Lecture 1 -Fundamentals of Free-electron Laser

Yen-Chieh Huang 黃衍介*

ychuang@ee.nthu.edu.tw, tel: 886-3-5162340, fax: 886-3-5162330

清華大學電機工程學系/光電研究所/物理系

Department of Electrical Engineering/Institute of Photonics Technologies/
*Department of Physics

National Tsinghua University, Hsinchu, Taiwan

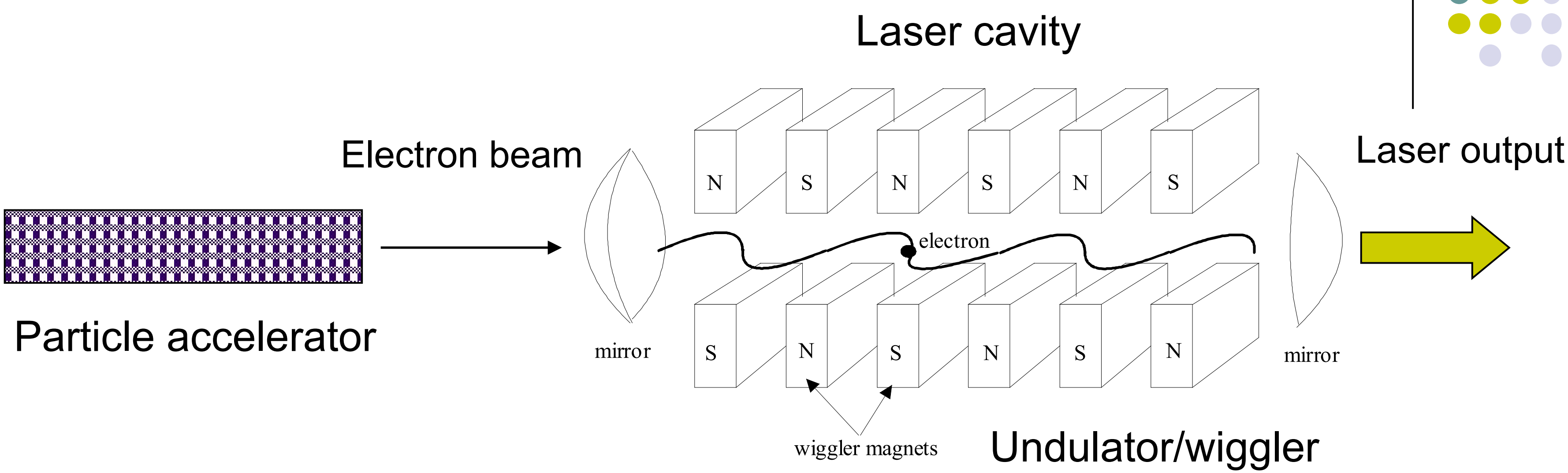
Outlines

1. Spontaneous emission – Compton scattering/Thompson scattering/undulator radiation

2. Stimulated emission – wave/particle energy exchange $\rightarrow$ laser gain

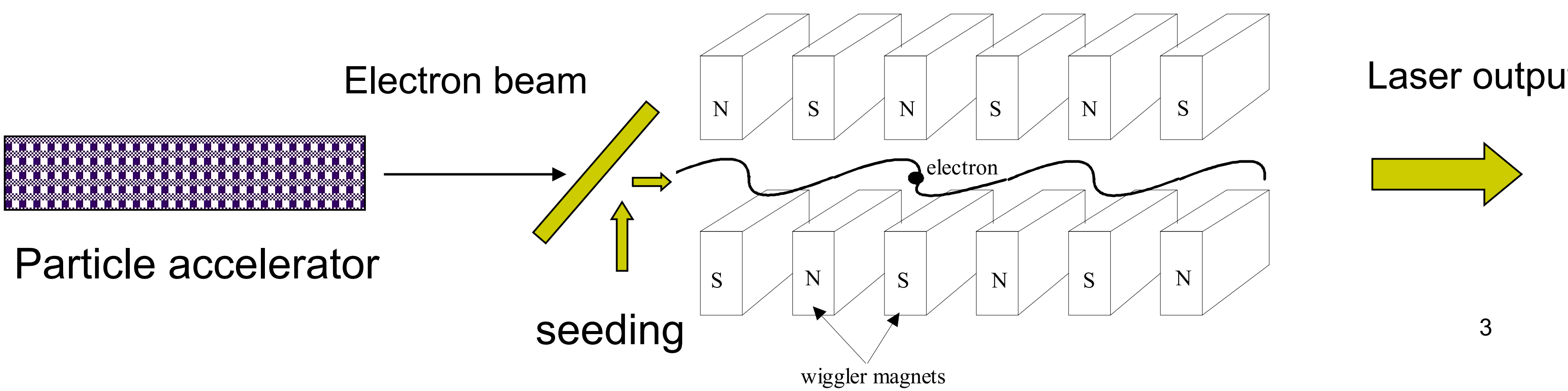
3. Requirements for FEL Oscillator: buildup time, energy spread, emittance, saturation power, etc.

Low gain → free-electron Laser Oscillator

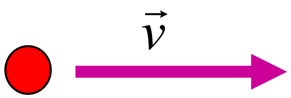


High gain → free-electron Laser Amplifier

Self-amplified Spontaneous emission (SASE) FEL



Parameters in Relativistic Mechanics



Moving particle

Lorentz factor $\gamma \equiv \dfrac{1}{\sqrt{1-\beta^2}}$

where $\beta \equiv v/c$, with c = speed of light in vacuum.

Electron mass $m = \gamma m_0$, m_0 = electron rest mass

Electron momentum: $p = mv = \gamma m_0 v$

Total electron energy: $\gamma m_0 c^2 = \sqrt{m_0^2 c^4 + p^2 c^2}$, $m_0 c^2$ = electron rest energy ~ 0.5 MeV

In laboratory frame: length L

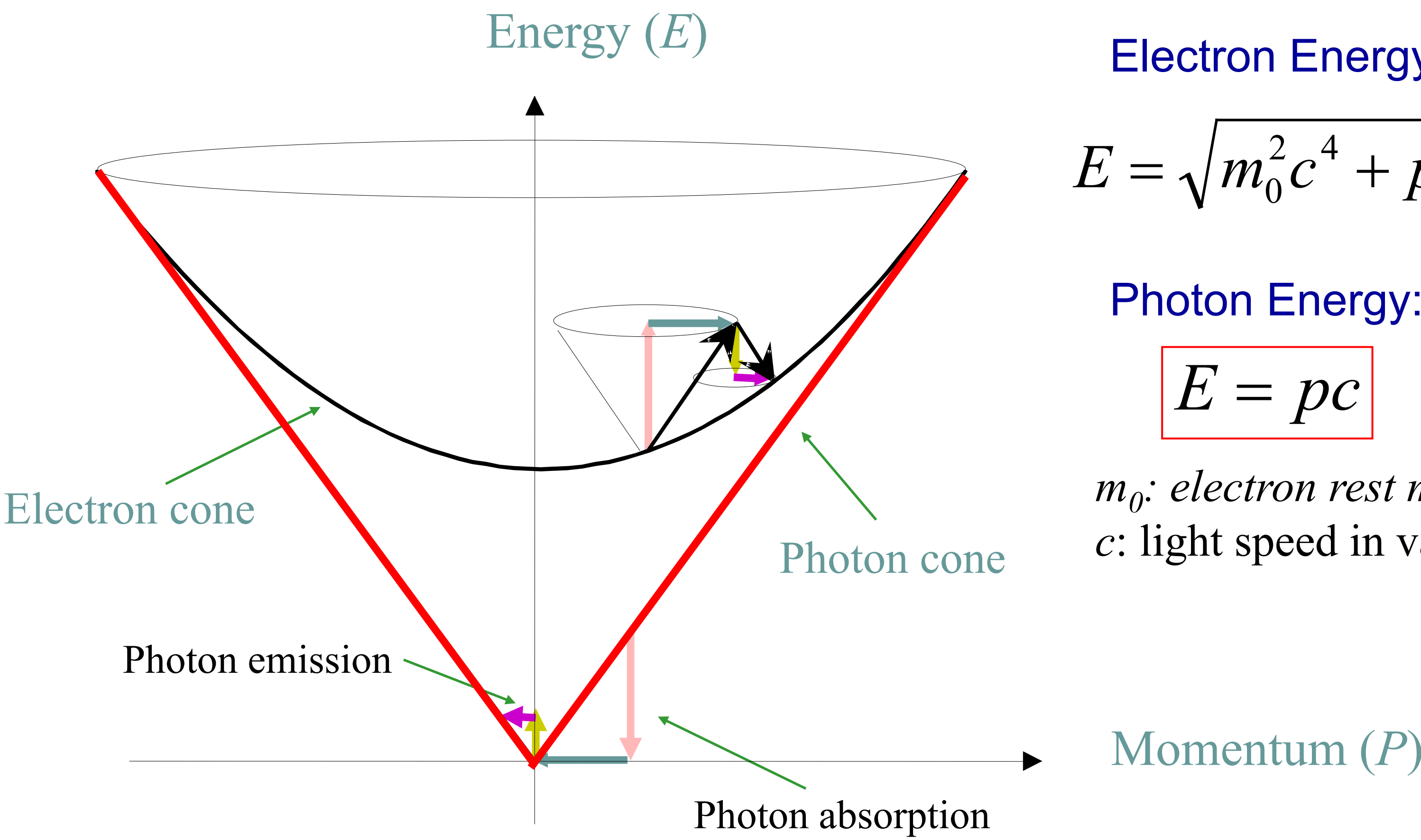
In electron frame: length $L/\gamma \leftarrow$ Lorentz contraction

In the relativistic regime $\beta \equiv v/c \sim< 1$

$$\gamma \equiv \frac{1}{\sqrt{1-\beta^2}} >> 1 \Rightarrow \frac{1}{\beta} \sim 1 + \frac{1}{2\gamma^2} \Rightarrow \beta \sim 1 - \frac{1}{2\gamma^2}$$

Photon-electron Energy Exchange in Free Space

requirements: energy conversation & momentum conservation



Electron Energy:

$$E = \sqrt{m_0^2 c^4 + p^2 c^2}$$

Photon Energy:

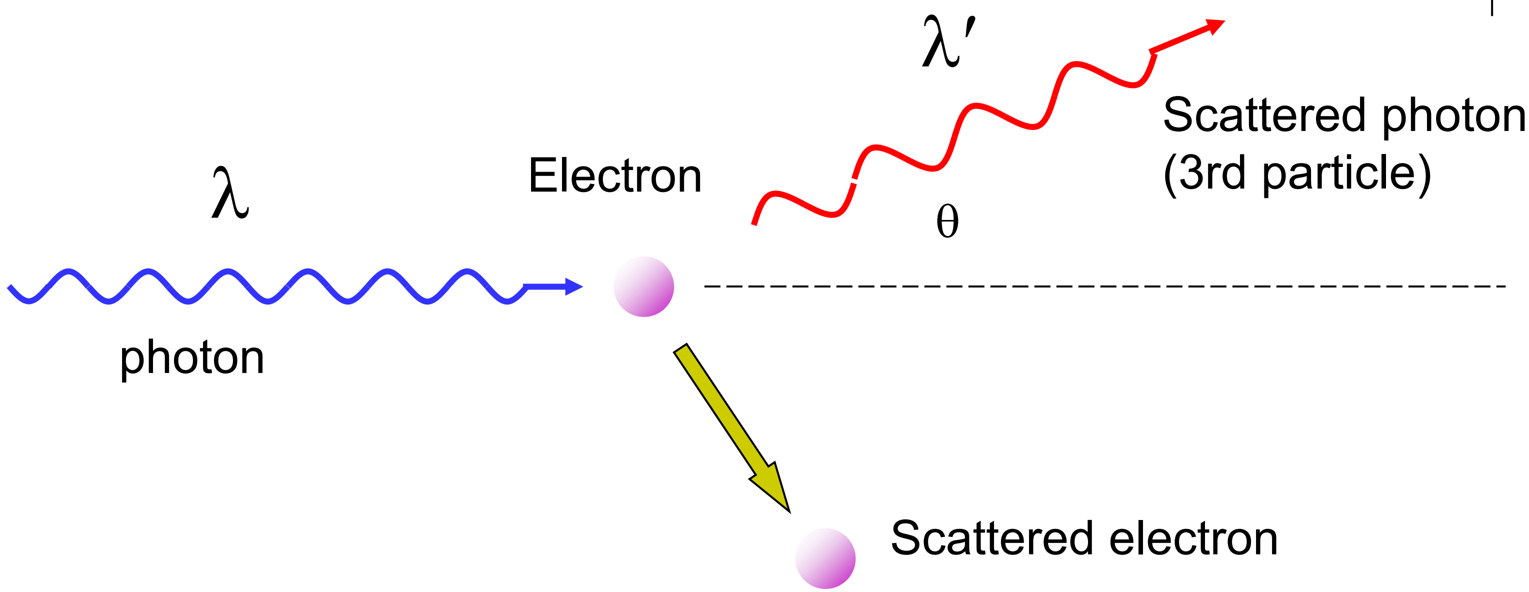
$$E = pc$$

m_0: *electron rest mass*
c: light speed in vacuum

Energy-momentum diagram of Compton Scattering

Photon-electron energy exchange is prohibited in a vacuum unless a third particle exists or is created

Compton Scattering

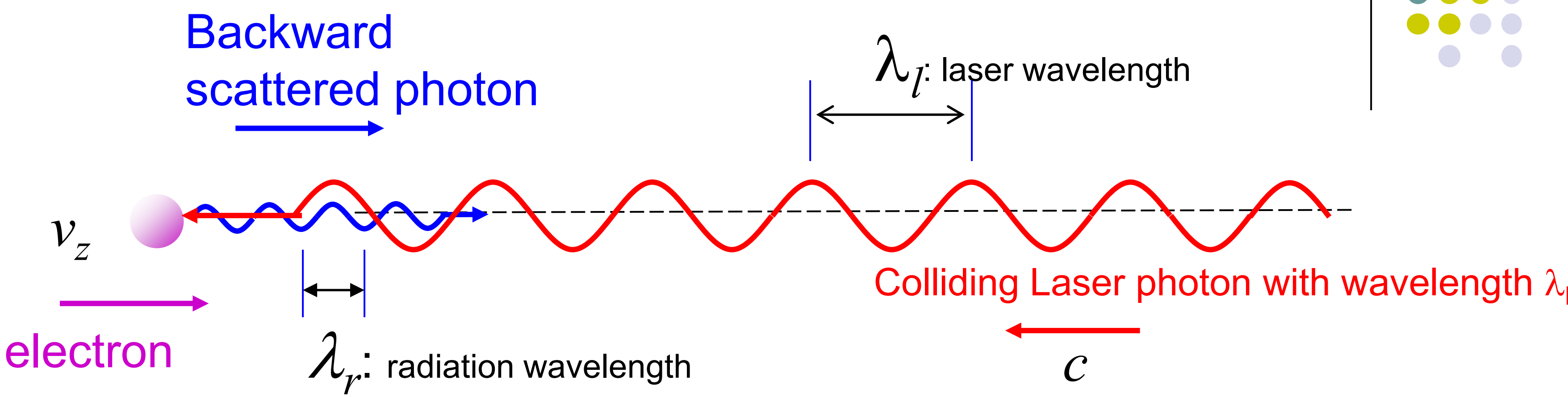


λ: wavelength
h: Planck's constant
m_0: electron rest mass
c: vacuum wave speed
p: momentum

Compton Effect $$\lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\theta)$$

Thomson Back Scattering:

Compton scattering with electron energy loss much less than the photon energy



Double Doppler shift

$$f_r = f_l \sqrt{\frac{1+\beta_z}{1-\beta_z}} \cdot \sqrt{\frac{1+\beta_z}{1-\beta_z}} = f_e' \times \sqrt{\frac{1+\beta_z}{1-\beta_z}} \Rightarrow \lambda_r = \frac{\lambda_l}{4\gamma_z^2}$$

Radiation frequency in the lab (f_r)

Doppler shift seen by a lab observer

f_e' : Doppler shifted laser frequency seen by electron

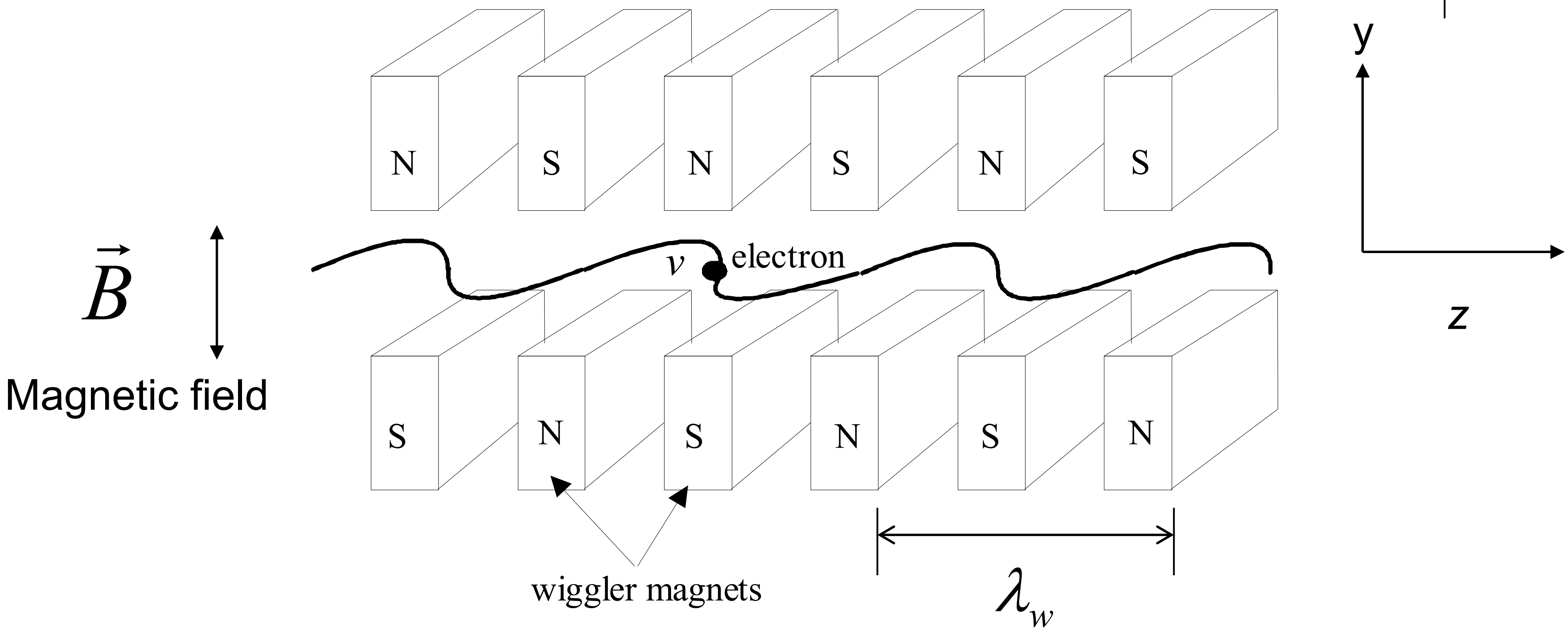
Given λ = 800 nm (Ti:sapphire laser), $\gamma \sim \gamma_z = 45$ (23 MeV beam), $\lambda_r = 1$ Å (hard x-ray!)

Longitudinal Lorentz factor $\gamma_z \equiv \dfrac{1}{\sqrt{1-\beta_z^2}}$

where $\beta_z \equiv v_z / c$

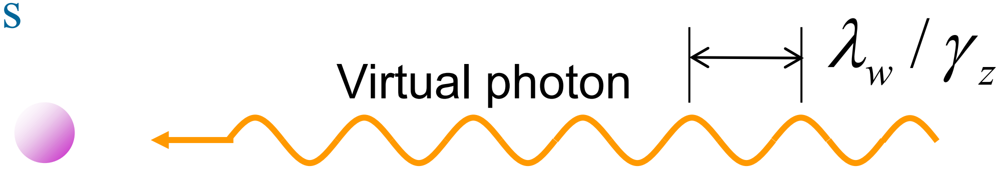
Undulator Radiation

In laboratory frame



In electron rest frame, the electron sees a "wave" with fields:

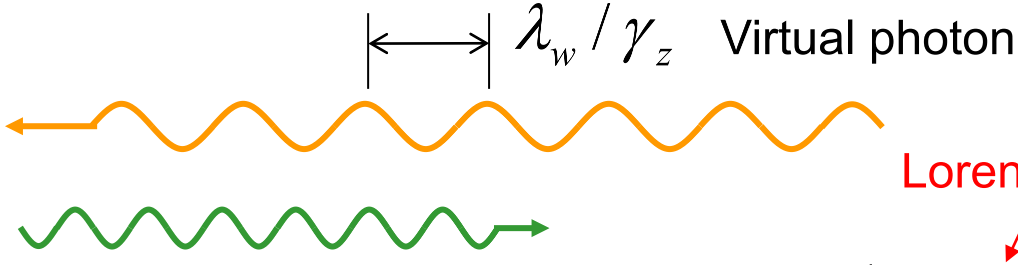
$$\vec{E}' = \gamma\vec{\beta} \times \vec{B}, \vec{B}' = \gamma\vec{B}$$



Spontaneous Undulator Radiation

Electron Rest Frame

electron

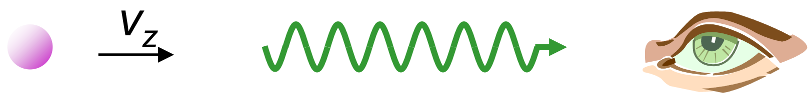


Lorentz contraction

Electron oscillation frequency in the electron frame

$$f' = (T')^{-1} = \left(\frac{\lambda_w / \gamma_z}{v_z} \right)^{-1}$$

Laboratory Frame



Doppler Shift $\quad f = f' \sqrt{\dfrac{1+\beta_z}{1-\beta_z}}$

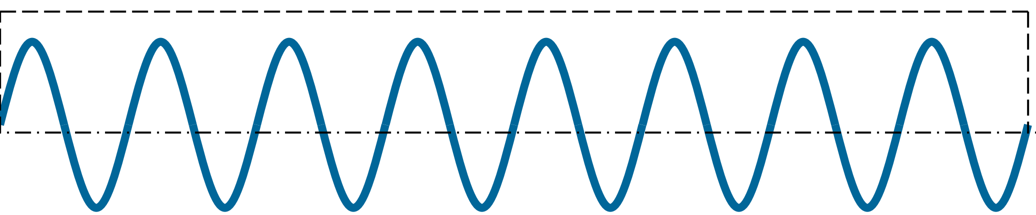
$$\lambda = \lambda_w \left(\frac{1}{\beta_z} - 1 \right) \approx \frac{\lambda_w}{2\gamma_z^2}$$

For $\lambda_w \sim 1$ cm, 100 MeV($\gamma_z \sim 200$), $\Rightarrow \lambda = 125$ nm

"Cheap" long-wavelength virtual photon $\Rightarrow$ expensive short-wavelength photon

Spontaneous Undulator Radiation

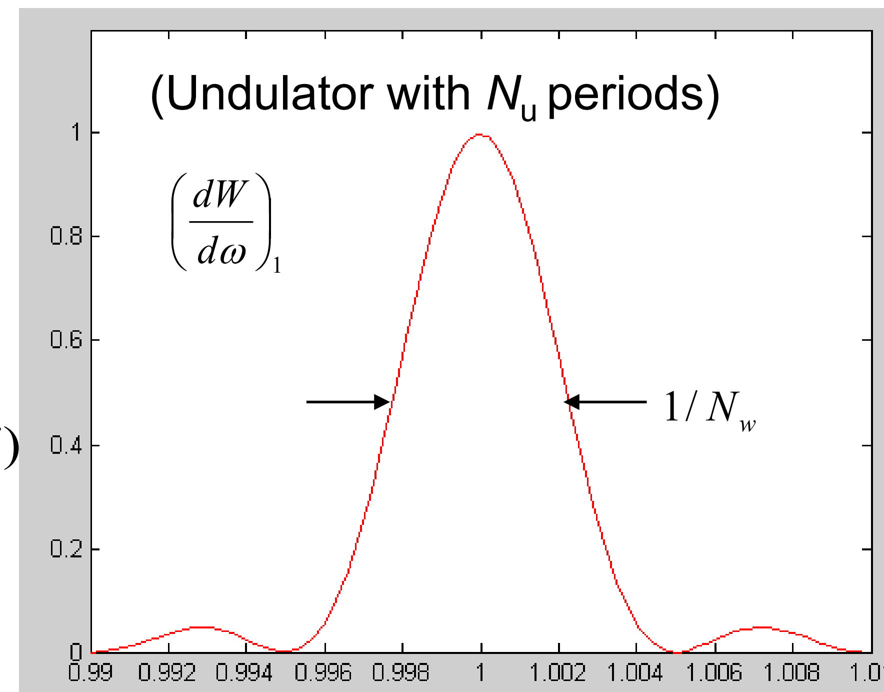
$$\tau = \frac{L_r}{c}, L_r = N_w \lambda_r$$



$$e^{i\omega_r t} \times \mathrm{rect}[\frac{t}{\tau}]$$

N_w: *number of undulator periods*
L_r: slippage distance
τ: radiation pulse length
ω_r: resonant radiation frequency

Spectral-energy Lineshape Function



ω/ω_r

Radiation Spectrum

Lemma

Rectangular function: $\mathrm{rect}[t] = \begin{cases} 1 & \text{for } |t| \le 1/2 \\ 0 & \text{otherwise} \end{cases}$

Fourier Transform $\{\mathrm{rect}[t]\} = \dfrac{\sin(\omega/2)}{\omega/2} = \mathrm{sinc}(f)$

So,

Fourier Transform $\{e^{i\omega_r t} \times \mathrm{rect}[\frac{t}{\tau}]\}$

$$\propto \frac{\sin[2N_w(\omega/\omega_r - 1)]}{2N_w(\omega/\omega_r - 1)}$$

Spectral energy $\left(\dfrac{dW}{d\omega}\right)_1 \propto \{\mathrm{sinc}[2N_w(\omega/\omega_r - 1)/\pi]\}^2$

Effect of Magnetic field on e^- Quiver Motion

A general assumption: a relativistic beam $\gamma >> 1$

Assume a planar/linear wiggler with a wiggler field of $\vec{B} = \hat{y}\sqrt{2}B_{rms}\sin k_w z$

Begin with the Lorentz force equation $\frac{d\vec{p}}{dt} = e\vec{v}\times\vec{B}$, where $\vec{p} = \gamma m_0 \vec{v}$

$$v_x \approx \frac{-\sqrt{2}ca_w}{\gamma}\cos(k_w z) = \frac{-\sqrt{2}ca_w}{\gamma}\cos(\underline{k_w v_z t})$$

Wiggler wavenumber

$$k_w = 2\pi/\lambda_w$$

$$v_z = \sqrt{v^2 - v_x^2} \approx v - \frac{a_w^2}{2\gamma^2}\frac{c}{\beta}\cos(2k_w z)$$

$$= v - \frac{a_w^2}{2\gamma^2}\frac{c}{\beta}\cos(\underline{2k_w v_z t})$$

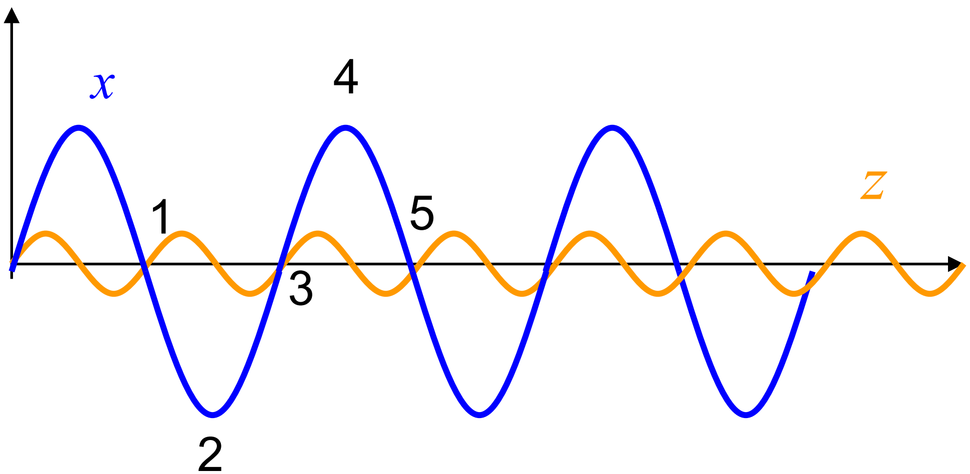
√

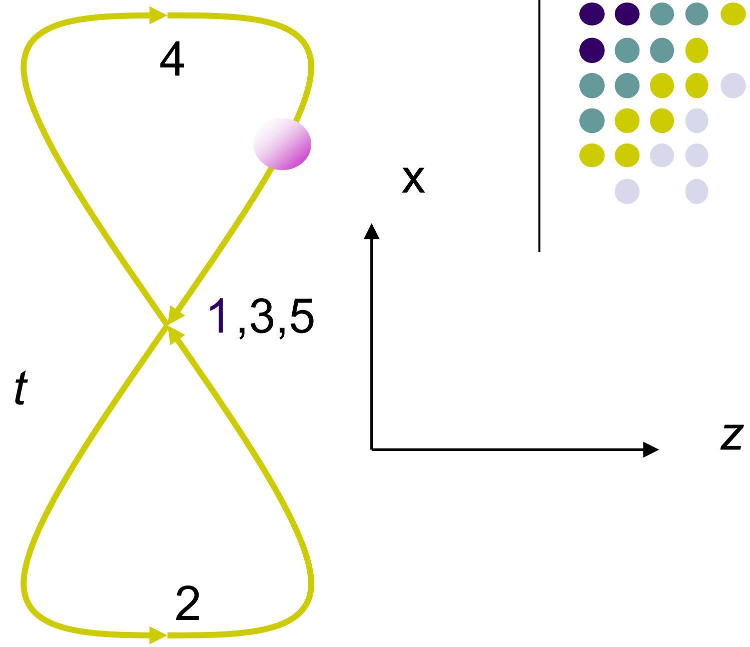
where $a_w = \frac{eB_{rms}}{m_0 c k_w}$

Wiggler parameter

In the Electron Rest Frame

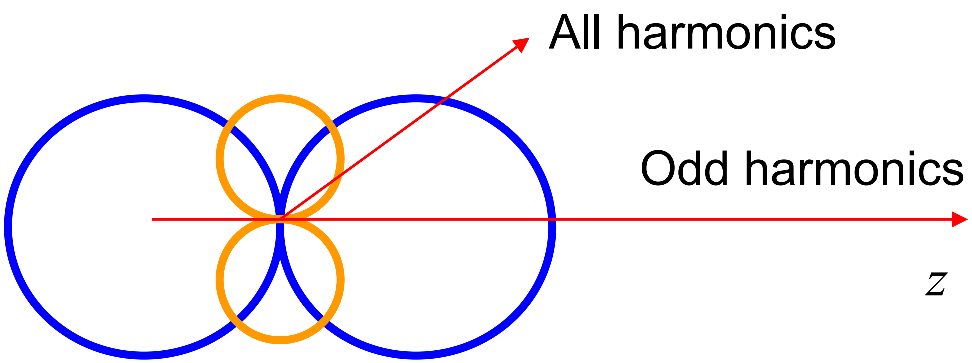
figure-8 motion





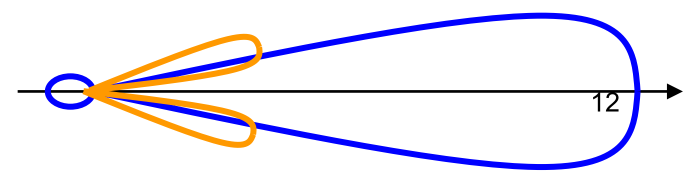
$\vec{v}_{z,2\omega} \times \vec{B}_{y,\omega}$ generates $\vec{v}_{x,3\omega}$, $\vec{v}_{x,3\omega} \times \vec{B}_{y,\omega}$ generates $\vec{v}_{z,4\omega}$ ....

Dipole radiation pattern in the electron rest frame



Dipole radiation pattern in the laboratory frame

($\gamma >> 1$)



Undulator Radiation Wavelength

Because

$$\gamma_z \equiv \frac{1}{\sqrt{1-\beta_z^2}} = \frac{1}{\sqrt{1-v_z^2/c^2}}, \text{ and } \quad \lambda \approx \frac{\lambda_w}{2\gamma_z^2}$$

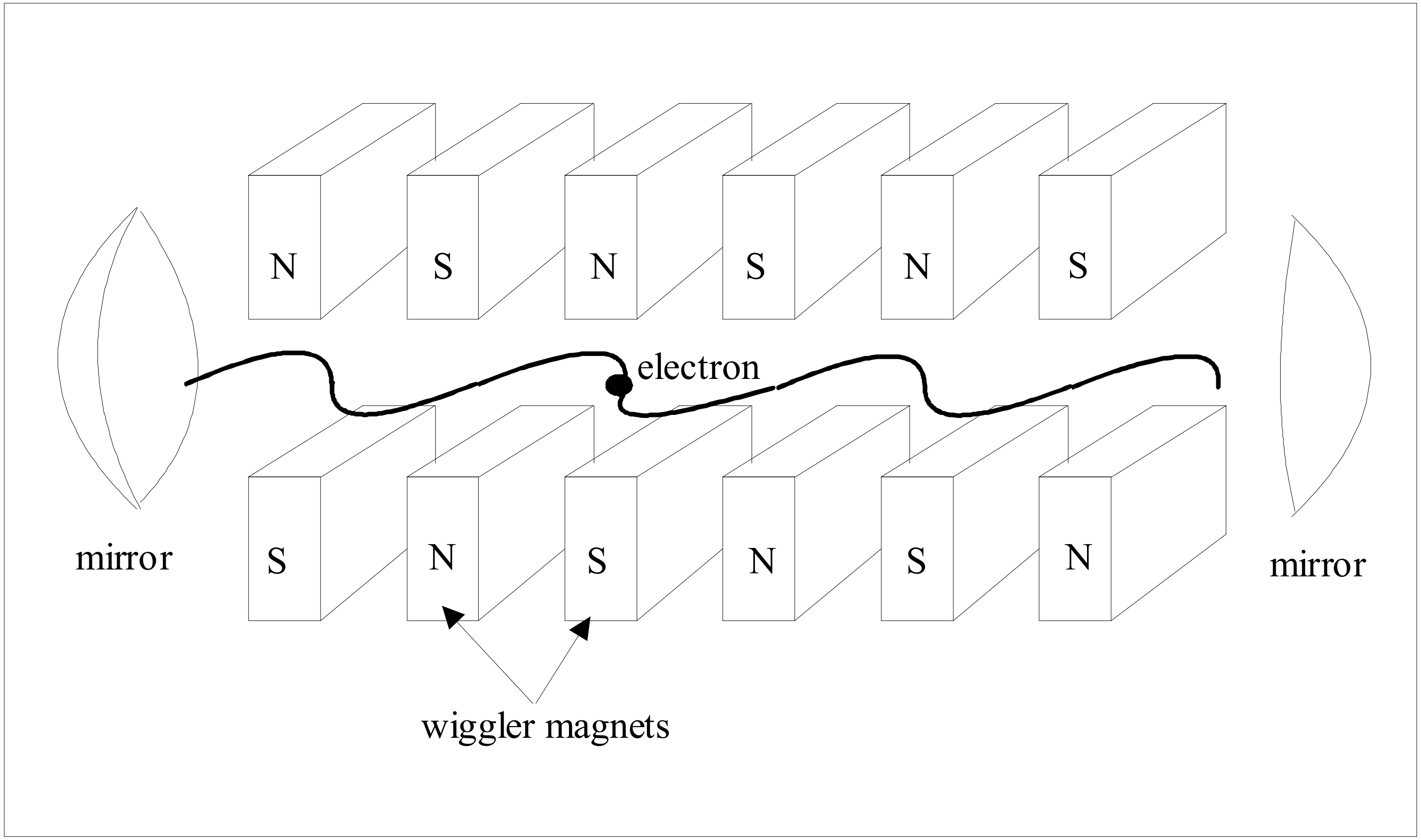
$$\Rightarrow \frac{1}{\gamma_z^2} = \frac{1+a_w^2}{\gamma^2}$$ where $$a_w = 0.093 B_{rms}(\text{kgauss}) \times \lambda_w(\text{cm})$$

is called the *wiggler/ undulaotor parameter*

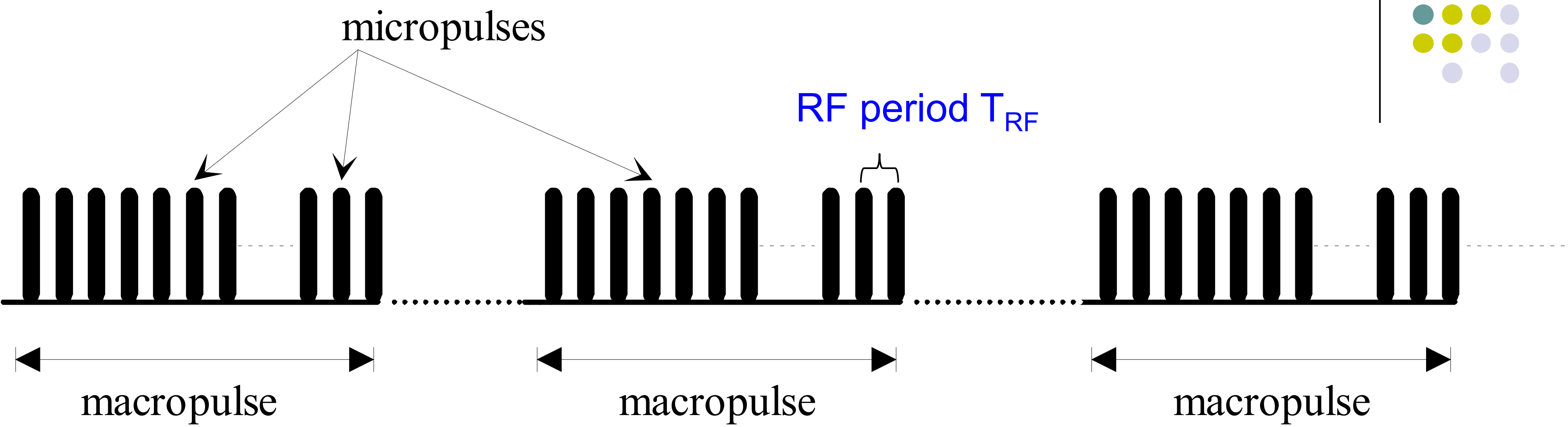
$$\Rightarrow \lambda = \frac{1+a_w^2}{2\gamma^2}\lambda_w \quad \text{(FEL synchronism condition)}$$

Undulator radiation wavelength can be tuned by magnetic field *B*, wiggler period λ_w, and electron energy γ

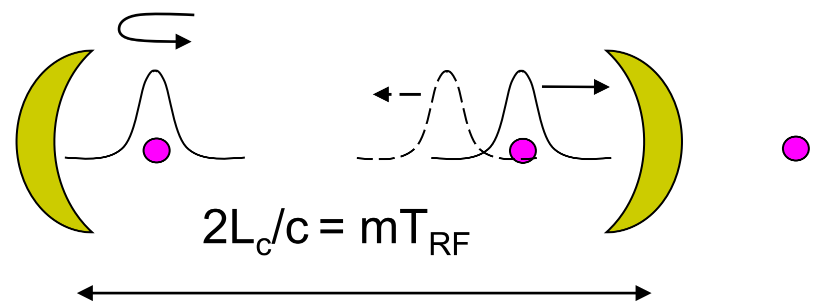
A Free-electron Laser Oscillator



Pulse Structure of a RF Linac-driven FEL Oscillator



Taking SLAC S-band RF as an example:
1. RF frequency = 2.856 GHz
2. Micropulse length ~ 10 ps
3. Macropulse length: 1-5 μs
4. Macropulse repetition rate: 10-100 Hz



* Macropulse length > laser buildup time

Oscillation condition: $E_0 e^{-j\phi} e^{g_{th}L_c - 2\alpha L_c} = E_0$

$g_{th}L_c$: 1-way threshold gain, $2\alpha L_c$: roundtrip loss, ϕ = roundtrip phase

(1) Threshold condition: gain = loss $g_{th} = 2\alpha$ (2) Phase condition: $\phi = 2m\pi$

Electron-Wave Energy Exchange

$$\frac{dK}{dt} = e\vec{v} \cdot \vec{E}$$ K: electron kinetic energy

Wave Amplification $$\Delta W = \int \vec{F} \cdot \vec{v} dt = e\int_{\tau = L/v_{//}} \vec{E} \cdot \vec{v} dt < 0$$

Particle Acceleration $$\Delta W = e\int_{\tau = L/v_{//}} \vec{E} \cdot \vec{v} dt > 0$$

Transverse Coupling
(fast wave, Eg. Compton/Thomson/undulator radiation etc.)

$$\Delta W = e\int_{\tau = L/v_{//}} \vec{E}_{\perp} \cdot \vec{v}_{\perp} dt$$

Longitudinal Coupling
(slow wave, Eg. Smith-Purcell radiator, Traveling wave tube, backward-wave oscillator etc.)

$$\Delta W = e\int_{\tau = L/v_{//}} \vec{E}_{//} \cdot \vec{v}_{//} dt$$

Resonant Interaction between Electron and Field

To have FEL gain

$$\Delta W = e\int_{\tau=L_w/v_z} \vec{E}\cdot\vec{v}\,dt < 0$$

L_W is the length of the wiggler

For $E_x = E_0\cos(\omega t - kv_z t + \phi)$ and $v_x = \dfrac{-\sqrt{2}c_0 a_w}{\gamma}\cos(k_w v_z t)$

$$\Rightarrow\quad \vec{E}\cdot\vec{v} \propto \cos\{[k-(k+k_w)\beta_z]ct+\phi\} + \cos\{[k-(k-k_w)\beta_z]ct+\phi\}$$

Ψ:pondermotive phase

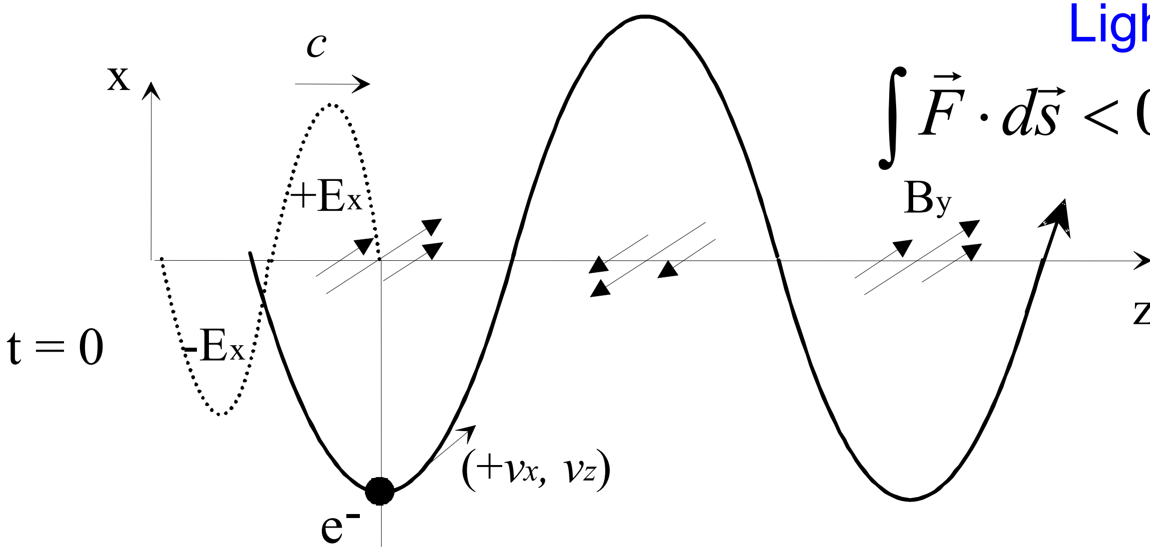
whether $\vec{E}\cdot\vec{v} > 0$ (radiation) or $\vec{E}\cdot\vec{v} < 0$ (particle acceleration) depends on ϕ

To have appreciable value in $\int_{\tau=L_w/v_z} \vec{E}\cdot\vec{v}\,dt$

$$k-(k+k_w)\beta_z = 0 \;\Rightarrow\; \lambda = \frac{1+a_w^2}{2\gamma^2}\lambda_w$$ The FEL synchronism condition

$$k-(k-k_w)\beta_z = 0 \;\Rightarrow\; \beta_z \equiv v_z/c_0 > 1$$ Impossible in vacuum

Undulator Light Amplification



Light amplification

$$\int \vec{F}\cdot d\vec{s} < 0 \text{ or } \int \vec{v}\cdot\vec{E}dt > 0$$

$$\Delta W = e\int_{\tau=L/v_{//}} \vec{E}_{\perp}\cdot\vec{v}_{\perp}dt$$

$$\tau = \frac{\lambda_w/2 + \lambda_z/2}{c} = \frac{\lambda_w/2}{v_z}$$

light slips one wavelength ahead per wiggler period

$$\lambda = \lambda_w\left(\frac{1}{\beta_z} - 1\right) \approx \frac{\lambda_w}{2\gamma_z^2}$$

Pendulum Equation

The pondermotive (beat) phase $\psi = (k + k_w)z - \omega t$

was previously found from the beam-wave energy coupling equation

$$\frac{dK}{dt} = ev_x E_x = \frac{ec_0 a_w E_0}{\sqrt{2}\gamma} \cos\{\omega t - (k + k_w)z(t) + \phi\}$$

Take first derivative of ψ with respect to z and use the FEL synchronism condition to obtain

$$\frac{d\psi}{dz} = 2k_w \frac{\gamma - \gamma_r}{\gamma_r} = 2k_w \boxed{\frac{\Delta\gamma}{\gamma_r}}$$

where γ_r is the resonant particle energy satisfying the synchronism condition

$$\lambda = \lambda_w \frac{1 + a_w^2}{2\gamma_r^2} \quad \text{or} \quad k_w = k \frac{1 + a_w^2}{2\gamma_r^2}$$

A second derivative to the beat phase with respect to *z* gives the

pendulum equation

$$\frac{d^2\psi}{dz^2} = -k_\psi^2 \sin\psi$$

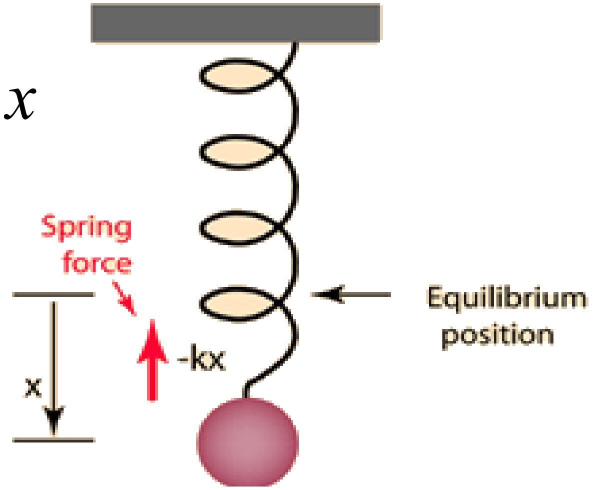
where $$k_\psi^2 = \left[\frac{e}{\gamma_r m_0 c_0}\right]^2 \frac{\sqrt{2} B_{rms} E_0}{c_0} \equiv \frac{2\pi}{L_\psi}$$

L_ψ: synchrotron oscillation wavelength

For a small Ψ, $$\frac{d^2\psi}{dz^2} \sim -k_\psi^2 \psi$$

Recall the harmonic oscillator equation

$$\frac{d^2x}{dt^2} = -\omega_0^2 x$$

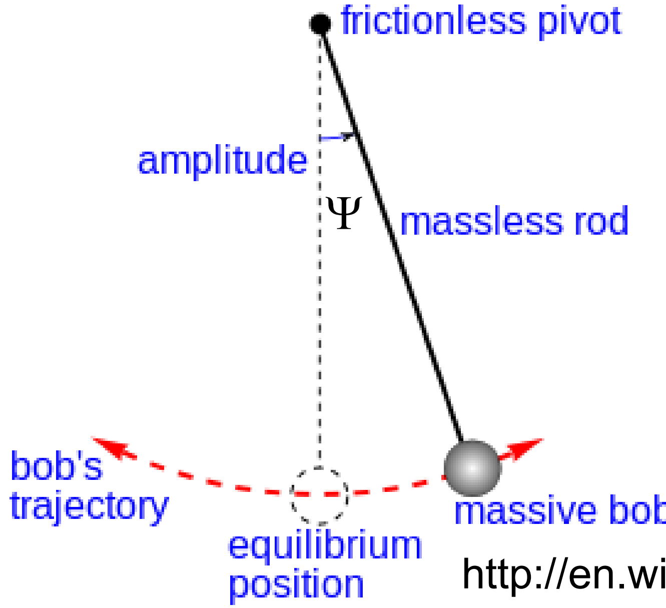


Particles oscillate, drift in the pondermotive phase .

http://hyperphysics.phy-astr.gsu.edu/hbase/oscda.html

pendulum equation

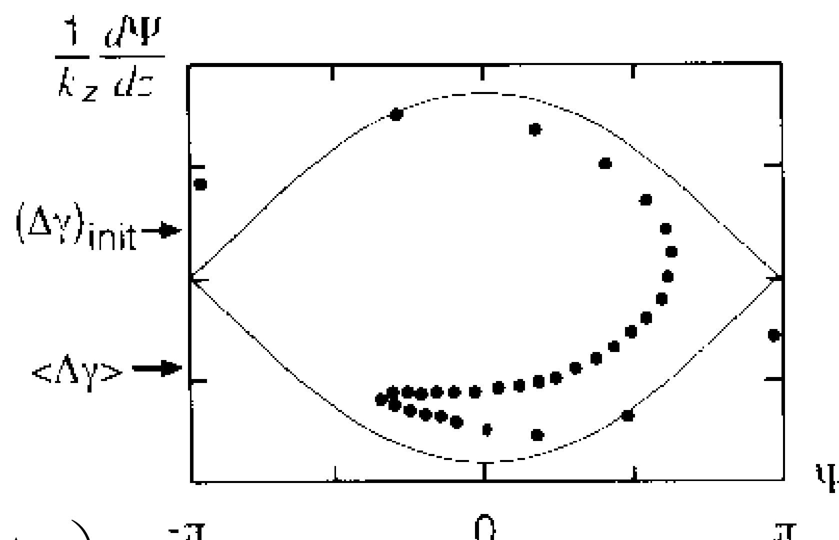
$$\frac{d^2\psi}{dz^2} = -k_\psi^2 \sin\psi$$



http://en.wikipedia.org/wiki/Pendulum

With the definition of k_ψ, the *phase diagram* can be plot from

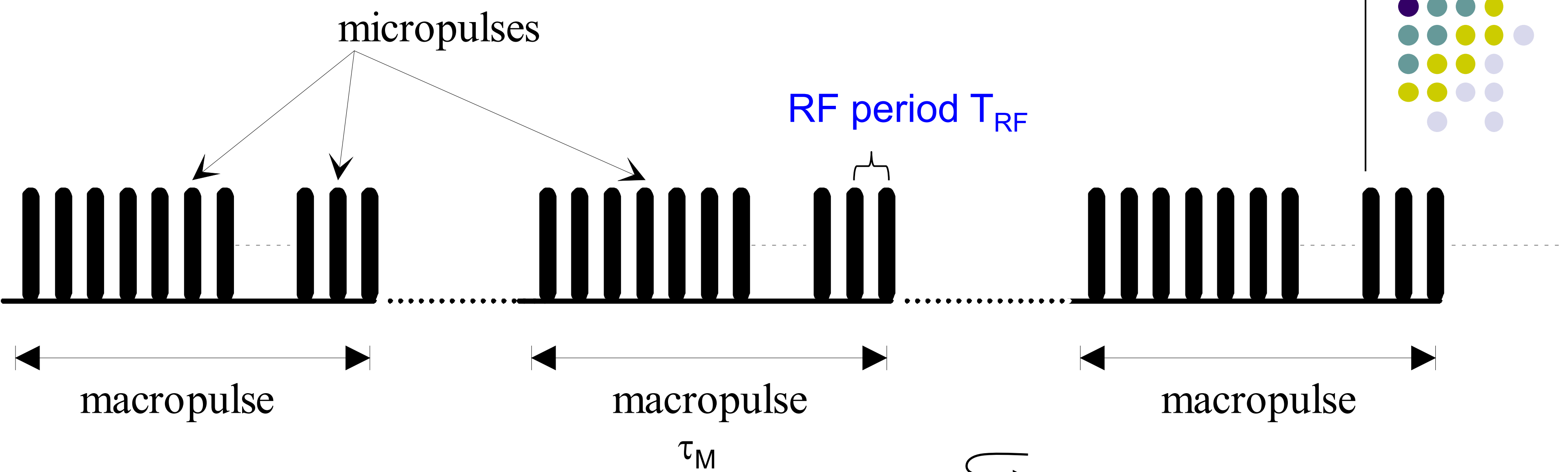
$$\frac{d\psi}{dz} = \pm\sqrt{2}k_\psi\sqrt{\cos\psi + 1} = 2k_w\frac{\Delta\gamma}{\gamma_r}$$



The bucket height = 4 k_ψ, and the maximum energy extraction occurs at half synchrotron wavelength: FEL length is ~$L_\psi/2$

The maximum energy efficiency for an FEL = $\left(\frac{\Delta\gamma}{\gamma_r}\right)_{max} = 1/(2N_w)$

FEL Buildup Time τ_B



Saturation power

Roundtrip net gain

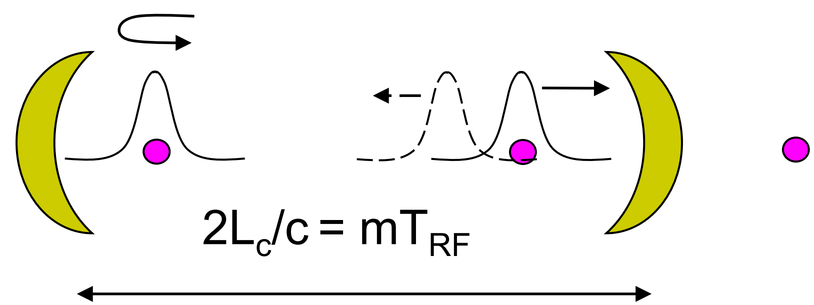
$$P_b \times \left[\frac{1}{2N_w}\right] = P_s \times \left[e^{(gL_c - 2\alpha L_c)}\right]^{\frac{\tau_B}{2L_c/c}}$$

\# of roundtrips

Beam power

Spontaneous radiation power

$$\left(\frac{\Delta\gamma}{\gamma_r}\right)_{max} = 1/(2N_w)$$



* Macropulse length > laser buildup time

$\tau_M > \tau_B$

FEL Gain

$$G = \frac{W_f - W_i}{W_i} = e^{gL_c}$$

To have gain

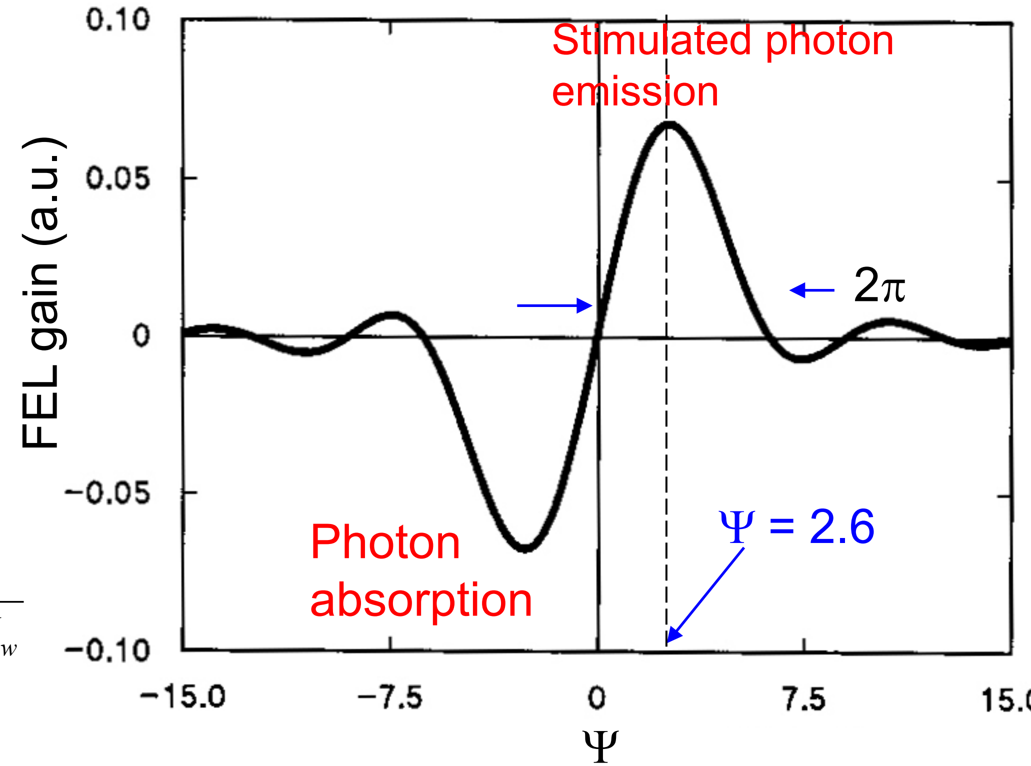
$$\Delta W = e\int_{\tau = L_w / v_z} \vec{E}\cdot\vec{v}\, dt < 0$$

L_W is the length of the wiggler

For $E_x = E_0 \cos(\omega t - k v_z t + \phi)$ and $v_x = \frac{-\sqrt{2} c a_w}{\gamma} \cos(k_w v_z t)$

whether $\vec{E}\cdot\vec{v} > 0$ (radiation) or $\vec{E}\cdot\vec{v} < 0$ (particle acceleration) depends on ϕ

- **Gain is small for short wavelength FEL**
- **Electron injection energy has to be detuned from synchronism**
- **Gain is proportional to injection current**
- **Energy spread can't exceed** $\frac{\Delta\gamma}{\gamma} < \frac{1}{2N_w}$ **where N_w is the number of wiggler period**



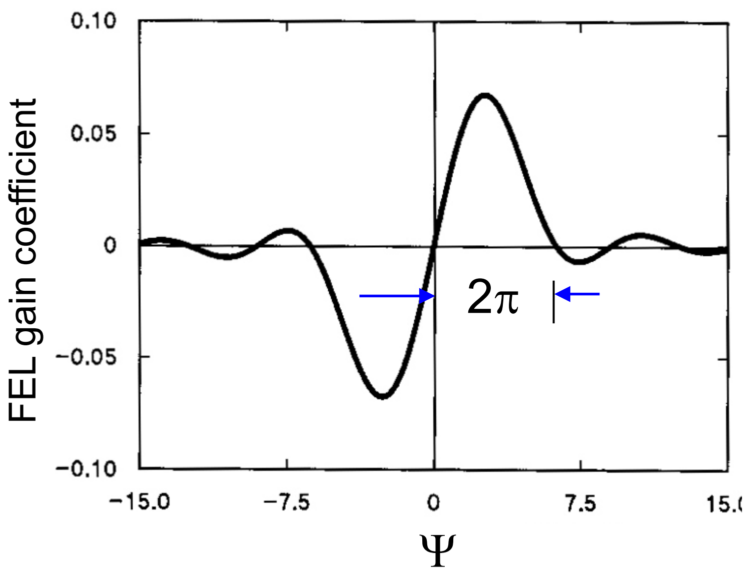
Energy Spread Requirement

Refer to the FEL gain curve, for an electron to contribute its energy to the FEL gain, the acceptance phase width has to be confined to 2π or

$$\Delta\Psi = \left|[\omega-(k+k_w)\bar{v}_z]\frac{L_w}{\bar{v}_z}\right|_{\max} = 2\pi \Rightarrow \left|[\frac{\omega}{\bar{v}_z}-(k+k_w)]L_w\right|_{\max} = 2\pi$$

$$\Rightarrow \left|\frac{d\psi}{dz}L_w = 2k_w\frac{\Delta\gamma}{\gamma_r}L_w\right| < 2\pi \Rightarrow \left|\frac{\Delta\gamma}{\gamma_r}\right| < \frac{1}{2N_w}$$

where N_w is the number of undulator periods



So, the energy spread of the electron beam for an FEL has to be less than $1/(2N_w)$

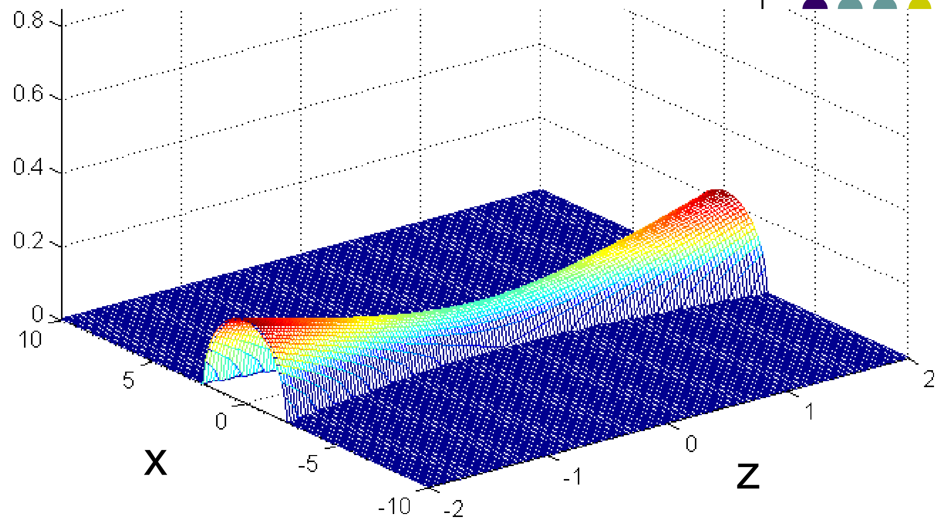
Emittance Requirement for an FEL

A Gaussian Laser Beam

Rayleigh range $z_{o,R} = \frac{\pi w_0^2}{\lambda}$

Far-field diffraction angle = $\theta \sim \frac{w_0}{z_R}$

The phase space (angle and beam size) area is $\pi \theta w_0 \sim \lambda$



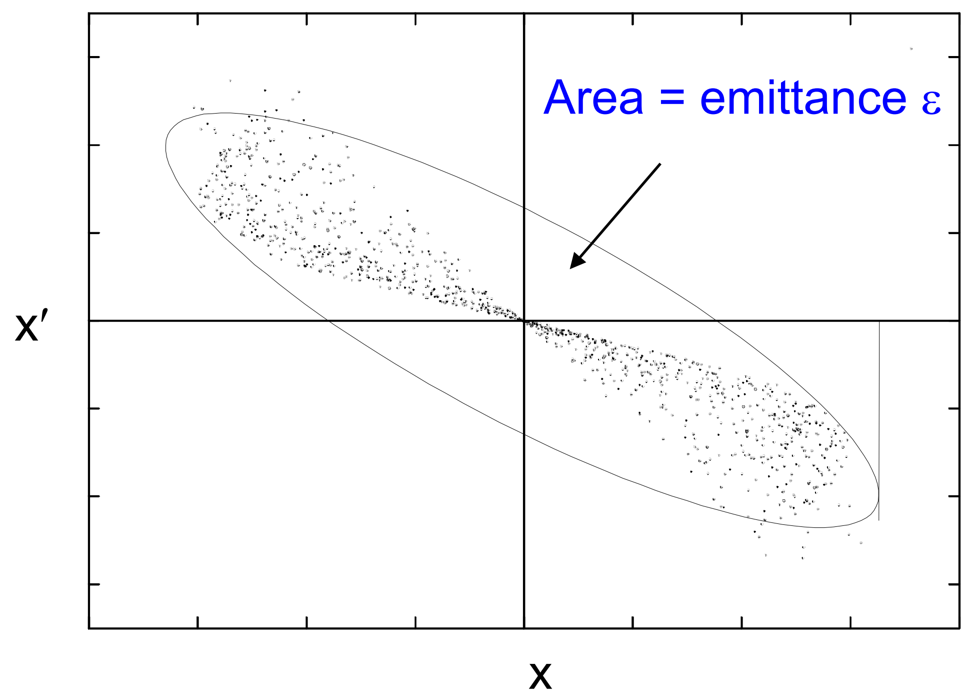
An Electron Beam

The phase space area is the beam's geometric emittance ε

To place an electron beam Inside an optical beam

$$\Rightarrow \quad \varepsilon < \lambda$$

Therefore long-wavelength FEL is more forgiving to e-beam quality



FEL Gain Bandwidth

The spectral bandwidth is defined by the variation of the spectral ratio

$$\left|\frac{\Delta\lambda}{\lambda}\right| = \left|\frac{\Delta\omega}{\omega}\right|$$

within the half width of the gain curve

$$\Delta\Psi = \left|\Omega\tau = [\omega - (k + k_w)\bar{v}_z]\frac{L}{\bar{v}_z}\right| < \pi$$

From the FEL synchronism condition $\lambda = \lambda_w \dfrac{1 + a_w^2}{2\gamma^2}$, it is straightforward to show

$$\left|\frac{\Delta\lambda}{\lambda}\right| = 2\left|\frac{\Delta\gamma}{\gamma}\right|$$

However the maximum allowed $\Delta\gamma/\gamma < 1/(2N_w)$ is obtained from the full width. For a half width

$$\left|\frac{\Delta\lambda}{\lambda}\right| = 2\left|\frac{\Delta\gamma}{\gamma}\right| < 2 \times \frac{1}{2N_w} \times \frac{1}{2} = \boxed{\frac{1}{2N_w}}$$

Characteristics of a Free-electron Laser

1. Laser: a coherent light source

2. Wavelength tunable:
   by varying the magnetic field and the electron energy

3. High peak power: GW-MW in 0.1~ 10 psec micropulse

4. High average power: kW in > ~ μsec macropulse

General Requirements for Building an FEL

Gain > loss

In particular

i. Electron energy spread $\Delta\gamma/\gamma < 1/2N_w$

ii. Electron emittance $\varepsilon < \lambda$

FEL Fundamentals (PART II)

Design Example for an FEL Oscillator

Yen-Chieh Huang 黃衍介
ychuang@ee.nthu.edu.tw, tel: 886-3-5162340, fax: 886-3-5162330

清華大學電機工程學系/光電研究所/物理系
Department of Electrical Engineering/Institute of Photonics Technologies/
Department of Physics
National Tsinghua University, Hsinchu, Taiwan

Outline

1. System Configuration

2. RF Electron Gun

3. Wiggler

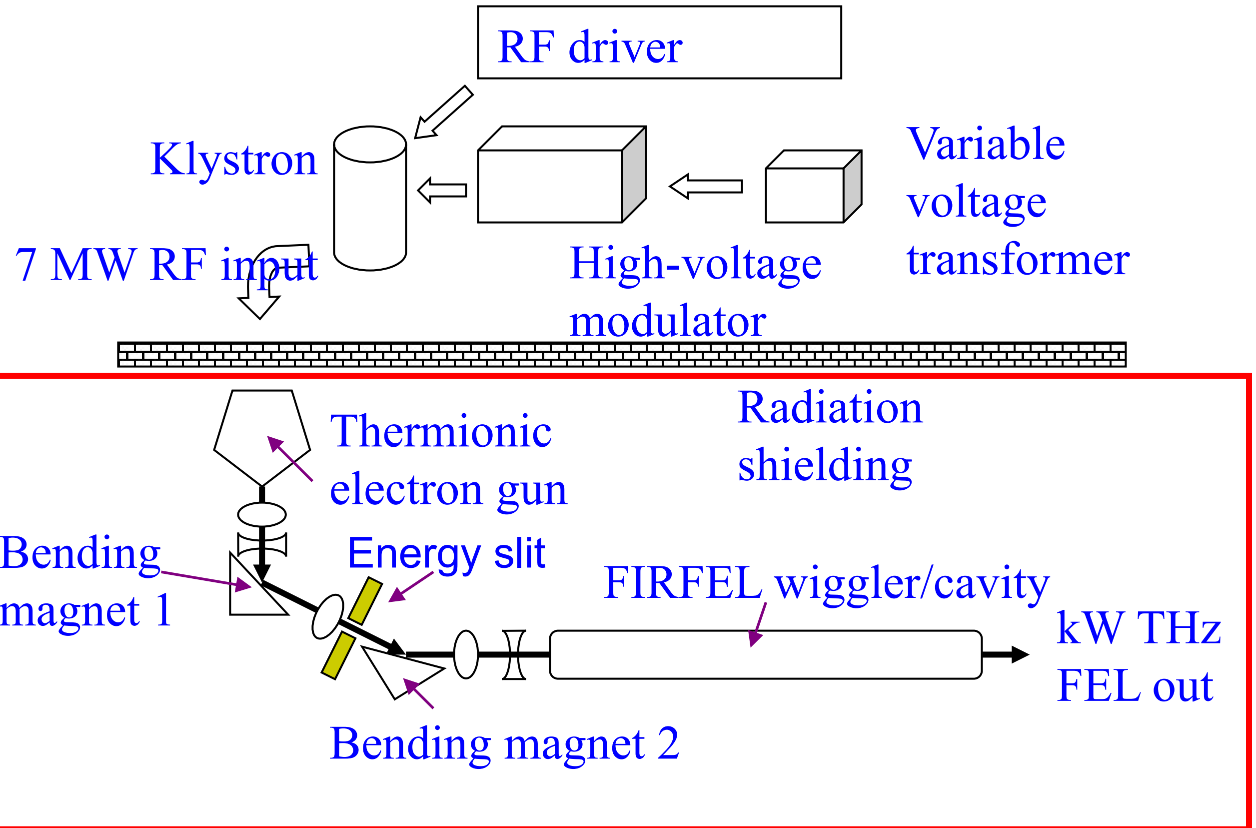
4. Laser Cavity

5. Radiation Measurement

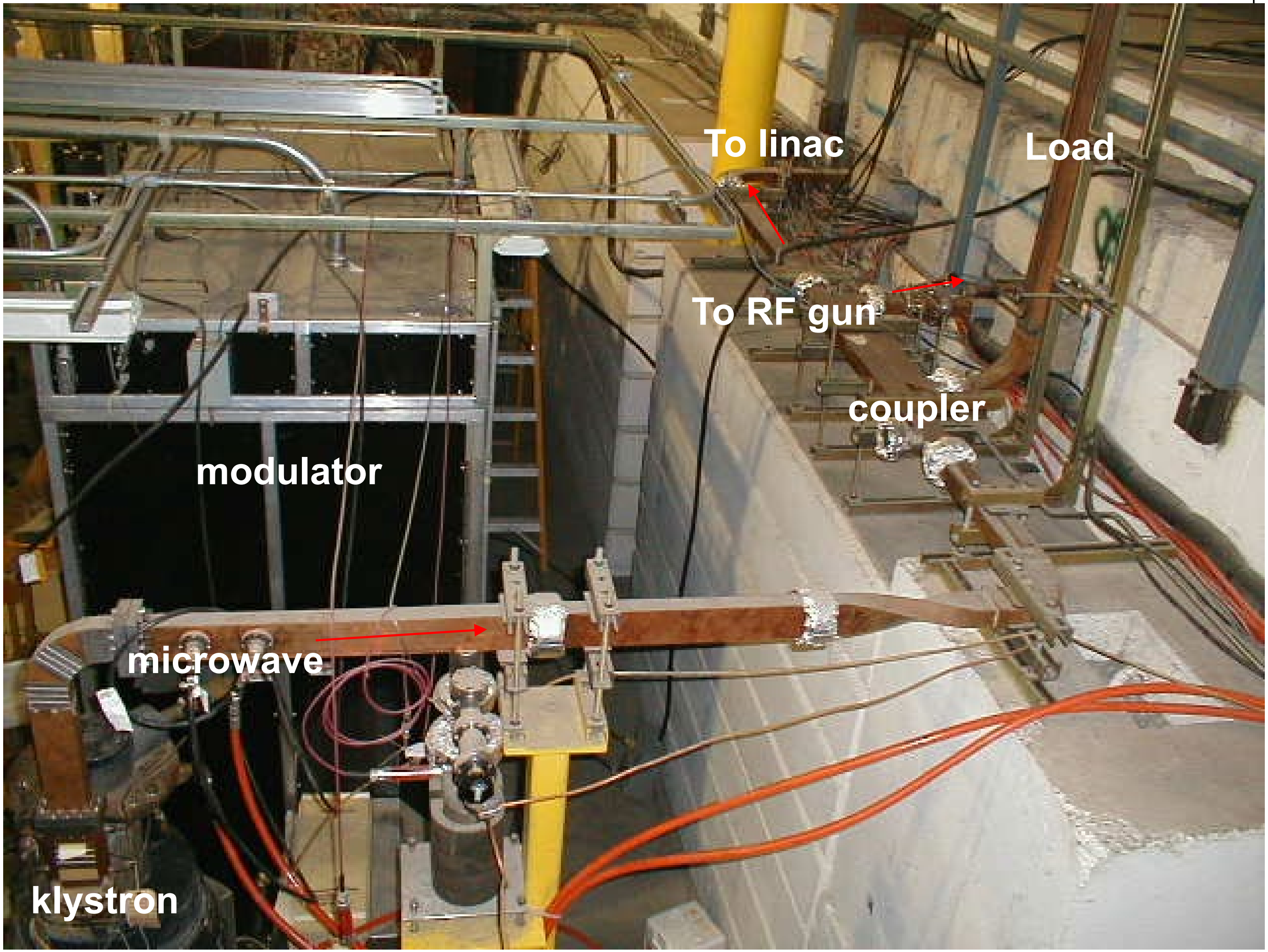
6. Perspectives

The Stanford $300k, 3.5 THz Compact FIRFEL

The RF/FIRFEL System Configuration

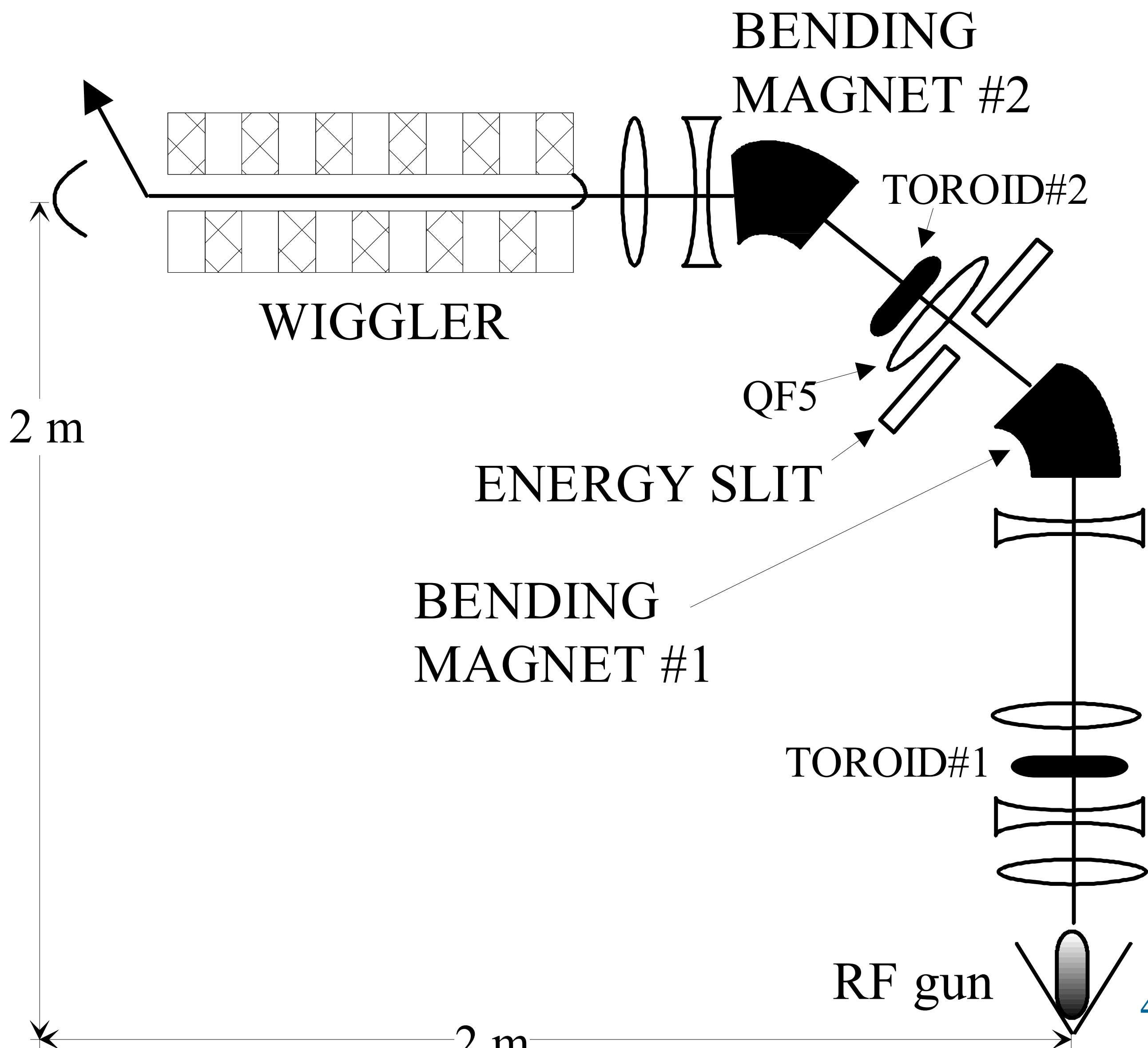


RF System



End Station III, HEPL, Stanford University

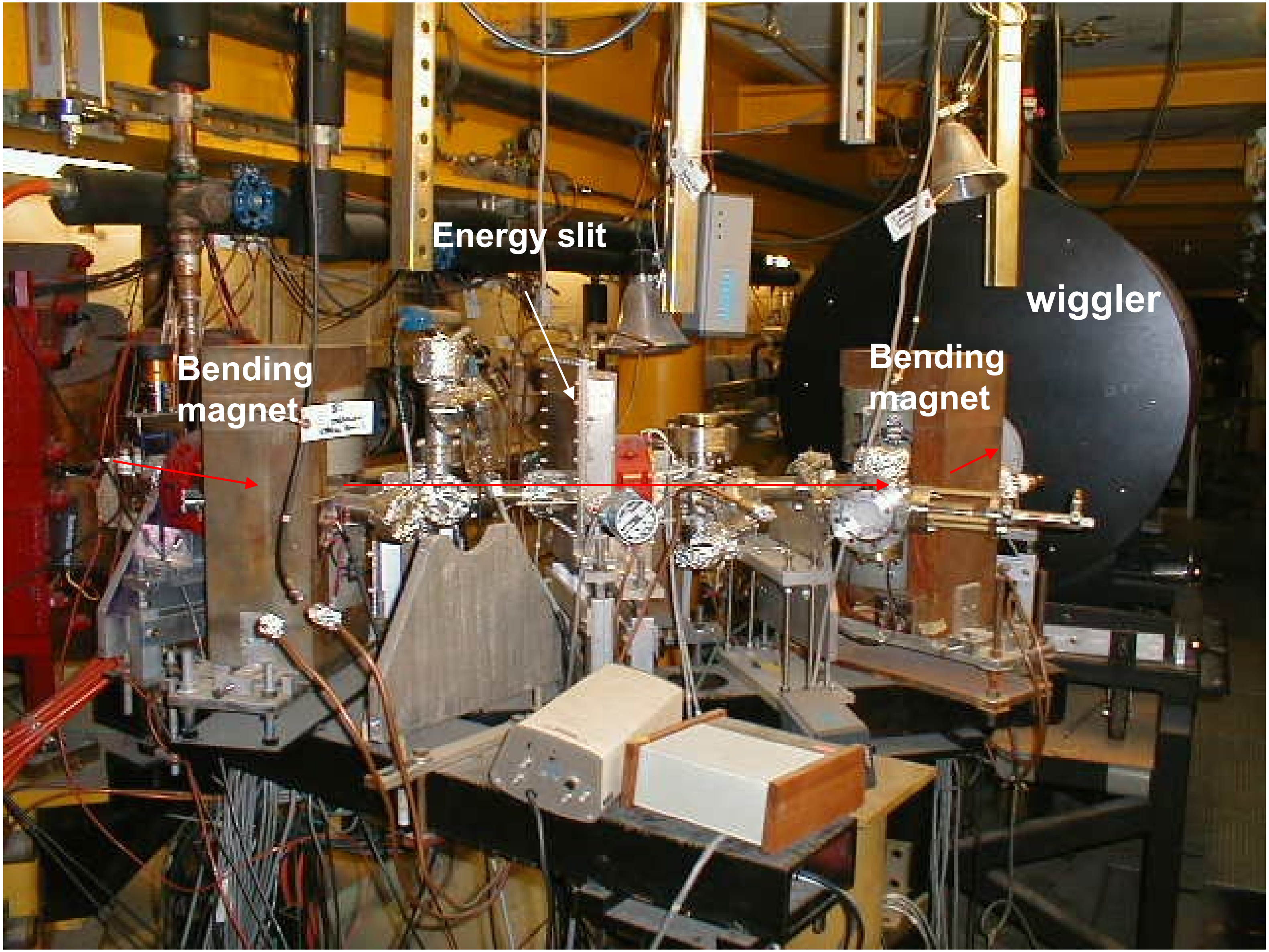
The FIRFEL Beamline



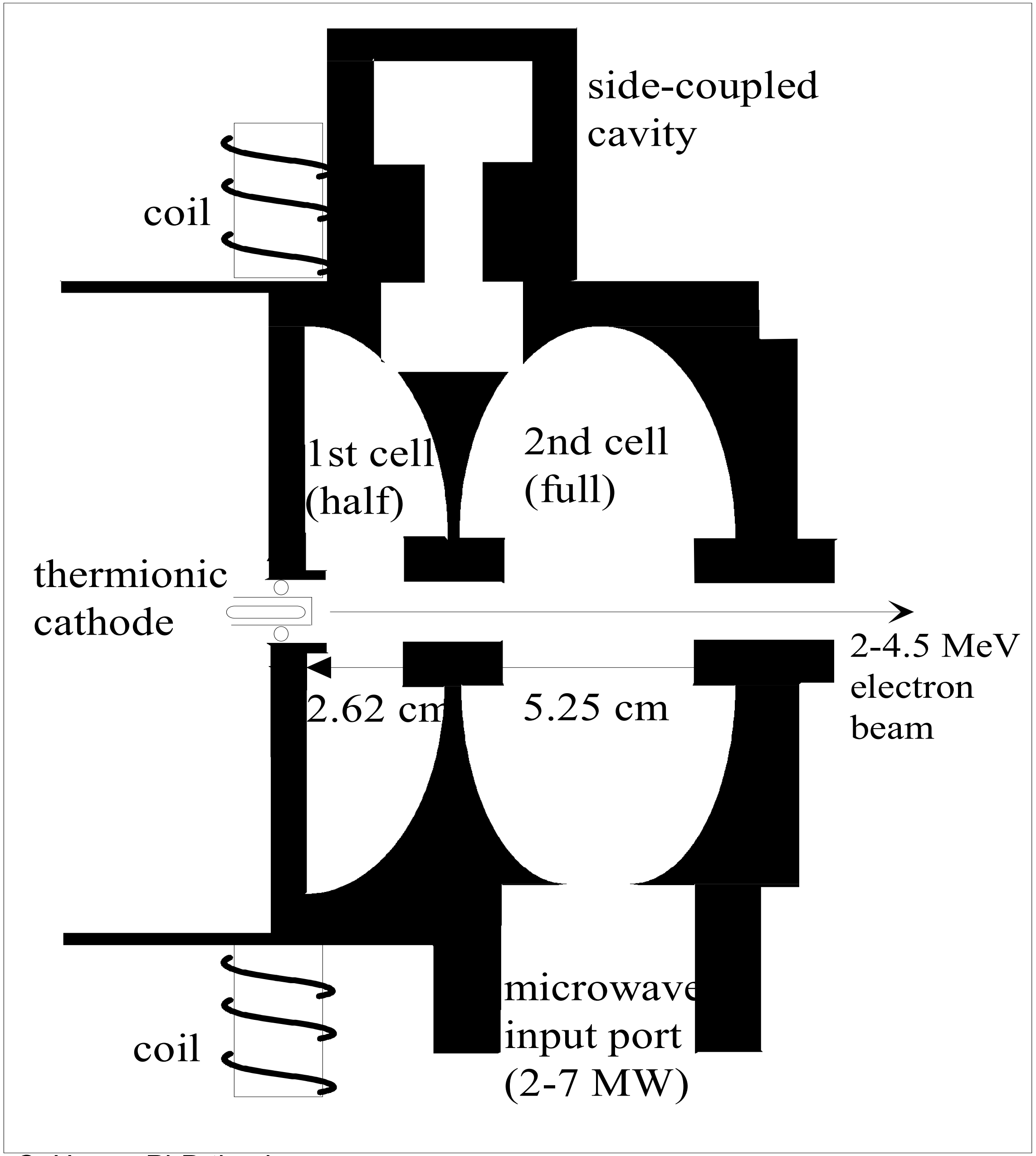
4 MeV electron gun[5]

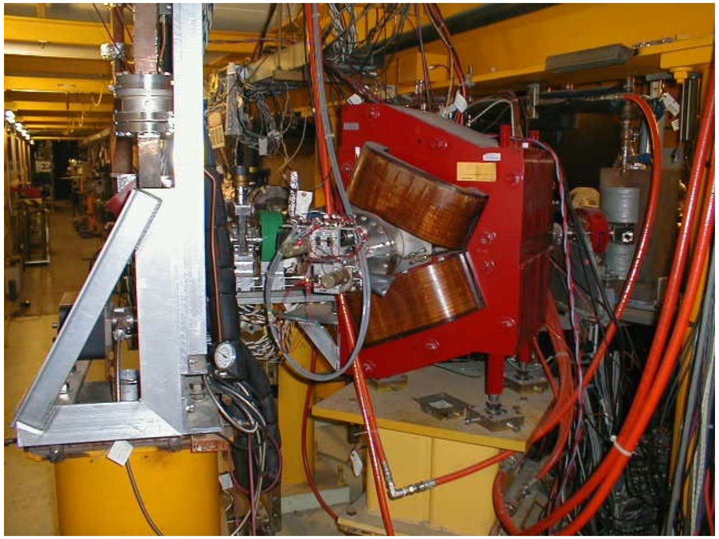
y. C. Huang PhD thesis

The Stanford FIRFEL



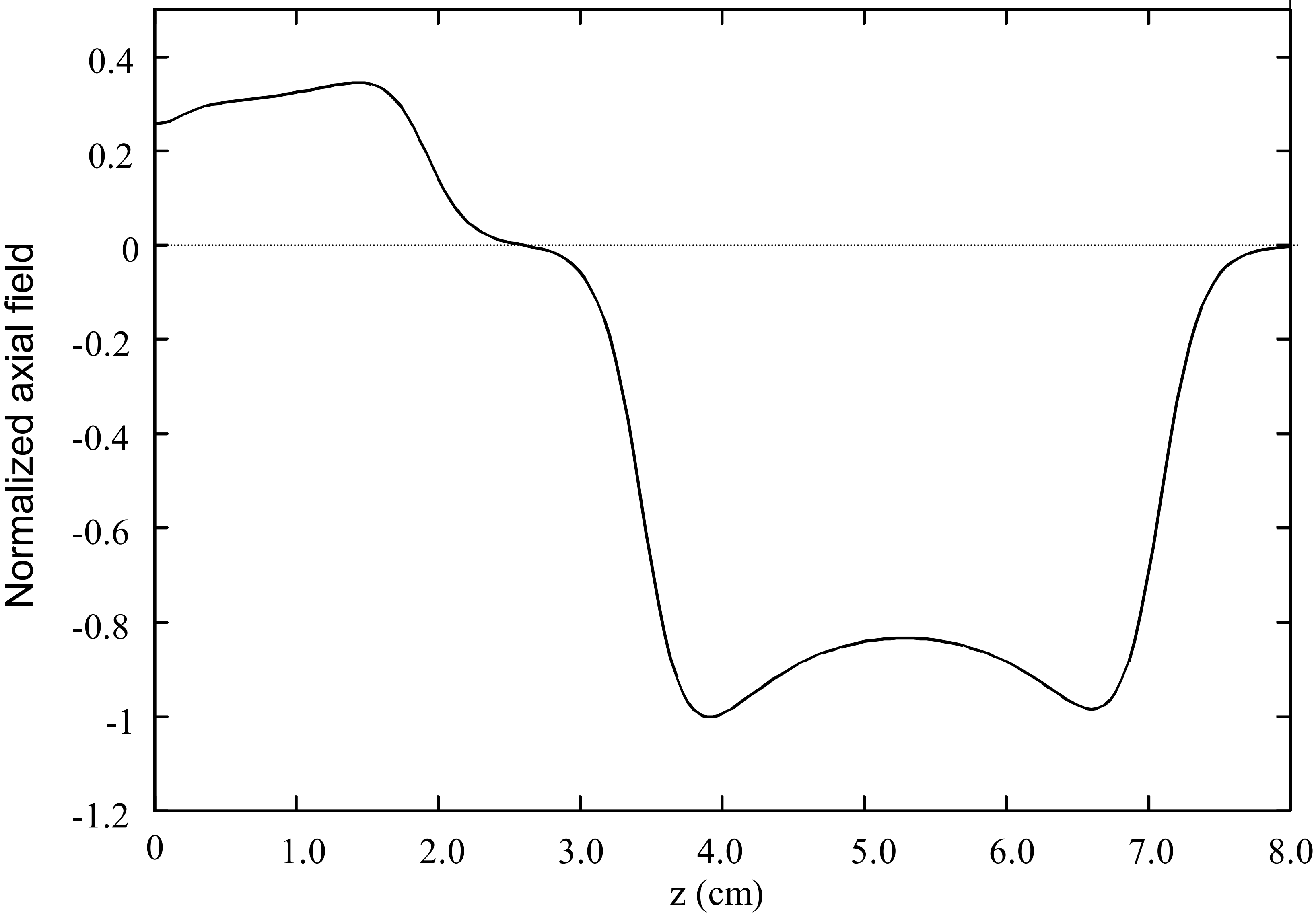
Microwave electron gun





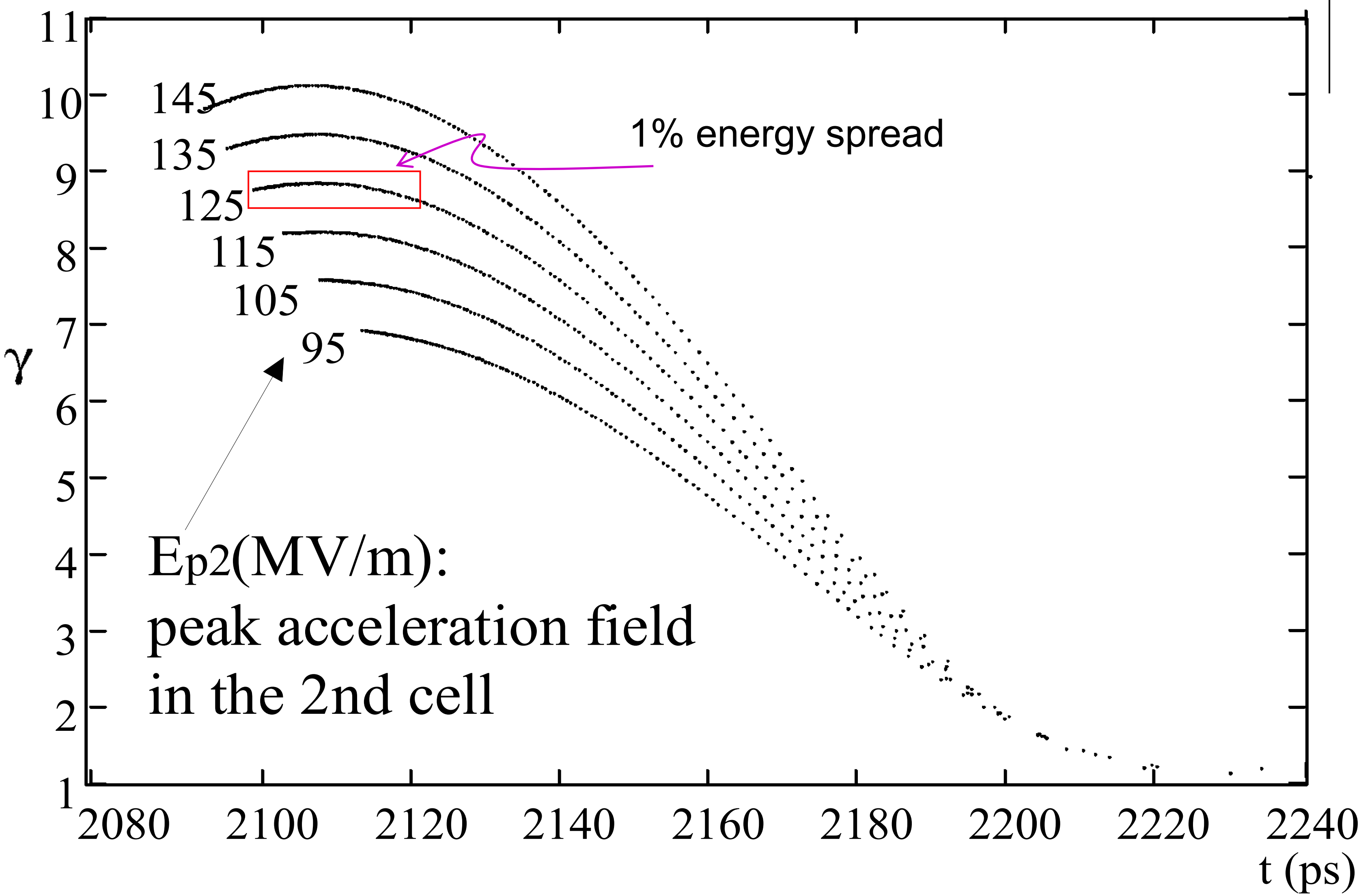
y. C. Huang PhD thesis

Axial Acceleration Field



y. C. Huang PhD thesis

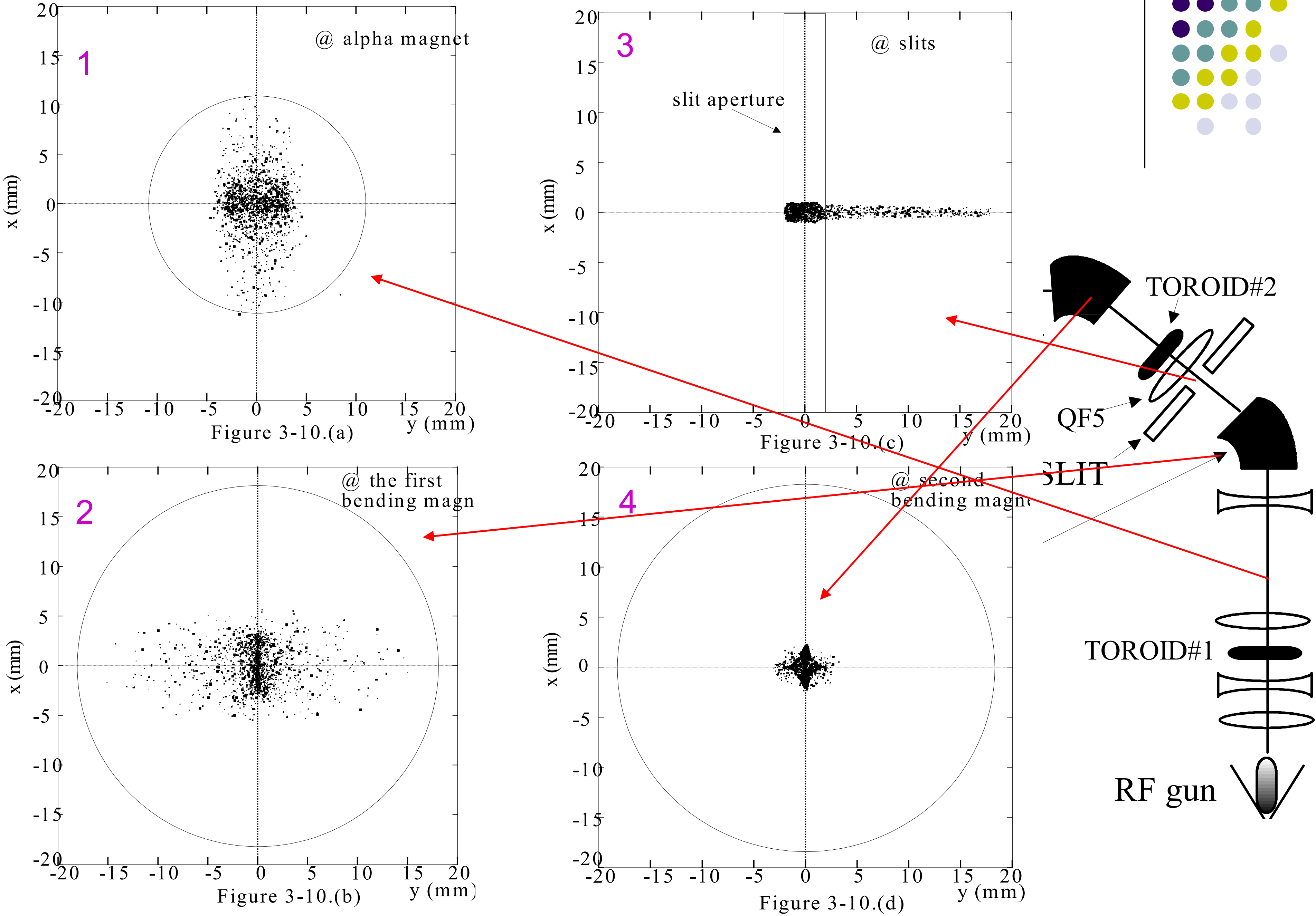
Electron Energy Spectrum from RF Gun



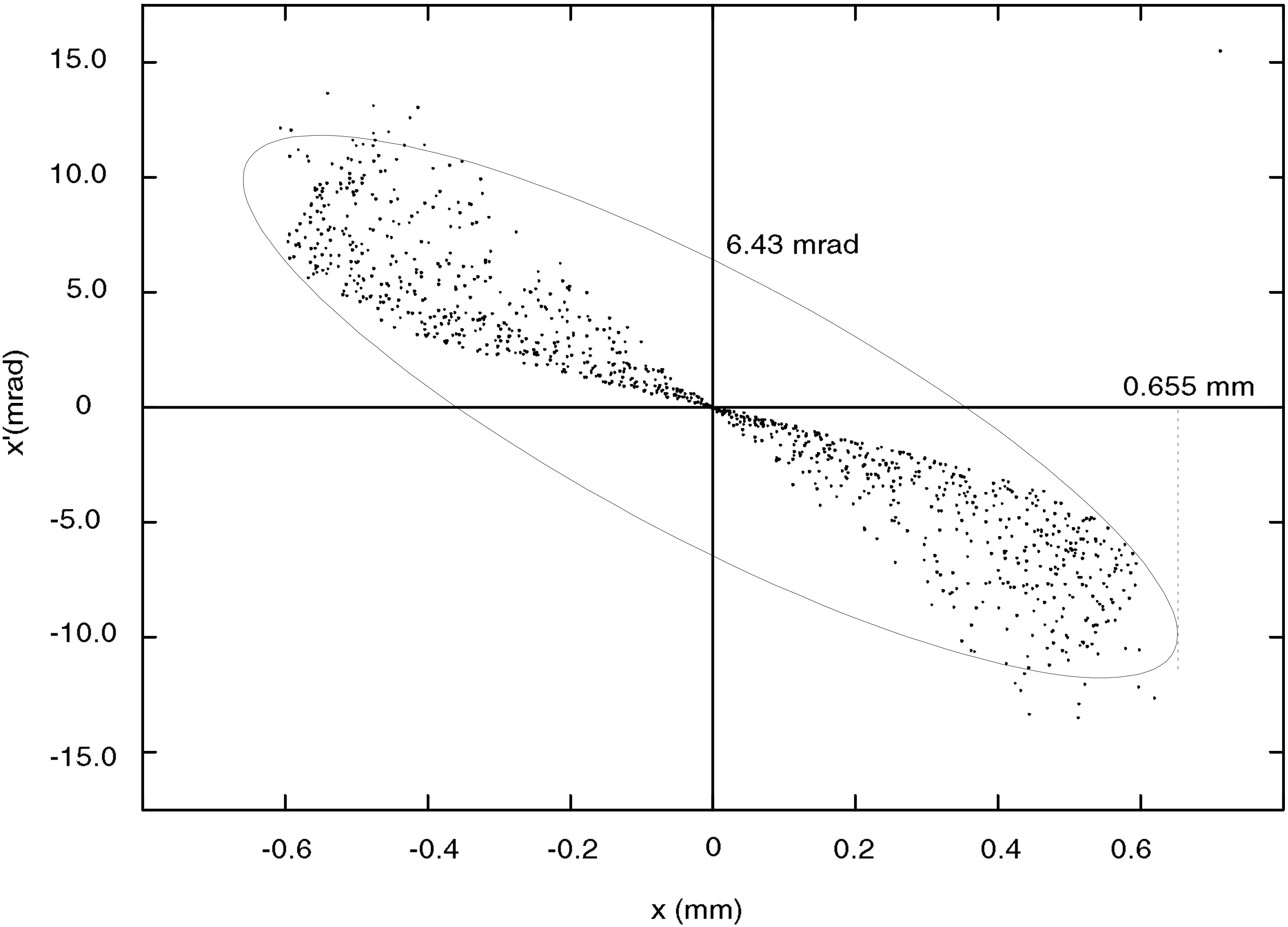
y. C. Huang PhD thesis

Energy Filtering

Y. C. Huang PhD thesis

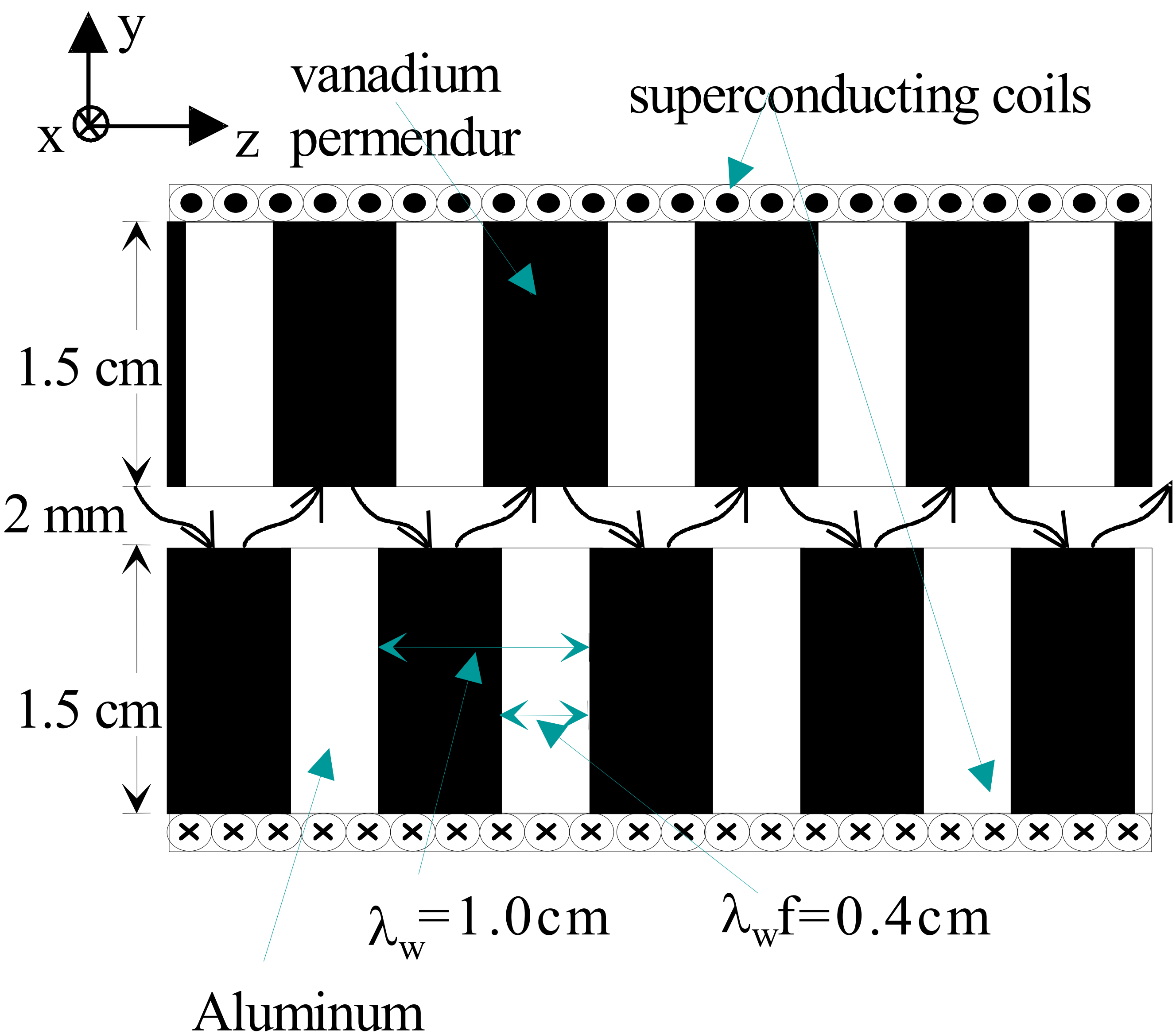


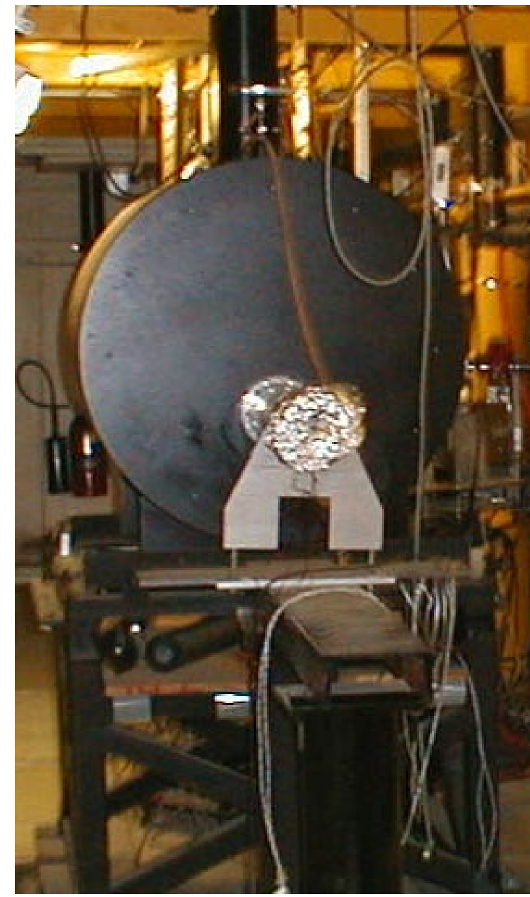
Geometric Emittance (4.2 π-mm-mrad for 90% particles)



Y. C. Huang PhD thesis

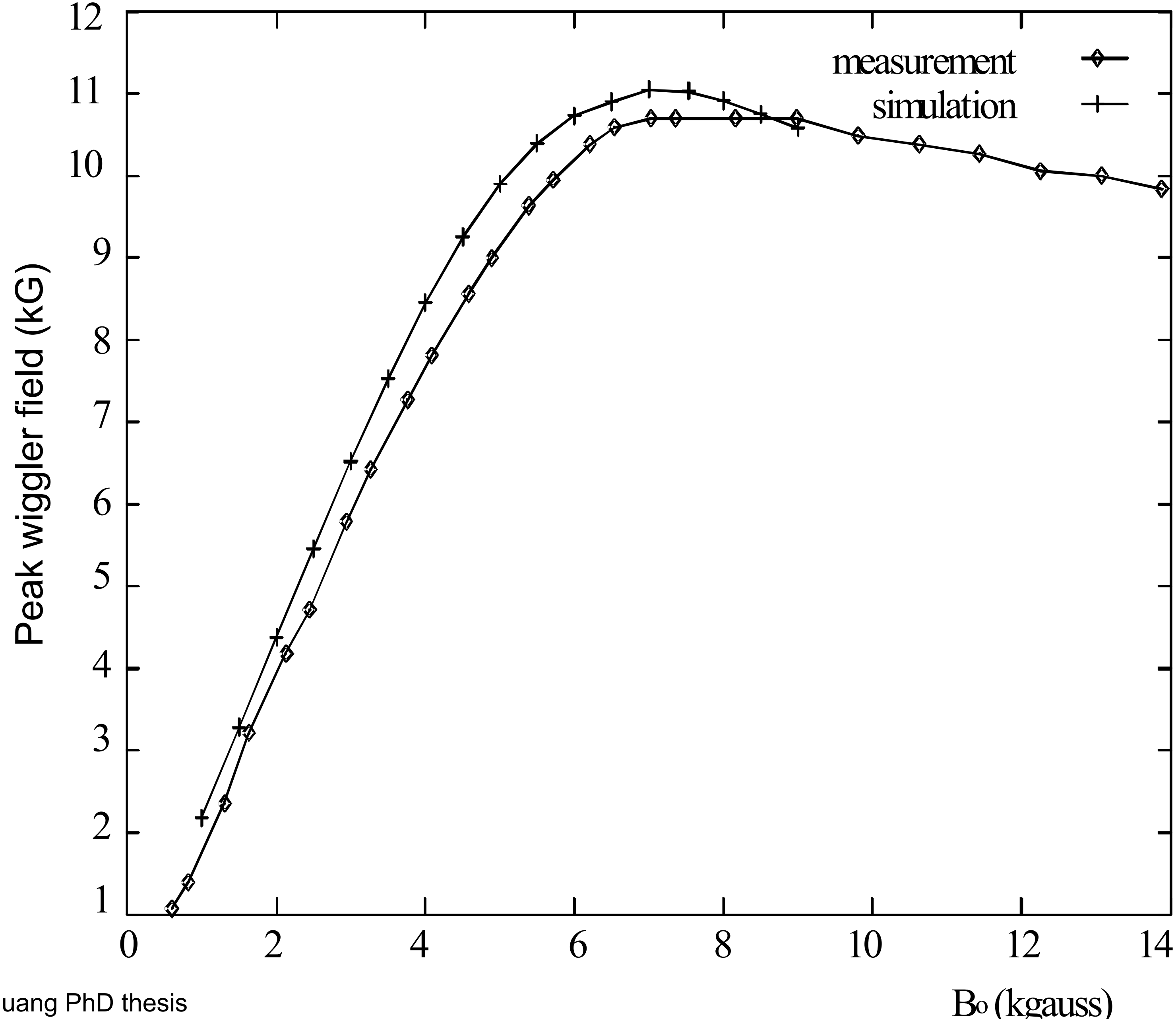
Superconducting Solenoid Wiggler





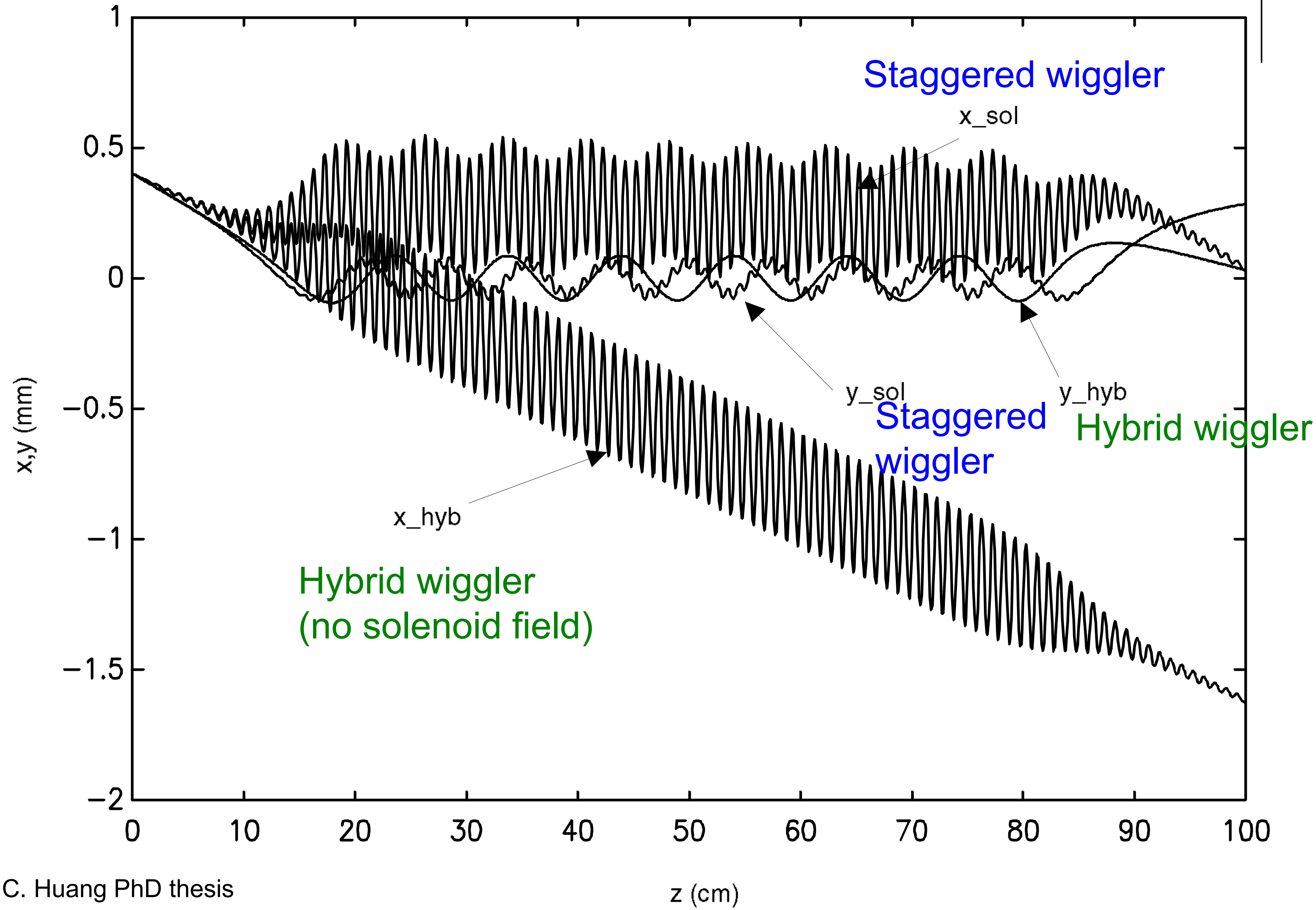
y. C. Huang PhD thesis

Peak Wiggler Field vs. Solenoid Field



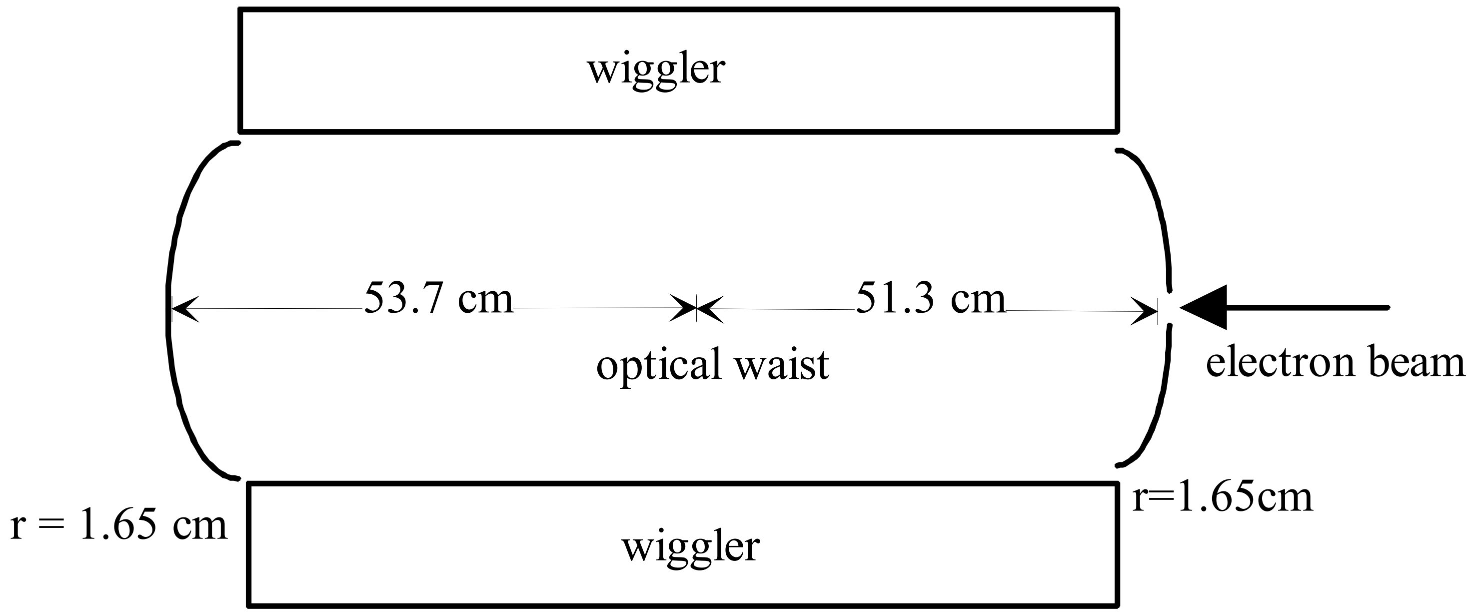
Y. C. Huang PhD thesis

Electron Trajectories



Y. C. Huang PhD thesis

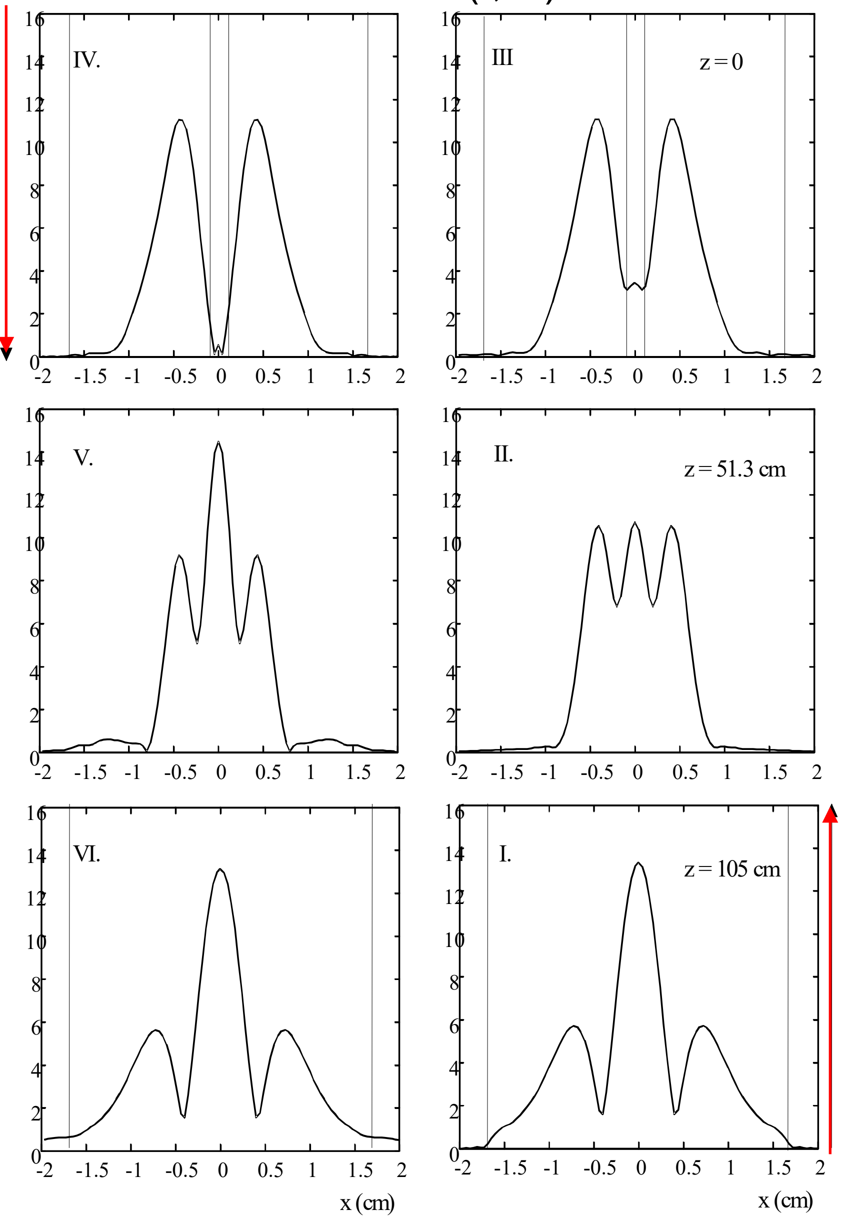
Laser Cavity Design

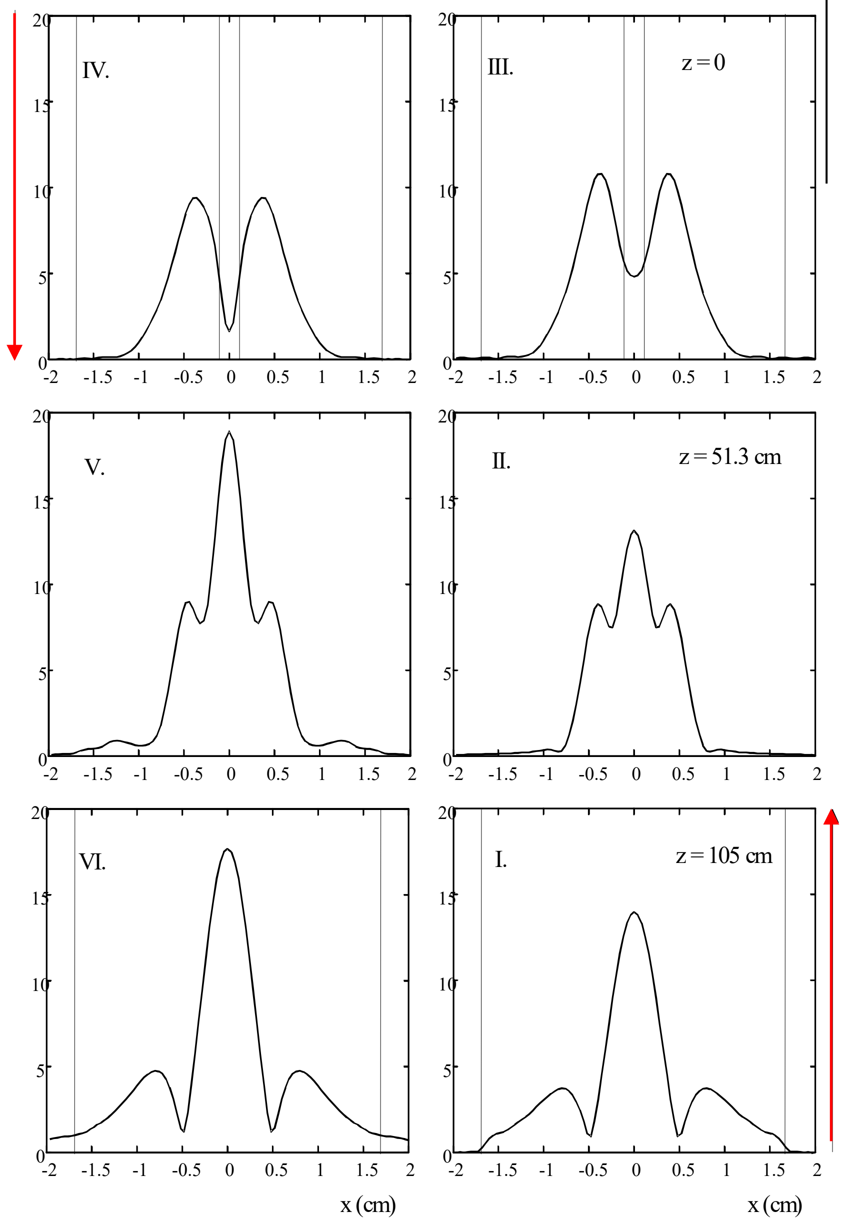


Rayleigh length Zr = 50.0 cm

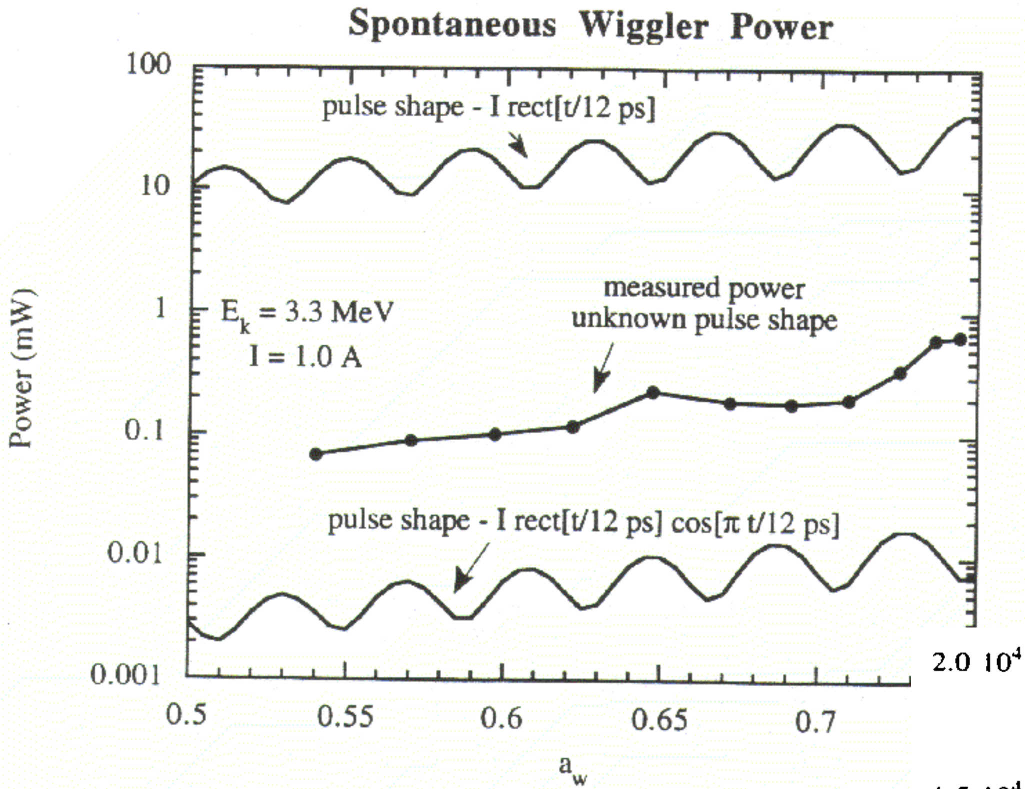
* The wiggler/waveguide has been rotated 90 degrees for clarity.

Cold Cavity Mode Evolution **Warm Cavity Mode Evolution**



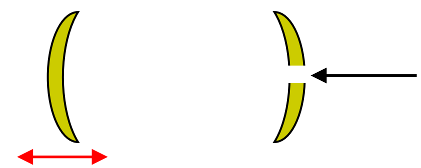


Y. C. Huang PhD thesis

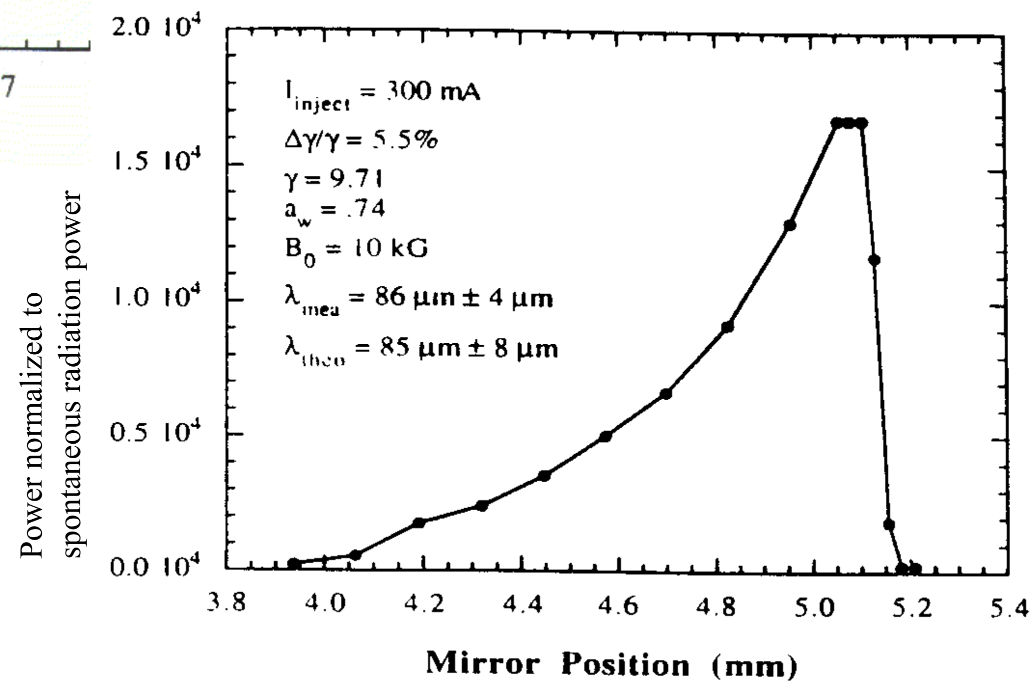


Magnetic field

← Spontaneous emission power



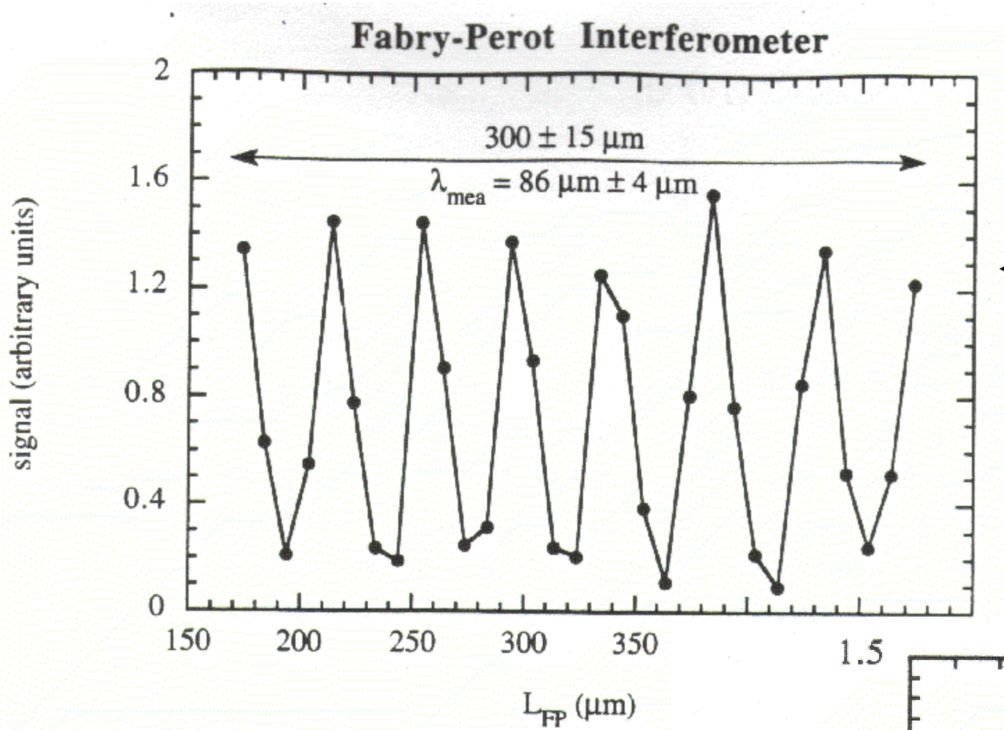
Stimulated emission power →



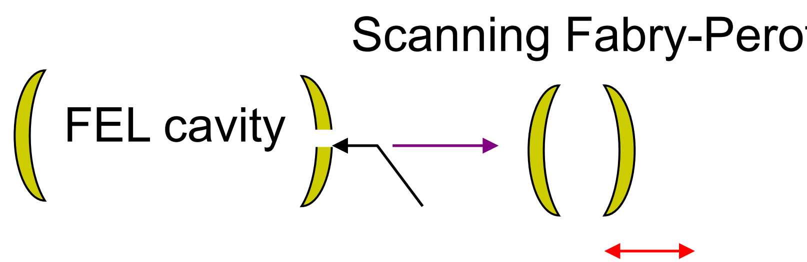
Power Measurement

J. F. Schmerge, J. Lewellen, Y.C. Huang, J. Feinstein, and R.H. Pantell, “The Free-electron Laser as Laboratory Instrument,” *IEEE J. Quantum Electronics*, vol. 31, NO. 6, June 1995, pp. 1166-1171.

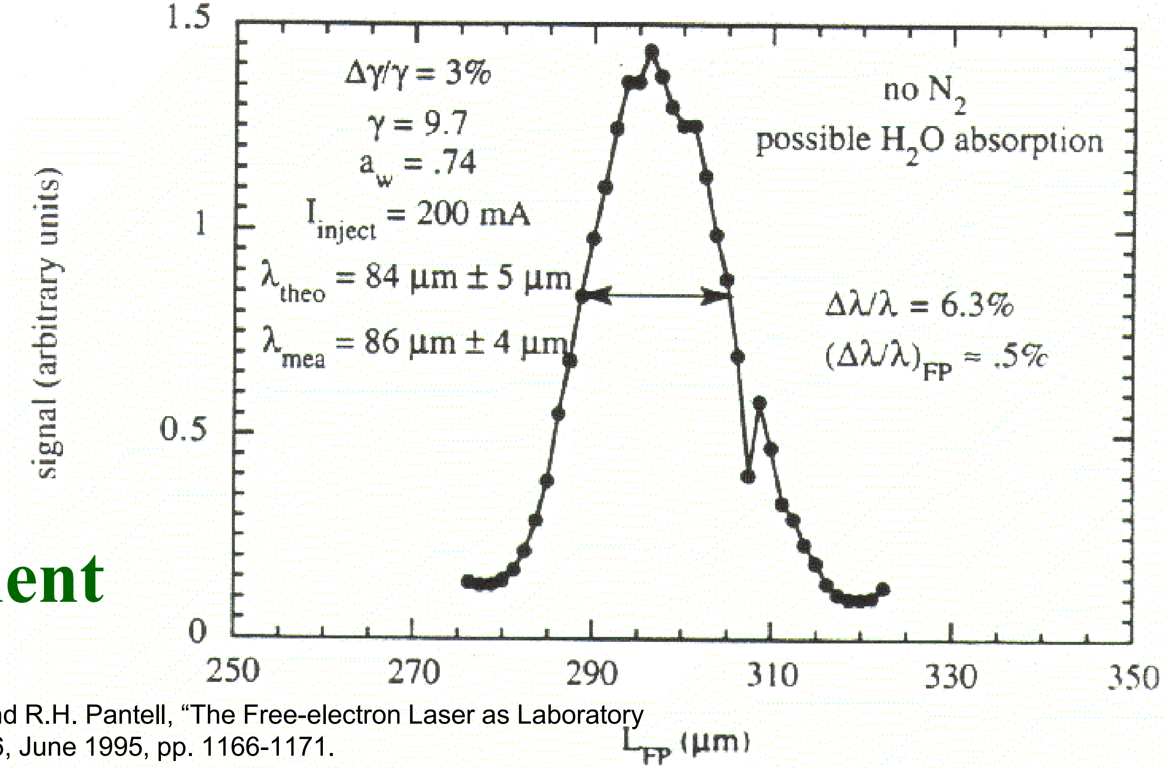
Spectral Measurement



Wavelength measurement



Spectral width



J. F. Schmerge, J. Lewellen, Y.C. Huang, J. Feinstein, and R.H. Pantell, "The Free-electron Laser as Laboratory Instrument," *IEEE J. Quantum Electronics*, vol. 31, NO. 6, June 1995, pp. 1166-1171.

Cost Breakdown

RF System:	$14.4 k
Electron beam:	$25 k
Vacuum System:	$32 k
Power Supplies:	$16 k
Optics:	$12 k
Wiggler:	$30 k
Superconducting Solenoid:	$34 k
Miscellaneous (cable, wire):	$5 k
Total:	**$298 k**