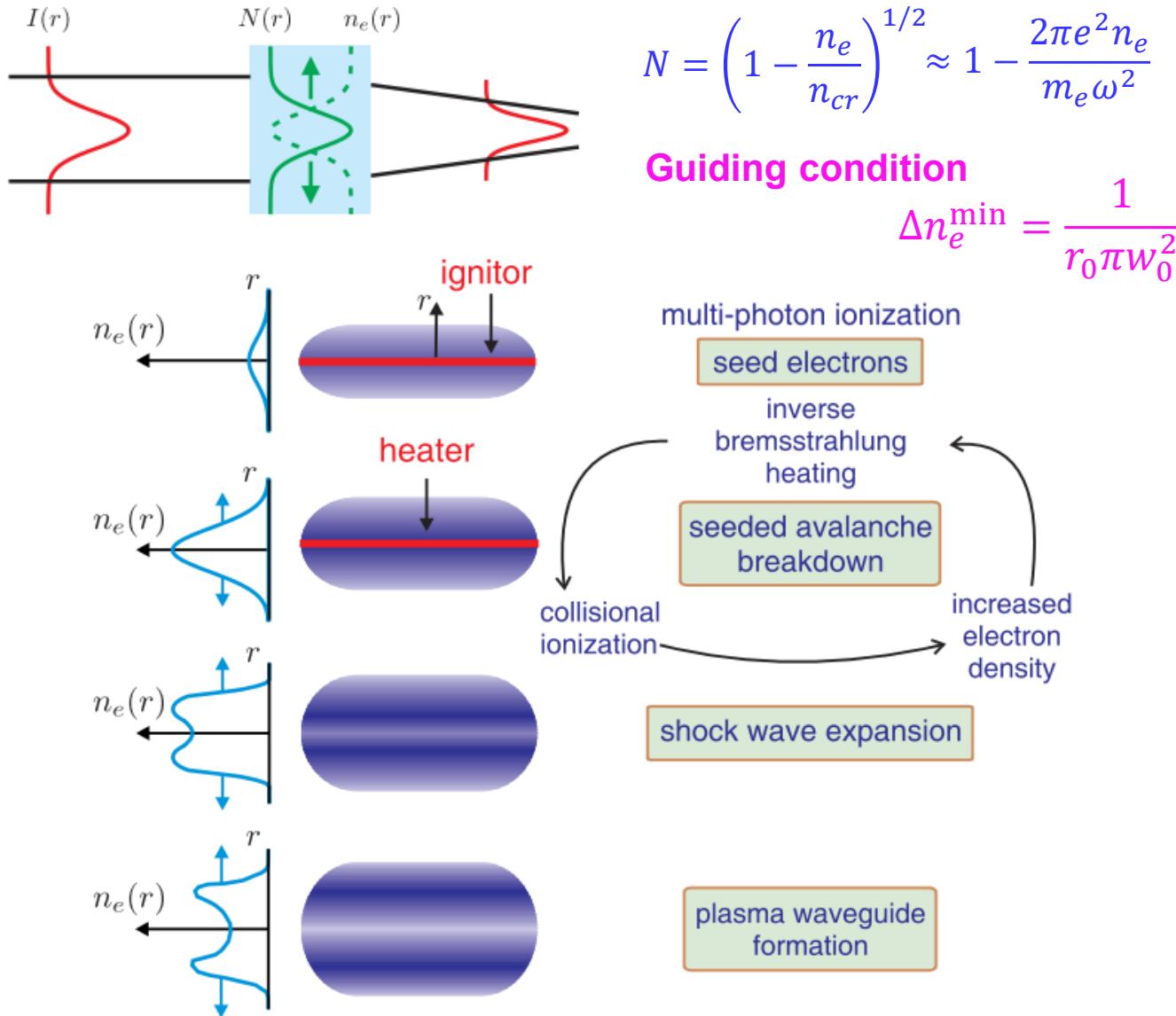


$$n = \left(1 - \frac{\omega_p^2}{\gamma \omega^2} \right)^{1/2}$$

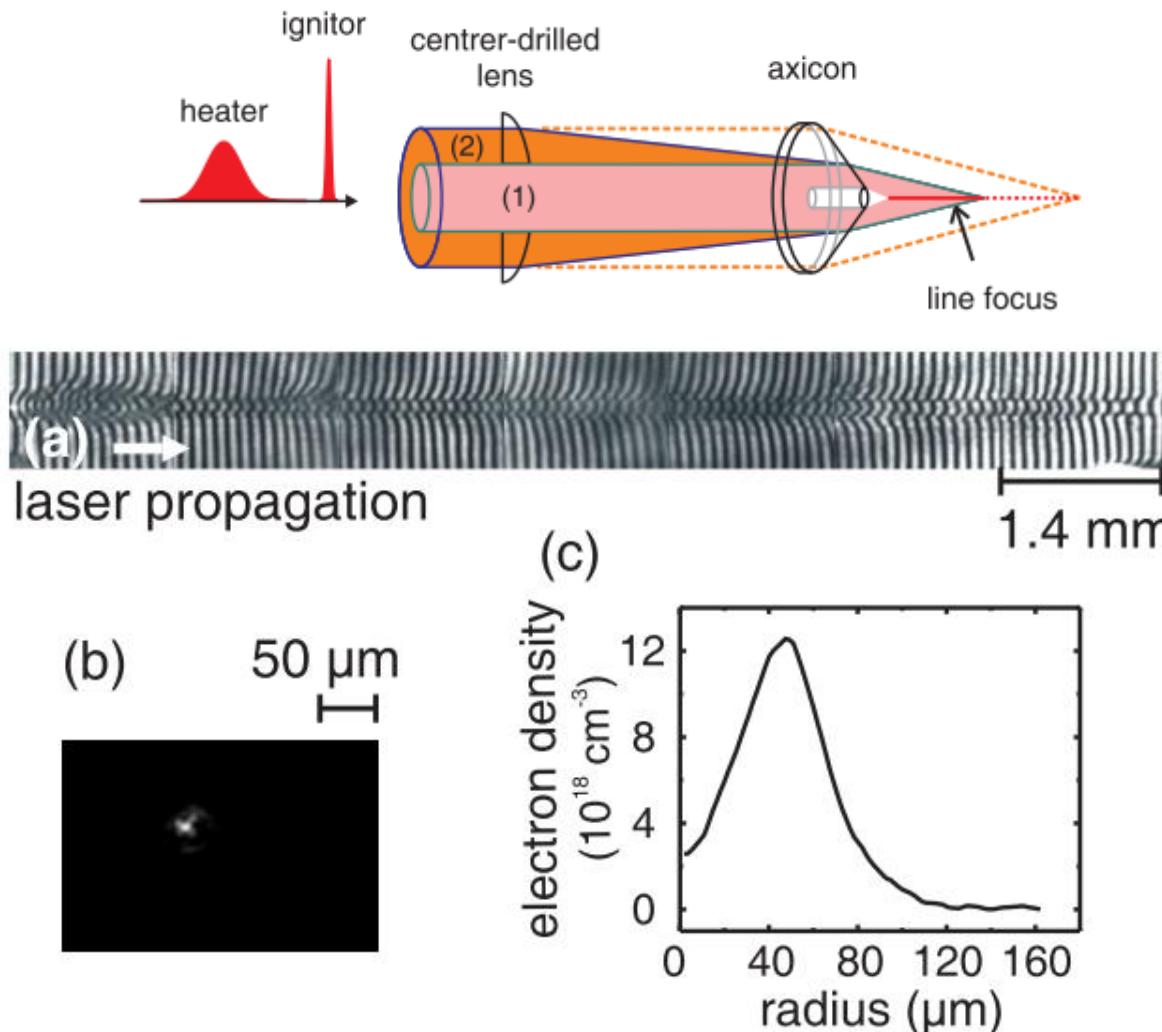
Critical power $P_c \approx 17 \left(\frac{\omega}{\omega_p} \right)^2 \text{ GW}$



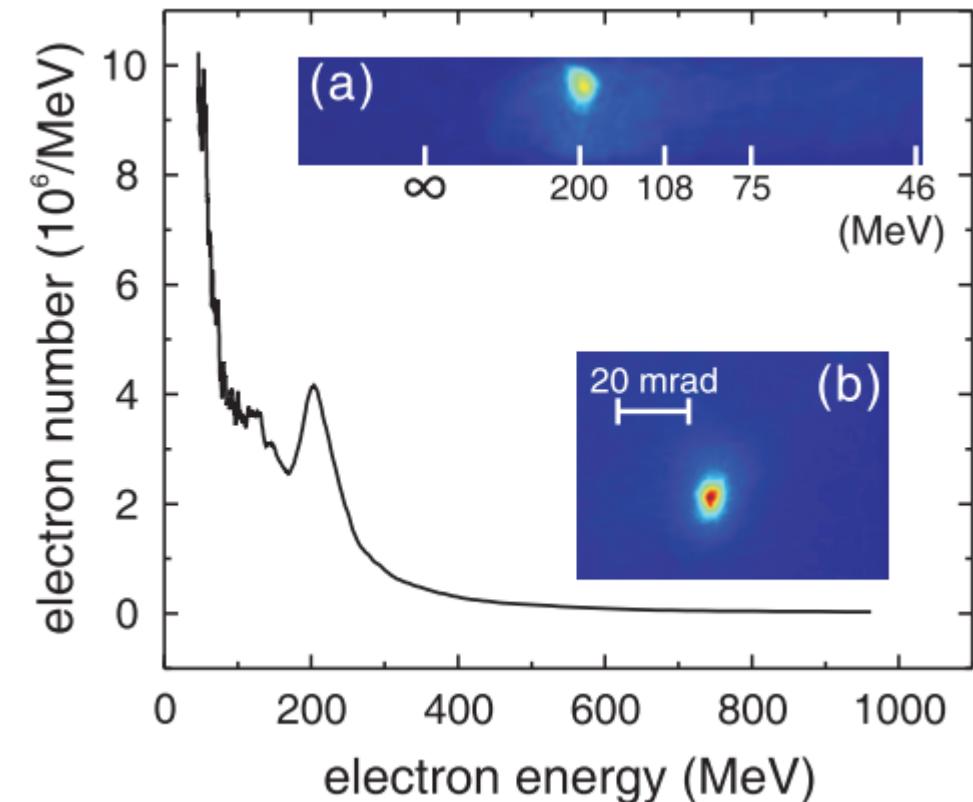
Hydrodynamic plasma waveguide



Hydrodynamic plasma waveguide

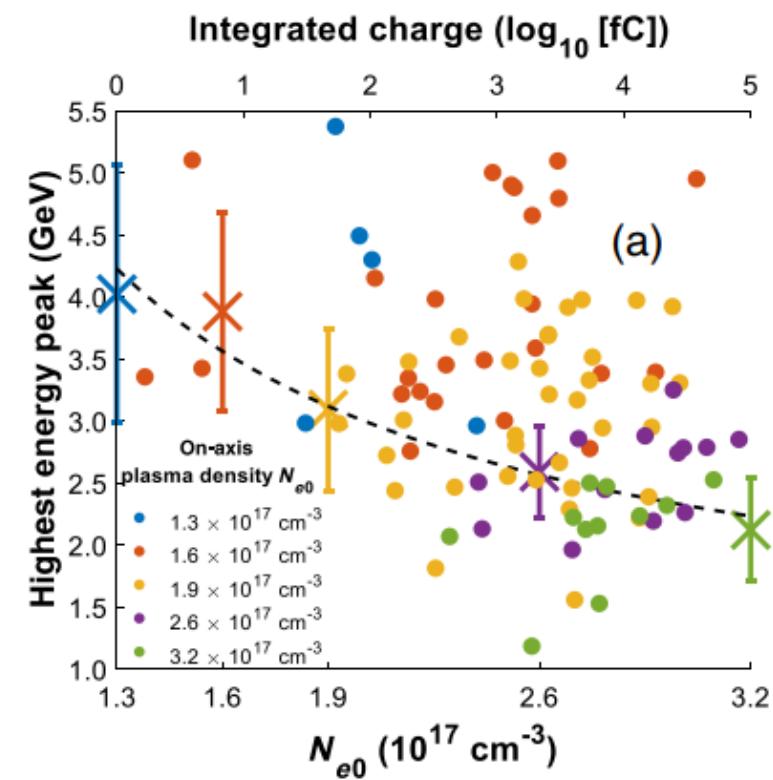
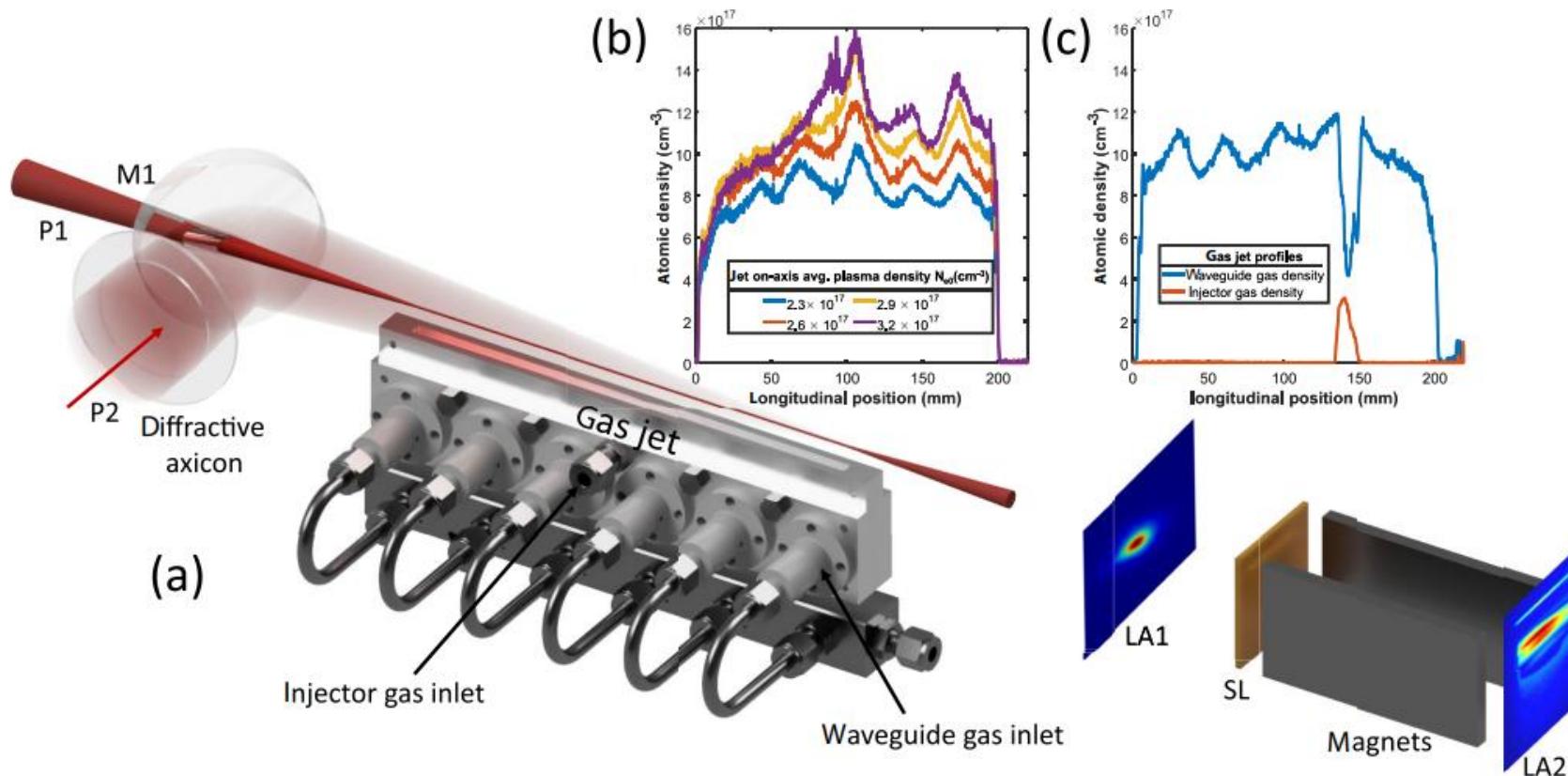


Mono-energetic electron beam at NCU



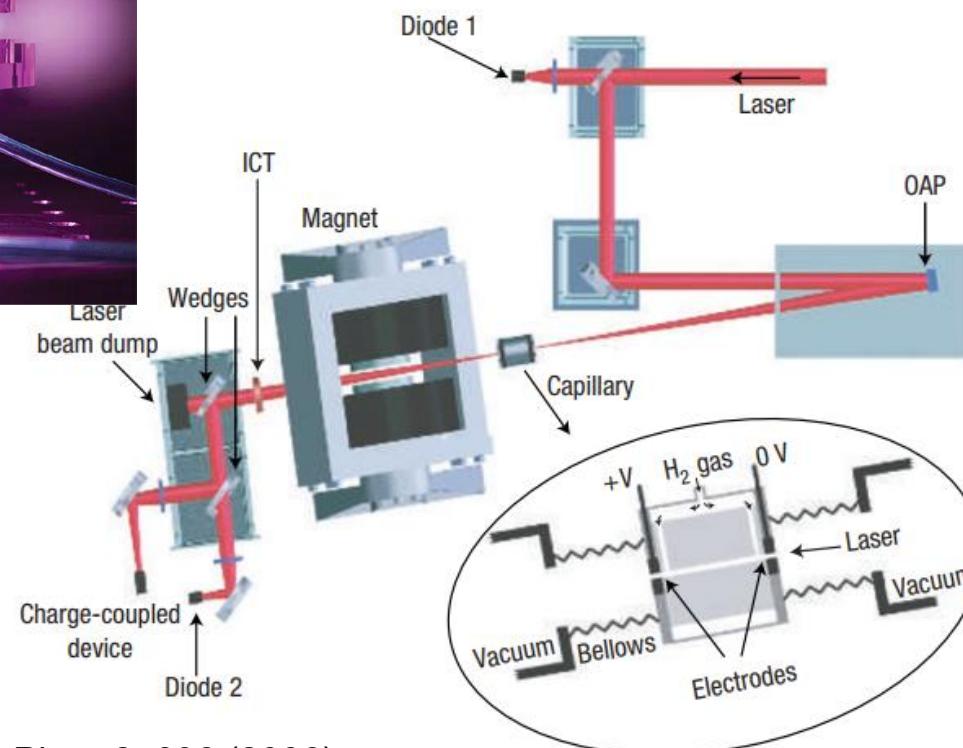
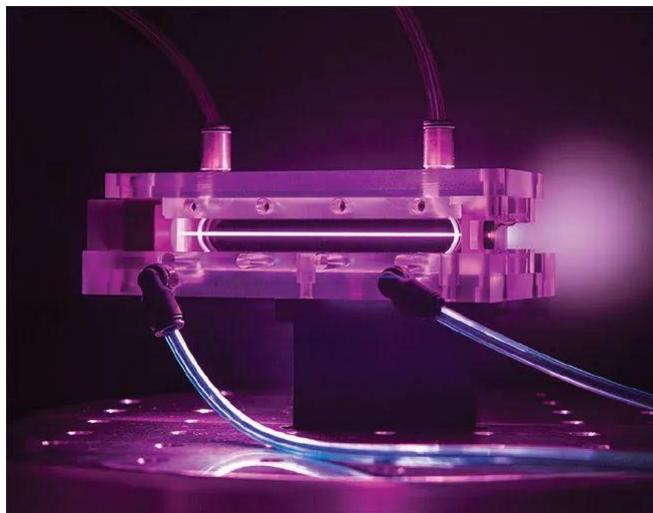
Hydrodynamic plasma waveguide

- Meter-long hydrodynamic plasma waveguide and multi-GeV electron acceleration has been demonstrated

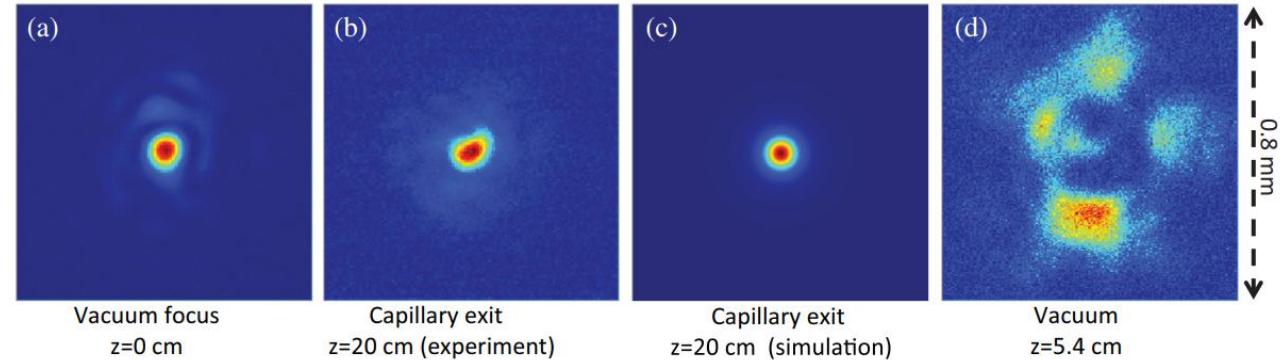


Capillary discharge waveguide

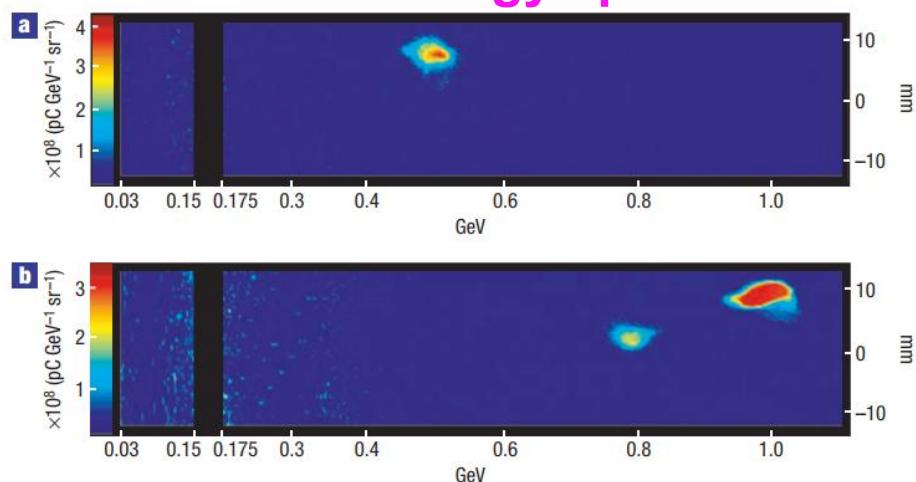
9 mm capillary discharge
at Berkley lab



Guiding:



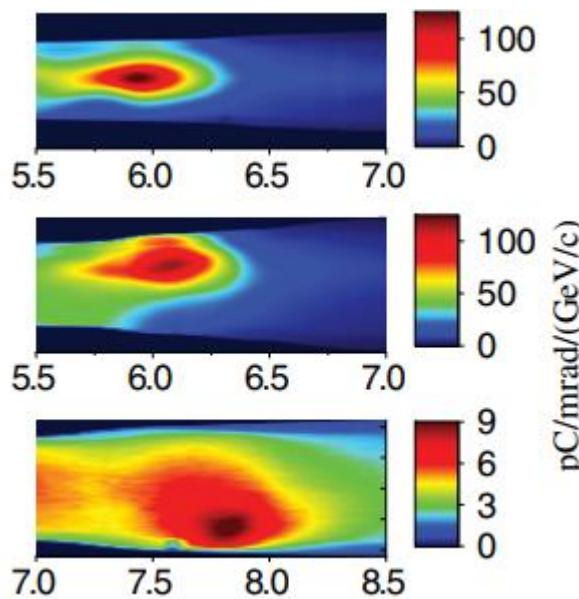
Electron energy spectra:



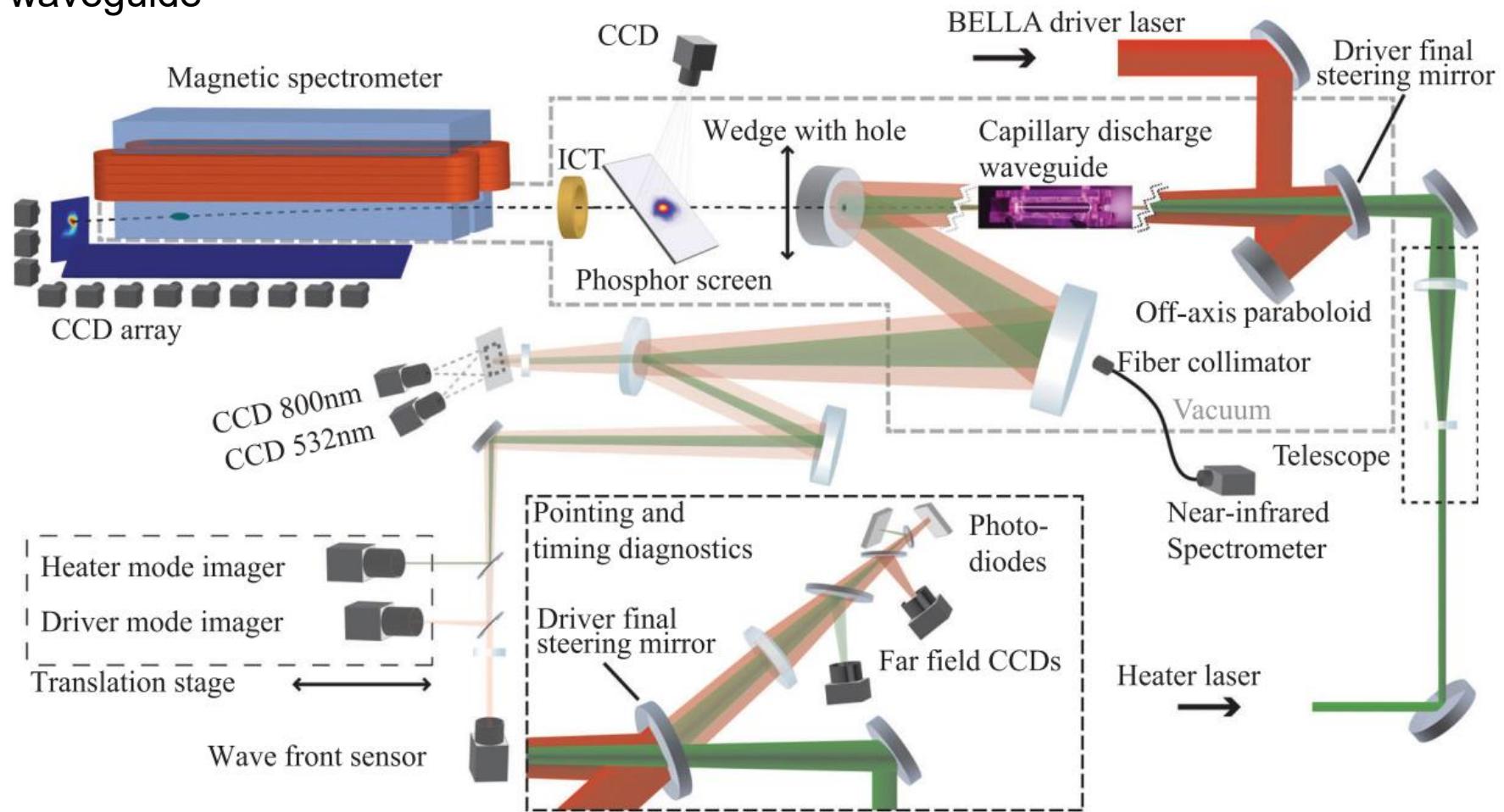
First demonstration of
GeV electron from LWFA

Capillary discharge waveguide

- BELLA laser beam: 31-J, 35-fs, 0.85-PW pulse
- 20-cm capillary discharge waveguide



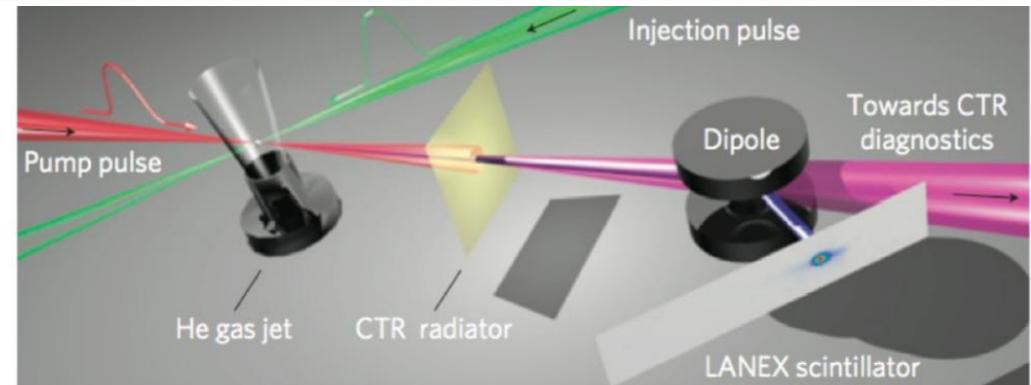
A. J. Gonsalves et al., *PRL*. **122**, 084801 (2019)



Laser plasma e-beams

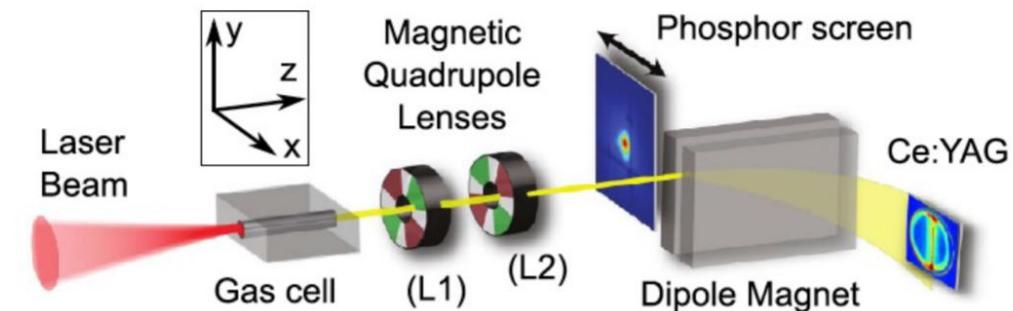
1. Ultra-short (~fs) bunches

CTR spectrum measurement
1.4 fs (RMS) duration; 4 kA



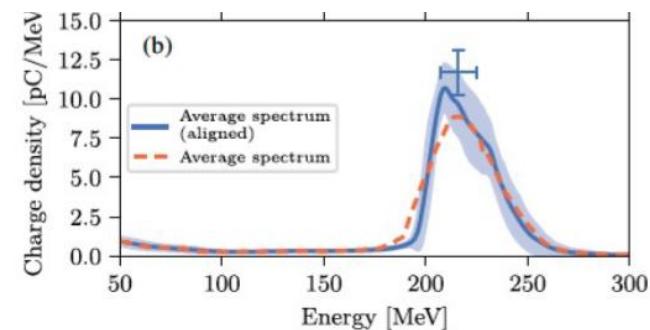
2. Sub-micron emittance

Quadrupole energy scan
0.2 μm normalized emittance at 245 MeV



3. High-charge operation

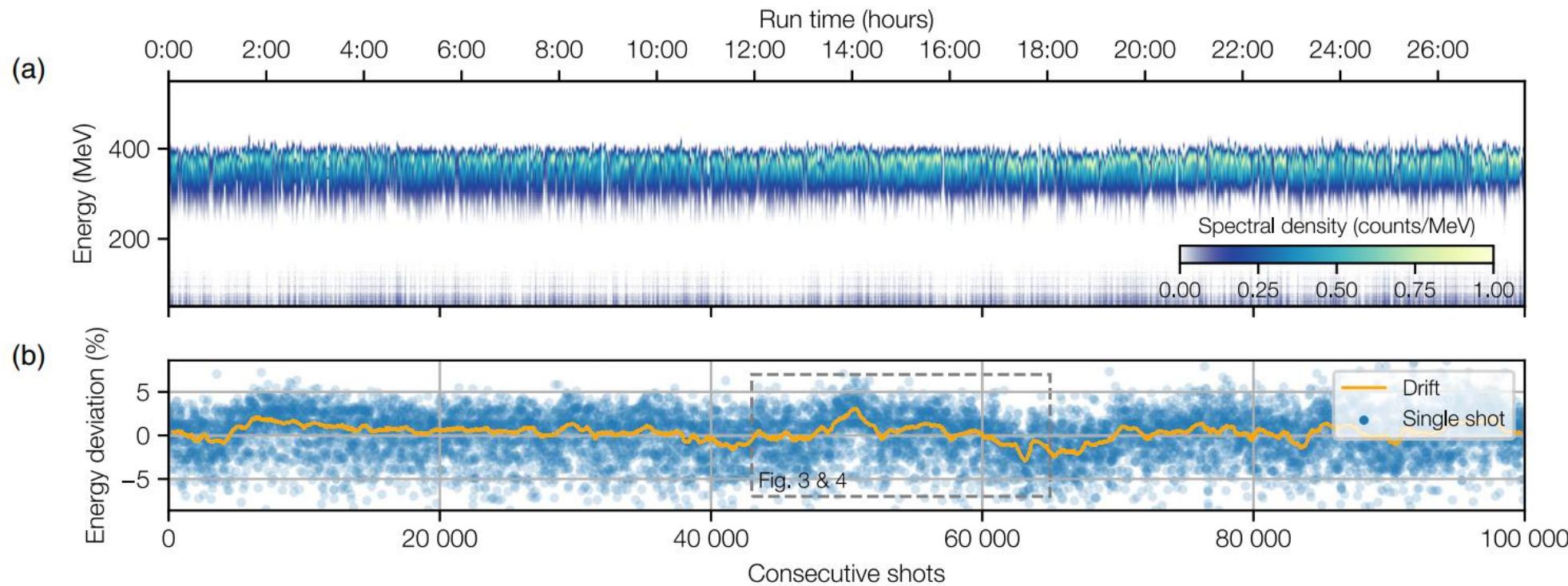
Density gradient injection achieved 338 pC
 $dE/E=15\%$; $I > 10$ kA



However, they are usually mutually exclusive

Demonstration of stable operation

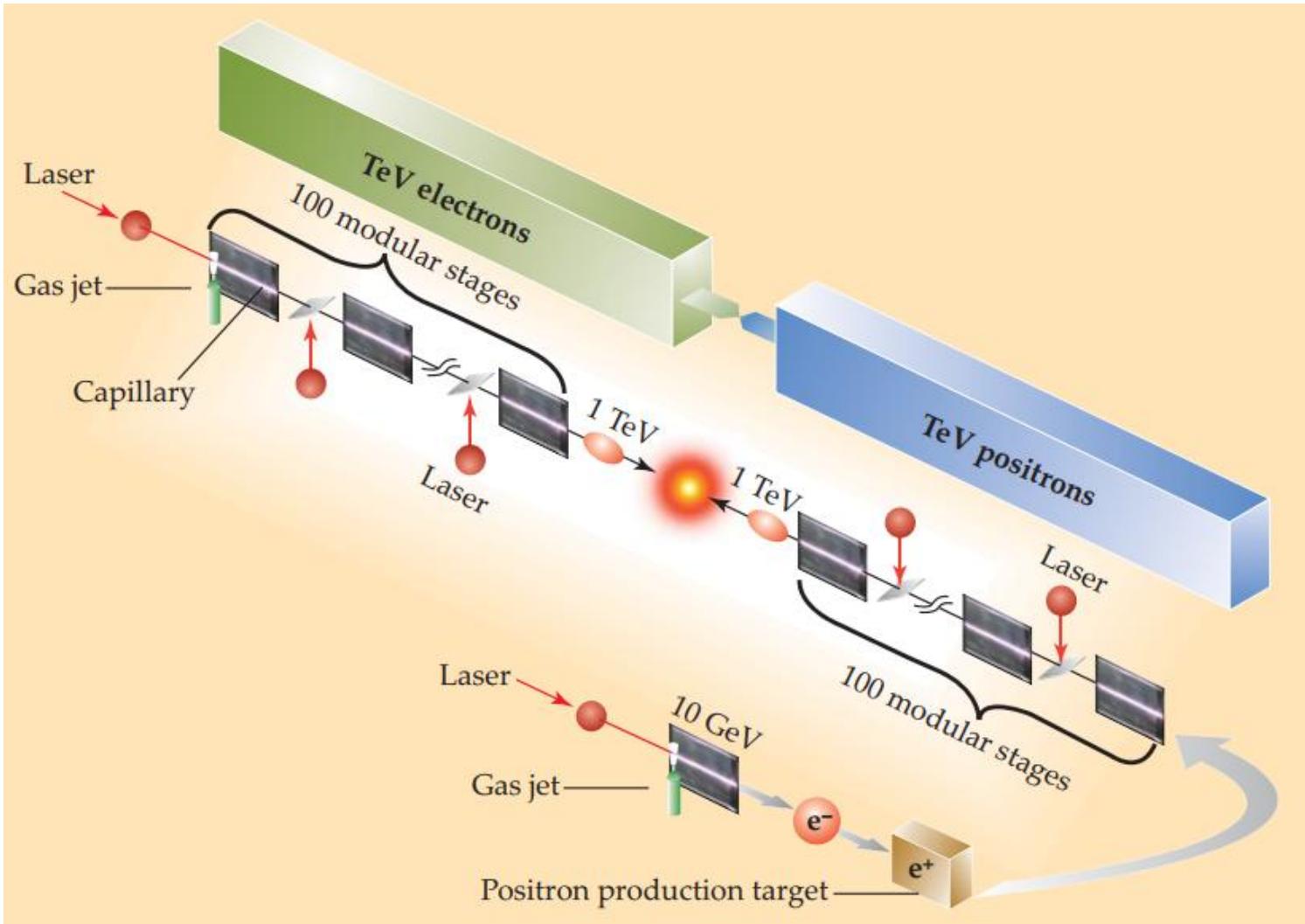
- A Milestone: Demonstrate the first continuous 24-hour operation of an LPA, producing 100,000 consecutive electron beams at a 1-Hz repetition rate.



- Using gas cell target, and active feedback control
- Energy fluctuations correlates with laser energy fluctuation and pointing instability

A. R. Maier et al., *PRX* **10**, 031039 (2020)

Application: TeV electron-positron collider

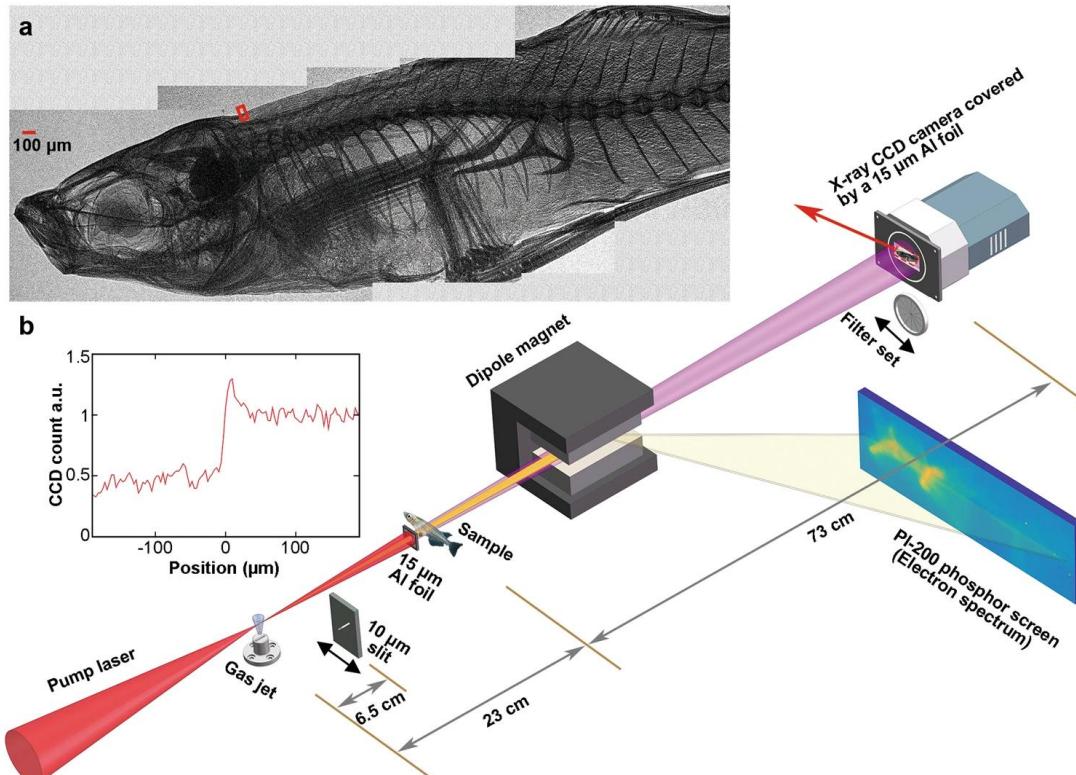


- Laser plasma accelerators can accelerate both electrons and positrons at high acceleration gradient
- **Scale through staging:** each arm with a string of acceleration modules
- This staging capability has been successfully shown in proof-of-concept experiments

Application: Tabletop femtosecond light sources

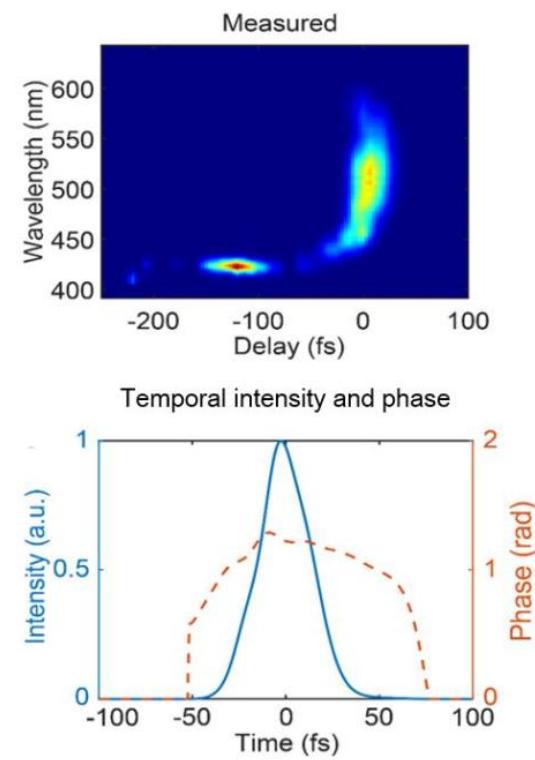
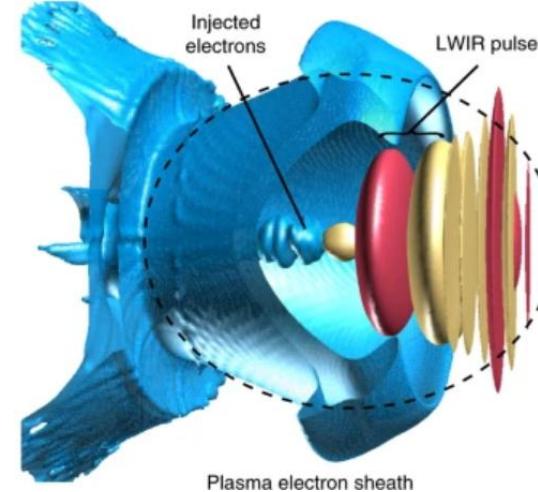
- Laser plasma accelerators can produce femtosecond hyperspectral radiation as an integral part of the laser-plasma interaction

X-rays (wavelength: 0.1 nm)



Scientific Reports 9, 7796 (2019)

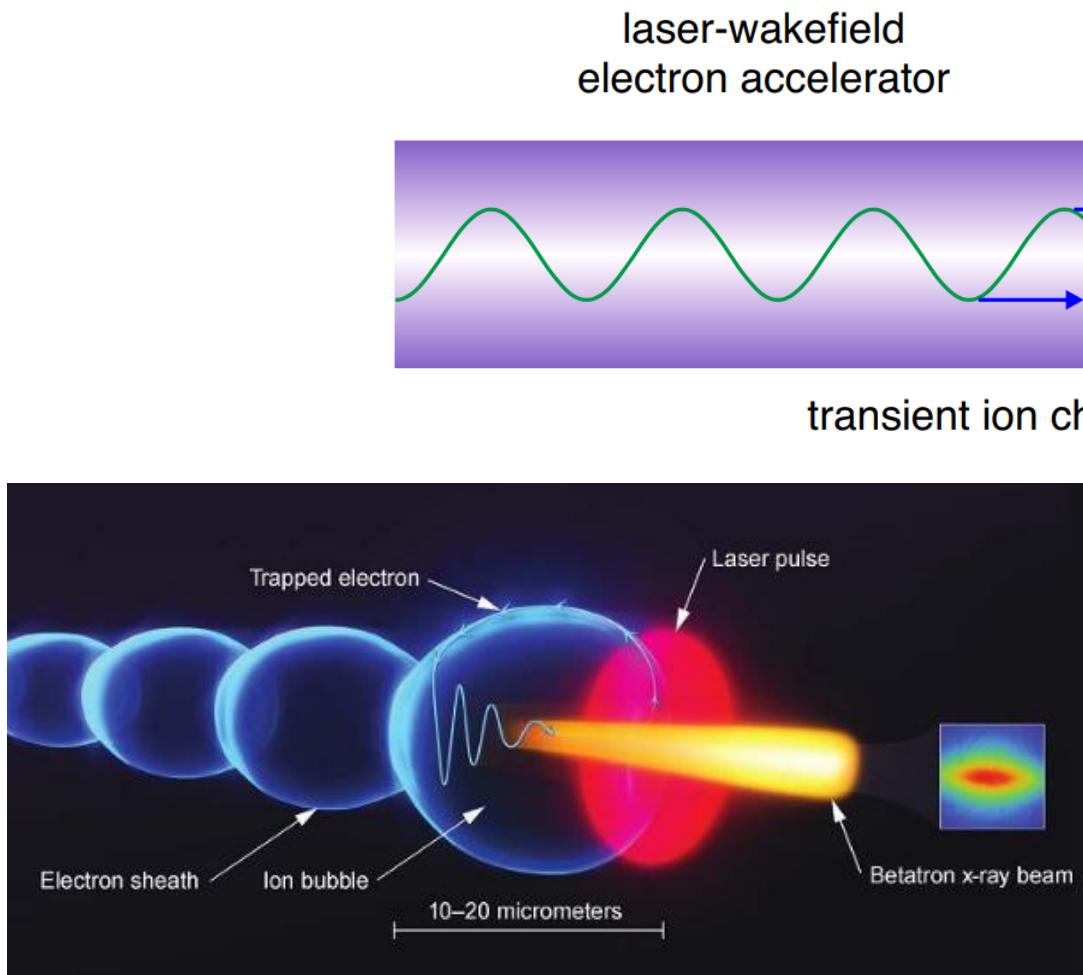
Single-cycle THz (wavelength: 10 μm)



Nat. Photonics 12, 489 (2018)

Nat. Comms. 11, 2787 (2020)

Betatron radiation from LWFA



period of betatron oscillation N_0

electron number N_e

$$\text{strength parameter } K = \frac{\gamma_b \omega_\beta r_0}{c}$$

$$\text{betatron oscillation frequency } \omega_\beta = \frac{\omega_p}{\sqrt{2\gamma_b}}$$

$$\text{x-ray critical frequency } \omega_c = \frac{3}{2} \frac{\gamma_b^3 r_0 \omega_\beta^2}{c}$$

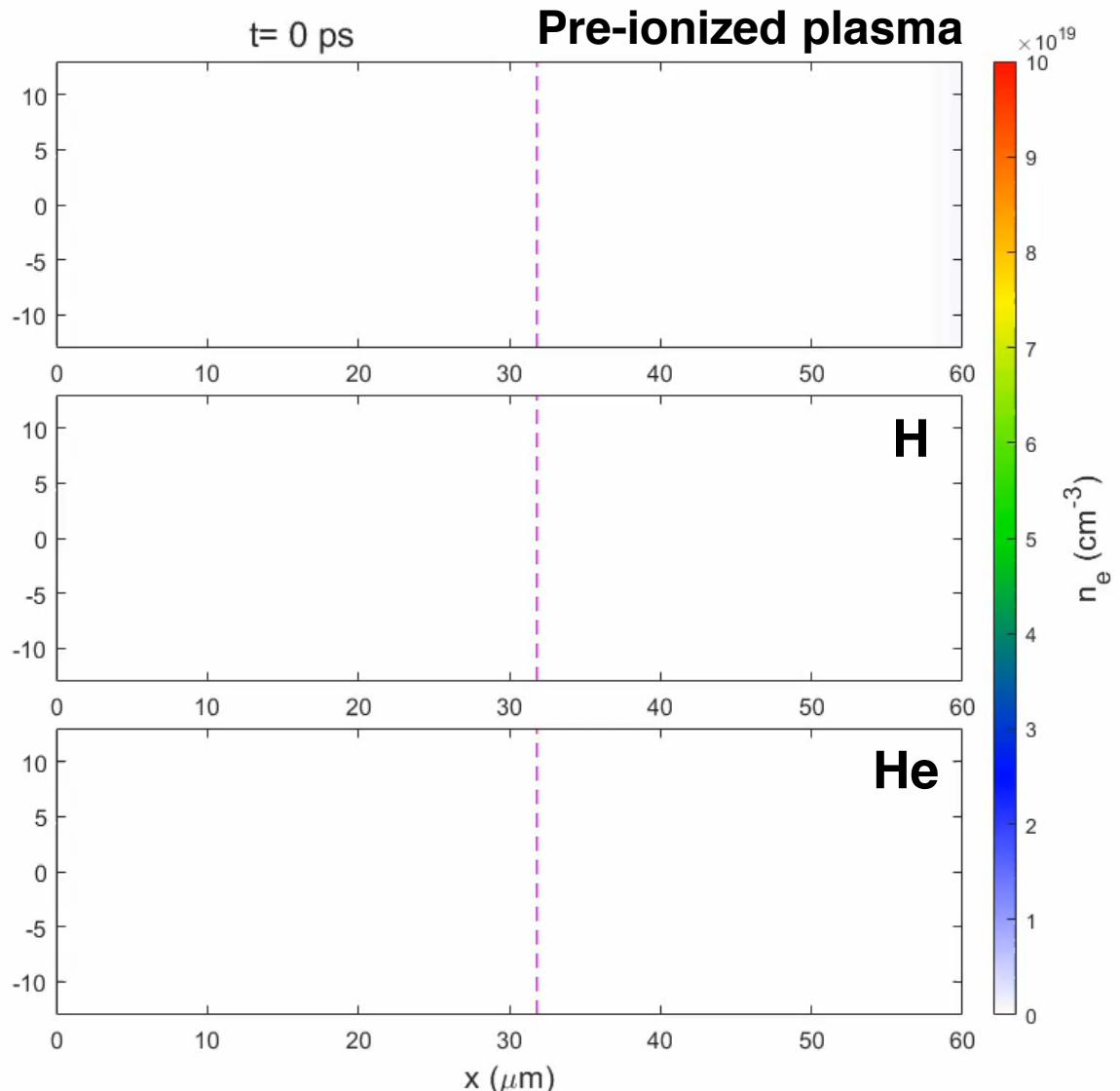
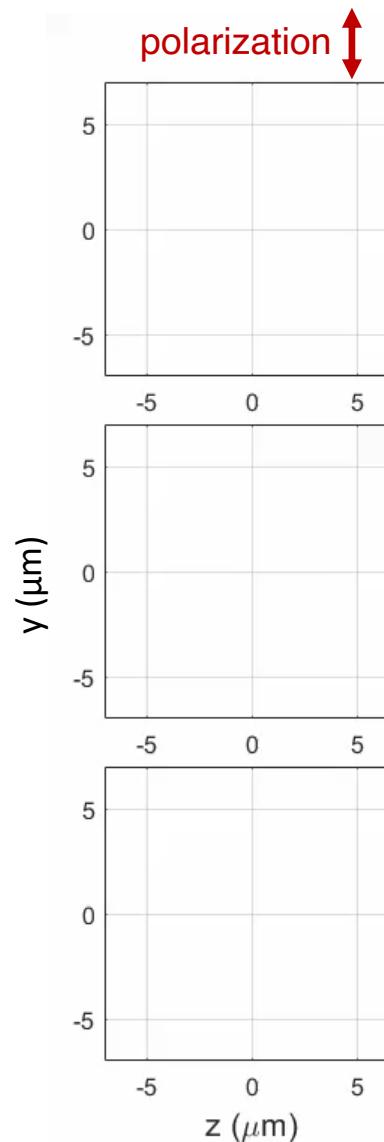
$$\text{x-ray photon number } N_c = 5.6 \times 10^{-3} N_0 N_e K$$

Simulation: Electron oscillation in LWFA

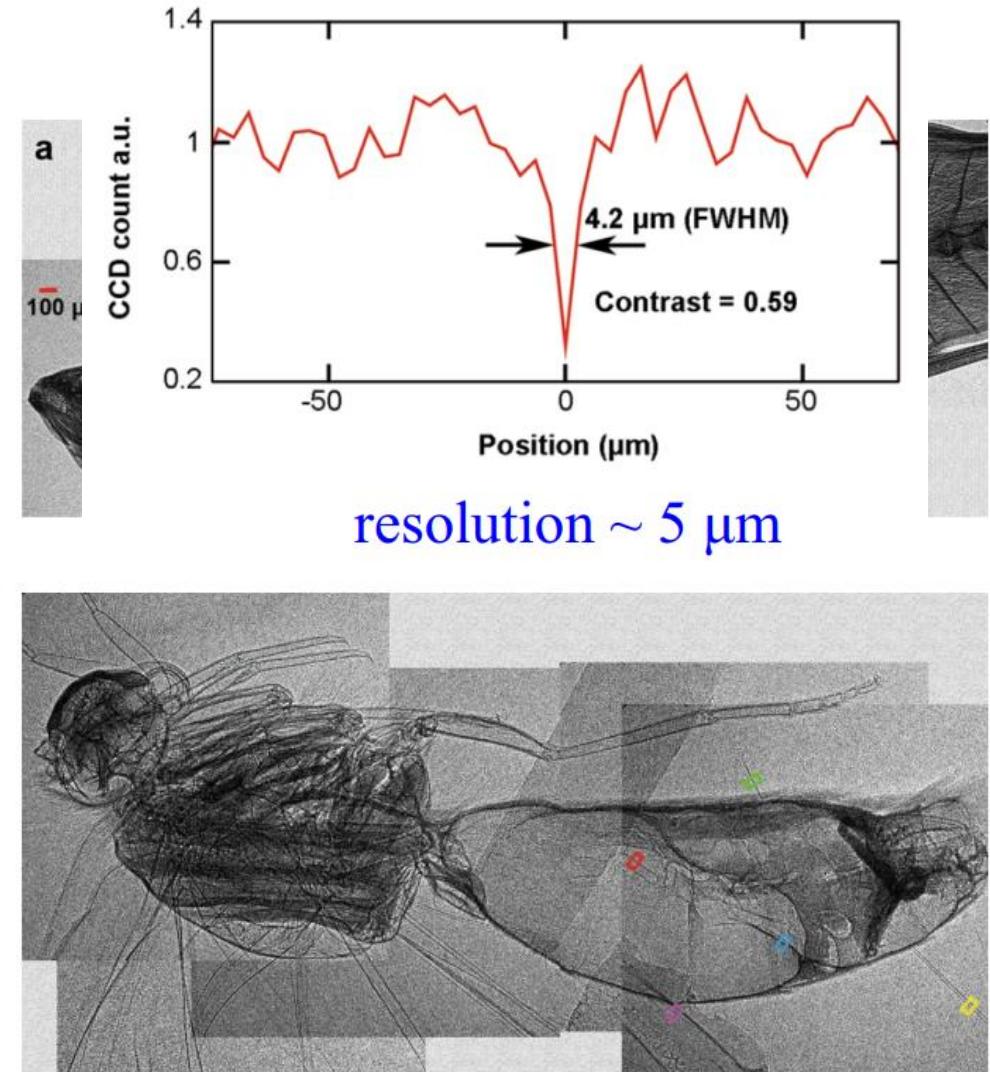
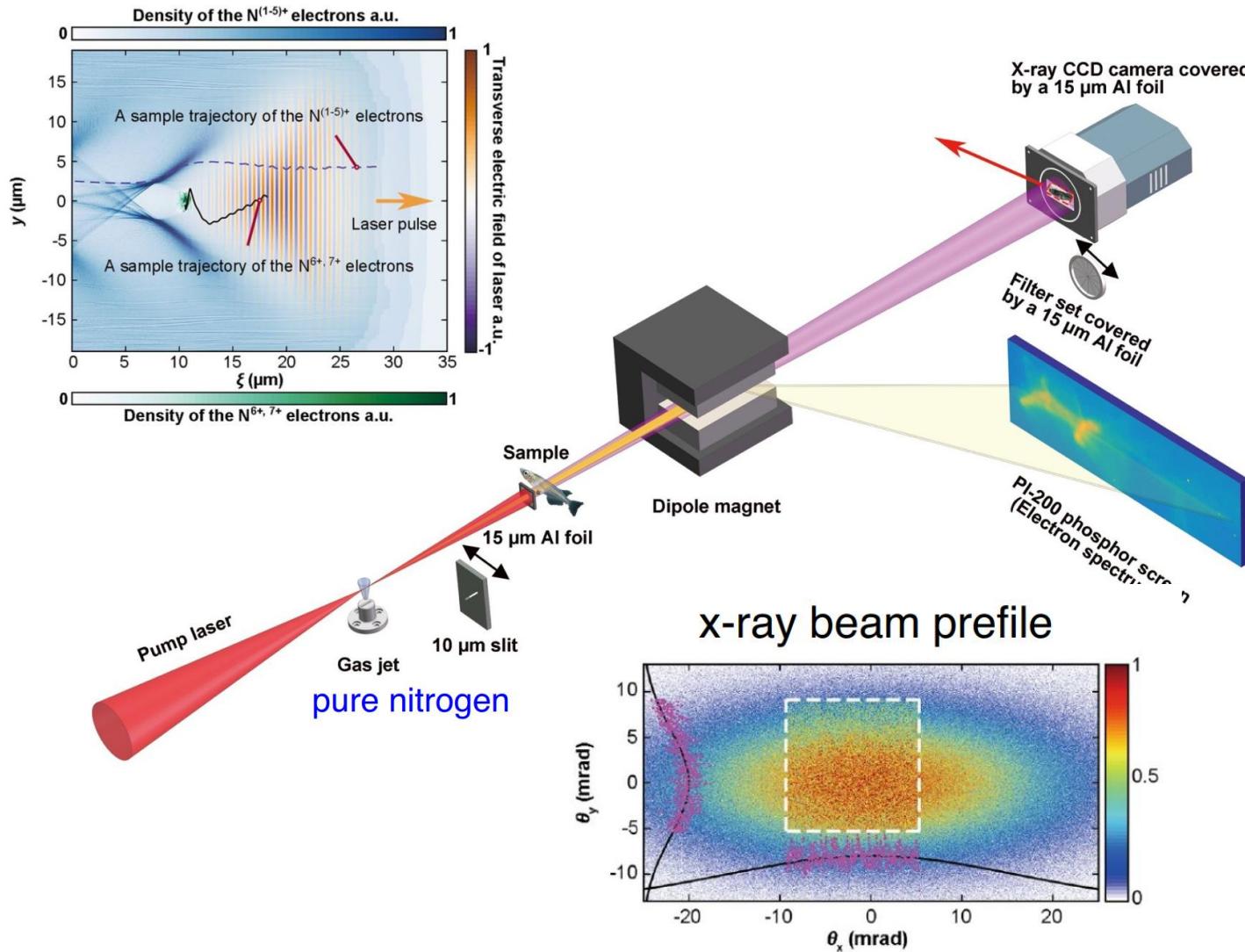
$n_p = 5 \times 10^{18} \text{ cm}^{-3}$
0.8 μm laser
15 fs pulse duration
Linear polarization
 $a_0 = 4$ ($I_0 \sim 3.5 \times 10^{19} \text{ W/cm}^2$)

Matched spot size:

$$w_0 = \frac{2\sqrt{a_0}c}{\omega_p} \approx 9.5 \mu\text{m}$$



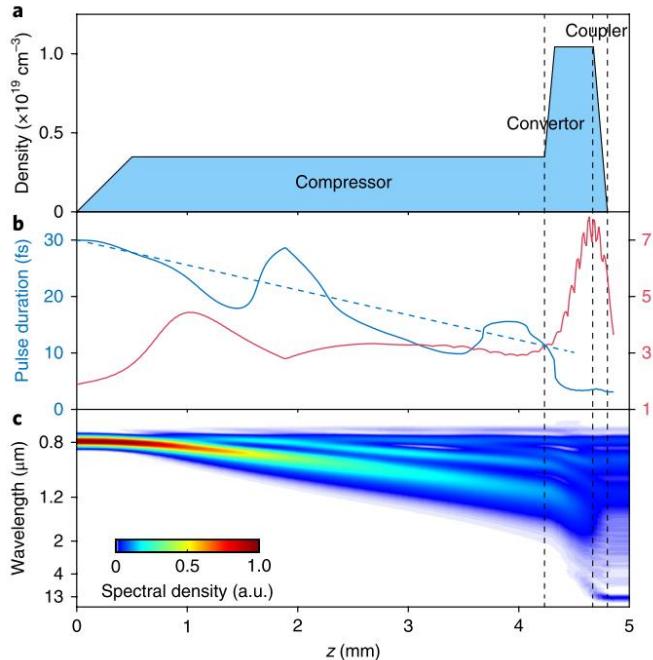
Ionization-injection betatron X-ray



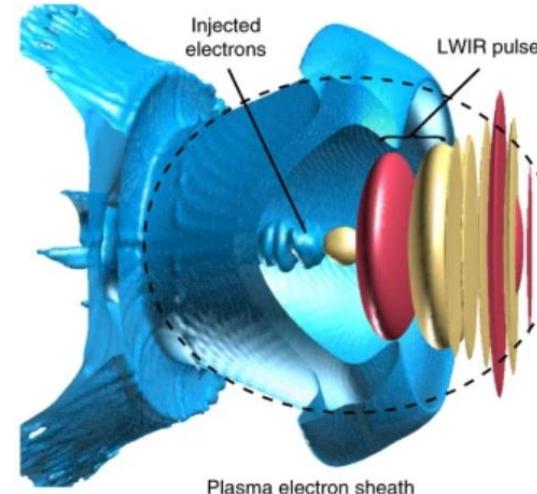
Long-wavelength-infrared (LWIR) pulse generation

- Based on the frequency downshifting process in a tailored plasma density structure, single-cycle long-wavelength infrared (LWIR) pulses with ultra-high intensity can be generated

Simulation

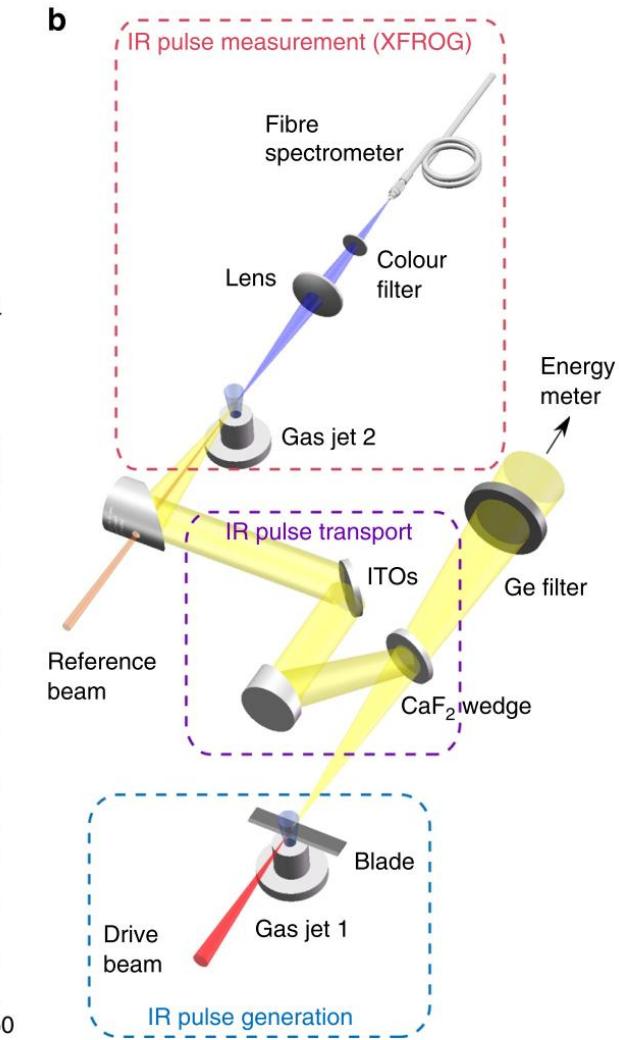
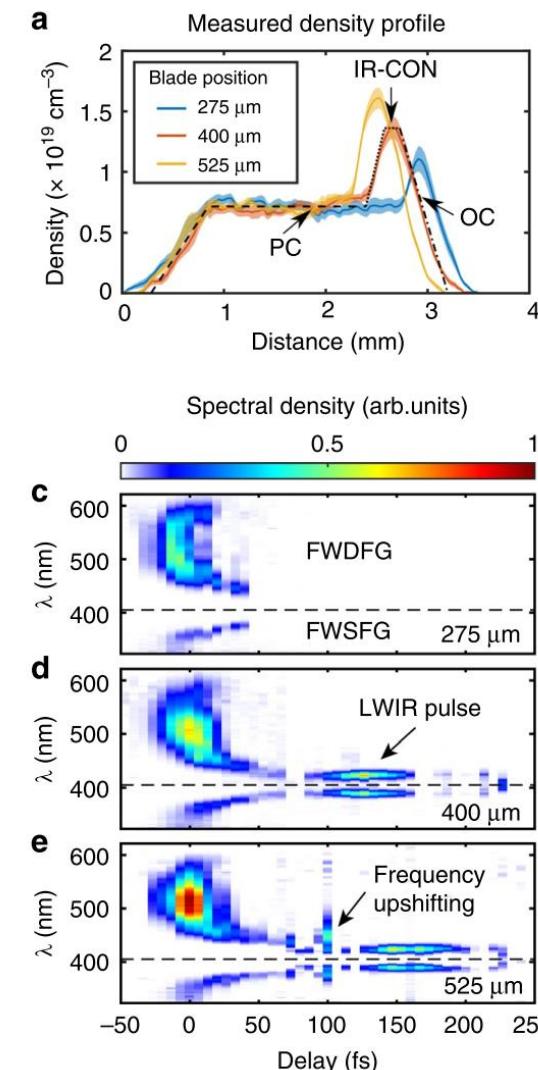


Nat. Photonics 12, 489 (2018)

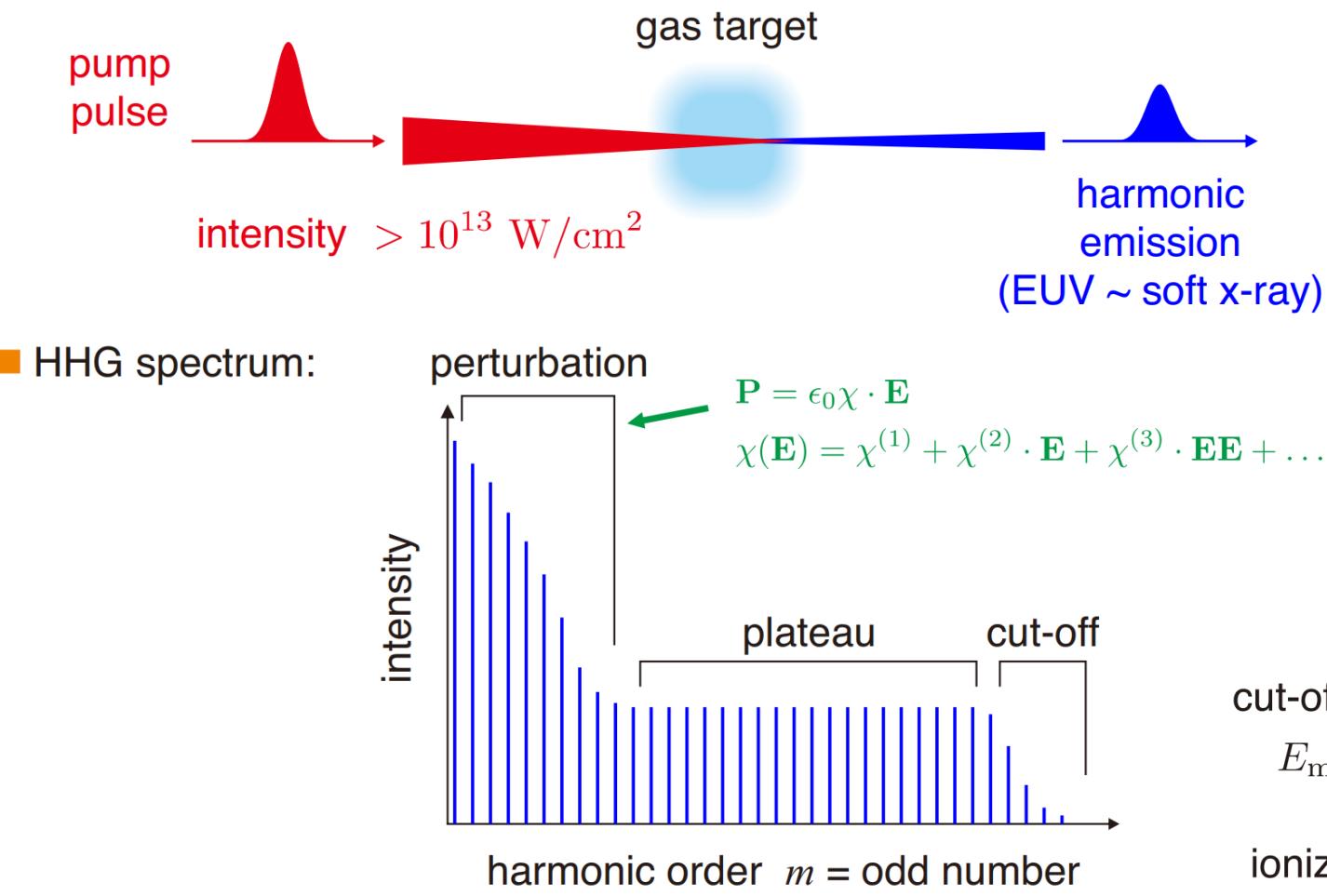


Experiment

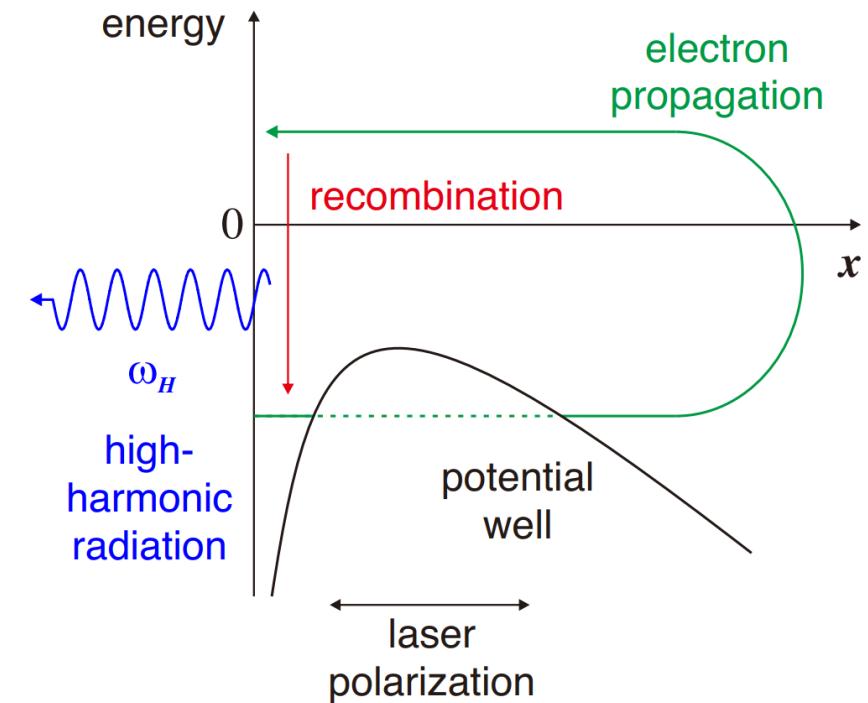
Nat. Photonics 12, 489 (2018)



High-harmonic generation (HHG)



Classical three-step model



cut-off energy:

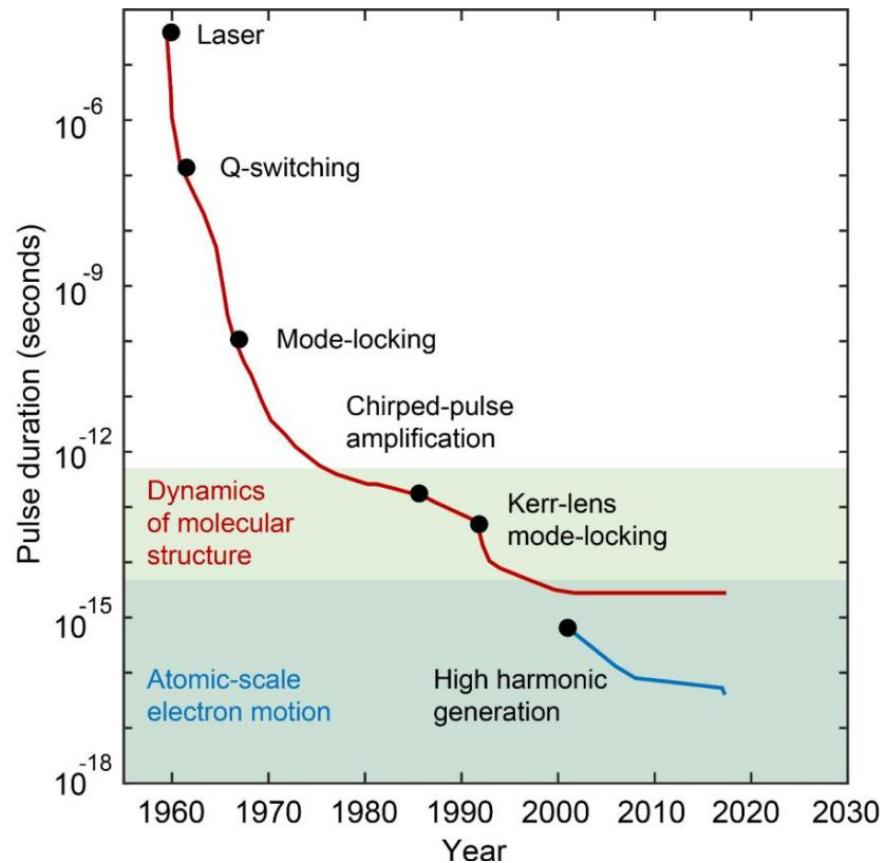
$$E_{\max} = I_p + 3.17 U_p$$

ionization potential

ponderomotive potential

Attosecond science

*"for experimental methods that generate **attosecond pulses of light** for the study of electron dynamics in matter"*



Pierre Agostini

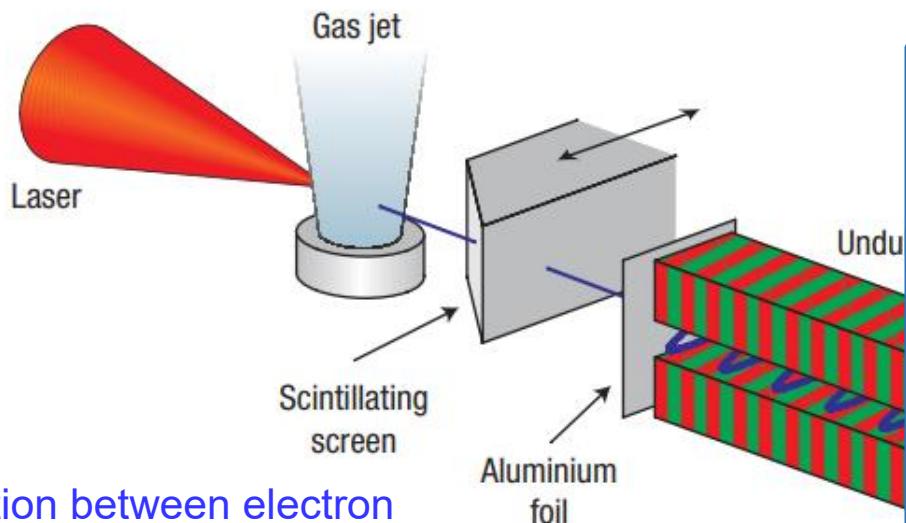
Ferenc Krausz

Anne L'Huillier

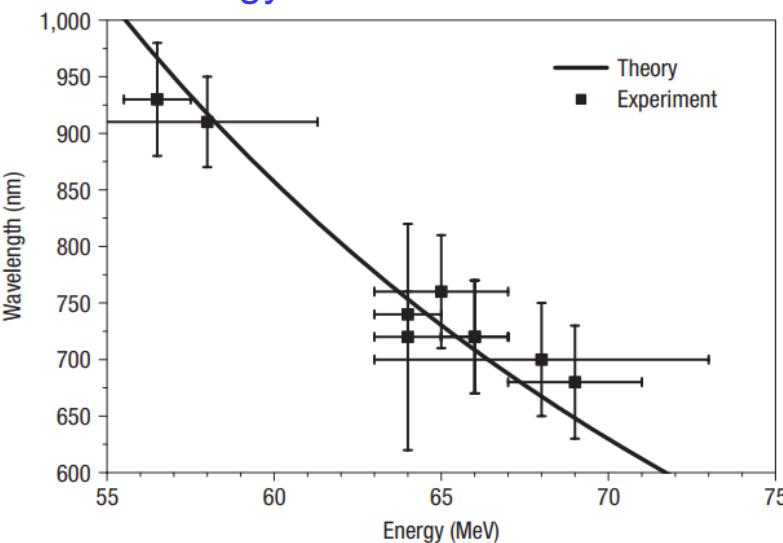


High Harmonic Generation to Attosecond Physics

Application: LWFA-driven Synchrotron and FEL



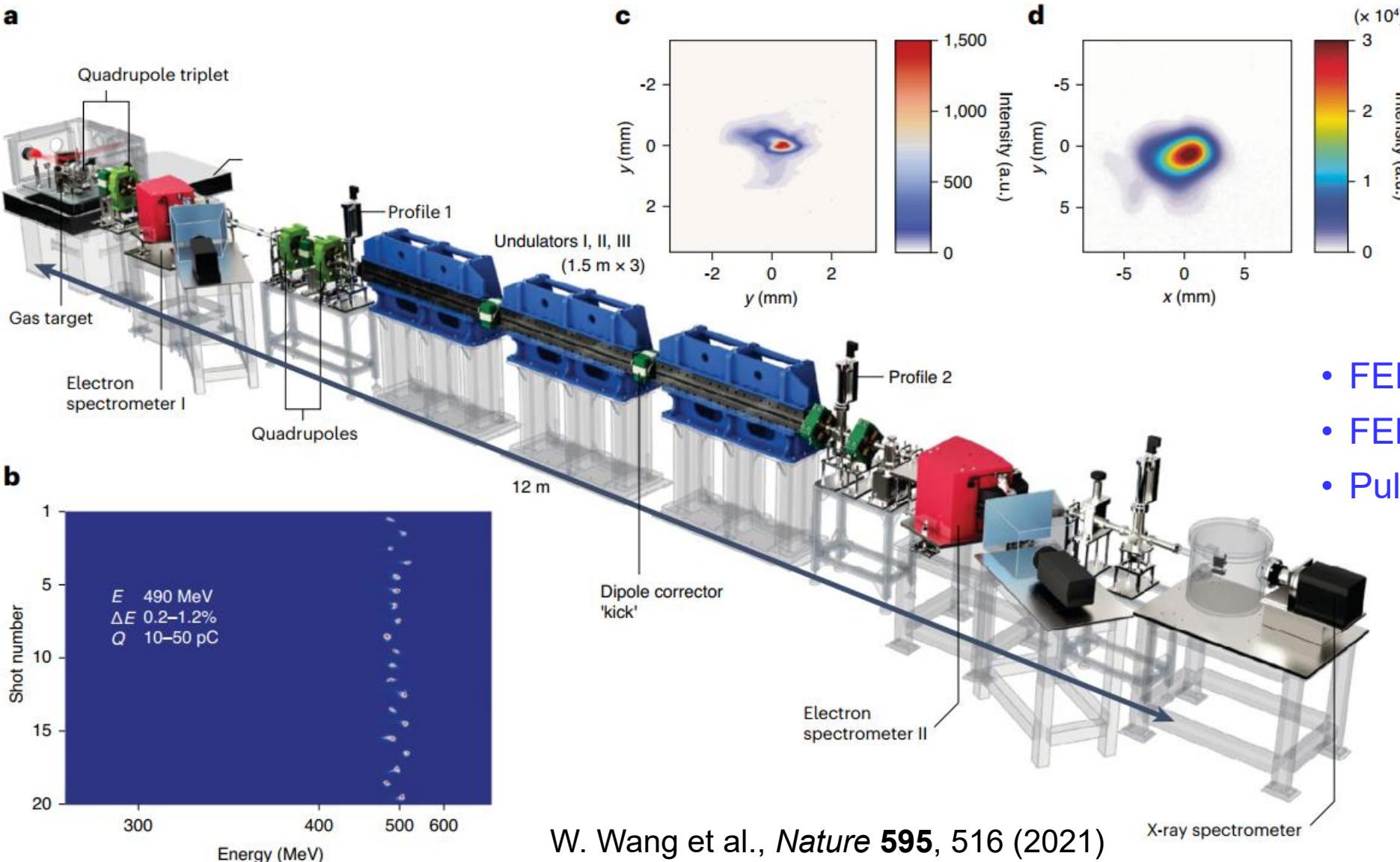
Correlation between electron energy and undulator radiation



Current limitations:

- 1. Energy spread ($\Delta E/E$):**
Typical LWFA beams have energy spreads of 1% to 5%. → 0.1% for FEL
- 2. High divergence**
Divergence increase when electron beams exit the plasma into a vacuum (often ~ 1 mrad).
- 3. Shot-to-shot fluctuation**
- 4. Low-repetition rate: 1-10 Hz**

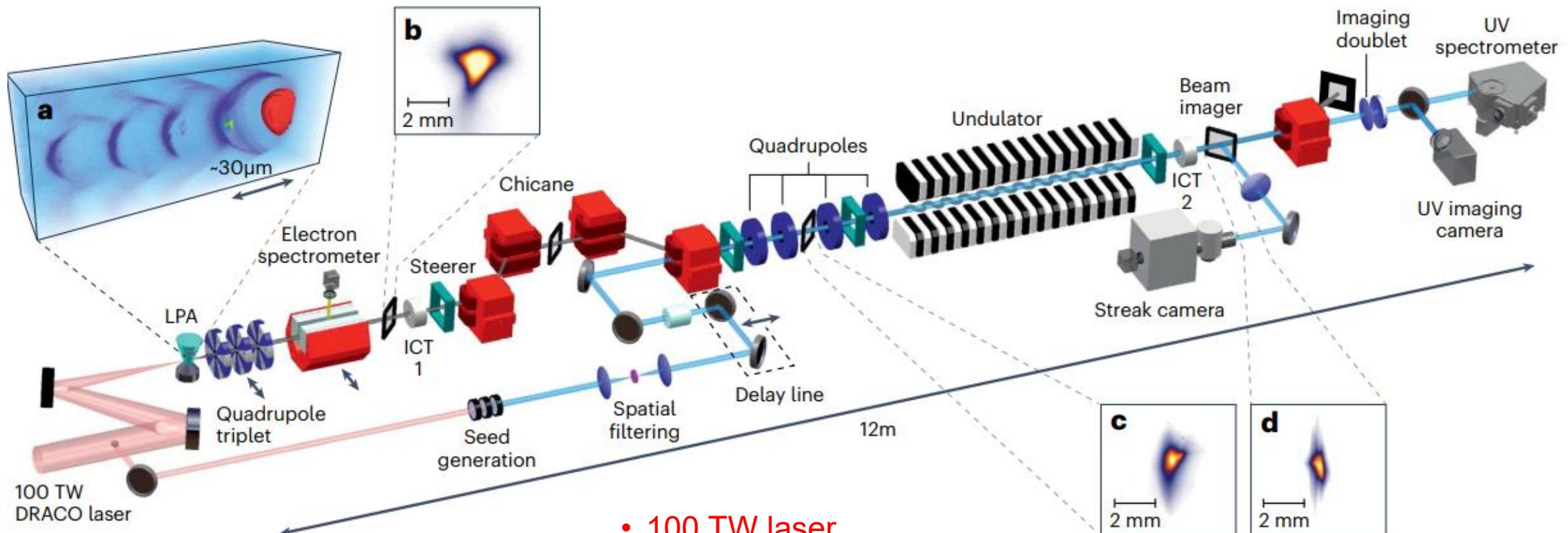
SASE LWFA-driven FEL



- 200 TW laser
- Mean energy: 490
- Energy spread: 0.5%

- FEL wavelength: 27 nm
- FEL repetition rate: 1–5 Hz
- Pulse energy: 0.15 μ J

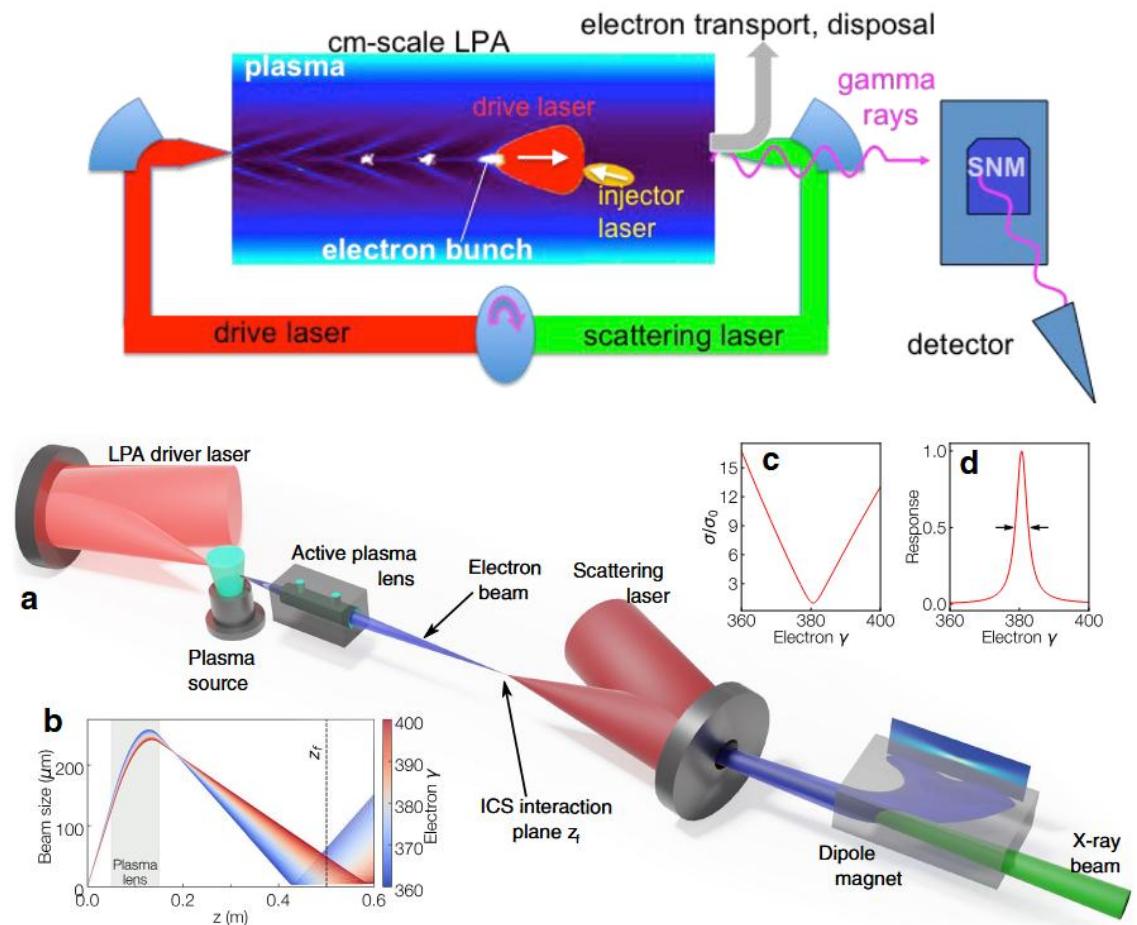
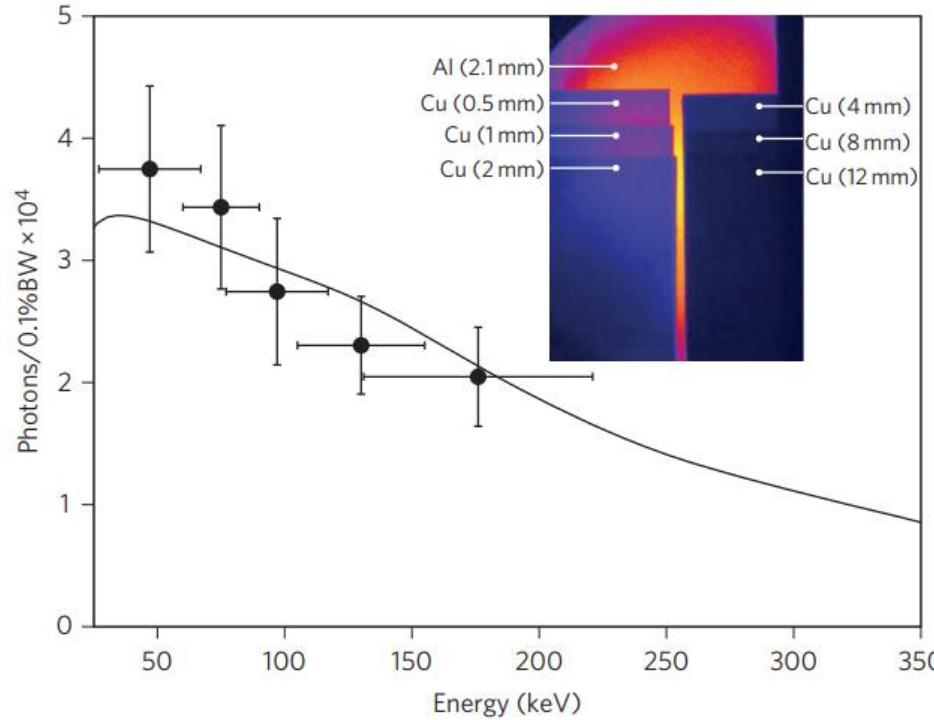
Seeded LWFA-driven FEL



- 100 TW laser
- Mean energy: 189
- Energy spread: 6.3%
- FEL wavelength: 275 nm
- FEL repetition rate: 0.1 Hz

Application: All-optical Gamma ray sources

- Multi-MeV gamma ray can be obtained from compact system by scattering electron beam against laser
 - Inverse Compton scattering



Applications of ultra-intense fields

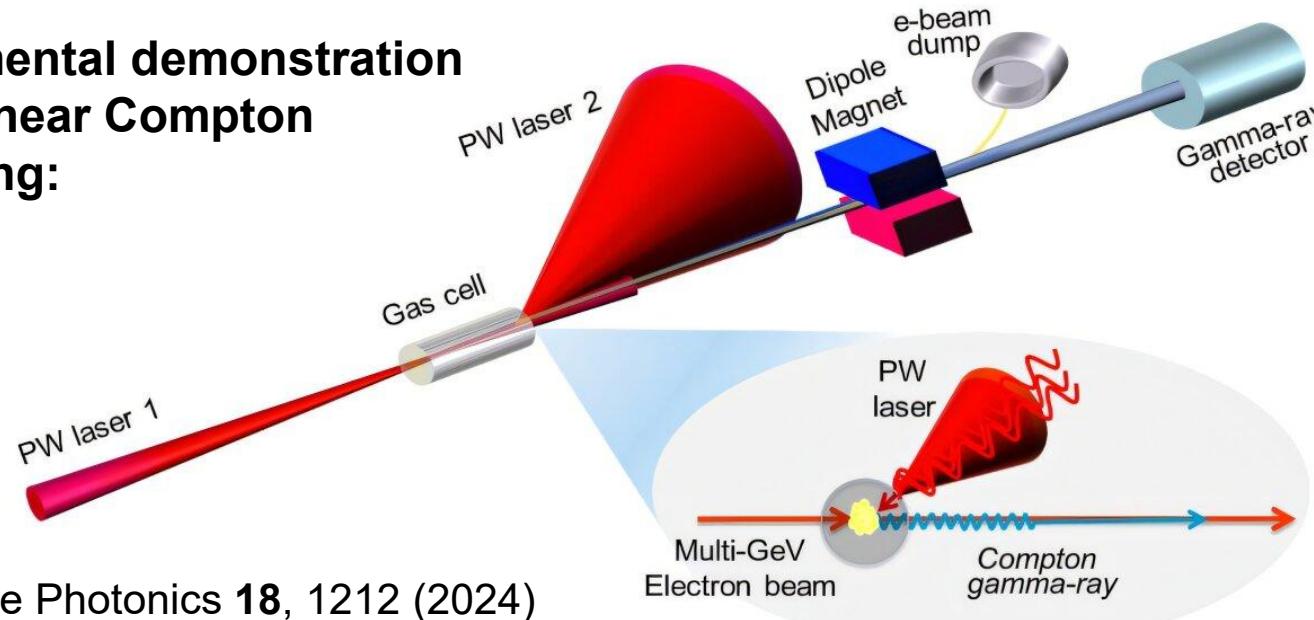
■ Strong-field quantum electrodynamics (QED)

- Nonlinear Compton scattering
- Electron-positron pair production in vacuum

Schwinger limit $\sim 10^{29}$ W/cm²

$$\frac{\text{Schwinger limit}}{\text{Highest laser intensity}} \sim 10^6$$

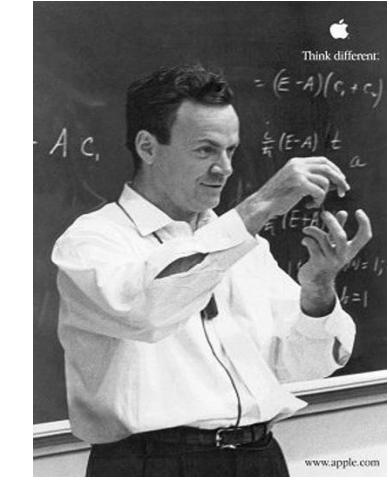
Experimental demonstration
of nonlinear Compton
scattering:



Nature Photonics 18, 1212 (2024)

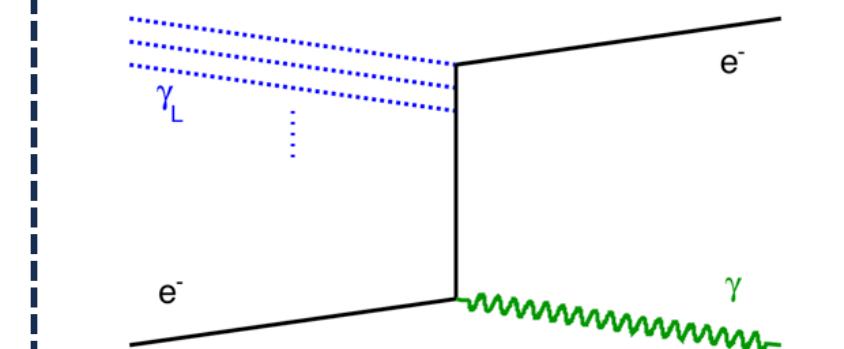


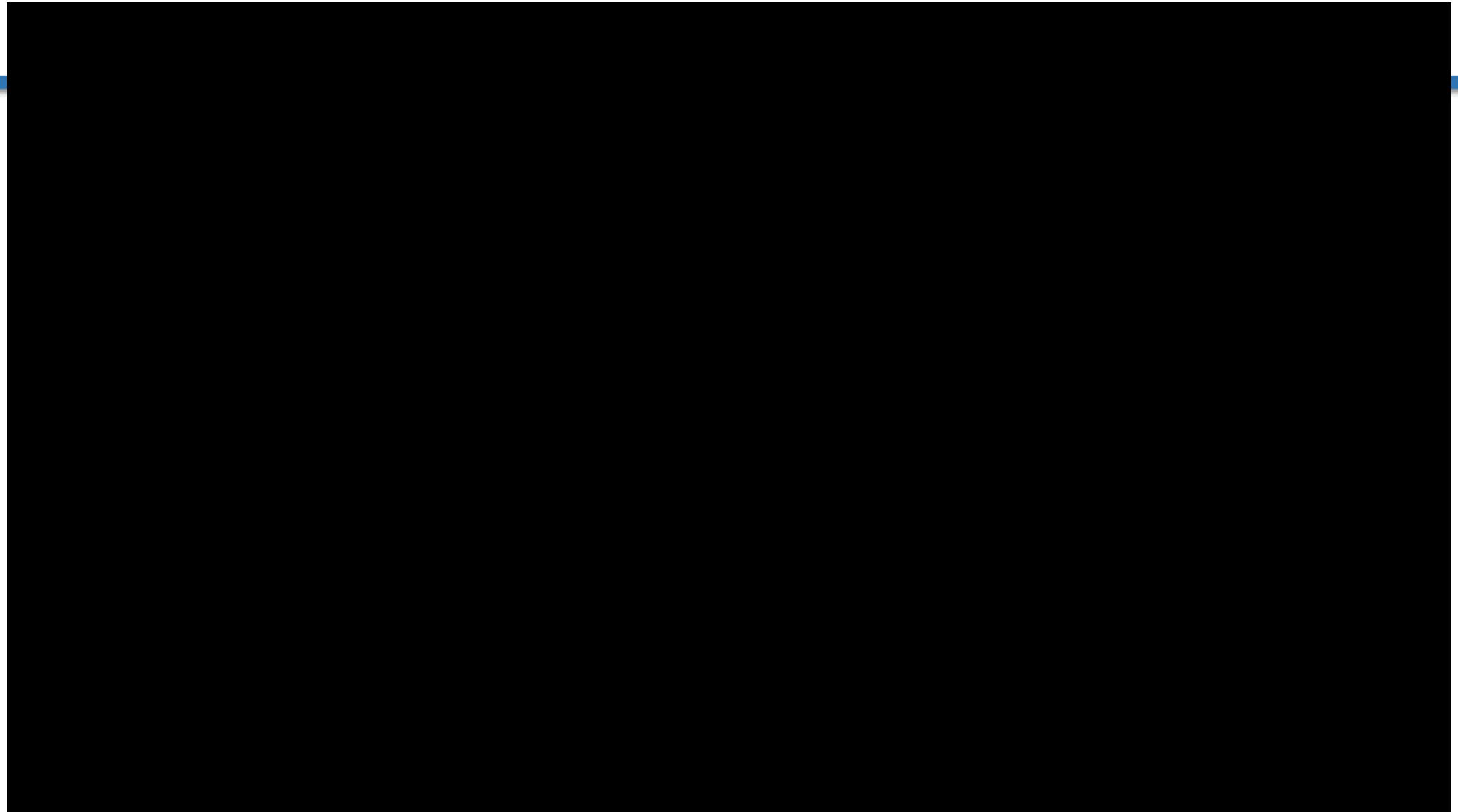
Julian Schwinger



Richard Feynman

Nonlinear Compton Scattering





(ZEUS Laser Animated Flythrough: <https://www.youtube.com/watch?v=3SYbLsSQBZE&t=207s>)

Summary

- Laser-driven plasma acceleration provides accelerating gradients (>100 GeV/m) that surpass conventional RF technology by orders of magnitude, reducing kilometer-scale machines to a single room.
- Successful demonstrated experiments, but challenges remain
 - Control Injection
 - Optical Guiding
 - Beam Quality
- The development of compact, laboratory-scale X-ray free-electron lasers democratizes access to ultrafast material and medical sciences.

Thanks for your attention !