



Summer School on Free Electron Laser 2024

自由電子雷射的調制器與發光器 (Modulators and radiators for Seeded FEL)

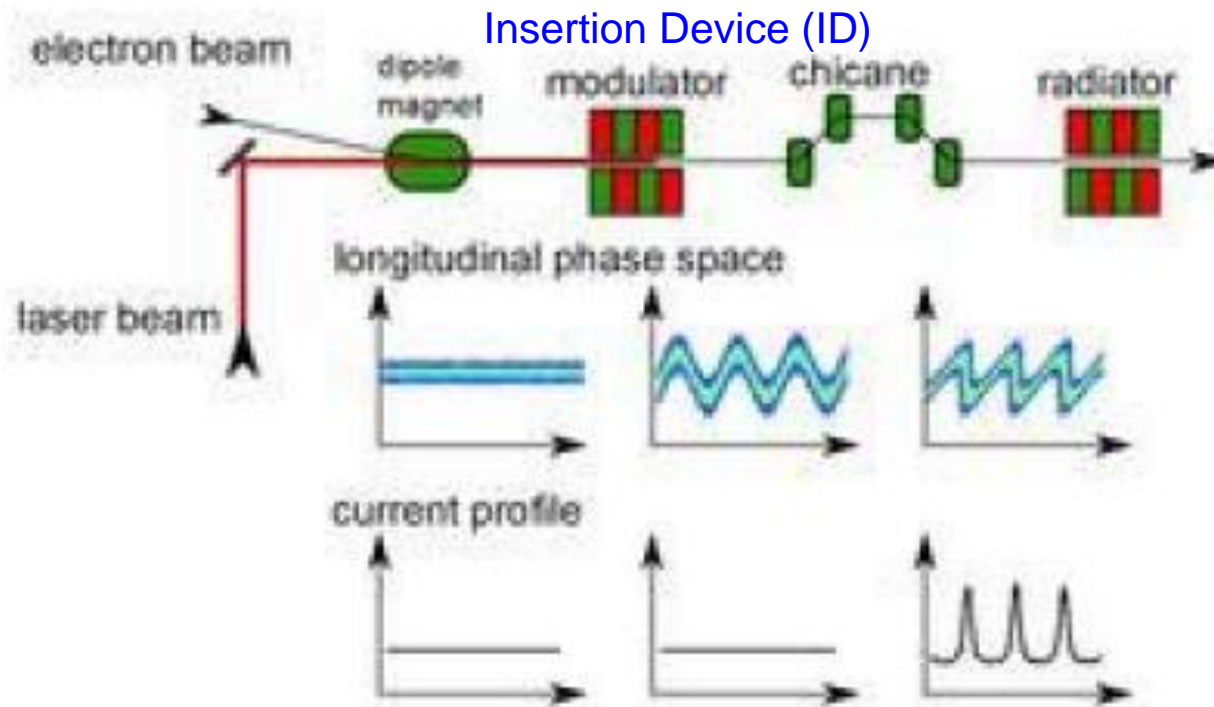
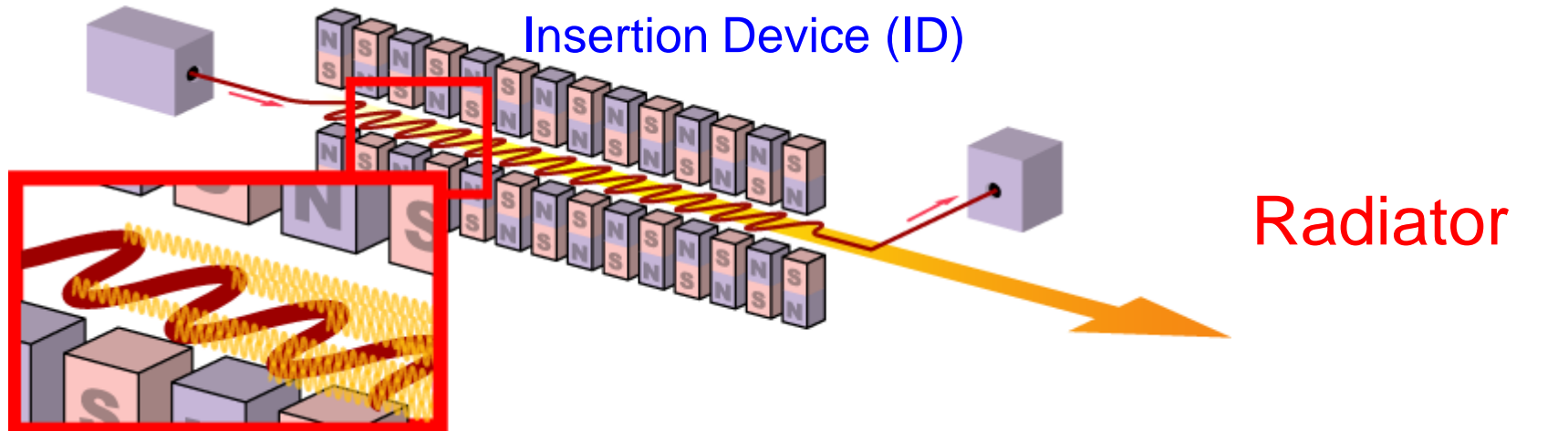
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July 16th, 2024 15:20 – 17:20

Modulators and Radiators for Seeded FEL

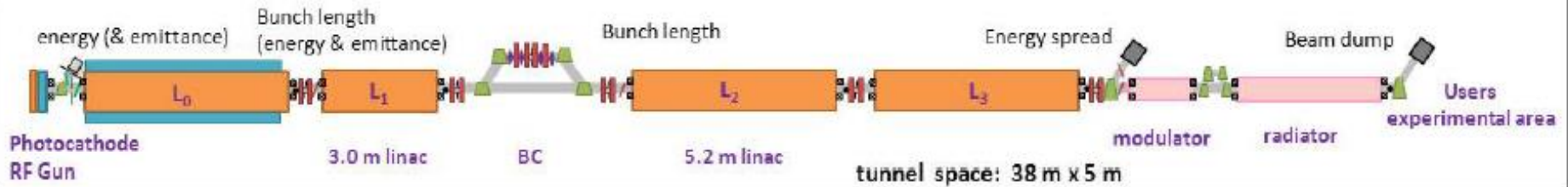




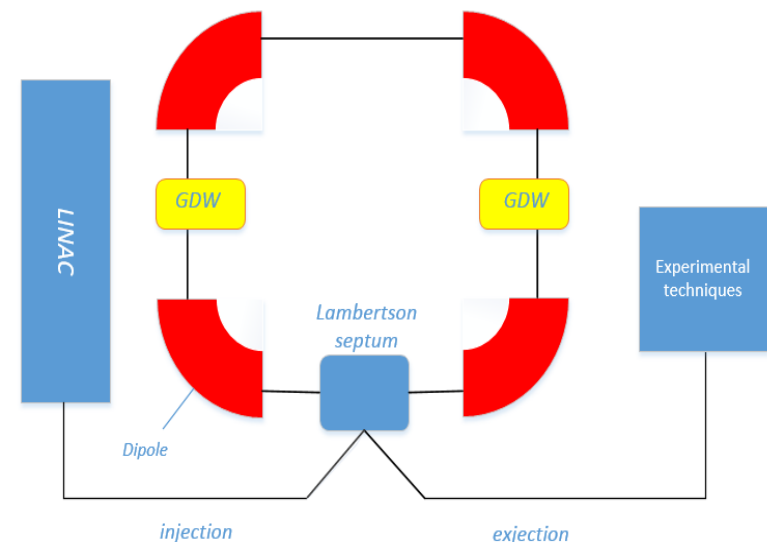
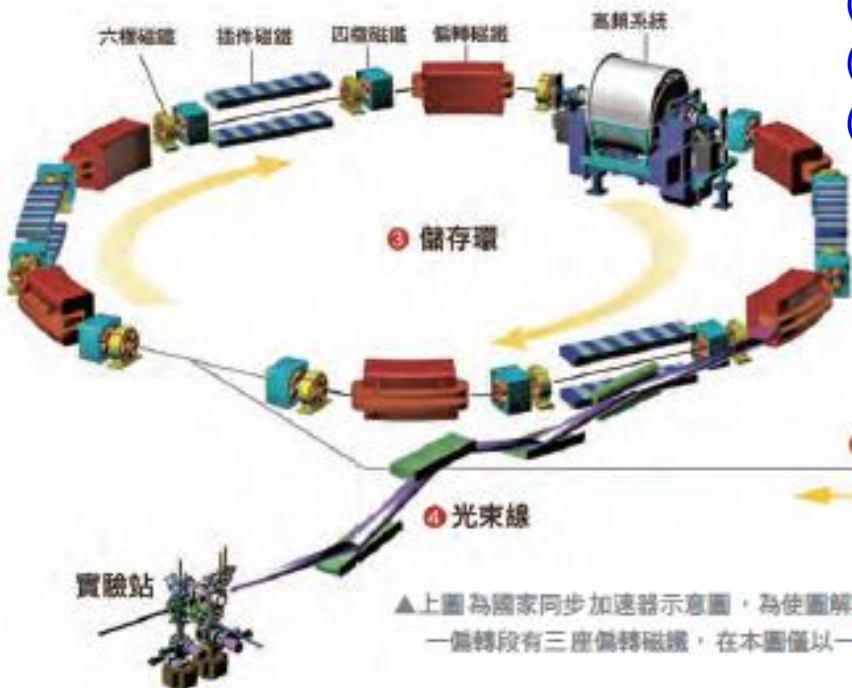
Outline

- **Application of Insertion Devices (ID)**
- **Introduction of ID**
 - **Wiggler (增頻磁鐵) & Undulator (聚頻磁鐵)**
 - **Development history**
- **Spectrum features & calculation**
 - **Photon Flux, Flux density, Brilliance**
 - **Photon Power, power density**
- **Example of the ID spectrum**
- **How to design and shimming ID**

Application of Insertion Device (ID)



- (1) Modulator or Radiator in FEL structure
- (2) Main light source in storage ring (SR) structure
- (3) Robinson wiggler to reduce emittance of SR
- (4) Gradient damping wiggler to vary damping partition number & momentum compaction factor of SR





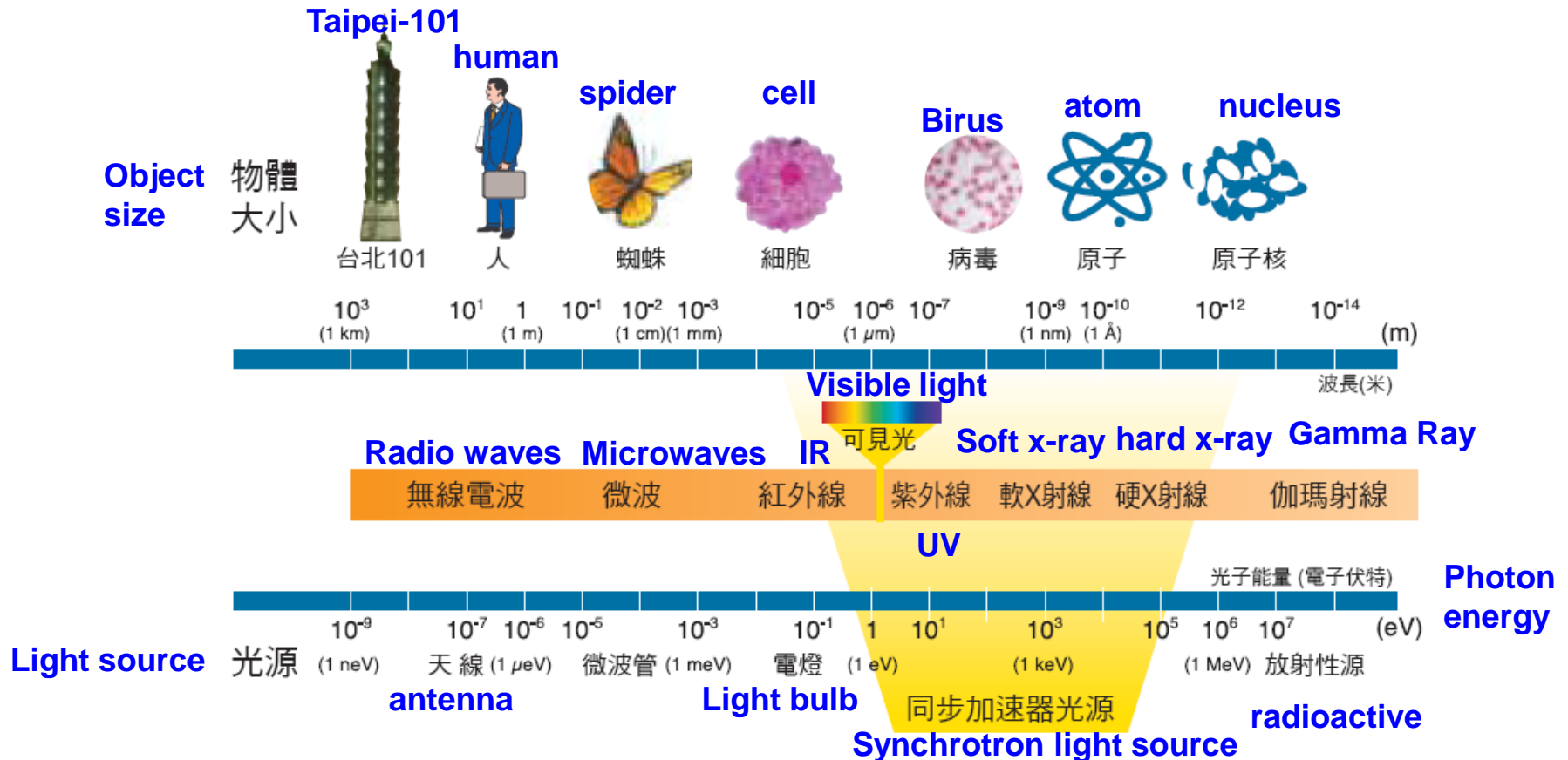
Introduction & history

- Insertion devices include the **wigglers** (增頻磁鐵) and **undulators** (聚頻磁鐵) that are magnetic devices producing a specially **periodic field variation**.
- They are all placed in the **straight sections** of storage ring.
- Wiggler spectrum at higher photon energies is smooth, similar to that of a bending magnet. The radiation intensity can be much higher as much as increased numbers of poles and **higher magnetic field** generate radiation with a **higher critical energy**.
- When the use of periodic magnets in a regime in which interference effects is coherent, and then the device is called “undulator”.
- The main radiation features of insertion devices are (1) higher photon energy, (2) higher flux and brightness, (3) different polarization characteristics.
- The theory behind undulators was developed by **Vitaly Ginzburg** in the USSR.
- First undulator was installed in a linac at Stanford, using it to generate millimetre wave radiation through to visible light in 1953.
- **First wiggler (undulator)** installed in storage ring at **SSRL (BINP)** around at 1979s.
- **Superconducting wavelength shifter**: are currently operating in several synchrotron radiation facilities: ESRF, UVSOR, PF and CAMAD (USA), NSRRC begin early 1980.
- **EPU** (APPLEII) solve the experimental problem of circular polarization light at 1994.
- **Superconducting wigglers**: are currently used in MAXLab, NSRRC, Diamond, ALBA,...
- **In-vacuum undulator**: are popular used in the new 3th generation light source.
- **Cryogenic permanent-magnet undulator**: ESRF & SPring8, Diamond, Soleil, NSRRC.
- **Superconducting Undulator**: In developing in NSRRC, ANKA, BASSY II, APS.



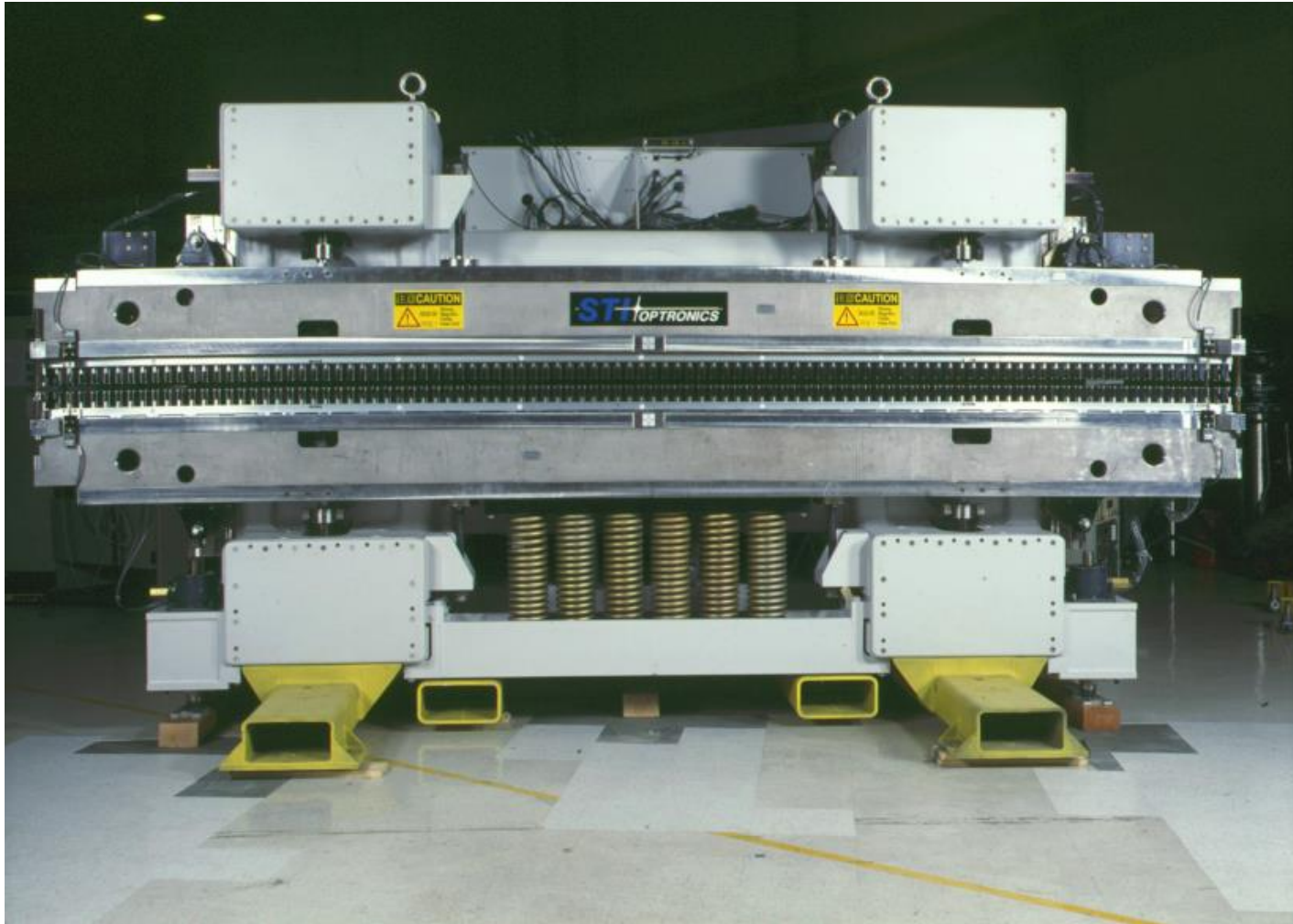
Members of the electromagnetic wave family

- In the mid-19th century, Maxwell organized the electromagnetic theory structure and established the electromagnetic wave theory (1865). Electromagnetic waves propagate at the speed of light, and "light" is a kind of electromagnetic wave.
- "Synchrotron Accelerator light source" is a continuous electromagnetic waves, covering infrared light, visible light, ultraviolet light, soft X-ray, hard X-ray and other bands.





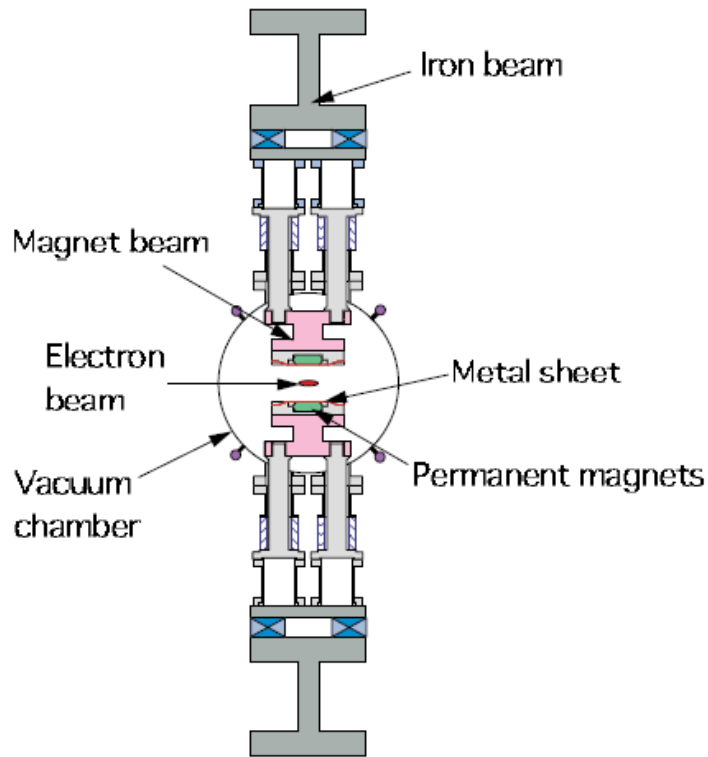
Out of vacuum planar undulator (U90)





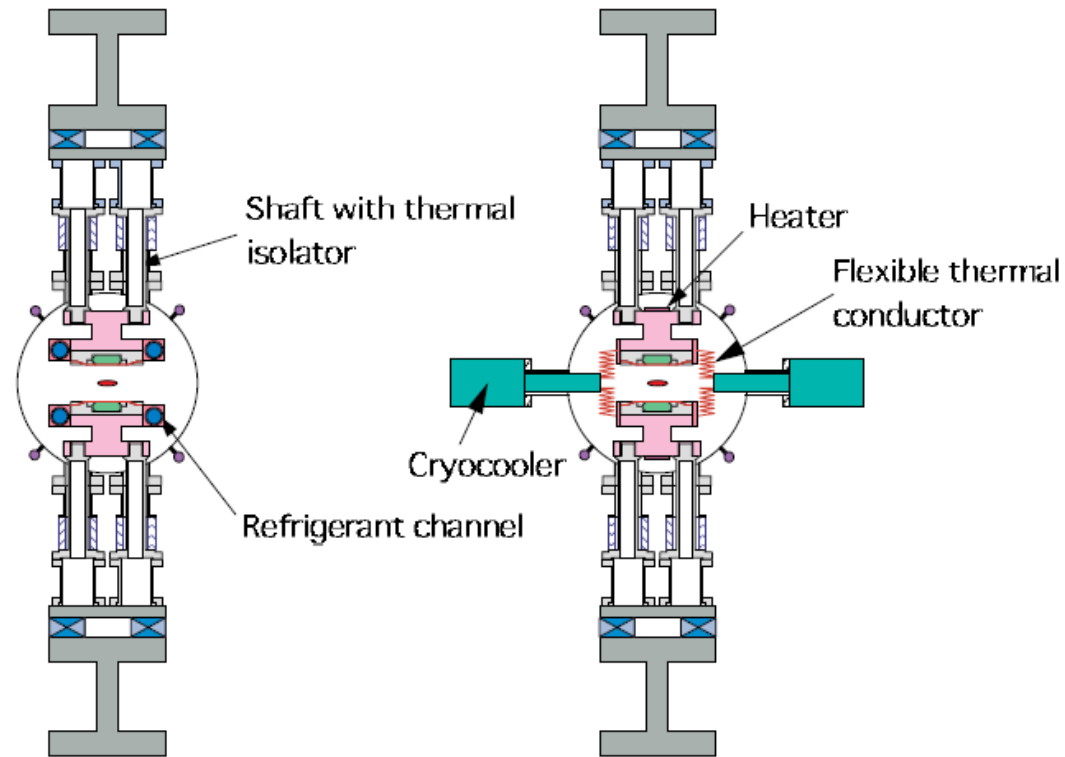
In-vacuum (IU) & cryogenic undulator (CU)

IU @Room Temperature



CU Below 77 K

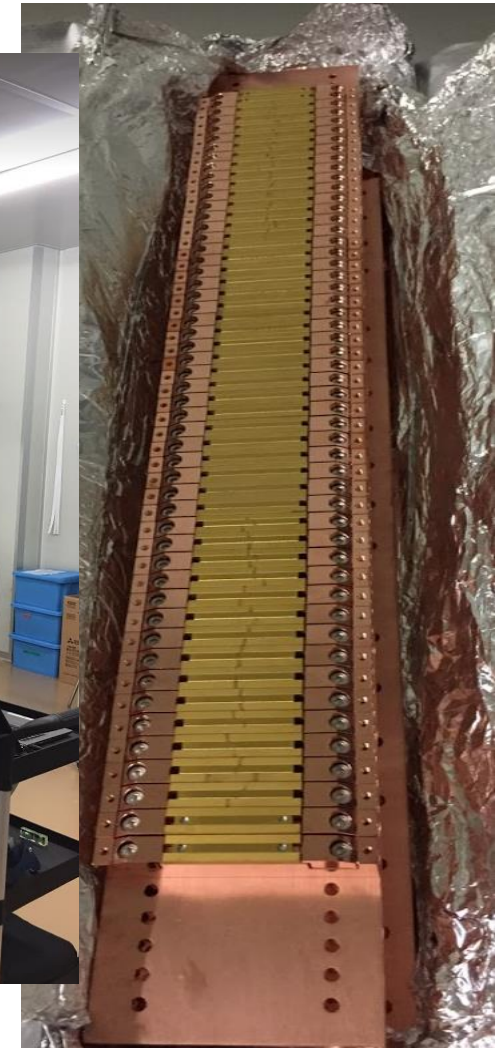
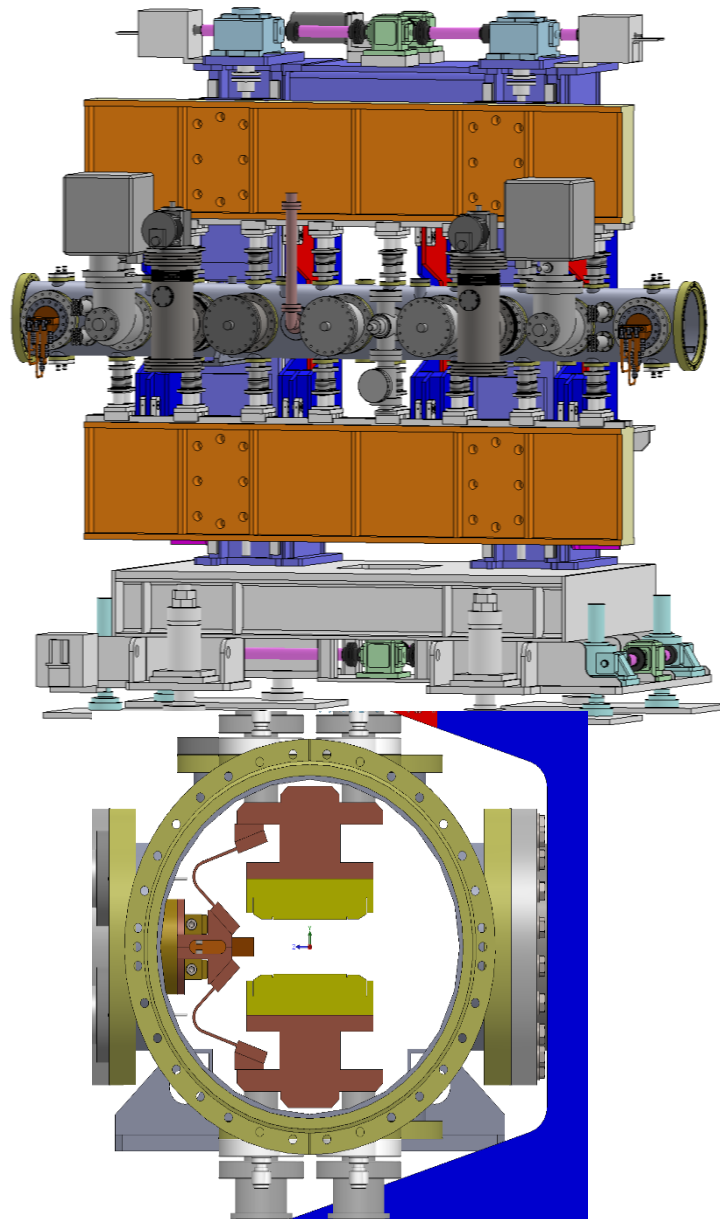
真空內聚頻磁鐵



- The cooling method of CU is (1) liquid nitrogen cryogenic system or (2) the cryocooler.
- The cooling method will depend on numbers of CU.



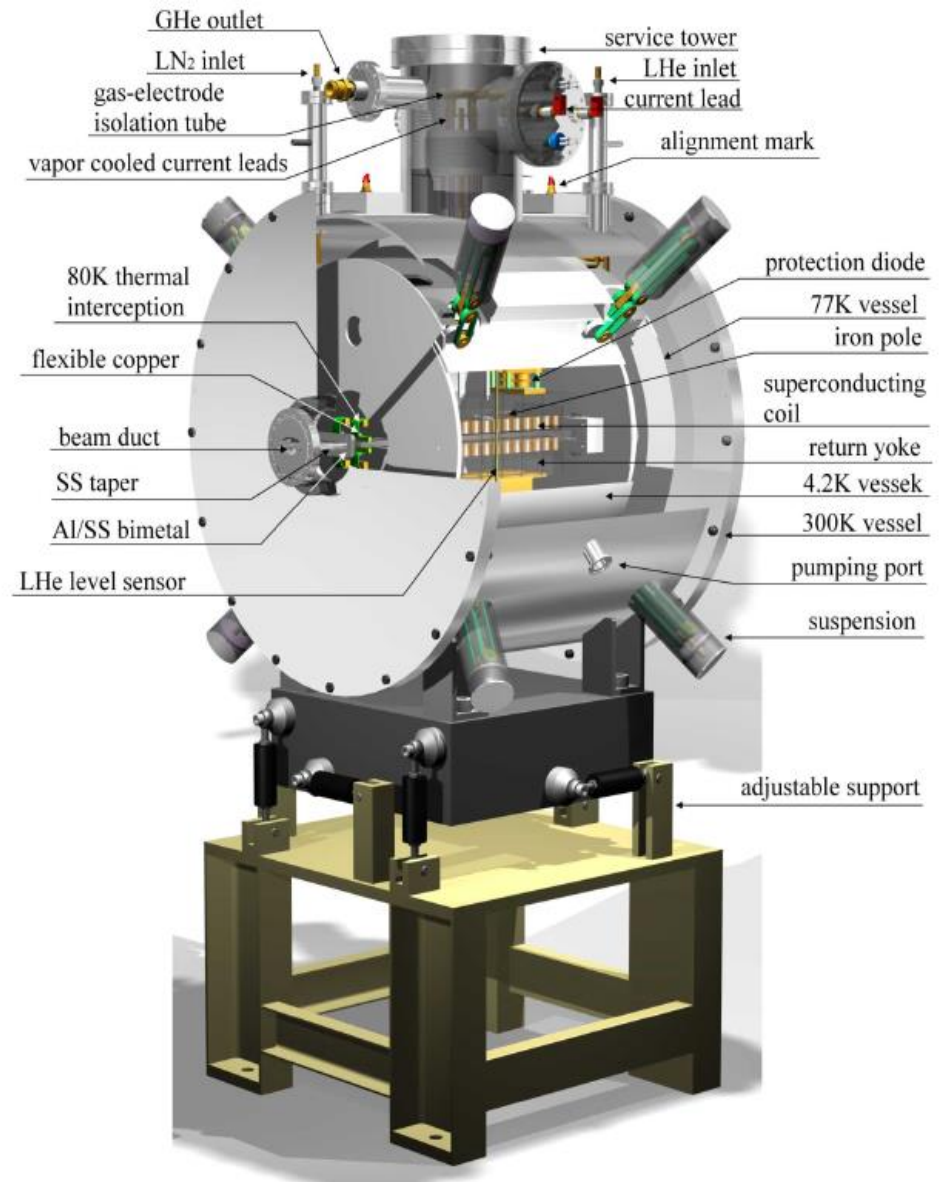
Cryogenic undulator (CU15)



- ◆ 0.6 m long prototype testing
- ◆ 2 m long CU15 will be finished before June 2019
- ◆ 200 W CH-110 cryocooler at 77K

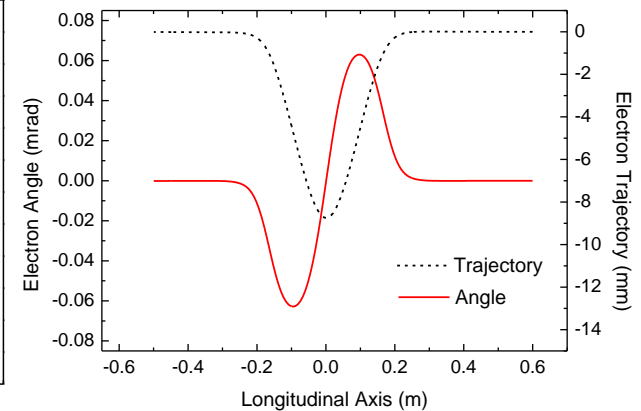
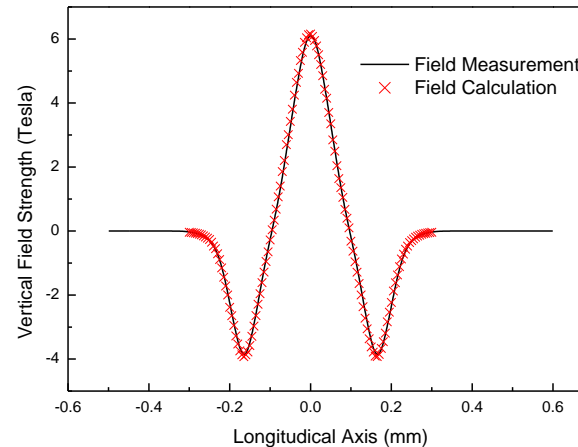
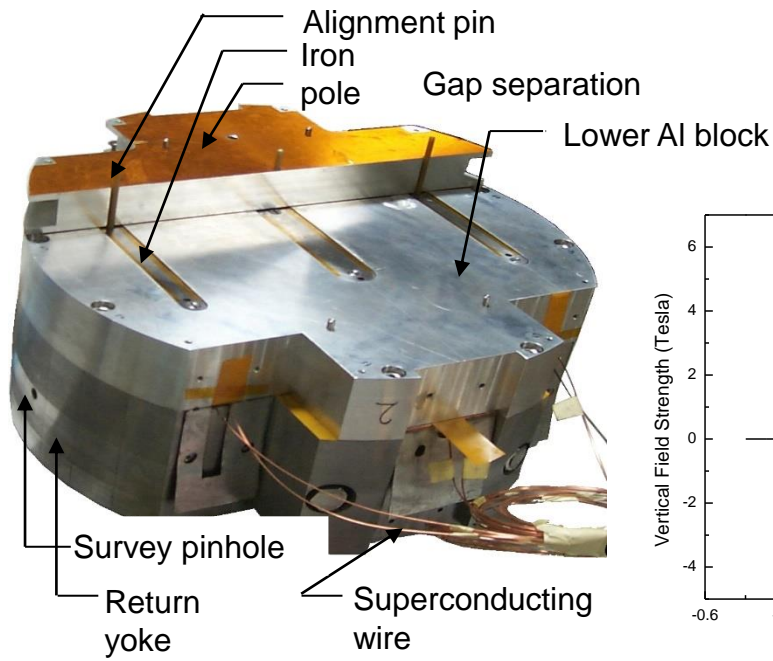
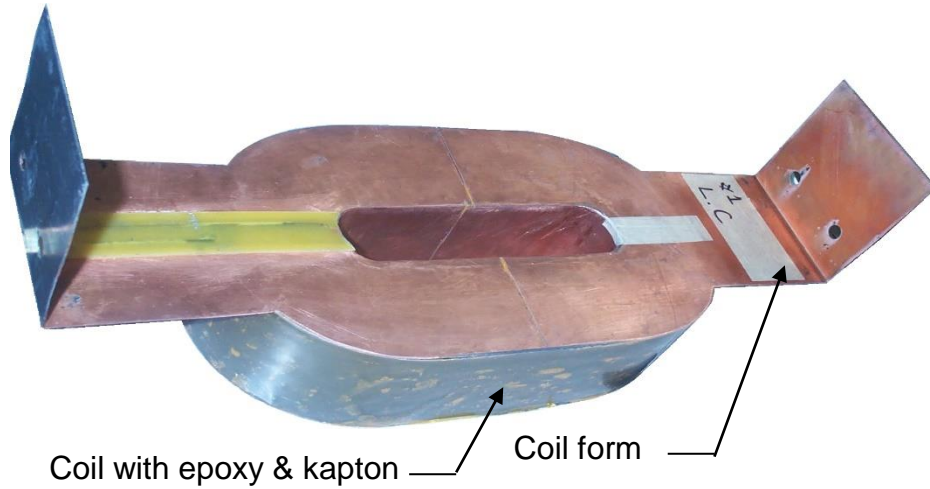


Superconducting ID (SW60) - Enhances photon energy



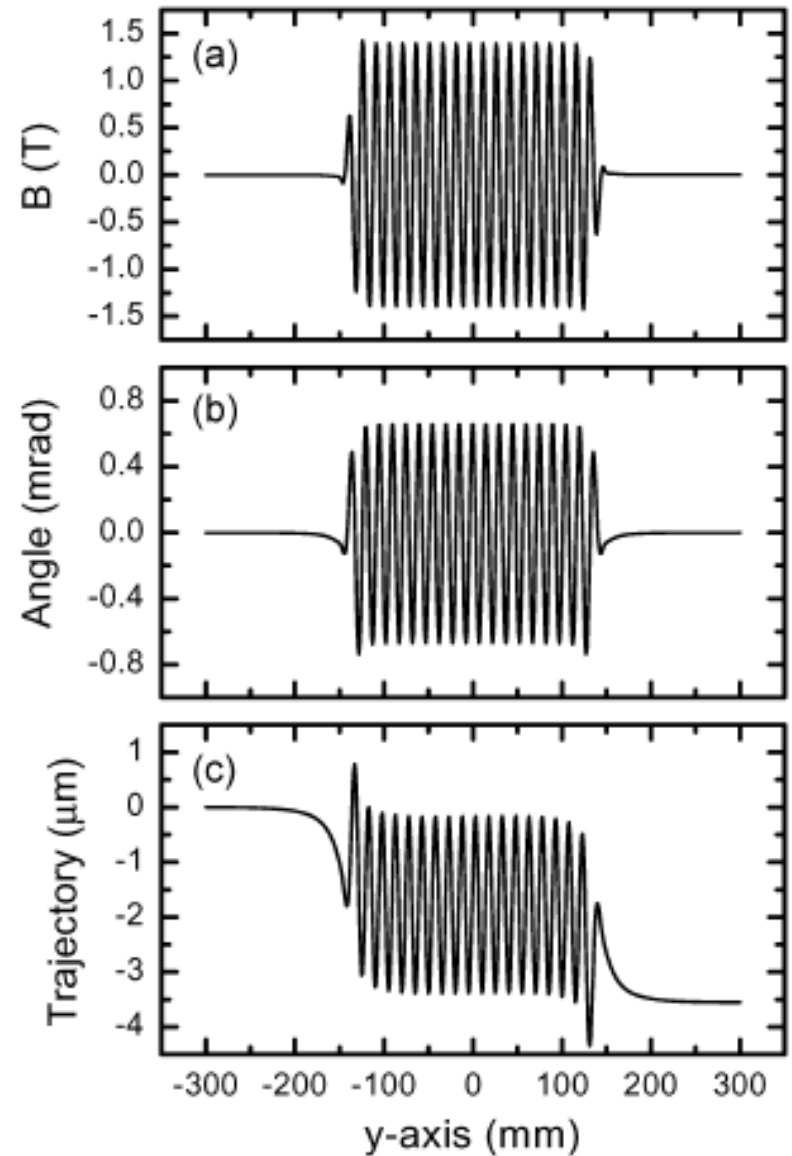
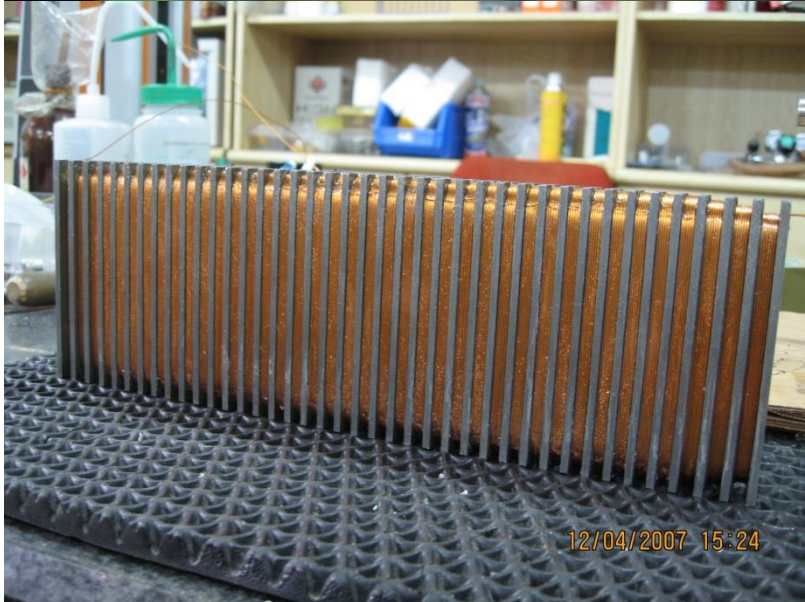


Superconducting wavelength shifter



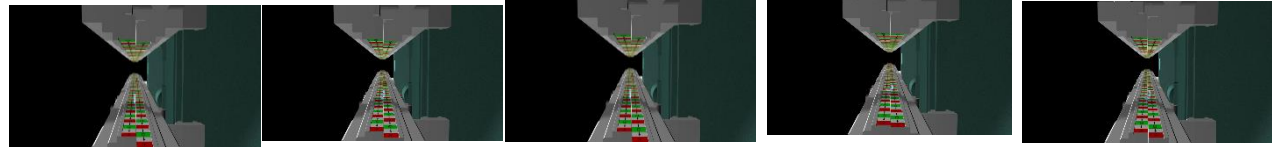


Superconducting undulator (SU15)

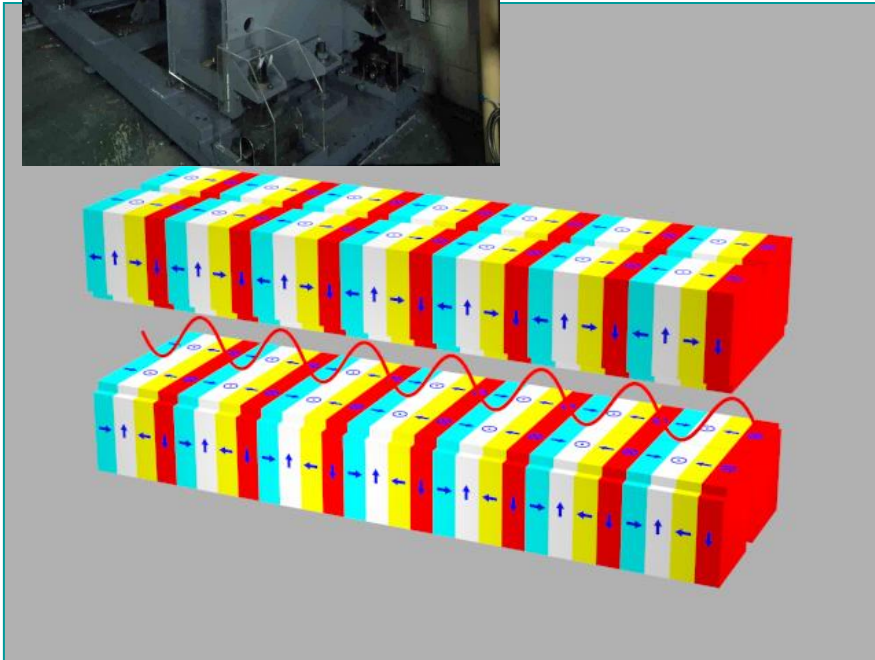
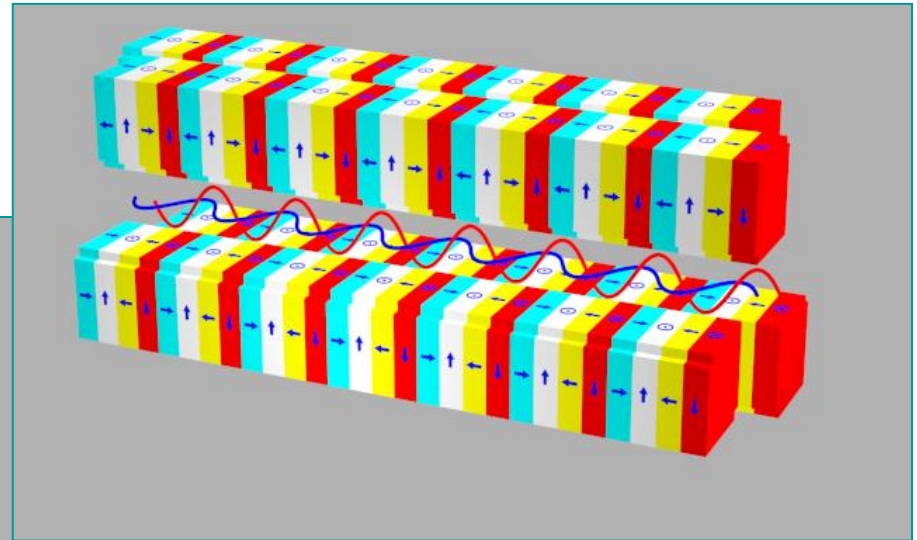




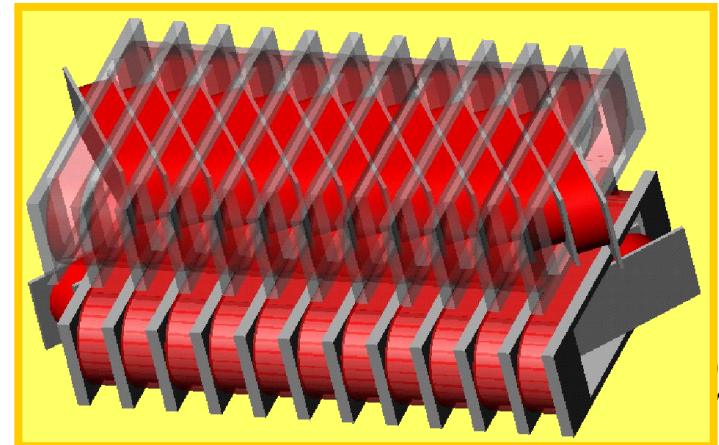
Produce Photon in various polarizations-Elliptically polarized Undulator



EPU56



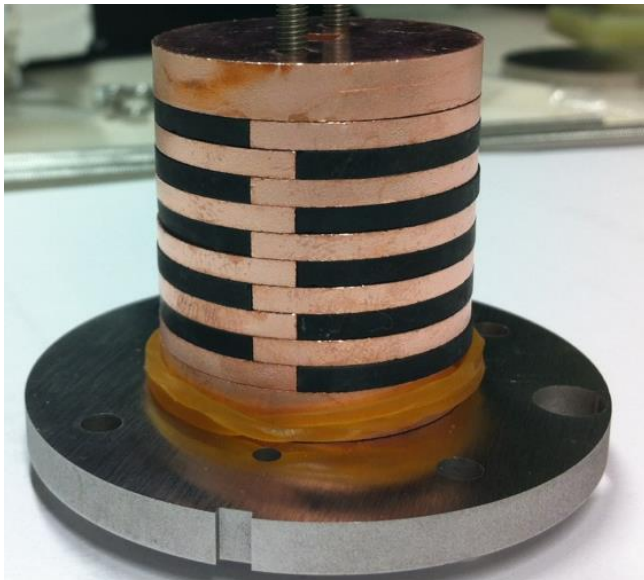
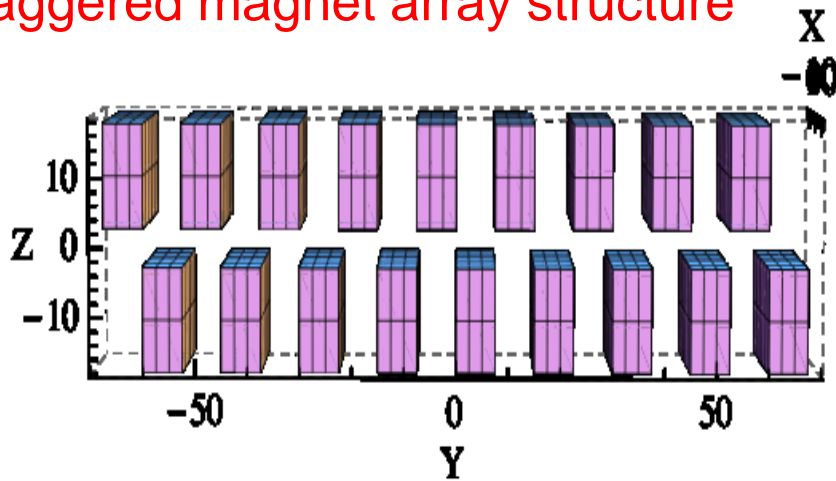
Superconducting EPU



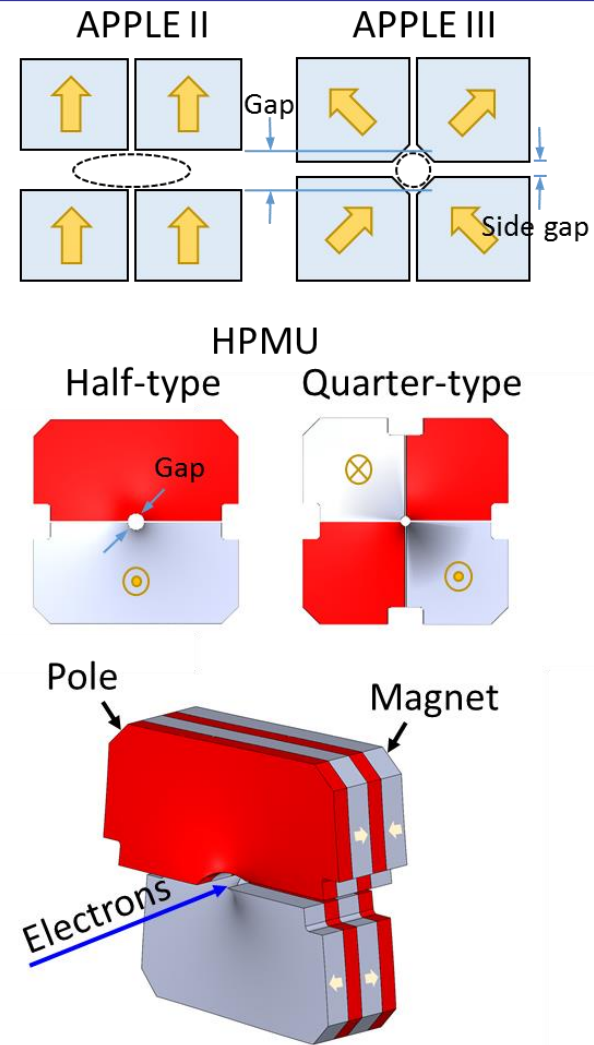


Staggered Undulator with magnetized Bulks

Staggered magnet array structure



➤ 10 Pole prototype structure without end pole optimization & using Field Cooling method.

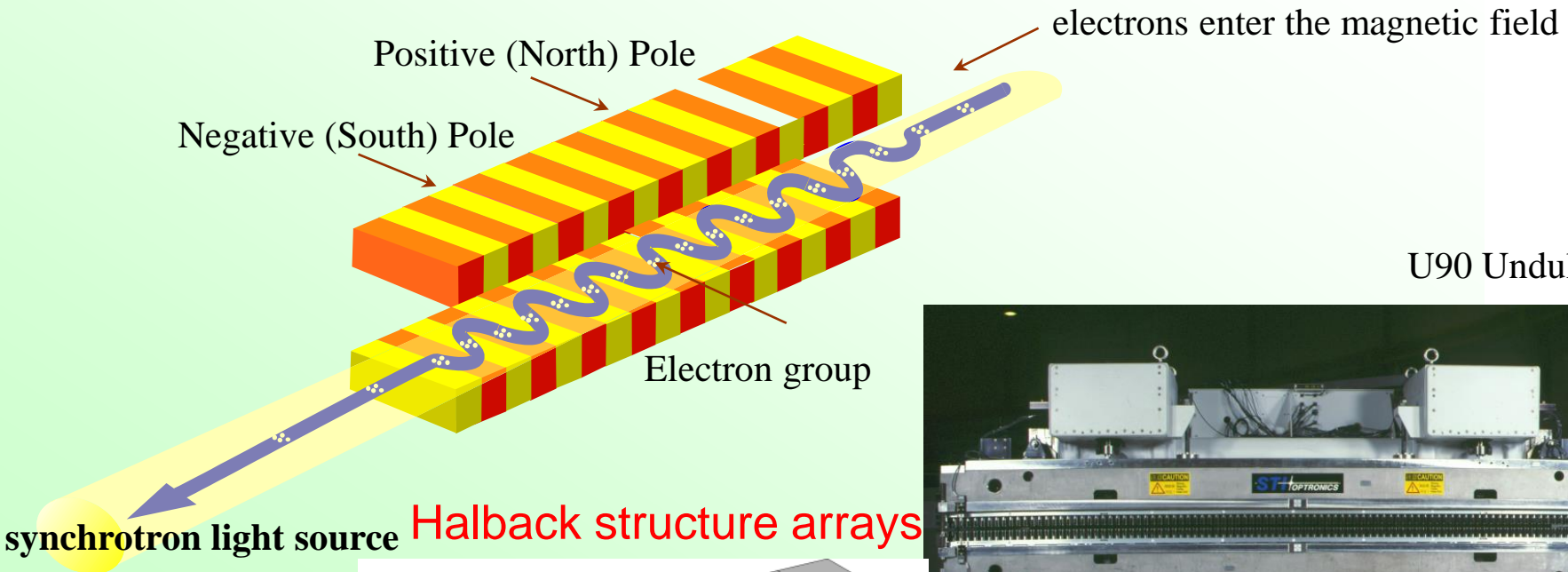


➤ The magnet flux density will depend on the trapped field of magnetized bulks.

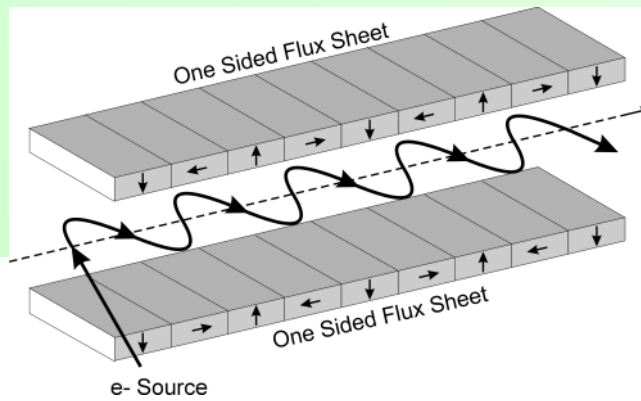
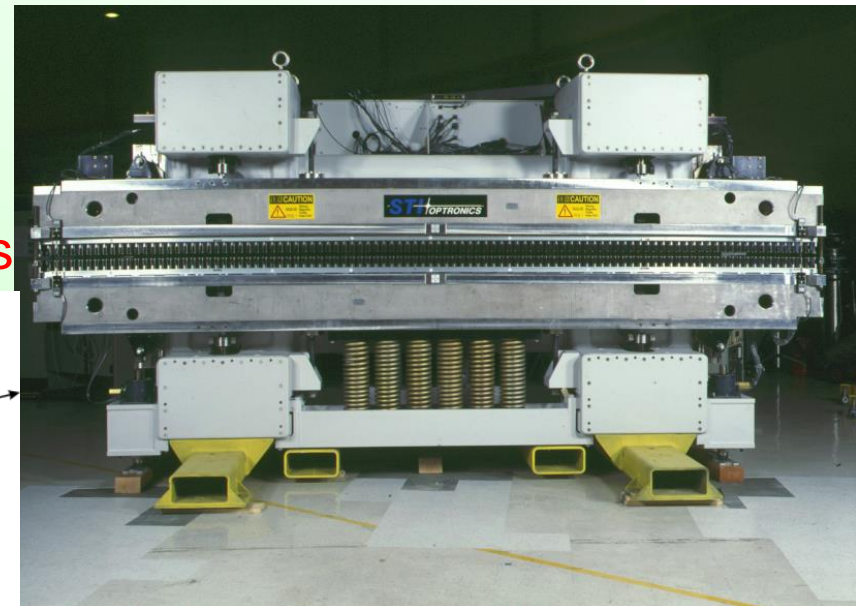


Synchrotron accelerator light source- Insertion Devices

- 一、Undulator: Focus light of the same frequency to increase the brightness of the light
- 二、Wiggler: Enhance more photon flux in higher frequency range
- 三、Wavelength shifter: Increase the frequency of light to a higher energy region



U90 Undulator





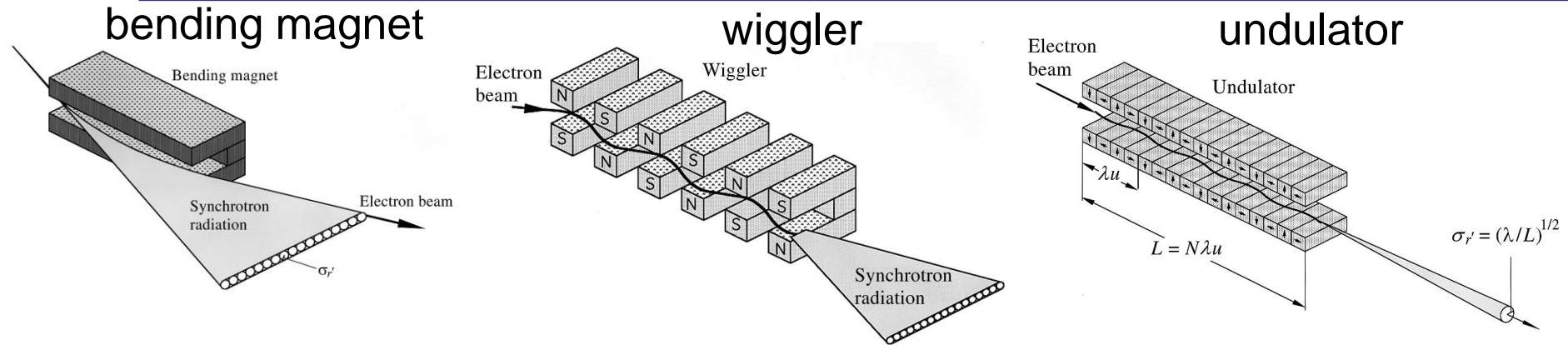
Different features in the insertion devices

Type	Electron Orbit	Electromagnetic field	Spectrum
$K \ll 1$ 偏向角 $\ll \gamma^{-1}$ Undulator			
$K \sim 1$ 偏向角 $\sim \gamma^{-1}$ Undulator			
$K \gg 1$ 偏向角 $\gg \gamma^{-1}$ Wiggler			

$$K = \frac{eB_o\lambda_o}{2\pi mc} = 0.934B_o[T]\lambda_o[cm]$$

K: Deflection parameters

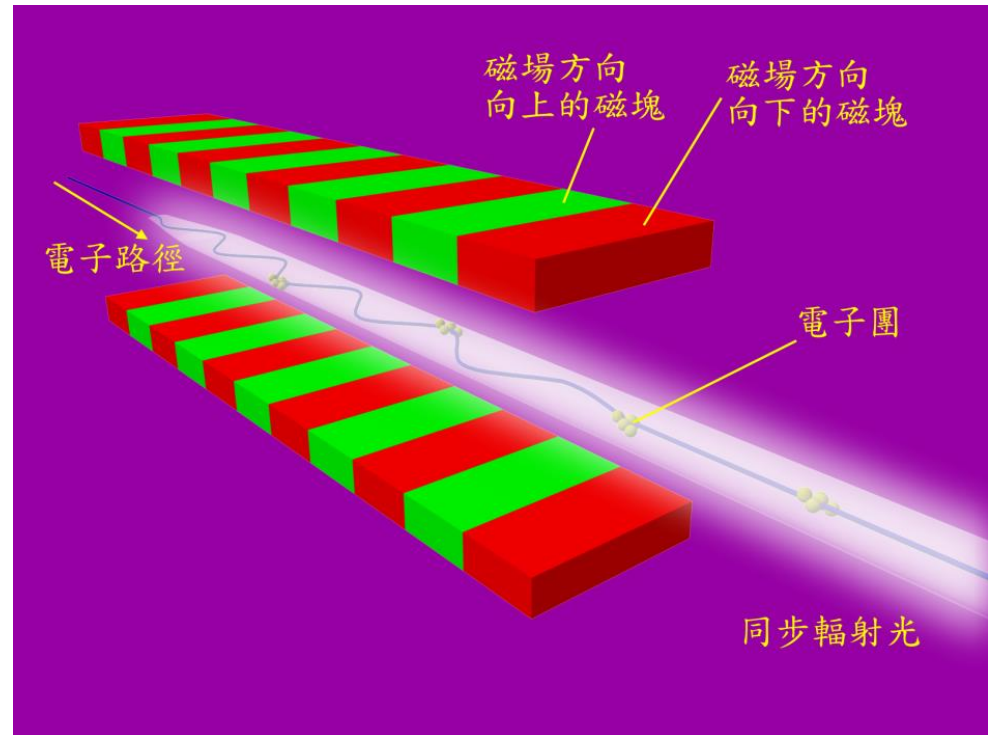
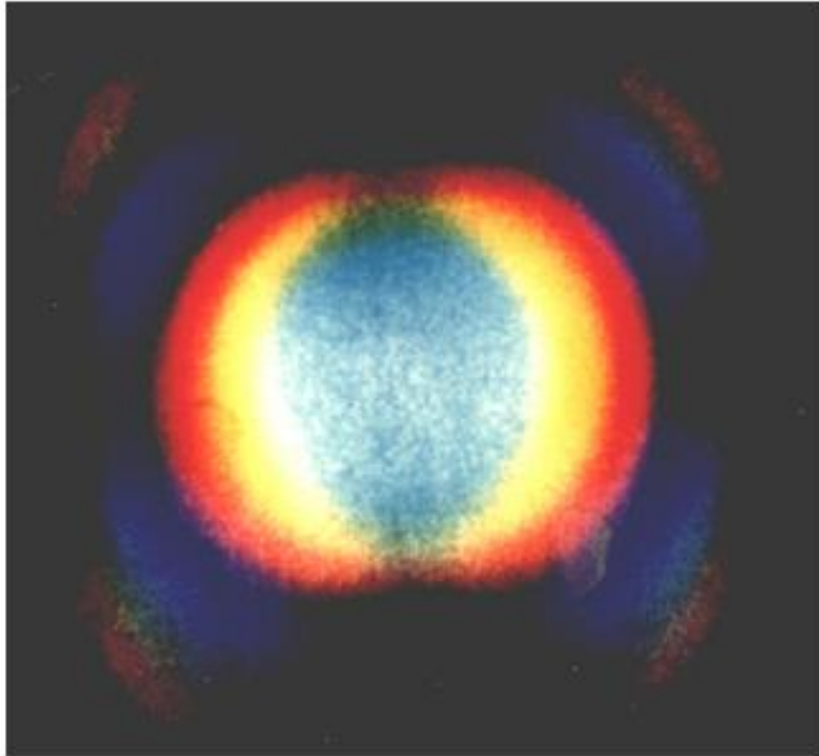
Basic features of the radiation from insertion devices



The synchrotron radiation emitted from (a) bending magnet, (b) wiggler, (c) undulator.

- ◆ The synchrotron radiation emitted from an electron beam which was bent in a spatially periodic sinusoidal field in an insertion device.
- ◆ An electron beam traveling in a curved path (Bending magnet) at nearly **the speed of light emits photons into a narrow cone of natural emission angle $\cong \gamma^{-1}$** .
- ◆ For the wiggler, the horizontal radiation cone become is $k\gamma^{-1}$ and the vertical cone is the same as that of the dipole magnet.
- ◆ For the **undulator**, the radiation cone in horizontal and vertical are all closed to be γ^{-1} .

Synchrotron Radiation from Insertion Devices

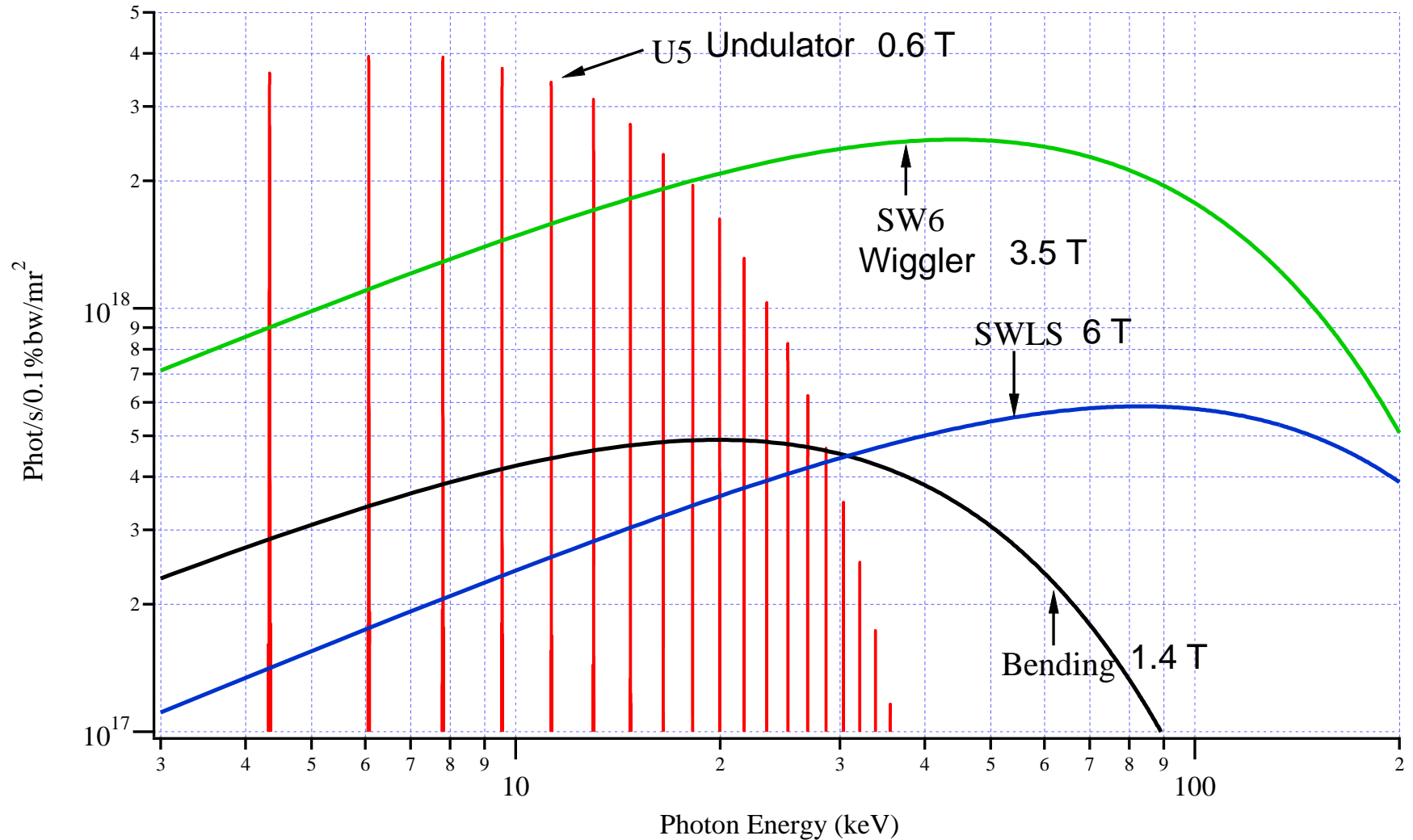


$$\lambda_p = \frac{\lambda_u}{2\gamma^2} \left(1 + \frac{K^2}{2} + \gamma^2 (\theta^2 + \psi^2) \right)$$

$$K = \frac{eB_o\lambda_o}{2\pi mc} = 0.934B_o[T]\lambda_o[cm]$$

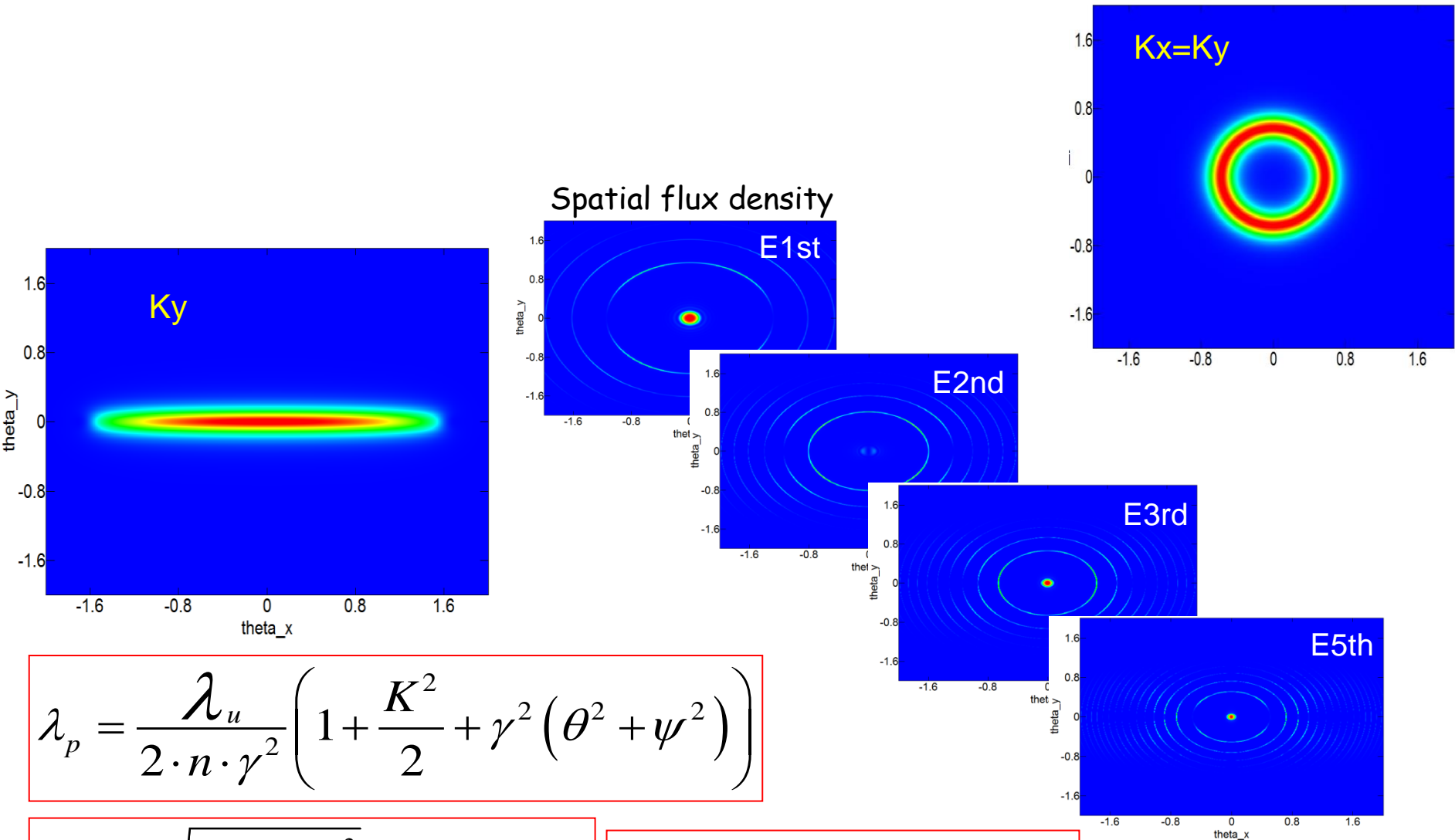


Spectrum of bending and insertion devices





Synchrotron Radiation from Insertion Devices



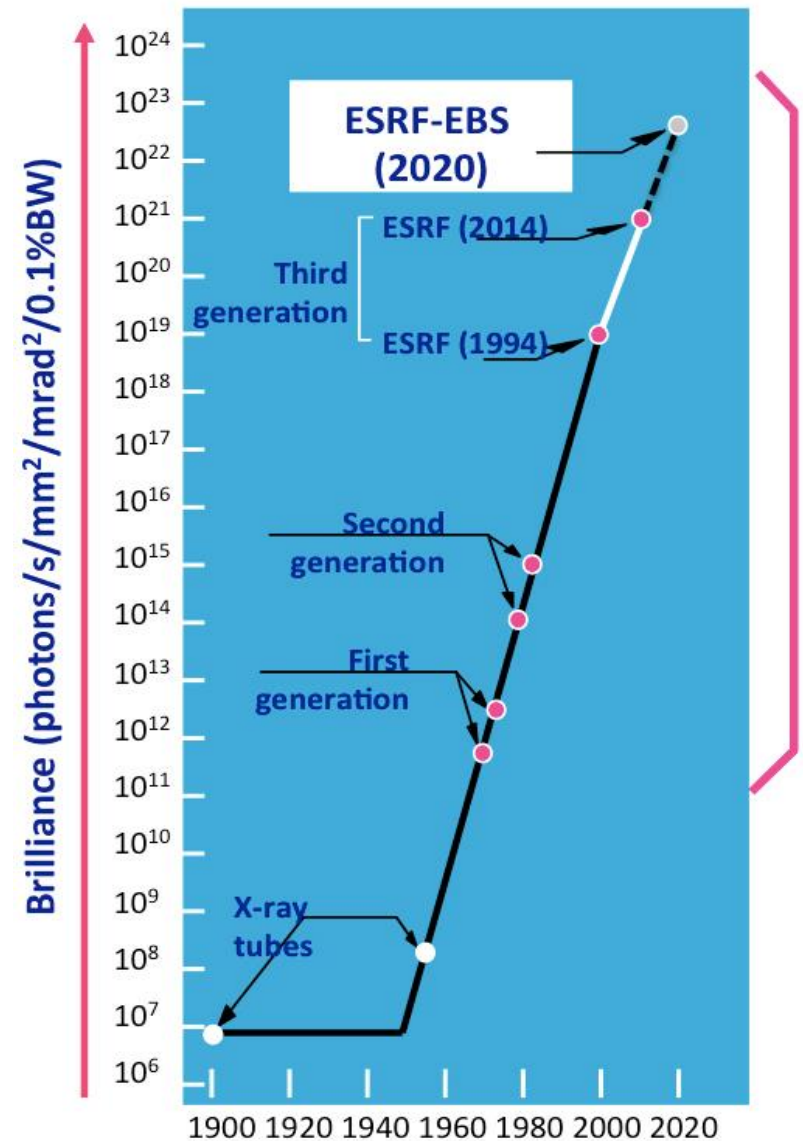
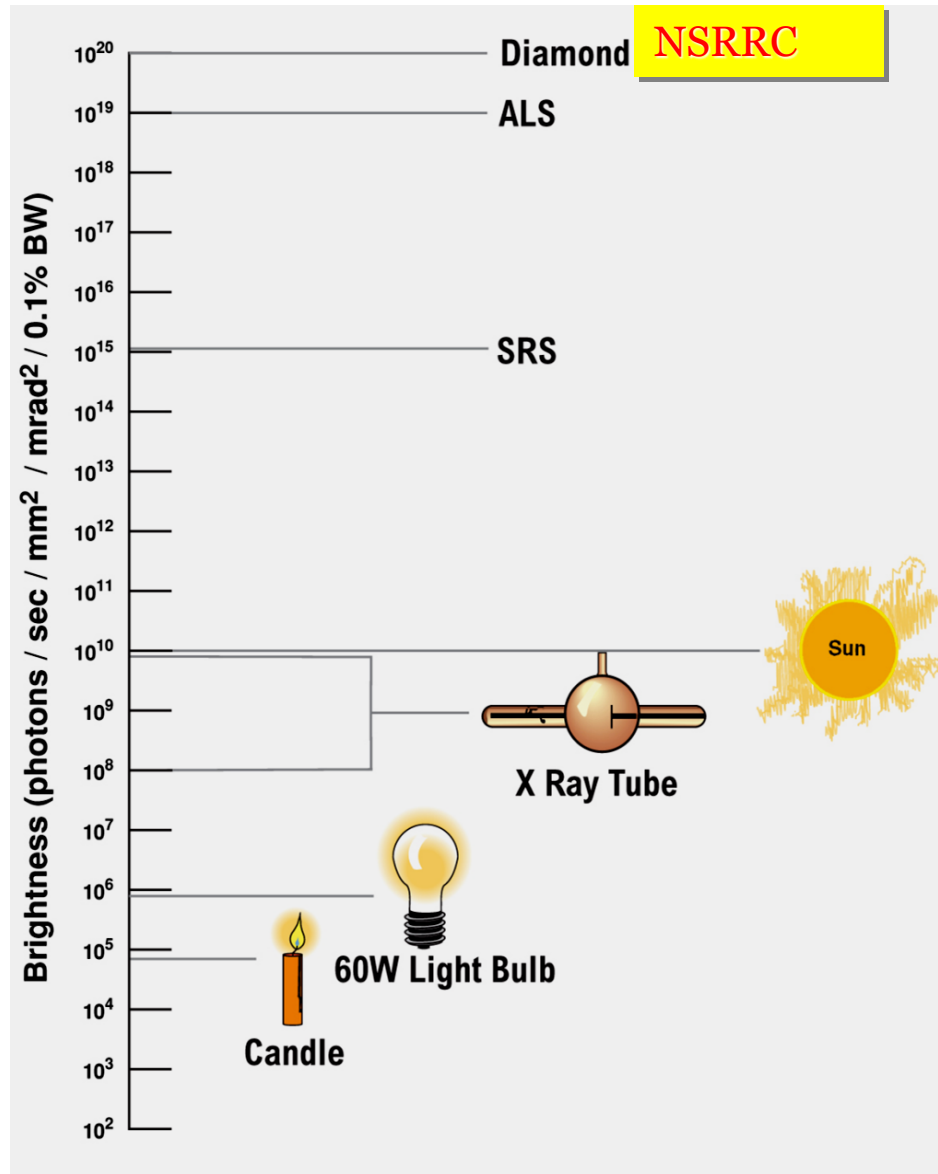
$$\lambda_p = \frac{\lambda_u}{2 \cdot n \cdot \gamma^2} \left(1 + \frac{K^2}{2} + \gamma^2 (\theta^2 + \psi^2) \right)$$

$$\theta_{n,l} = \frac{1}{\gamma} \sqrt{\frac{l}{n} \left(1 + \frac{K^2}{2} \right)}, (l = 1, 2, 3 \dots)$$

$$K = \frac{eB_o \lambda_o}{2\pi mc} = 0.934 B_o [T] \lambda_o [cm]$$



Comparison of spectrum





Field features of plan linear mode Insertion Devices

$$B_y(z) = B_0 \cos k_p z \quad \text{this is what we want}$$

Maxwell tells us what we can get! $B_y(y, z) = B_0 b(y) \cos k_p z$

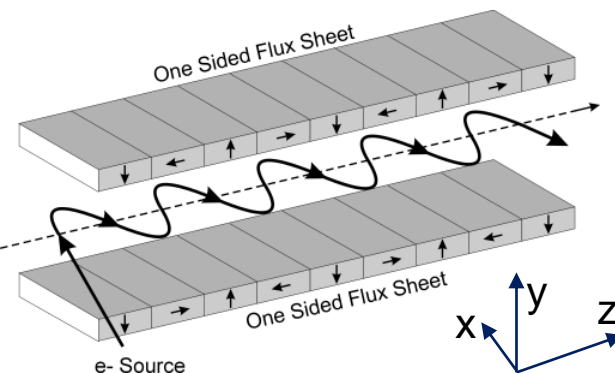
$$\nabla \times \mathbf{B} = 0 \quad \Rightarrow \quad \frac{\partial B_z}{\partial y} = \frac{\partial B_y}{\partial z} = -B_0 b(y) k_p \sin k_p z$$

and $B_y = -B_0 b(y) (1 - \cos k_p z)$

$$\nabla \cdot \mathbf{B} = 0 \quad \Rightarrow \quad \frac{\partial B_z}{\partial z} = -B_0 \frac{\partial b(y)}{\partial y} \cos k_p z$$

$\mathbf{B} \neq \mathbf{B}(\mathbf{x})$

form $\frac{\partial^2 B_z}{\partial y \partial z} \Rightarrow \frac{\partial^2 b(y)}{\partial^2 y} = k_p^2 b(y) \quad \Rightarrow \quad b(y) = a_1 \cosh k_p y + a_2 \sinh k_p y$



$$\begin{aligned} B_x &= 0 \\ B_y &= B_0 \cosh k_p y \cos k_p z \\ B_z &= -B_0 \sinh k_p y \sin k_p z \end{aligned}$$

Assume x-axis is infinite in plan undulator



Spectrum features & calculation

◆ Spectral/angular distribution

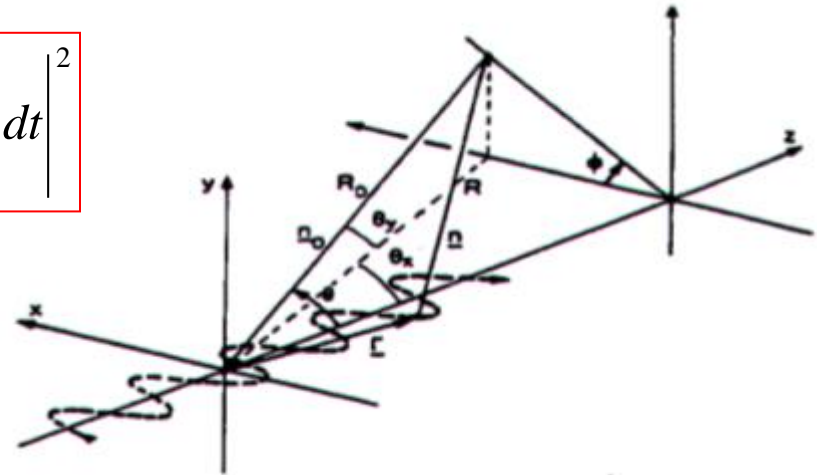
$$\frac{d^2 I}{d\omega d\Omega} = \frac{c}{4\pi^2} \left| \int_{-\infty}^{\infty} R E(t) e^{i\omega t} dt \right|^2 \quad E(t) = \frac{e}{\sqrt{4\pi\epsilon_0 c}} \left[\frac{\hat{n} \wedge \{(\hat{n} - \beta) \wedge \dot{\beta}\}}{(1 - \hat{n} \cdot \beta)^3 R} \right]_{t_{ret.}}$$

where $\hat{n} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$ is the unit vector from the point of emission to the observer (see Figure). The observer and emission times are related by:

$t = t_{ret.} + R/c$ where R is the distance between the emission and observer points, and hence:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2}{(4\pi\epsilon_0)4\pi^2 c} \left| \int_{-\infty}^{\infty} \frac{\hat{n} \wedge \{(\hat{n} - \beta) \wedge \dot{\beta}\}}{(1 - \hat{n} \cdot \beta)^2} e^{i\omega(t - \hat{n} \cdot r/c)} dt \right|^2$$

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2 \gamma^2 N^2}{(4\pi\epsilon_0) c} L(N\Delta\omega / \omega_1(\theta)) F_n(K, \theta, \phi)$$



Geometry for the analysis of undulator radiation

General radiation formula



Radiation from bending & wiggler magnet

In a **wiggler**, the deflection parameter K is large (typically $K \geq 10$) and photon radiation from different poles of the electron trajectory is enhanced incoherently. The angular density of flux is then given by $2N$ (N is the number of magnet periods) times the formula for bending magnets. *The angular distribution of radiation emitted by electrons that are moving through a **bending magnet**, following a circular trajectory in a horizontal plane is,*

$$\frac{d^2 \bar{B}(w)}{d\theta d\phi} = \frac{3\alpha\gamma^2}{4\pi^2} \frac{I}{e} \frac{\Delta w}{w} \left(\frac{\varepsilon}{\varepsilon_c} \right)^2 \left(1 + \gamma^2 \phi^2 \right)^2 \left[\overset{\text{Horizontal polarization}}{\overset{\sigma\text{-mode}}{K_{2/3}^2(\xi)}} + \frac{\gamma^2 \phi^2}{1 + \gamma^2 \phi^2} \overset{\text{Vertical polarization}}{\overset{\pi\text{-mode}}{K_{1/3}^2(\xi)}} \right]$$

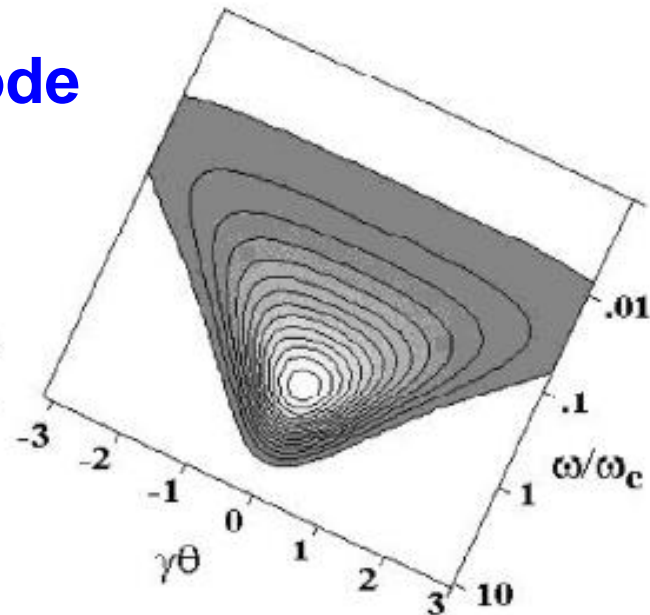
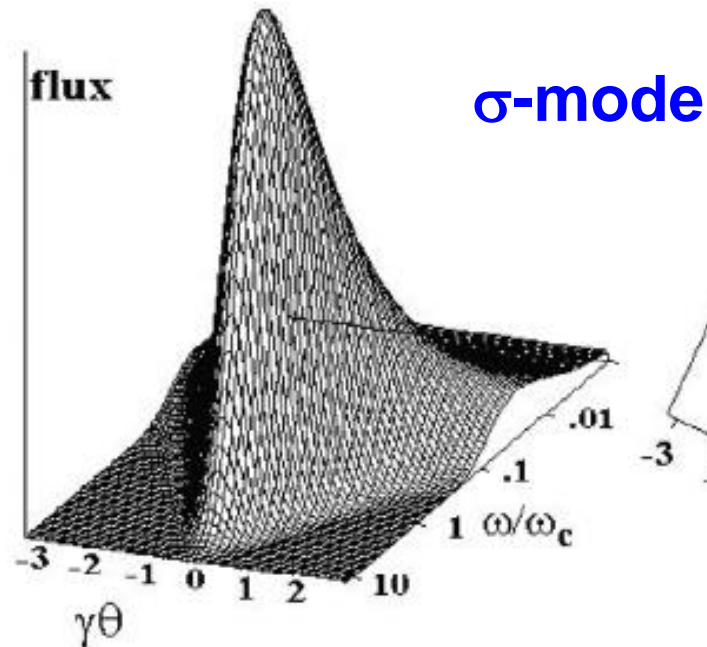
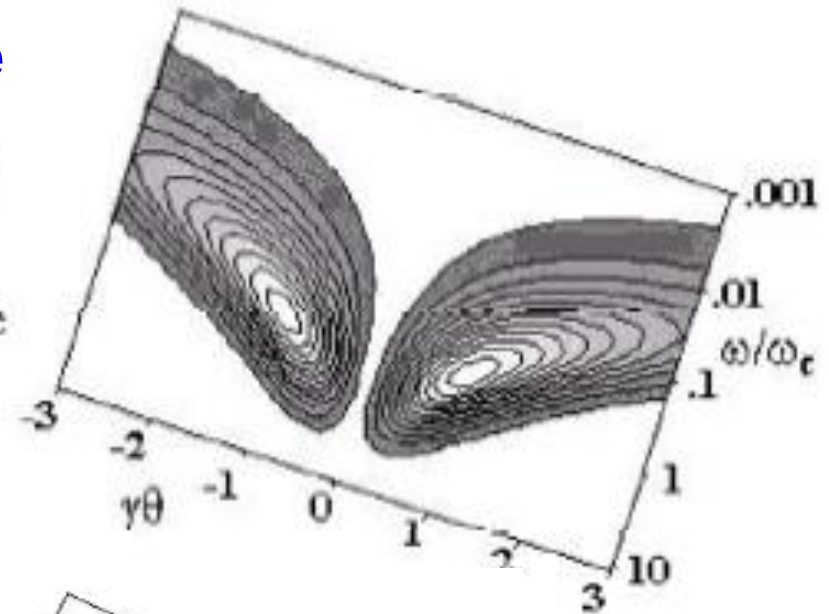
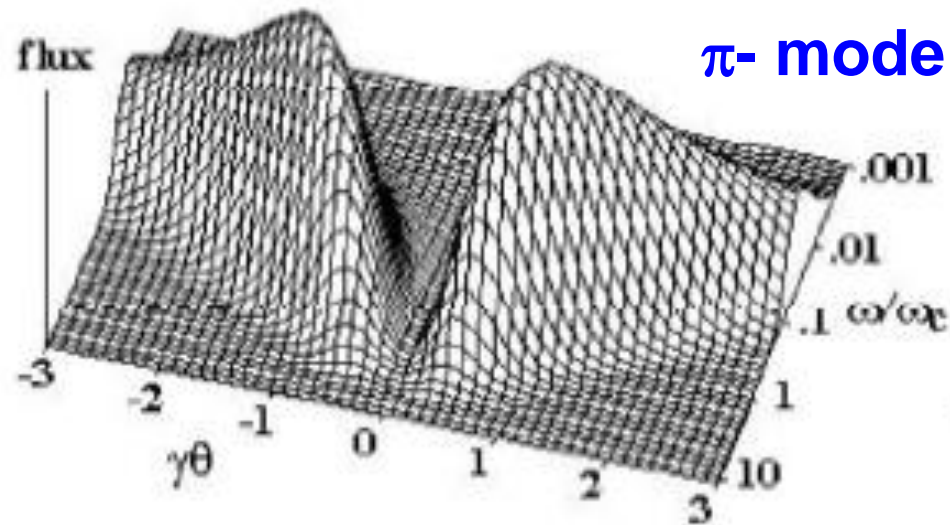
Where ε and ε_c are the photon energy and the photon critical energy, respectively; θ and ϕ are the observation angles in the horizontal and vertical directions, respectively; α is the fine-structure constant; I is the beam current; e is the electron charge; the subscripted K 's are modified Bessel functions of the second kind, and ξ is defined as

$$\varepsilon_c \text{ (keV)} = 0.665 E^2 \text{ (GeV)} B \text{ (T)}$$

$$\xi \equiv \left(\varepsilon / 2\varepsilon_c \right) \left(1 + \gamma^2 \phi^2 \right)^{3/2}$$

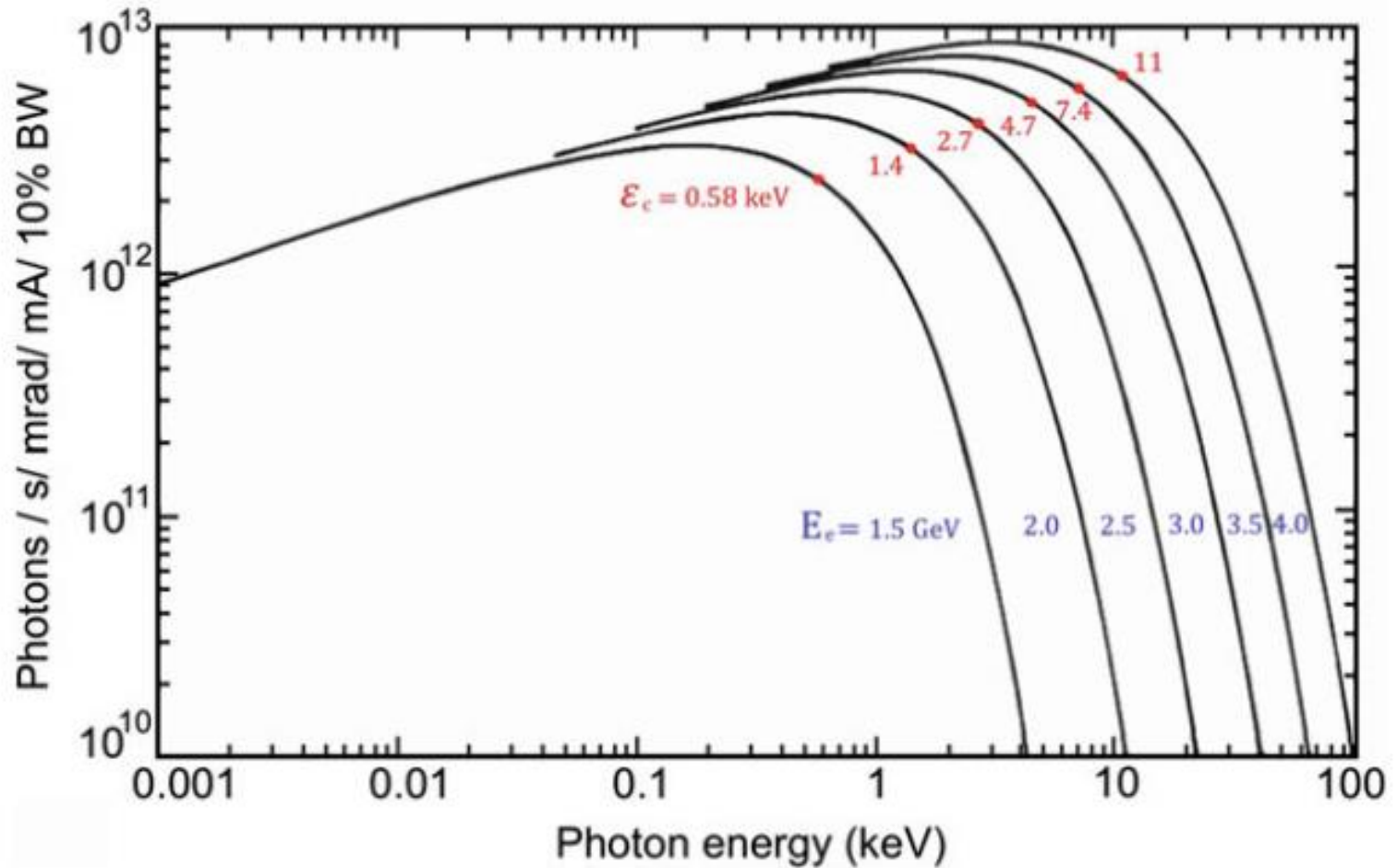
$$\varepsilon_c(\theta) = \varepsilon_c(0) \sqrt{1 - (\gamma\theta / K)^2}$$

Radiation distribution on π - σ mode





Photon spectrum from different electron energy



$$\varepsilon_c \text{ (keV)} = 0.665 E^2 \text{ (GeV)} B \text{ (T)}$$

$$\varepsilon_c(\theta) = \varepsilon_c(0) \sqrt{1 - (\gamma\theta / K)^2}$$



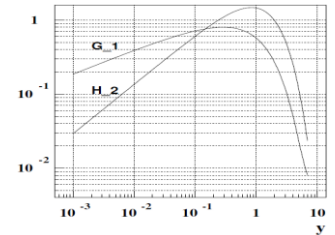
Flux calculation of bending magnet and wiggler

Flux Density $\frac{d^2 F(w)}{d\theta d\phi} [p / s / mrad^2] = 1.327 \times 10^{16} \frac{\Delta w}{w} E^2 [GeV] I [A] H_2(y)$

Flux Density distribution integrated over ϕ is given by

$$\frac{d F(w)}{d\theta} [p / s / mrad] = 2.457 \times 10^{16} \frac{\Delta w}{w} E [GeV] I [A] G_1(y)$$

At $\phi = 0$ $\frac{d^2 F(w)}{d\theta d\phi} = \frac{d F(w)}{d\theta} \cdot \frac{1}{\sigma_\phi \sqrt{2\pi}}$



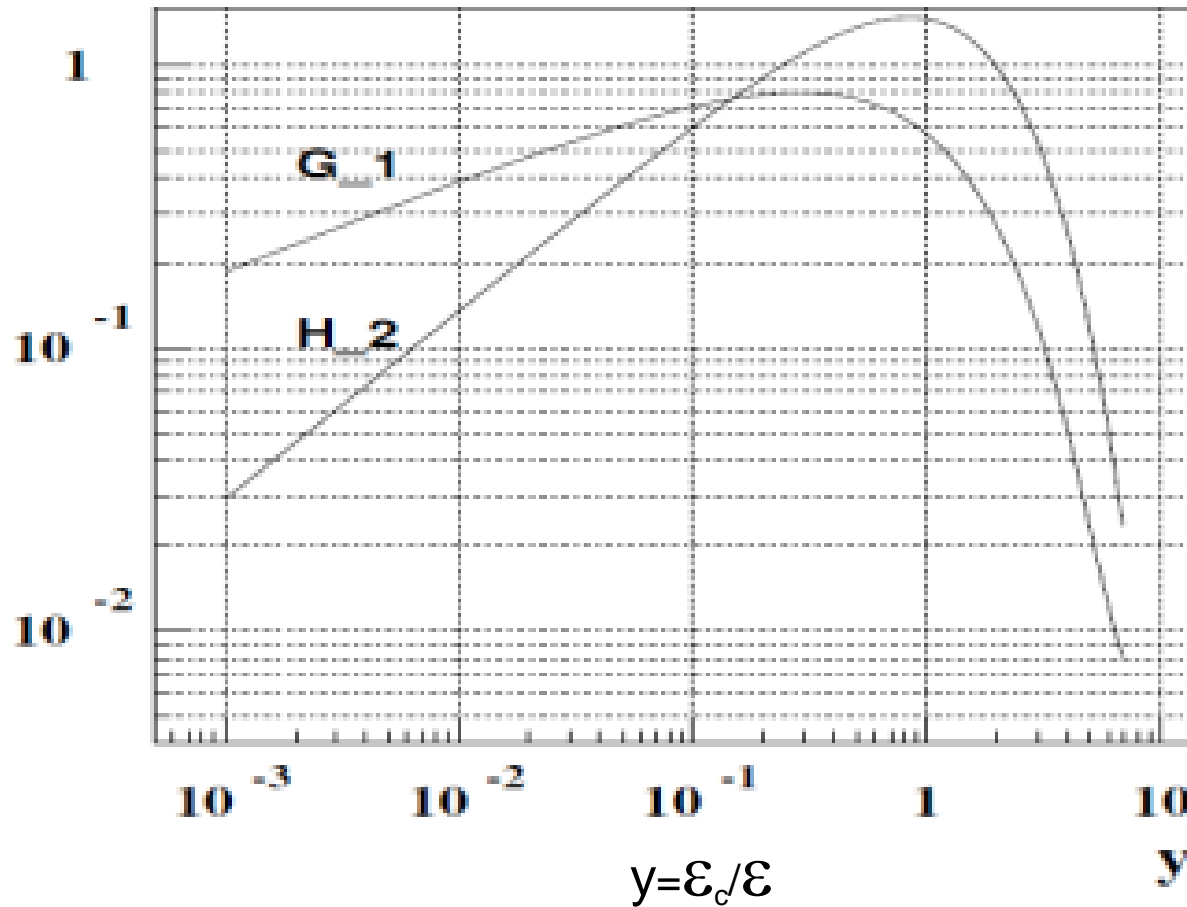
Therefore, the total flux $F(w)$ of bending radiation is integrated over θ . However, for **the wiggler radiation**, the total flux $F(w)$ will be multiplied by a factor of $2N$ (N is the period number).

$$\frac{d F(w)}{d\theta} [p / s / mrad] = 2.457 \times 10^{16} \frac{\Delta w}{w} E^{2N} [GeV] I [A] G_1(y)$$

Finally, the power density or total power is flux density or total flux multiply by photon energy, respectively.



Function G1(y) and H2(y) of the synchrotron radiation



$$\frac{d^2 F(w)}{d\theta d\phi} [p / s / mrad^2 / 0.1\% BW] = 1.327 \times 10^{13} E^2 [GeV] I [A] H_2(y)$$

$$\frac{d F(w)}{d\theta} [p / s / mrad / 0.1\% BW] = 2.457 \times 10^{13} E [GeV] I [A] G_1(y)$$



Electron motion in the Insertion Devices

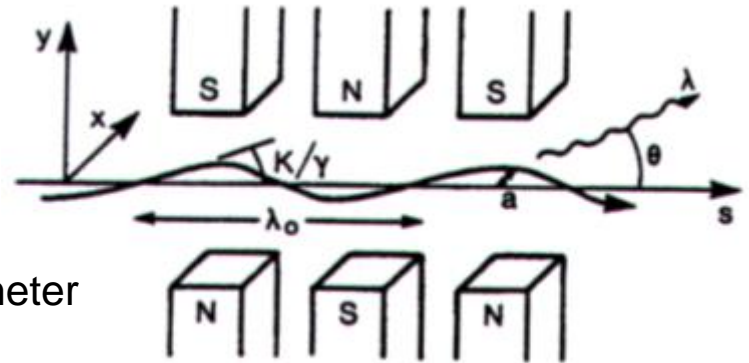
$B_y = B_o \sin(kz)$, where $k = 2\pi / \lambda_o$ and λ_o is the insertion device period length.

$$\ddot{x} = \frac{e}{\gamma m} (-\dot{z} B_y)$$

$$\ddot{z} = \frac{e}{\gamma m} (\dot{x} B_y)$$

$$\dot{x} = \frac{e B_o}{\gamma m} \frac{\cos(kz)}{k}$$

$$\beta_x = \dot{x} / c = \frac{K}{\gamma} \cos(kz)$$



where the dimensionless undulator or deflection parameter is defined as follows:

$$K = \frac{e B_o \lambda_o}{2\pi m c} = 0.934 B_o [T] \lambda_o [cm]$$

$$\beta_x^2 + \beta_z^2 = \beta^2 \quad (= \text{constant})$$

$$\beta_z \cong \beta \left(1 - \frac{K^2}{4\gamma^2} - \frac{K^2}{4\gamma^2} \cos 2kz \right)$$

First integral field $\int B ds = 0$ ($x'=0$) & Second integral field $\int (\int B ds') ds = 0$ ($x=0$)

The average velocity along the z-axis is thus: $\bar{\beta} \cong \beta \left(1 - \frac{K^2}{4\gamma^2} \right) \cong 1 - \frac{1}{2\gamma^2} - \frac{K^2}{4\gamma^2}$

We will only consider cases in which $K/\gamma \ll 1$ and so we can write to a good approximation that

$z = \bar{\beta} c t$ and $kz = \Omega t$ where $\Omega = 2\pi \bar{\beta} c / \lambda_o$. We have electron angle then:

$$\dot{x} = \frac{K}{\gamma} c \cos(\Omega t) \quad \text{which can be integrated directly to give e-trajectory: } x = \frac{K}{\gamma} \frac{c}{\Omega} \sin(\Omega t)$$

$$x' = \frac{K}{\gamma} \cos(\Omega t), \quad z = \bar{\beta} c t - \frac{K^2}{4\gamma^2} \frac{c}{2\Omega} \sin(2\Omega t), \quad z = \bar{\beta} c - \frac{K^2}{4\gamma^2} c \cos(2\Omega t), \quad x = \frac{K}{\gamma} \frac{\lambda_o}{2\pi}, \quad x' = \frac{K}{\gamma}$$



Photon Interference in undulator

In the time it takes the electron to move through one period length from point A to an equivalent point B ($\lambda_o / \beta c$) the wavefront from A has advanced by a distance λ_o / β and hence is ahead of the radiation emitted at point B by a distance d where:

$$d = \frac{\lambda_o}{\beta} - \lambda_o \cos \theta$$

and where θ is the angle of emission with respect to the electron beam axis. When this distance is equal to an integral number, n , of radiation wavelength there is constructive interference of the radiation from successive poles:

$$\frac{\lambda_o}{\beta} - \lambda_o \cos \theta = n\lambda$$

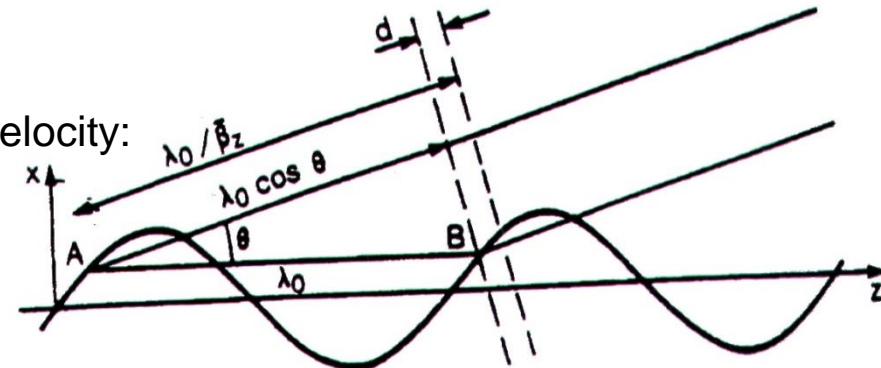
Inserting the expression for the average electron velocity:

$$\frac{1}{\beta} \cong 1 + \frac{1}{2\gamma^2} + \frac{K^2}{4\gamma^2}$$

results in the following interference condition:

$$\mathcal{E} [keV] = 0.95n \frac{E^2 [GeV]}{\lambda_o \left(1 + \frac{K^2}{2} + \gamma^2 (\theta^2 + \phi^2) \right)}$$

$$\mathcal{E} [eV] = \frac{1.2398}{\lambda_p (\mu m)}$$

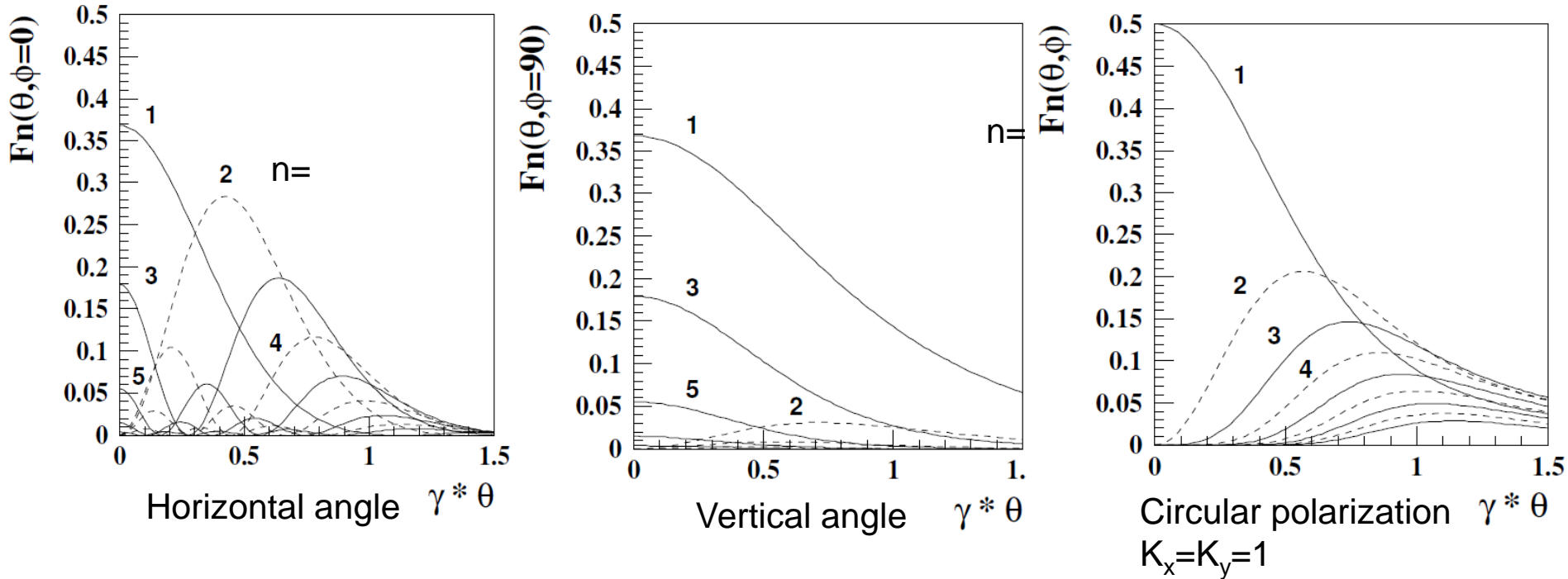


$K \gg 1$ wiggler, $K \approx 1$ undulator,

$$\lambda (A) = \frac{\lambda_o (mm)}{2n\gamma^2} \left(1 + \frac{K^2}{2} + \gamma^2 (\theta^2 + \phi^2) \right) = 1305.6 \frac{\lambda_p (m) \left(1 + \frac{K^2}{2} + \gamma^2 (\theta^2 + \phi^2) \right)}{nE (GeV)^2}$$



Angular flux density from undulator-I



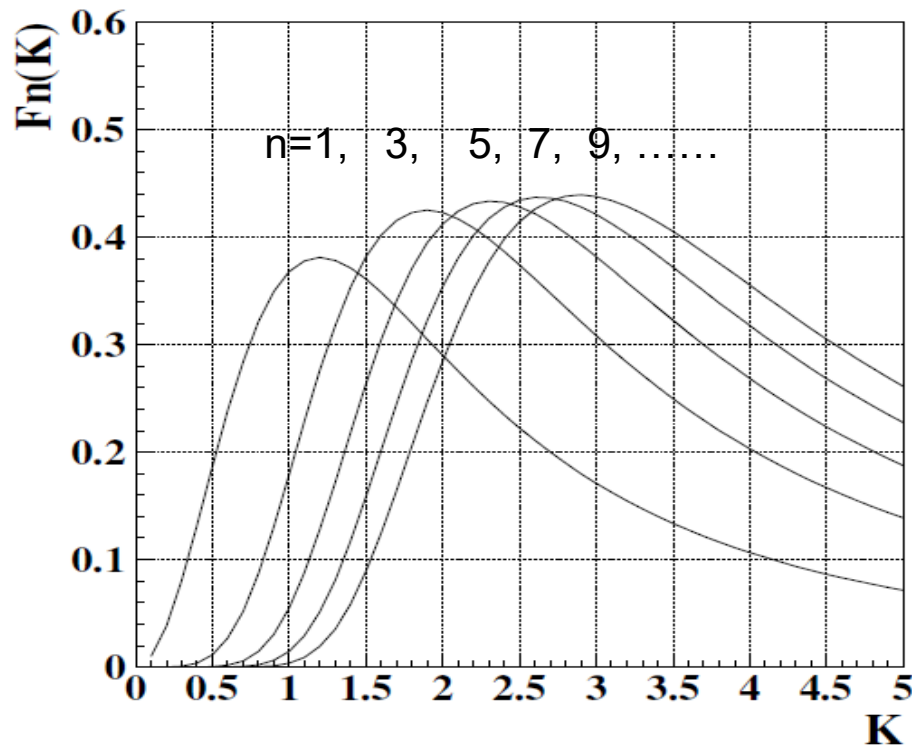
F function of Angular flux density in the horizontal (left) and vertical (right) planes for the case $K = 1$. $K = 0.934 B_o [T] \lambda_o [cm]$

$$F_n(K) = \frac{n^2 K^2}{(1 + K^2 / 2)^2} \left[J_{\frac{n+1}{2}}(Z) - J_{\frac{n-1}{2}}(Z) \right]^2 \quad Z = \frac{nK^2}{4(1 + K^2 / 2)}$$

On-axis ($\theta = \phi = 0$)



Angular flux density from undulator-2



On-axis angular flux density function

$$\bar{\beta} = \left. \frac{d^2 \dot{n}}{d\omega / \omega \cdot d\Omega} \right|_{\theta=0} [p / s / rad^2 / 0.1\% BW] = 1.744 \cdot 10^{20} N^2 E^2 [GeV] F_n(K) I_b(A)$$

$$\bar{\beta} = \left. \frac{d^2 \dot{n}}{d\Omega} \right|_{\theta=0} [p / s / rad^2] = 1.744 \cdot 10^{23} N^2 E^2 [GeV] F_n(K) I_b(A)$$

in practical units of photons/s/mrad²/0.1% bandwidth:

If $k \leq 2$ only $n=1,3,5,7,9,11,13,15$;

$k \leq 1$ only $n=1,3,5,7$; $k \leq 0.5$ only $n=1,3$

$k \leq 0.25$ only $n=1$

If $n=1 \rightarrow K_{\min}=0.15$; $n=3 \rightarrow K_{\min}=0.5$;

$n=5 \rightarrow K_{\min}=0.75$; $n=7 \rightarrow K_{\min}=1$;

$n=9 \rightarrow K_{\min}=1.2$; $n=11 \rightarrow K_{\min}=1.4$

$$\mathcal{E} [keV] = 0.95n \frac{E^2 [GeV]}{\lambda_o \left(1 + \frac{K^2}{2} + \gamma^2 (\theta^2 + \phi^2) \right)}$$

$$K = \frac{eB_o \lambda_o}{2\pi mc} = 0.934 B_o [T] \lambda_o [cm]$$



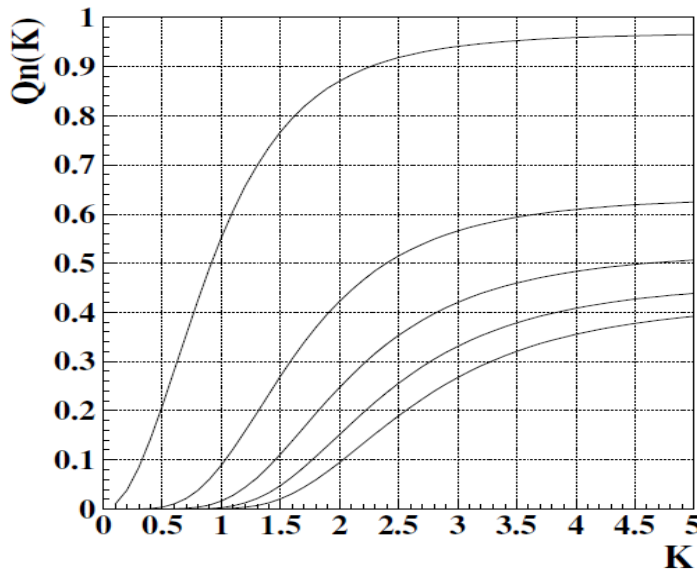
Total flux from undulator

◆ Total flux

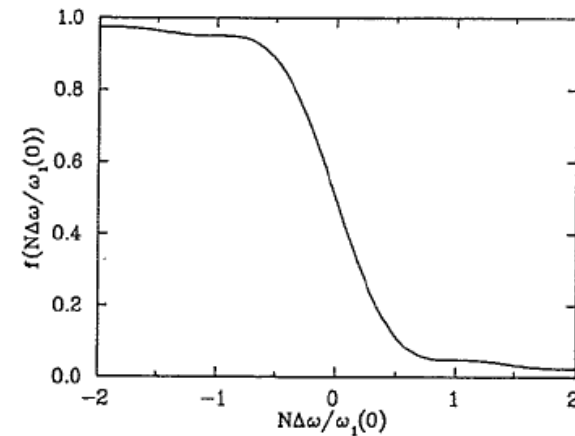
We obtain the total flux in the **central cone** in practical units of flux is **photons/s/0.1% bandwidth**:

$$B = \bar{B} d\Omega = \frac{d\dot{n}}{d\omega/\omega} = 1.431 \cdot 10^{14} N Q_n(K) f(N\Delta\omega/\omega_1(0)) I_b$$

where $Q_n(K) = (1 + K^2/2) F_n(K)/n$. The flux function $Q_n(K)$ and the detuning function $f(N\Delta\omega/\omega_1(0))$. It can be seen that for zero detuning (i.e. $\omega = \omega_n(0)$) the flux is very close to half of the usually quoted result. Nearly twice as much flux can be obtained however by a small detuning to lower frequency by approximately



$Q_n(k)$: Undulator flux function



Undulator flux function as function of detuning



Radiation power from insertion devices

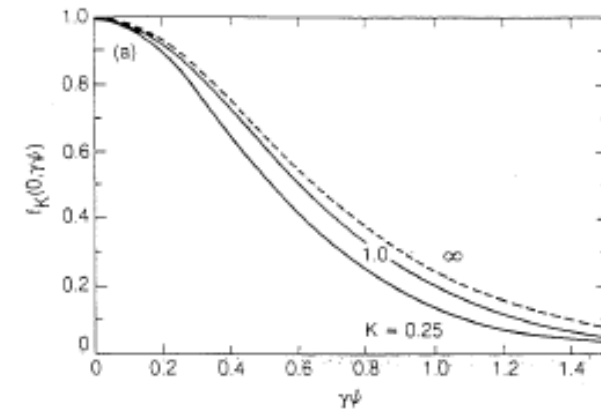
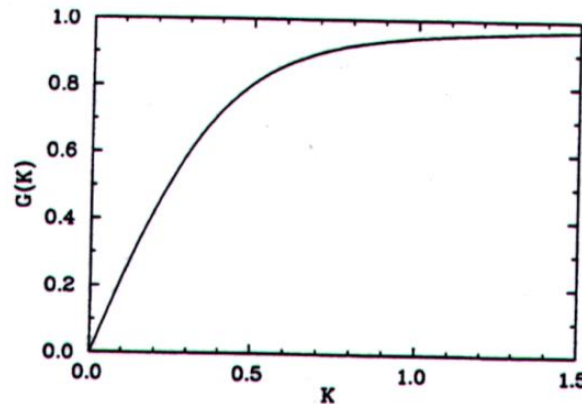
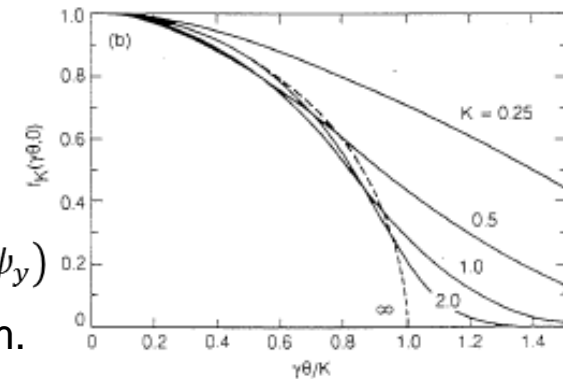
◆ Power and power density

$$\frac{d^2 \dot{n}}{d\omega / \omega \cdot d\Omega} \times E_P = \frac{dP}{d\Omega} [W / mrad^2] = 10.84 E^4 [GeV] B_o N I_b G(K) f_K(\theta_x, \theta_y)$$

where $G(K) = \frac{K(K^6 + \frac{24}{7}K^4 + 4K^2 + \frac{16}{7})}{(1 + K^2)^{7/2}}$ and

$$f_K(\theta_x, \theta_y) = \frac{16K}{7\pi G(K)} \int_{-\pi}^{\pi} \sin^2 \alpha \left[\frac{1}{D^3} - \frac{4(\gamma\theta_x - K \cos \alpha)^2}{D^5} \right] d\alpha, \text{ as obtained by Kim.}$$

$$f_K(\theta_x, \psi_y)$$



- Total power on ID

$$P_{tot} [kW] = 0.633 \cdot E^2 [GeV] B_o^2 L I_b$$

- Total power on Bending magnet

$$P_{tot} [kW] = 0.633 \cdot E^2 [GeV] 2B_o^2 L I_b$$



Definition of Radiation brilliance

◆ Brightness

We will obtain the brightness in practical units is **photon/s/mm²/mrad²/0.1% bandwidth**. Photon flux unit is **photon/s/0.1% bandwidth**.

$$Brilliance(\beta) = \frac{\text{photon flux}(B)}{4\pi^2 \sigma_x \sigma_y \sigma'_x \sigma'_y \sigma_{BW}/E} \quad \frac{\sigma_{BW}}{E} \approx \sqrt{\left(\frac{0.85}{nN_w}\right)^2 + \left(\frac{\sigma_E}{E}\right)^2}$$

$$\sigma_x = \sqrt{\varepsilon_x \beta_x + \eta^2 (\sigma_E/E)^2 + \sigma_{ph}^2} \quad \sigma'_x = \sqrt{\varepsilon_x \gamma_x + \eta'^2 (\sigma_E/E)^2 + \sigma_{ph}'^2}$$

$$\sigma_y = \sqrt{\varepsilon_y \beta_y + \sigma_{ph}^2} \quad \sigma'_y = \sqrt{\varepsilon_y \gamma_y + \sigma_{ph}'^2}$$

$$\gamma_{x,y}(s) = (1 + \alpha_{x,y}(s)^2) / \beta_{x,y}(s), \quad \alpha_{x,y}(s) = -\frac{1}{2} \frac{d\beta_{x,y}(s)}{ds}, \quad \beta_{x,y}(s) = \beta_{x,y}(0) + \frac{s^2}{\beta_{x,y}(0)}$$

The diffraction-limited source size (rms) $\sigma_{ph} \sigma_{ph}' = \frac{\lambda}{4\pi}$

corresponding to the angular divergence of ID $\sigma_{ph}' = \sqrt{\lambda_n / L} = \frac{1}{\gamma} \sqrt{\frac{1 + k^2/2}{2N \cdot n}}$

λ is the photon wavelength and λ_u is the undulator periodic length.

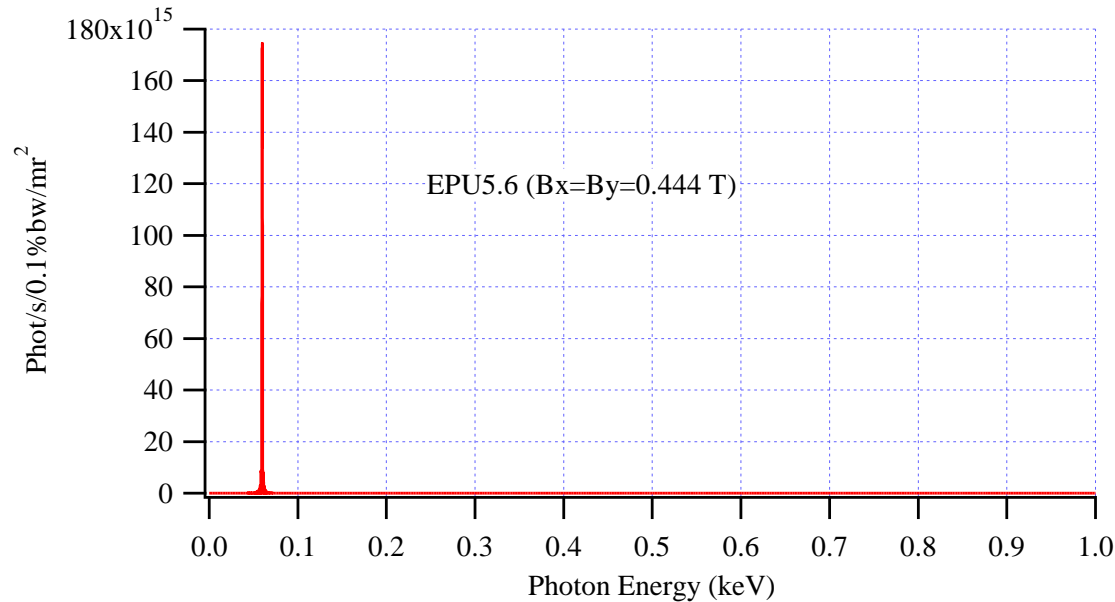
corresponding to the angular divergence of bending magnet $\sigma_{ph}'(mr) = 0.48 \frac{C(y)}{E(GeV)}$



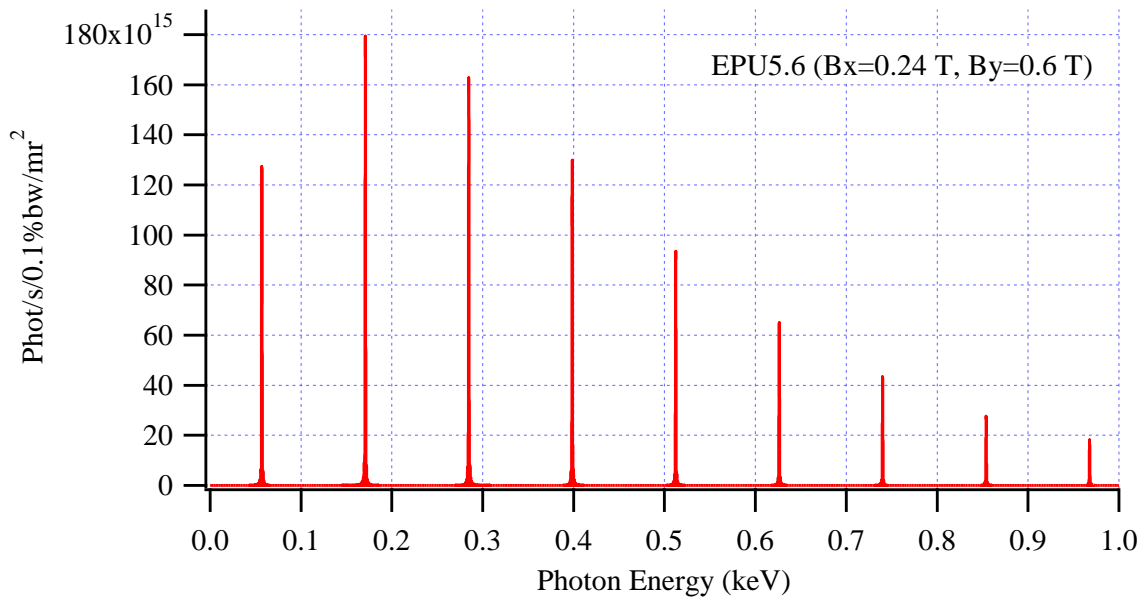
Example of the characteristics of ID spectrum



Features of elliptically polarized undulator (EPU)



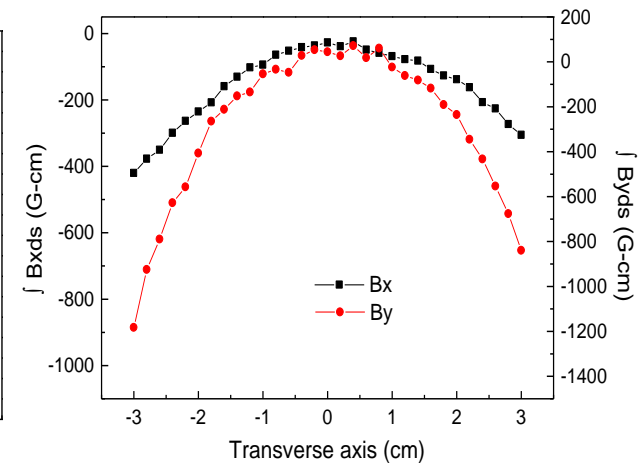
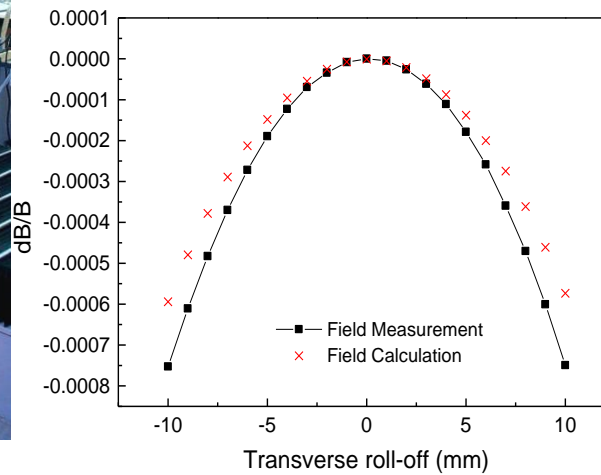
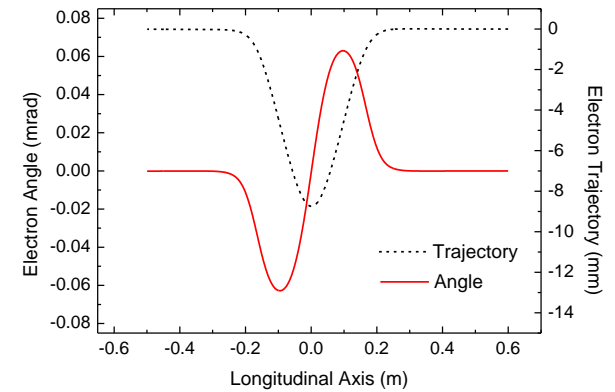
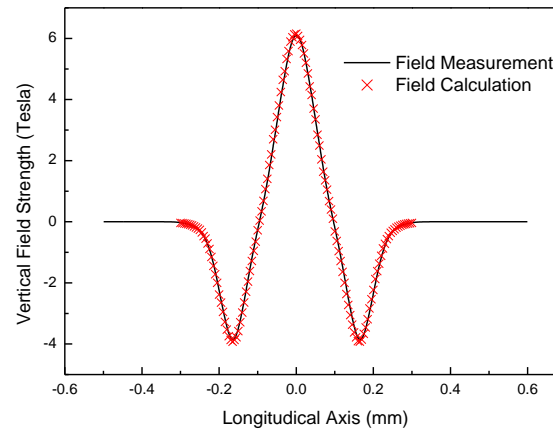
Circular polarization n=1,
higher harmonic spectrum is zero



Elliptical polarization for fundamental
and higher harmonic spectrum

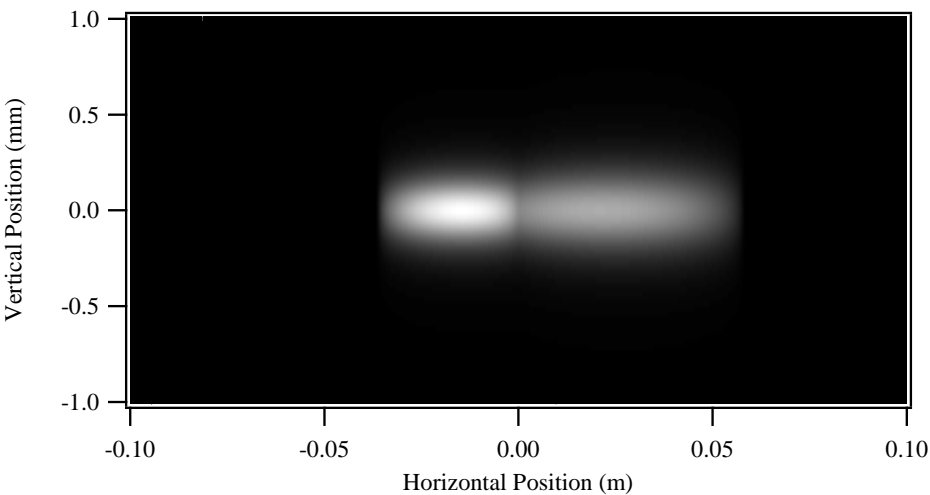


Wavelength shifter (SWLS) with 6 T-example

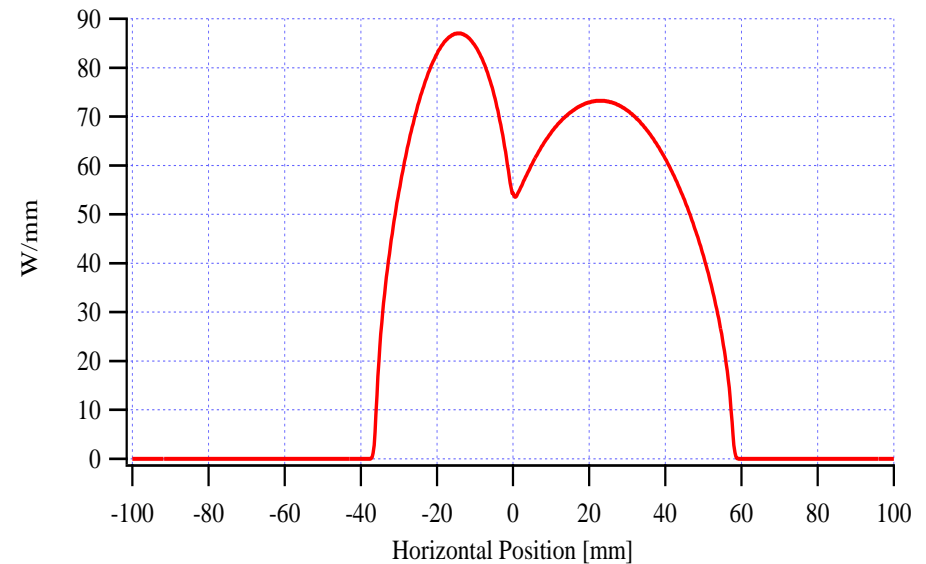
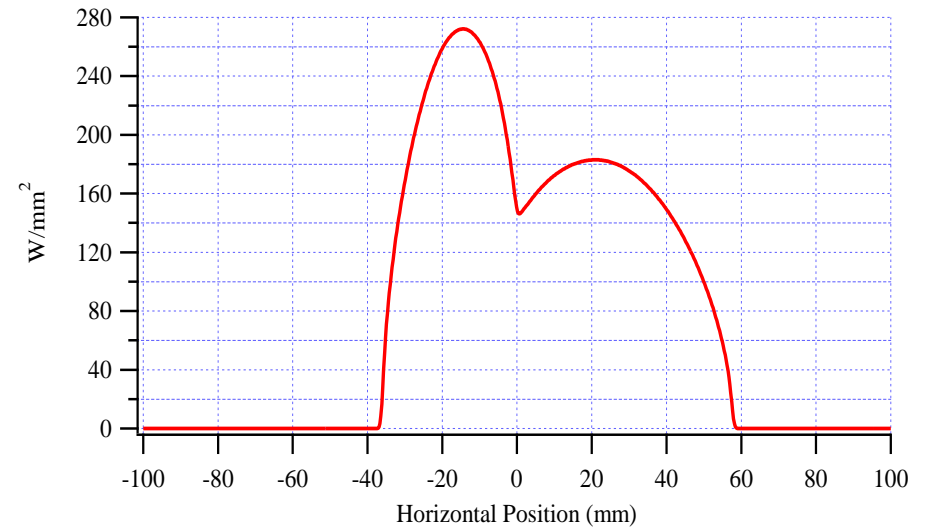
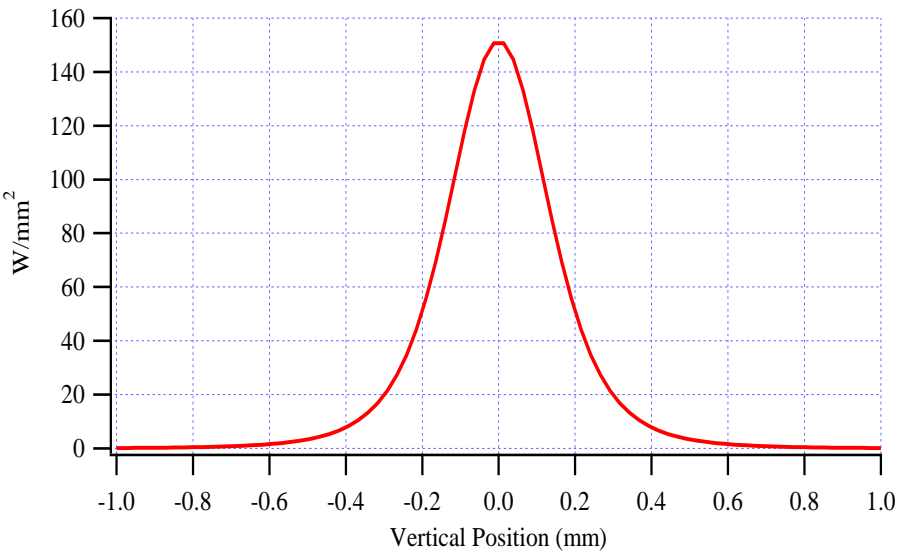




SWLS Power density calculation (total 5.96 kW)

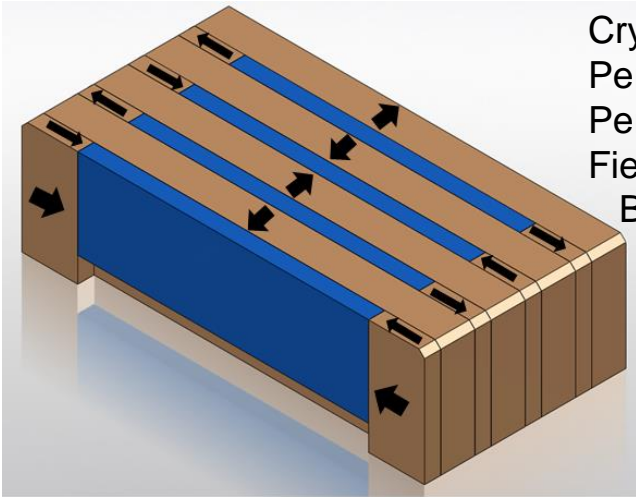


Away from source 0.7 m

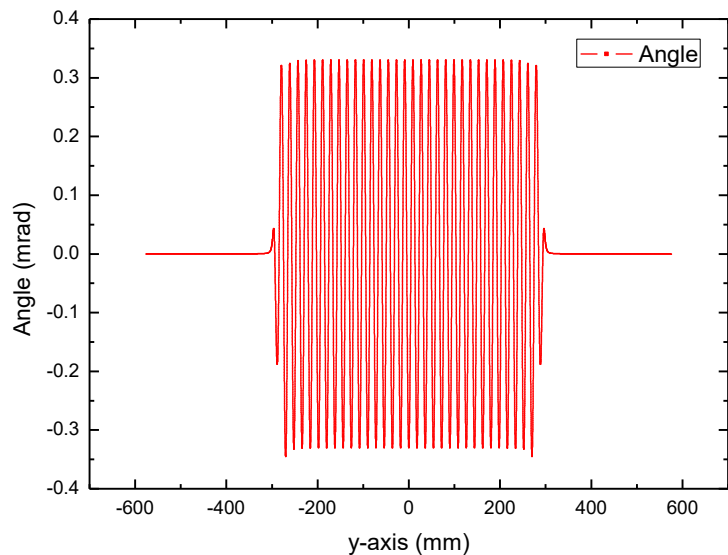
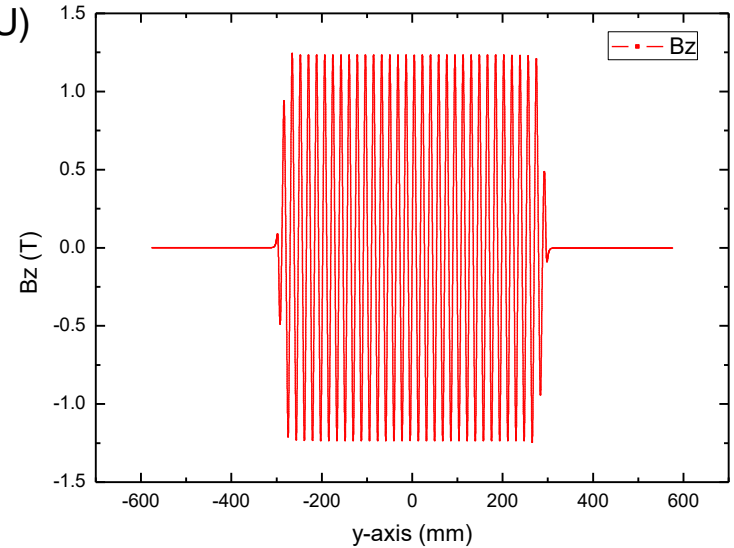




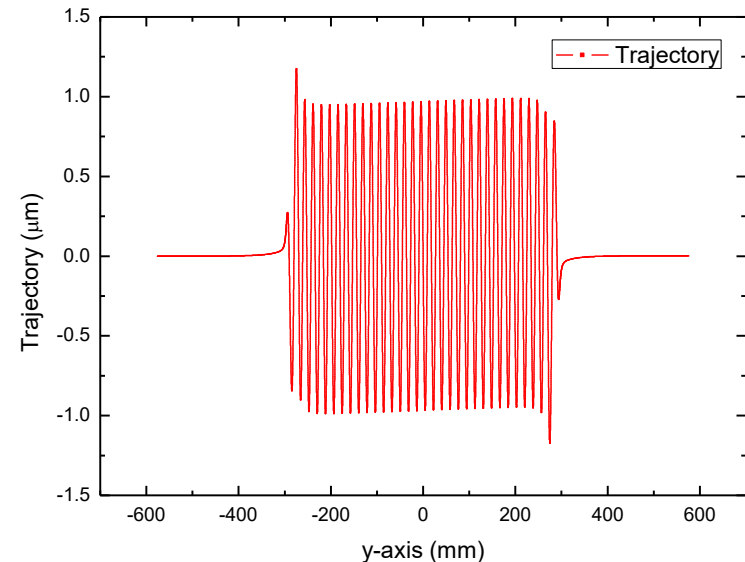
Field, first & second field integral of CU18



Cryogenic undulator (CU)
 Period length: 18 mm,
 Period number: 170,
 Field strength: 1.23 T,
 $B_r = 1.5 \text{ T @ } 135\text{K}$



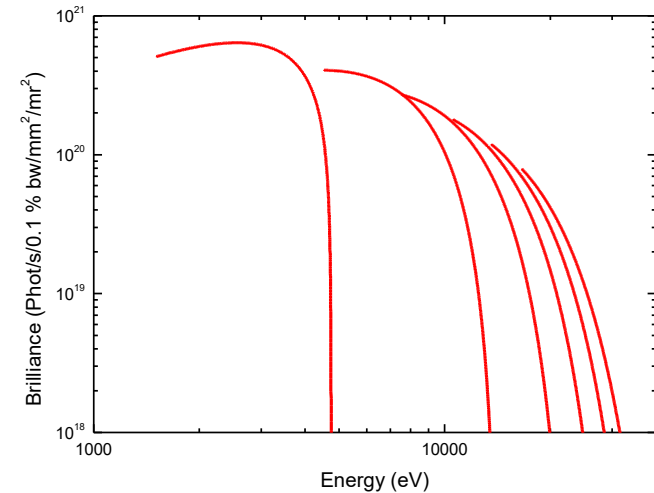
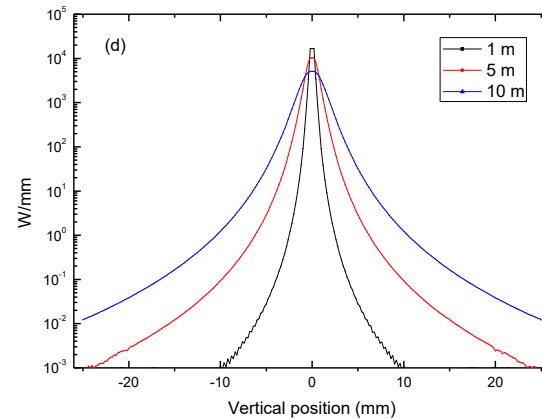
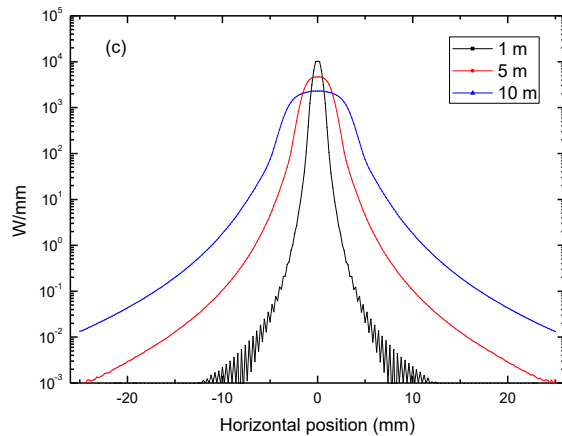
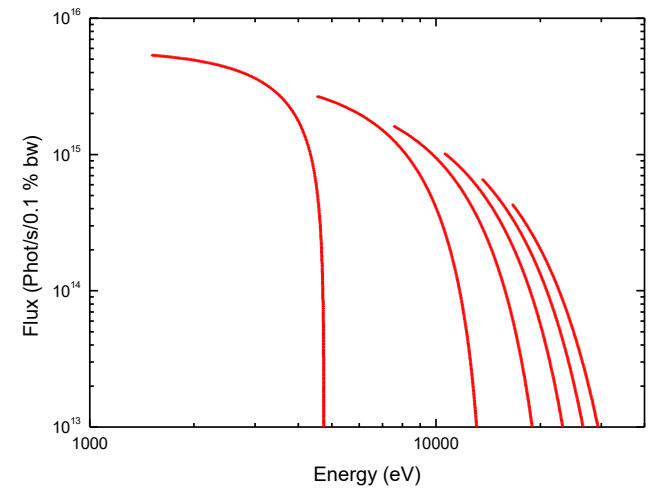
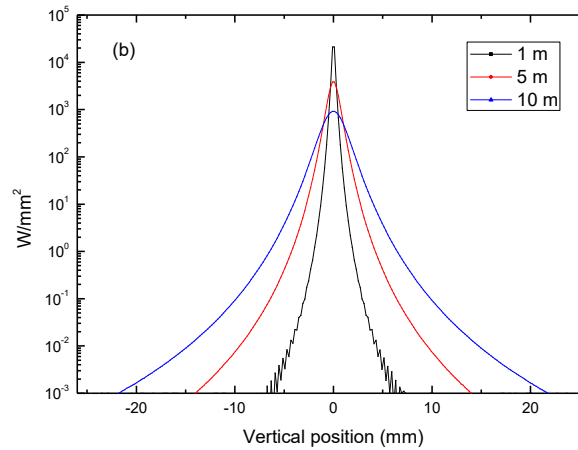
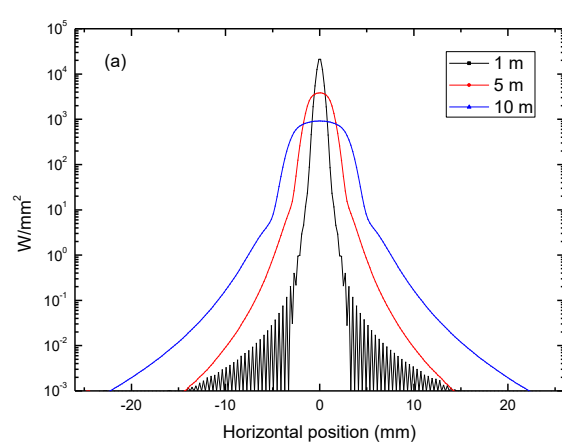
$$x' = \frac{K}{\gamma} \cos(\Omega t),$$



$$x = \frac{K}{\gamma} \frac{\lambda_u}{2\pi} \sin(\Omega t)$$



Spectra and power distribution of CU18

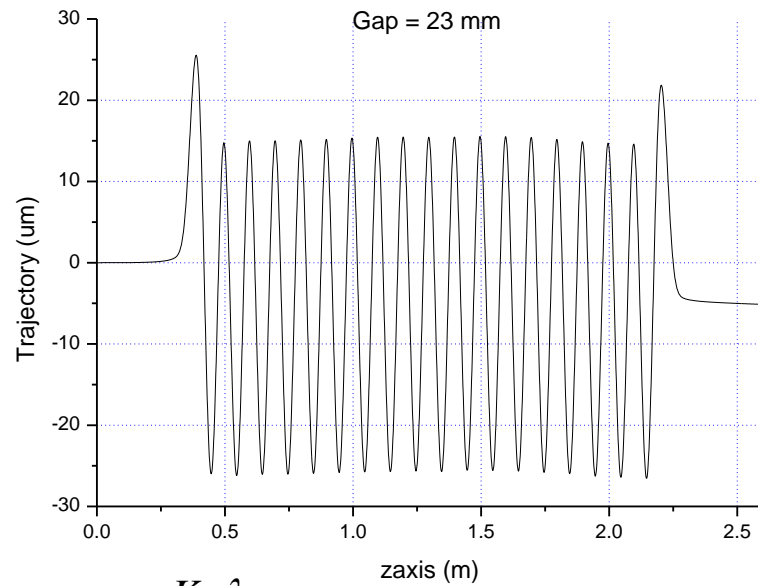
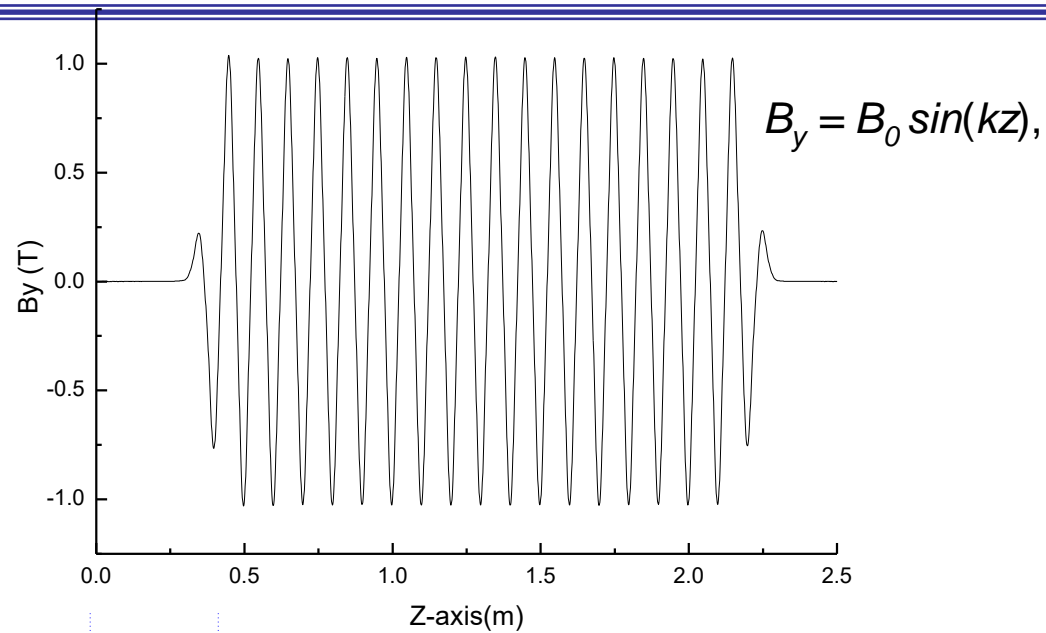


power distribution

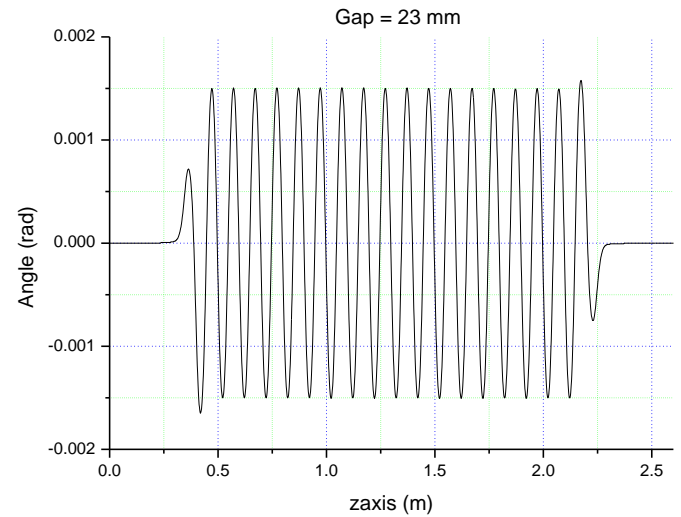
spectrum



U100 field distribution- example



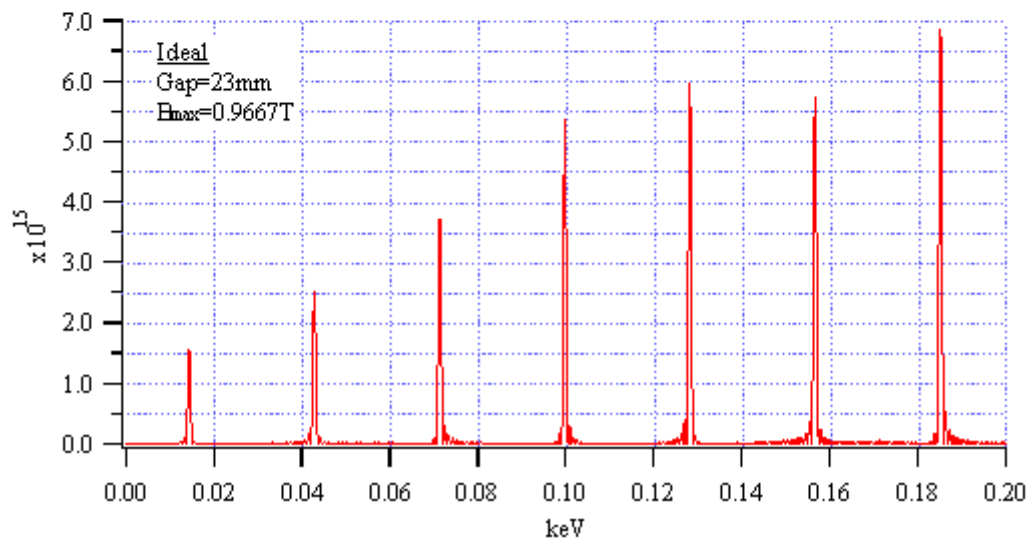
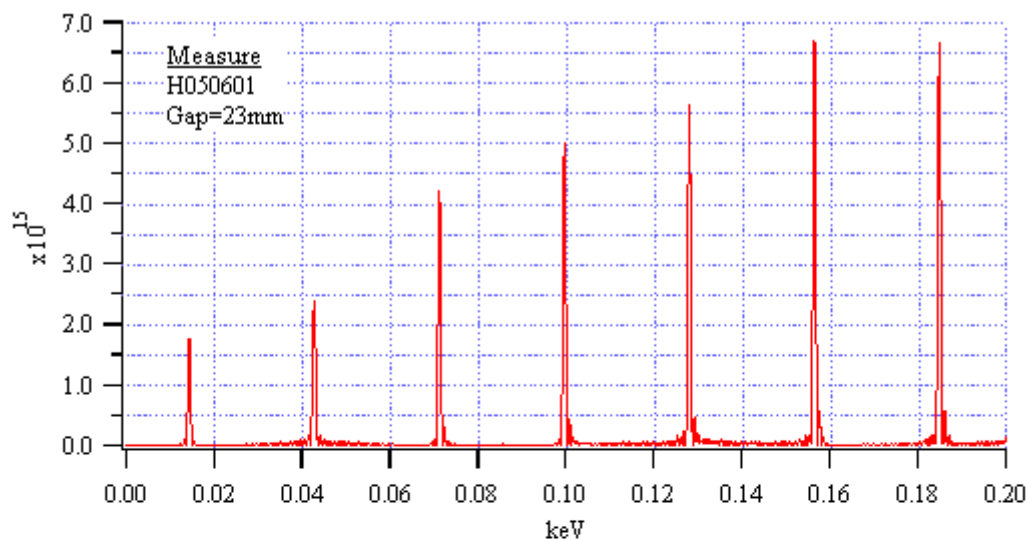
$$x = \frac{K}{\gamma} \frac{\lambda_u}{2\pi} \sin(\Omega t)$$



$$x' = \frac{K}{\gamma} \cos(\Omega t),$$

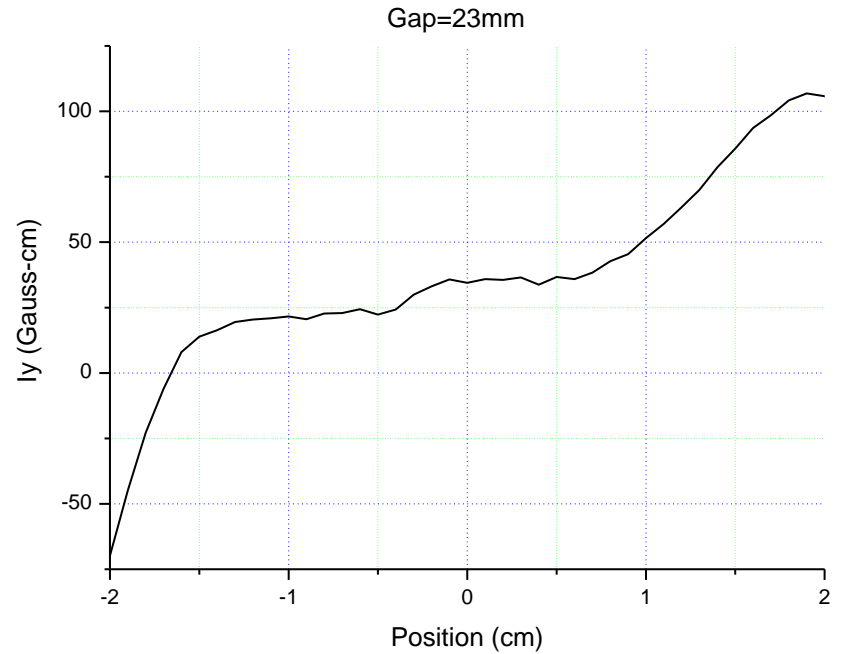
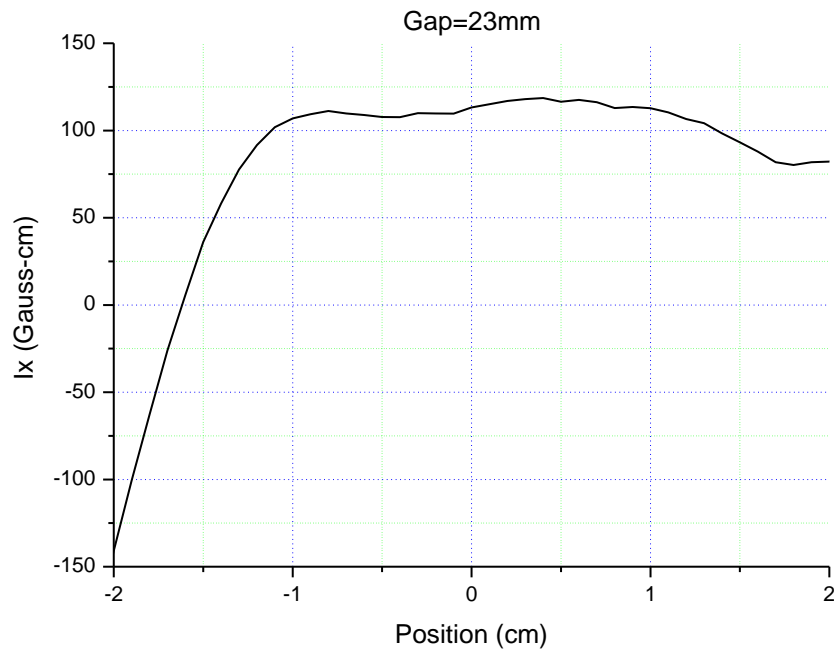


U100 spectrum- example





U100 Integral multipole- example



First Integral (electron angle) distribution in the axis of horizontal and vertical



How to design and shimming ID



Spectra calculation code

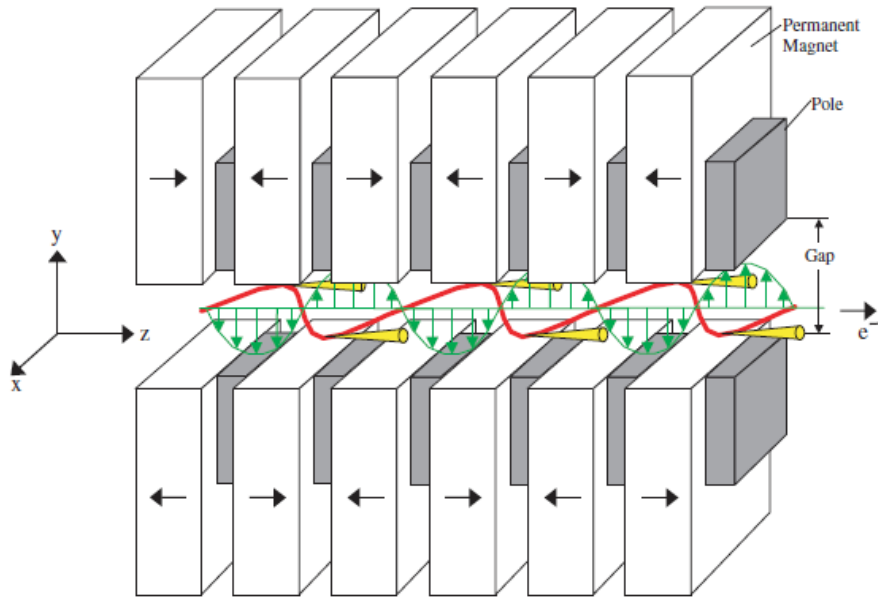
Advantages	Disadvantages
SRW (ESFR):	
<ul style="list-style-type: none">* User friendly package;* Associated with slit for beam line design;* Easy to do data process and data analysis;* Calculation spectrum & power distribution;* For simple field calculation;* Fast calculation for FFT analysis spectrum* Run in PC	<ul style="list-style-type: none">* Training course needed for familiarization;* Documentation is not clear;* Large computer needed;* Program is not yet completed;* Some parameters are not included;* Can down load from ESRF website
Spectra (SPing8):	
<ul style="list-style-type: none">* User friendly package* Calculation spectrum & power distribution* Easy to put parameters and data process* Taking into account different bata function* Fast calculation* Run in PC	<ul style="list-style-type: none">* Training course needed for familiarization;* Large use of memory;* Documentation is not clear;* Program is not yet completed;* Can down load from SPring8 website



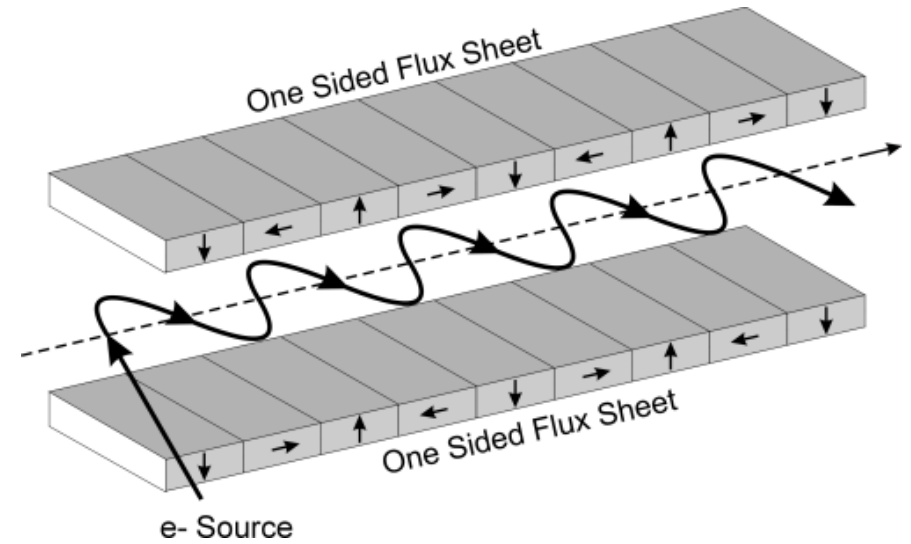
Magnet computation codes for magnet design

Advantages	Disadvantages
TOSCA:	
<ul style="list-style-type: none">* Full three dimensional package;* Accurate prediction of distribution and strength in 3D;* Extensive pre/post-processing;* Multipole function and Fast calculation* For static & DC & AC field calculation* Run in PC or workstation	<ul style="list-style-type: none">* Training course needed for familiarization;* Expensive to purchase;* Large computer needed.* Large use of memory.* Cpu time is hours for non-linear 3D problem.* It can be run combined field
RADIA:	
<ul style="list-style-type: none">* Full three dimensional package* Accurate prediction of distribution and strength in 3D* With quick-time to view and rotate 3D structure* Easy to build model with mathematic* Easy to perform data analysis and data plot* Run in PC	<ul style="list-style-type: none">* Larger computer needed* Large use of memory* Be careful to make segmentation* Only DC field calculation* Can down load from ESRF website

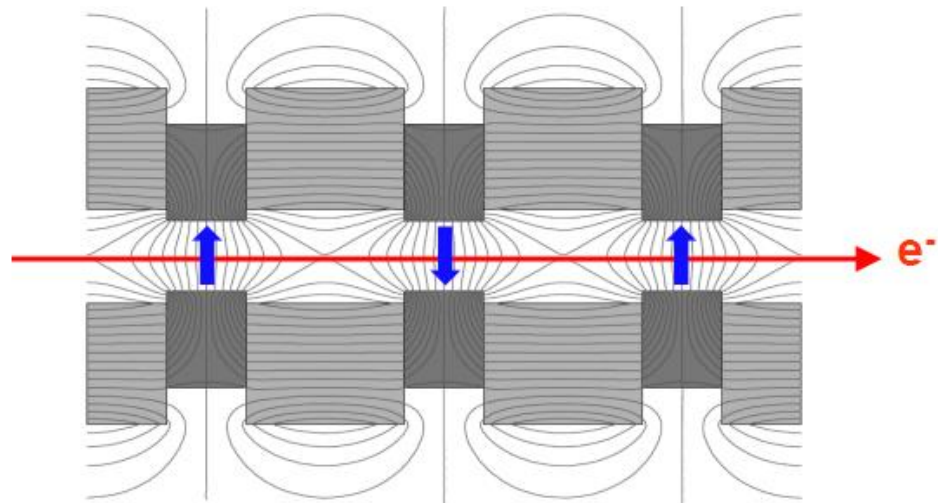
Magnet circuit type



Hybrid structure



Pure structure





Peak field calculation on pure and hybrid magnet

- Pure structure magnet array

$$B_0[T] = 1.895(e^{-\pi g/\lambda_u})$$

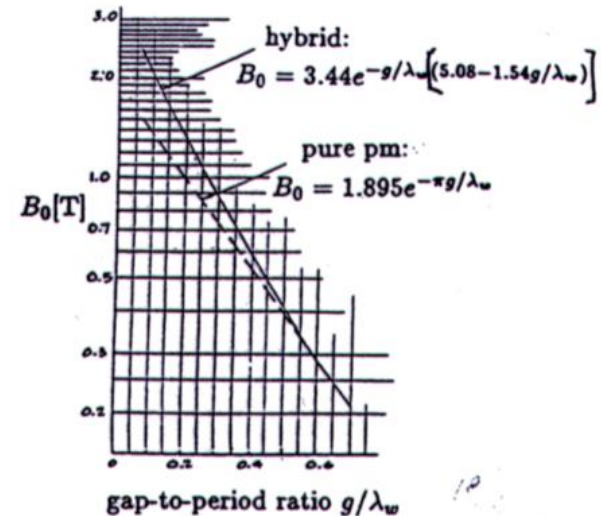
- Hybrid structure magnet array

- samarium-cobalt magnet

$$B_0[T] = 3.33 \exp \left[-\frac{g}{\lambda_u} \left(5.47 - 1.8 \frac{g}{\lambda_u} \right) \right]$$

- Neodymium-iron boron magnet

$$B_0[T] = 3.44 \exp \left[-\frac{g}{\lambda_u} \left[5.08 - 1.54 \frac{g}{\lambda_u} \right] \right] \quad 0.085 < \frac{g}{\lambda_u} < 0.8$$

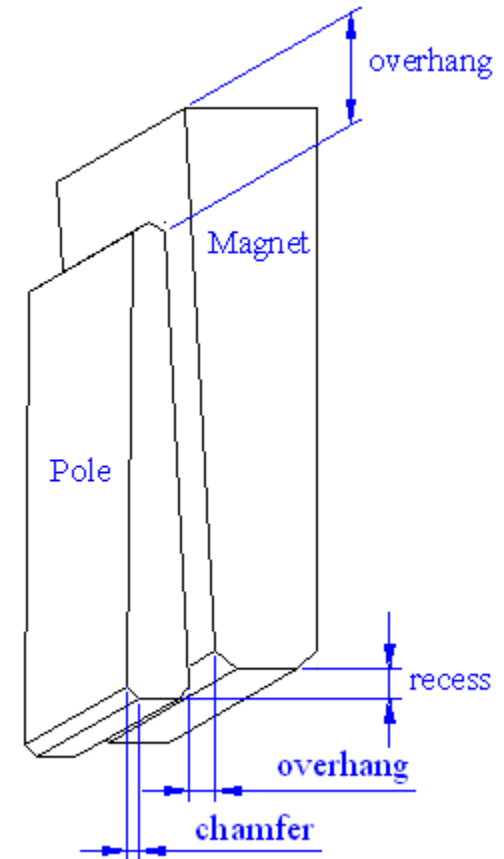


Attainable on-axis field in pure PM and hybrid insertion devices ($B_r = 1.1$ T, $H_{pm} = -0.8H_c$)

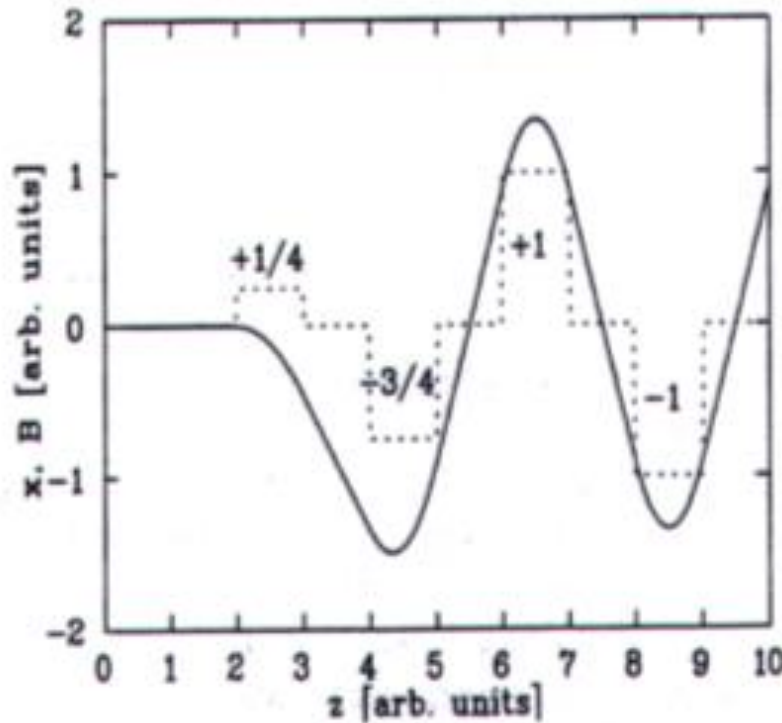
$$0.07 < \frac{g}{\lambda_u} < 0.7$$

Design criteria of IDs

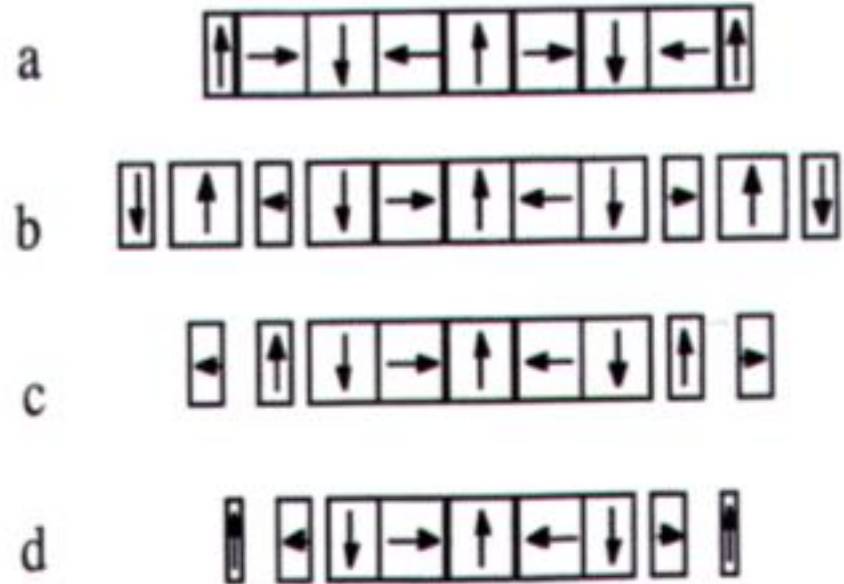
- ◆ **Wedged-poles** were shaped with a thicker cross section at pole tip.
- ◆ **Chamfers** are used to reduce local saturation and demagnetizing field
- ◆ **Vertical recess** to minimize on-axis field strength variation.
- ◆ **Magnet overhang** reduces 3-D leakage flux and roll-off is slower.
- ◆ **Different thickness of magnet block sizes** with partial strength on the both end poles.
- ◆ **0.5 mm thickness shim** at magnet edge increase vertical field roll-off.
- ◆ Two **rows of trim magnets** for B_y and B_x multipole field shimming.
- ◆ **Magnet & iron shim pieces** for trajectory and spectrum phase shimming.
- ◆ **Longitudinal distance** between each end pole, the **pole height**, and **pole tilt** can be adjustable.



End pole design-I



Sequence of magnet poles (dotted line) resulting in no offset between the electron trajectory (solid line) and the magnet axis.



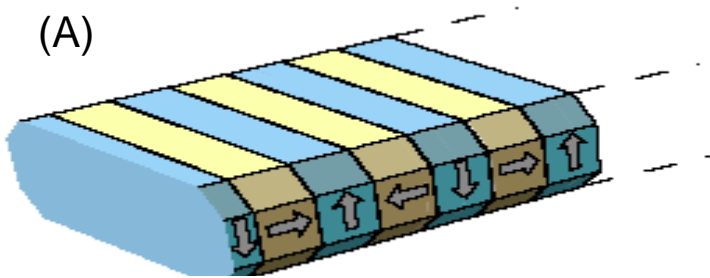
Various end-sequences for the pure-permanent magnet structure

The criteria of ID design:

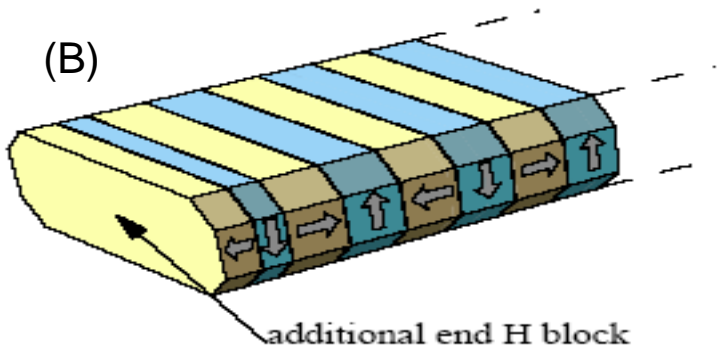
1. First integral field $\int B ds = 0$
2. Second integral field $\int (\int B ds') ds = 0$

End pole design-II

(A)

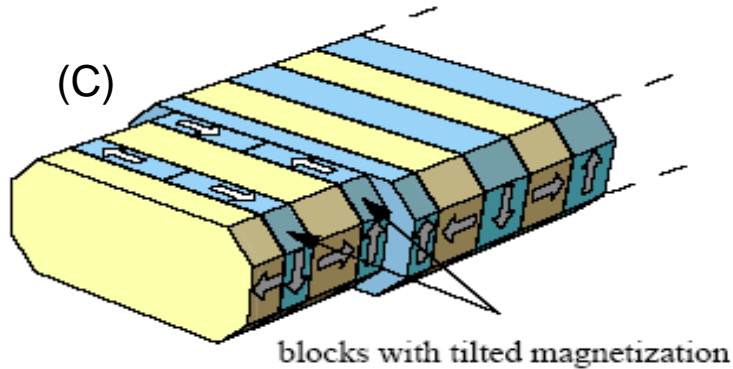


(B)

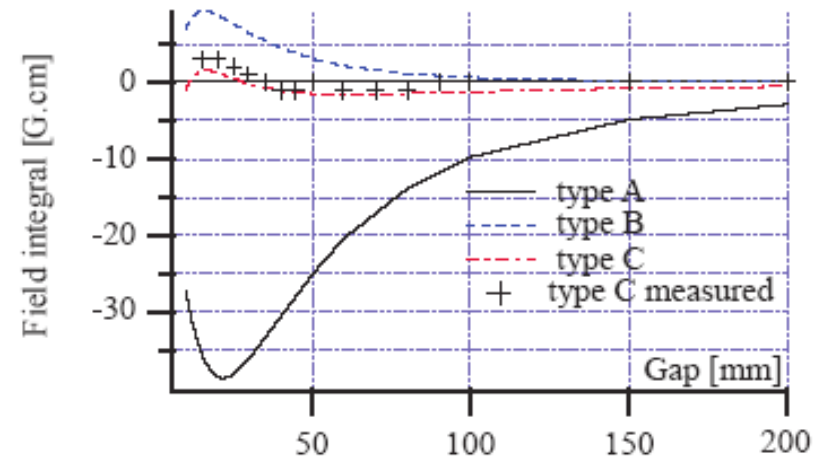
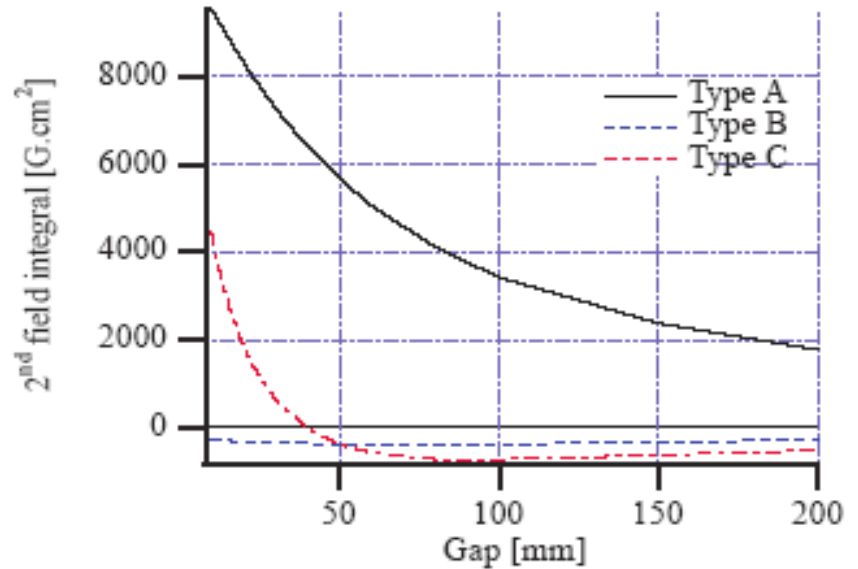


Reduce integral field strength with gap

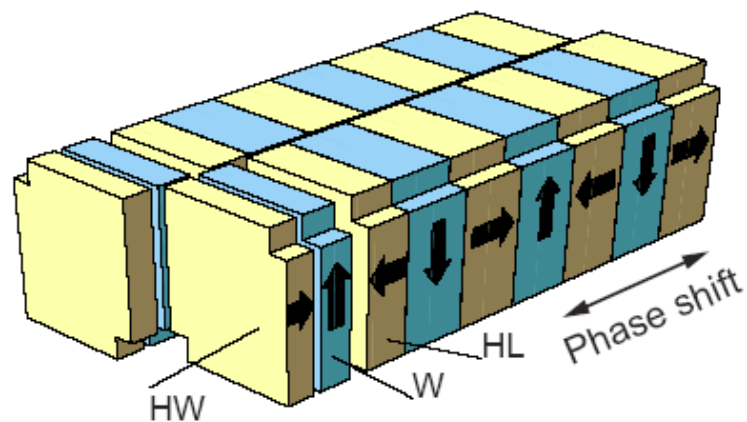
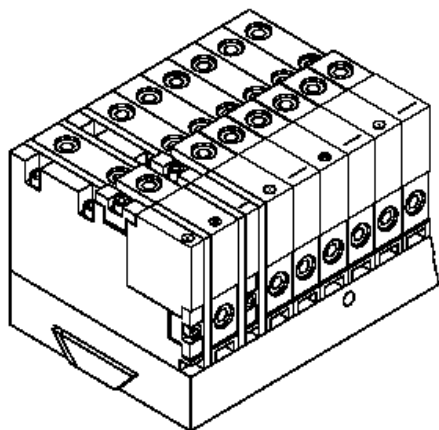
(C)



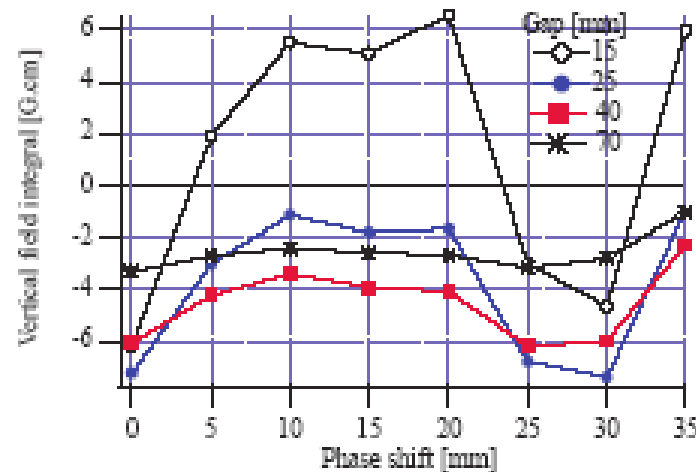
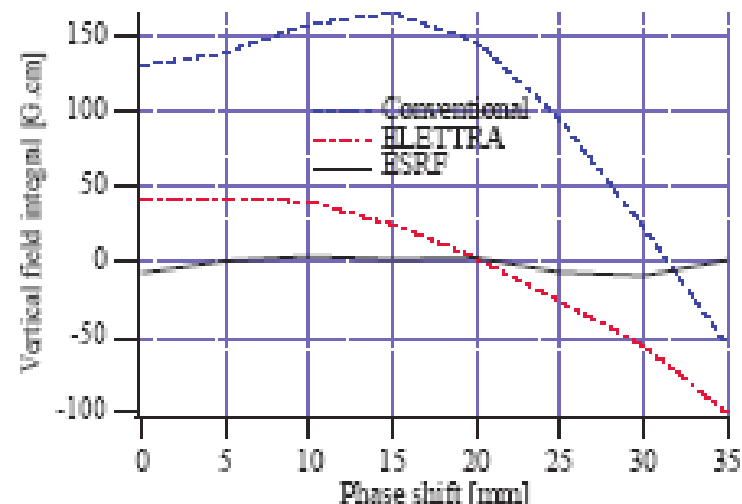
Reduce integral field strength with gap and phase



Apple II End pole design



The magnets of type HL, W and HW have the same cross-section but a different longitudinal dimension. The air gap is 5 mm (2 mm) between the HL and W (W and HW) magnet blocks.



Reduce integral field strength with gap and phase & the second field integral



Phase error calculation for shimming methods

$$\Theta(z) = \frac{2\pi}{\lambda} \left(\frac{z}{2\gamma^2} - \frac{\int x'^2 dz}{2} \right)$$

where $x' = dx/dz$ represents the electron angle with respect to the undulator z-axis, λ is the photon radiation fundamental wavelength, and γ denotes the relativistic velocity. In the ideal undulator device, the phase at each pole should be a perfect linear variation and the phase error is zero.

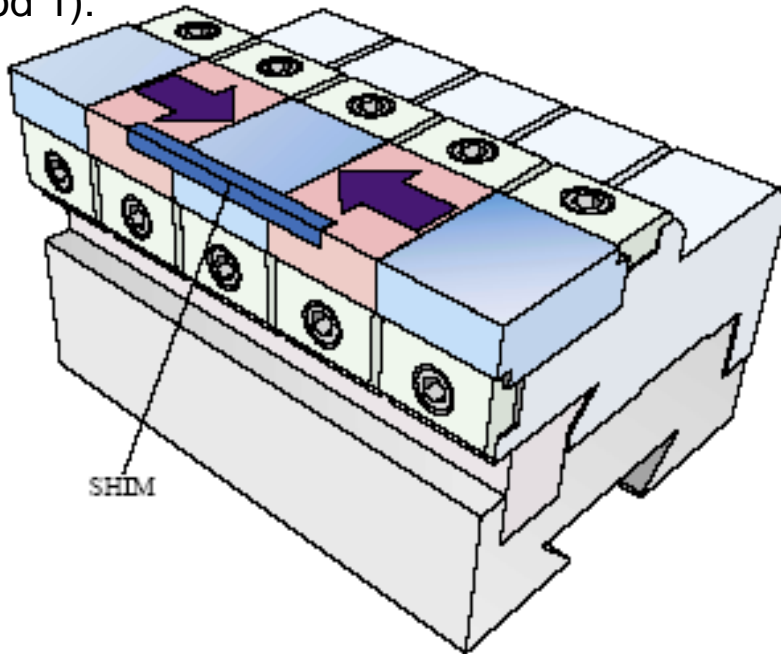
However for a real undulator, the phase error $\Delta\Theta$ is not zero and can be obtained by subtracting the two optimum linear fits of the real and ideal field

$$I = I_0 e^{-(n\Delta\Theta_{rms})^2}$$

Where I and I_0 represent the spectrum flux intensities with and without phase error.

Dynamic aperture shimming methods on EPU

(Method 1):



$$\int_{-\infty}^{\infty} (B_x + i B_y) dz \cong \sum_{n=0}^n (b_n + i a_n) (x + iy)^n.$$

Where a_n and b_n denote the integral normal and skew components.

The shimming method has been studied to re-enlarge the dynamic aperture with the addition of a multipole field component. Such shims are placed on each of the four magnet arrays. They are designed based on the criteria of correcting the tune shift vs. x .

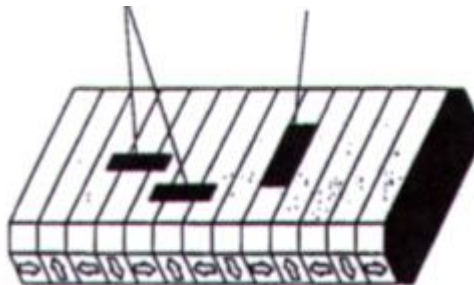
(Method 2):

Using multi filament flat wire on the surface of the EPU vacuum chamber to compensate for the multipole error which is induced from dynamic integral field.

Multipole & spectrum shimming method

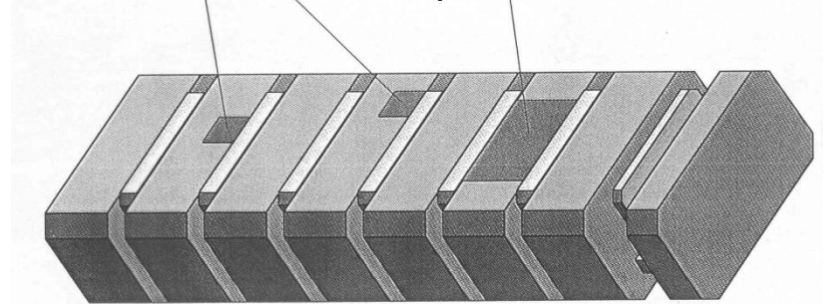
- Measuring the individual permanent magnet block and then arranging them by sorting block in the structure.
- Measuring the integral field strength of each block which on the keeper to reduce the mechanical error.
- Swapping blocks after assembly and field measurement.
- Using the thin iron pieces or permanent magnet pieces on magnet to correct the multipole and spectrum shimming.

Multipole shim Spectrum shim



Permanent magnet

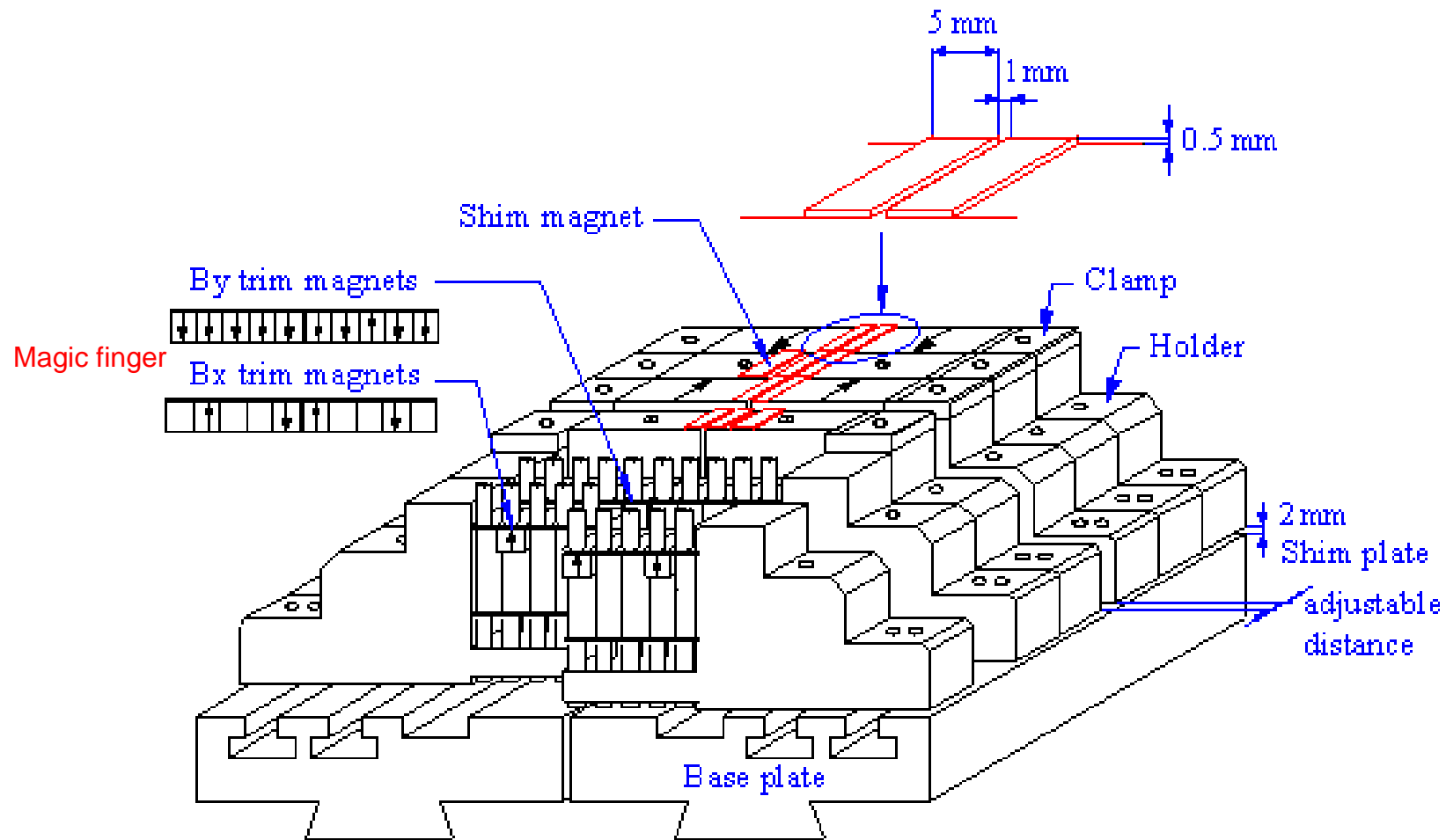
Multipole shim Spectrum shim



Hybrid magnet

Method of magnetic shimming to improve the magnetic field quality

Field quality control by various methods



$$\int_{-\infty}^{\infty} (B_x + i B_y) dz \cong \sum_{n=0}^n (b_n + i a_n) (x + iy)^n.$$

Where a_n and b_n denote the integral normal and skew components.



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