



Lecture 3 - Alternative Concepts for Future FEL

黃衍介 Yen-Chieh Huang

ychuang@ee.nthu.edu.tw, tel: +886-3-5162340, fax: +886-3-5162330

High-energy OPTics & Electronics (HOPE) Laboratory
Department of Electrical Engineering/Institute of Photonics
Technologies/Department of Physics, National Tsinghua University, Hsinchu
30013, Taiwan

Outline

1. Ultra-compact FEL
2. Chirped-pulse FEL

3. Laser-accelerator driven FEL
4. Photonic-chip FEL

Small \approxeq Economic \approxeq Stable

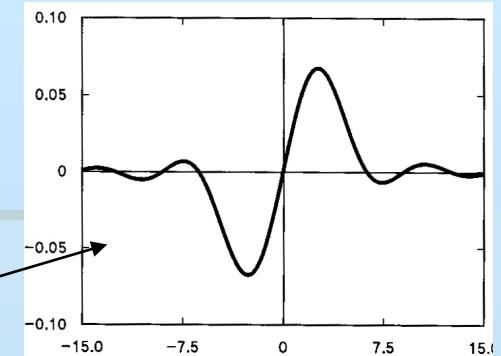
outline

1. Reducing beam energy \rightarrow **THz FEL** \rightarrow reducing both accelerator/undulator length
2. Increasing acceleration gradient \rightarrow **laser-driven particle accelerator** \rightarrow shortening accelerator with fixed beam energy
3. Increasing startup power \rightarrow **particle bunching** \rightarrow shortening undulator length to reach saturation power
4. Reducing beam energy and undulator period \rightarrow reducing both accelerator/undulator lengths for XFEL \rightarrow **quantum FEL vs. classic FEL**

FELO Small Signal Gain – lasing requirement: gain > loss

$$G - 1 = 2\pi \frac{a_u^2 e L_u^2 N_u}{\epsilon_0 \gamma^3 m_0 c_0^3} J_e \times F(\xi) \times g(\theta) \propto \frac{1}{\gamma^3}$$

↑ ↑ ↑
Factor ~ 1 Gain function



Shortening an FEL \rightarrow reducing γ (beam energy) \rightarrow shortening accelerator

Recall the FEL Synchronism Condition $\lambda_r = \lambda_u \frac{1 + a_u^2}{2\gamma^2}$

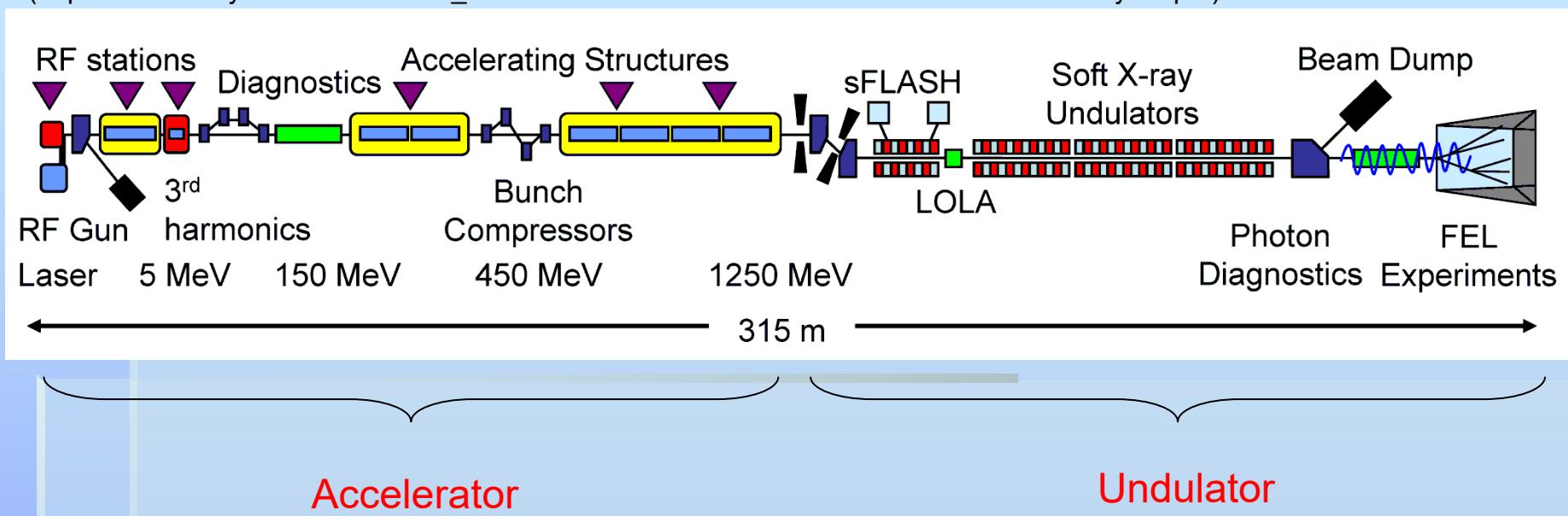
undulator period: λ_u

Undulator parameter $a_w = \frac{eB_{u,rms}}{m_0 c_0 k_w} = \frac{eA_{u,rms}}{m_0 c_0} = 0.093 B_{u,rms} (\text{kG}) \times \lambda_u (\text{cm})$

Reducing $\gamma \rightarrow$ THz FEL

SASE FEL

(https://flash.desy.de/sites2009/site_vuvfel/content/e66400/infoboxContent66401/FLASHlayout.pdf)



Goal: shortening a FEL for a given radiation wavelength λ_r

Shortening an FEL = Shortening Accelerator + Shortening Undulator

Shortening Accelerator for same γ (1) – Laser plasma accelerator

increasing **acceleration gradient** \Rightarrow laser driven particle acceleration \rightarrow shortening accelerator for same γ

Laser/plasma wakefield Accelerator

$$\tau_{laser} < \lambda_p / c$$

For $r \sim \lambda_p \sim 20 \mu\text{m}$

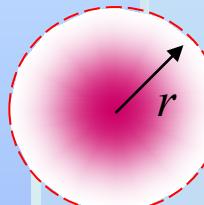
$$N \sim 5 \times 10^{17}/\text{c.c.}$$

$$\text{Gauss Law} \rightarrow E_a \sim \frac{Q}{4\pi\epsilon_0 r^2} = \frac{rNe}{3\epsilon_0}$$

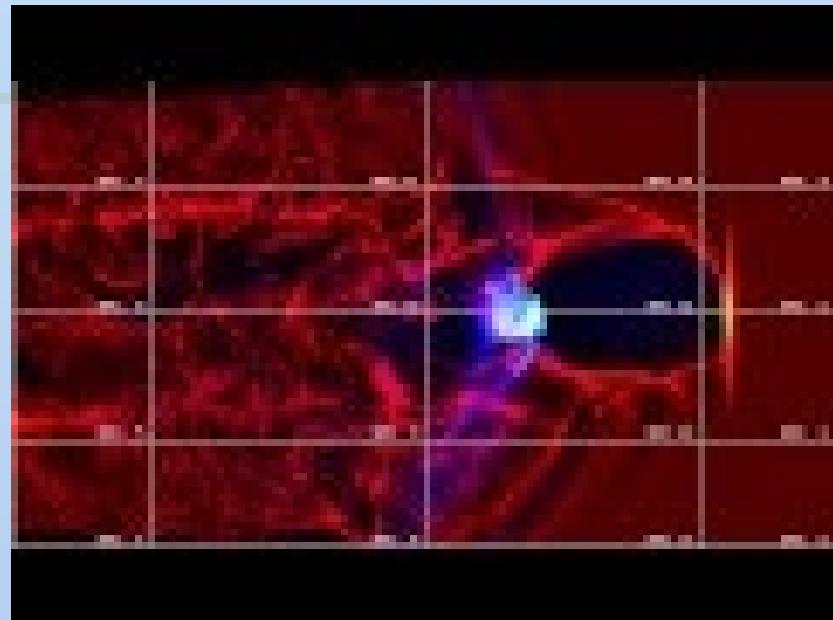
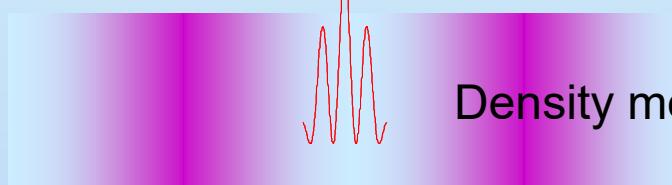
$$\sim 100 \text{ GV/m}$$

Short laser pulse

Density modulated plasma



km accelerator for XFEL reduces to m long!



Shortening Accelerator for same γ (2) – laser plasma accelerator

COMPACT X-RAY SOURCES

Towards a table-top free-electron laser

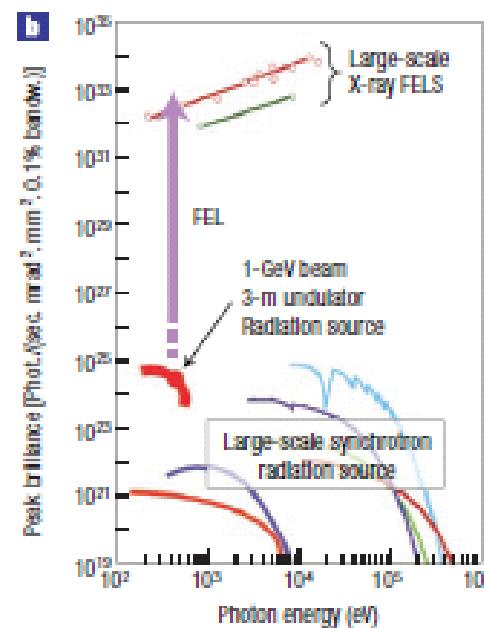
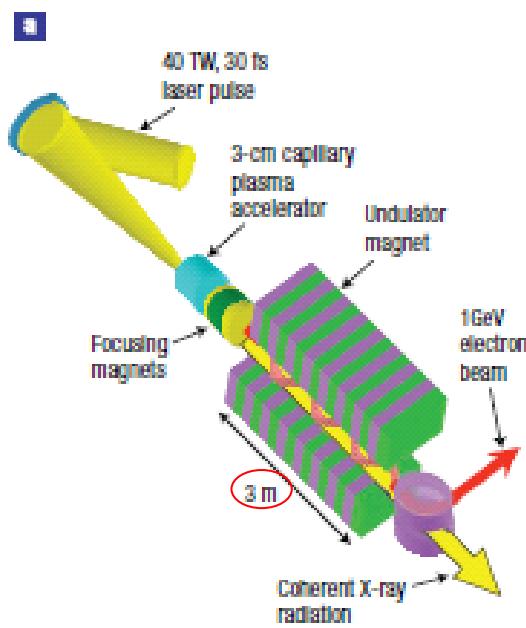
Synchrotron radiation generated using an electron beam from a laser-driven accelerator opens the possibility of building an X-ray free-electron laser hundreds of times smaller than conventional facilities currently under construction.

Kazuhisa Nakajima

is at the High Energy Accelerator Research Organization, 1-1 Oho, Tsukuba, Ibaraki 305-0081, Japan.

e-mail: nakajima@post.kek.jp

Synchrotron radiation sources have become an indispensable tool in a wide range of disciplines, including physics, biology, materials science, chemistry and medicine. The reason they are so useful is the high intensity of X-rays they produce — generated when the path of a beam of electrons moving at relativistic speeds is bent by a periodic magnetic field — in comparison with other X-ray sources. Such utility is expected to grow still further with the development of X-ray free-electron lasers



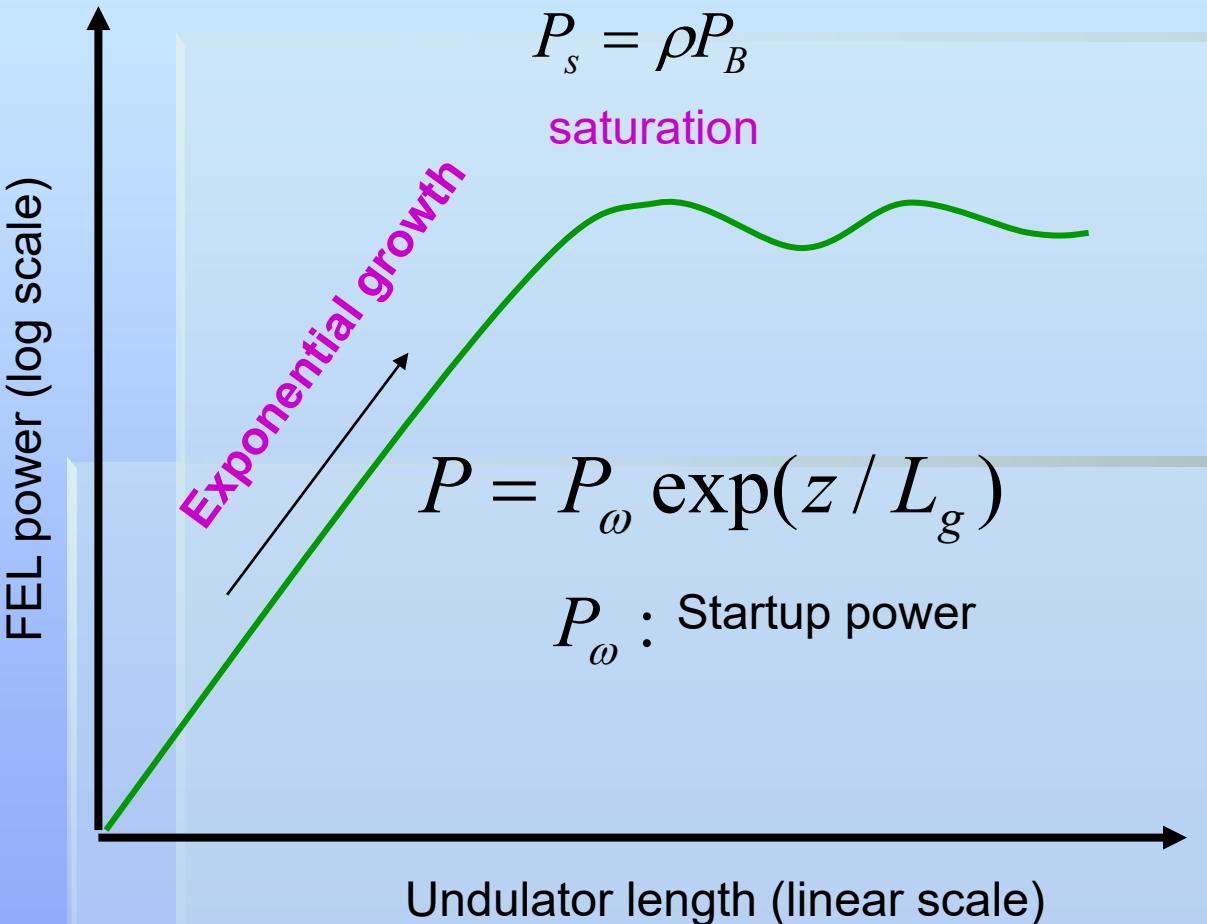
Shortening Accelerator for same γ (3) – laser plasma accelerator

TABLE I. Parameters of different scenarios for realizing free-electron lasers from laser-driven plasma-based accelerators in the XUV and x-ray domains with the corresponding FEL quantities

by combining them with conventional undulators. The scenarios X-FEL 1 and 2 highlight the possibility to produce strong FEL radiation from a laser-plasma accelerator. The most challenging condition is the relative energy spread of the electron beam, which has to be decreased to attain the x-ray part of the spectrum. In addition, high-quality electron beams at the GeV level are required [1 GeV electron beams have already been generated (Leemans *et al.*, 2006) but not with the same quality, i.e., energy spread and stability, as at the 100 MeV level]. The notion of slice energy spread and slice

Pulse energy	50 μ J	25 μ J
rms pulse duration	2 fs	2 fs

Typical SASE FEL Buildup Curve



Gain length

$$L_g = \frac{\lambda_u}{4\pi\sqrt{3}\rho}$$

Pierce parameter

$$\rho = 1.78 \times 10^{-5} \frac{A_u^{2/3}}{\gamma} \times \lambda_u^{2/3} [cm] n_e^{1/3} [cm^{-3}]$$

$A_u = a_u$ undulator parameter
for helical undulator

Beam Power

$$P_B = IV_B$$

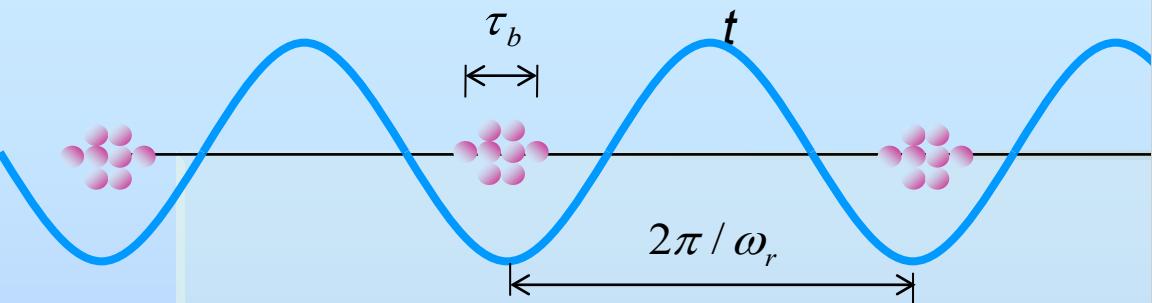
Shortening Undulator: large startup P_ω

$$\left. \begin{array}{l} P_s = \rho P_B \text{ Saturation power} \\ P(z) = P_\omega \exp(z / L_g) \end{array} \right\} \text{Undulator length } L_u = L_g \ln(\rho P_B / P_\omega)$$

To shorten z with constant saturation power P_s , **large** startup power $P_\omega \uparrow$ and **short** gain length $L_g \downarrow$ are preferred.

The startup power (mostly from spontaneous radiation) is related to the **spectral content** of the electron distribution

CSR from Periodic Bunches (comb-like pulses)



Spectral Energy

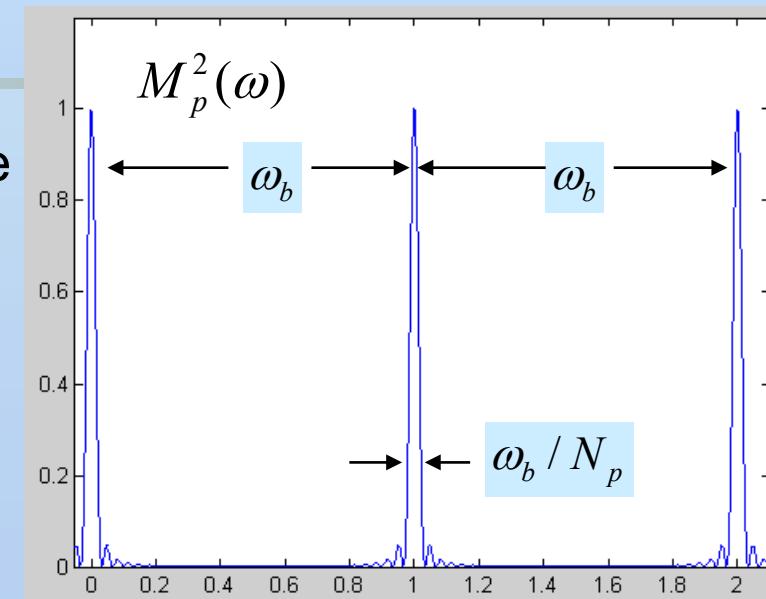
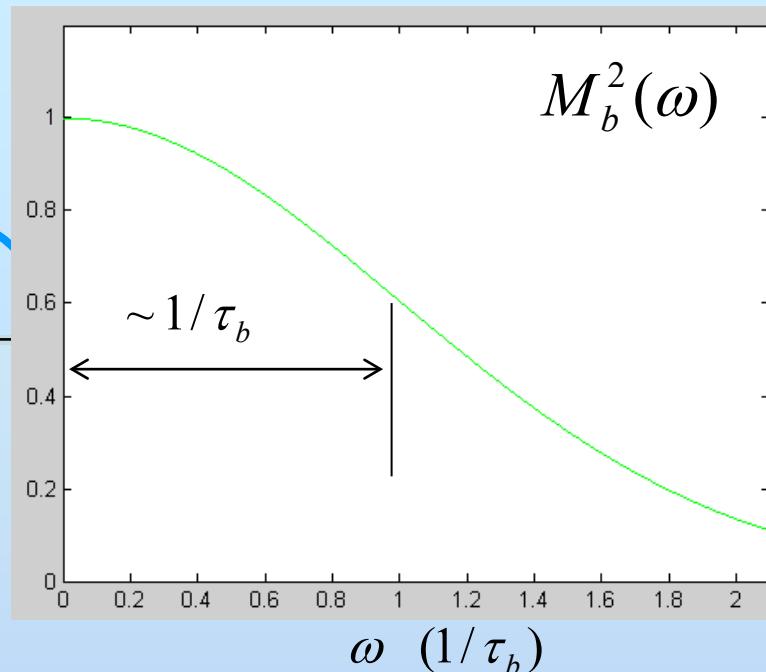
$$\left(\frac{dW}{d\omega}\right)_{SR} = \left(\frac{dW}{d\omega}\right)_1 [N_b^2 M_b^2(\omega) \times N_p^2 M_p^2(\omega)]$$

$M_b(\omega)$:Fourier transform of the bunch shape

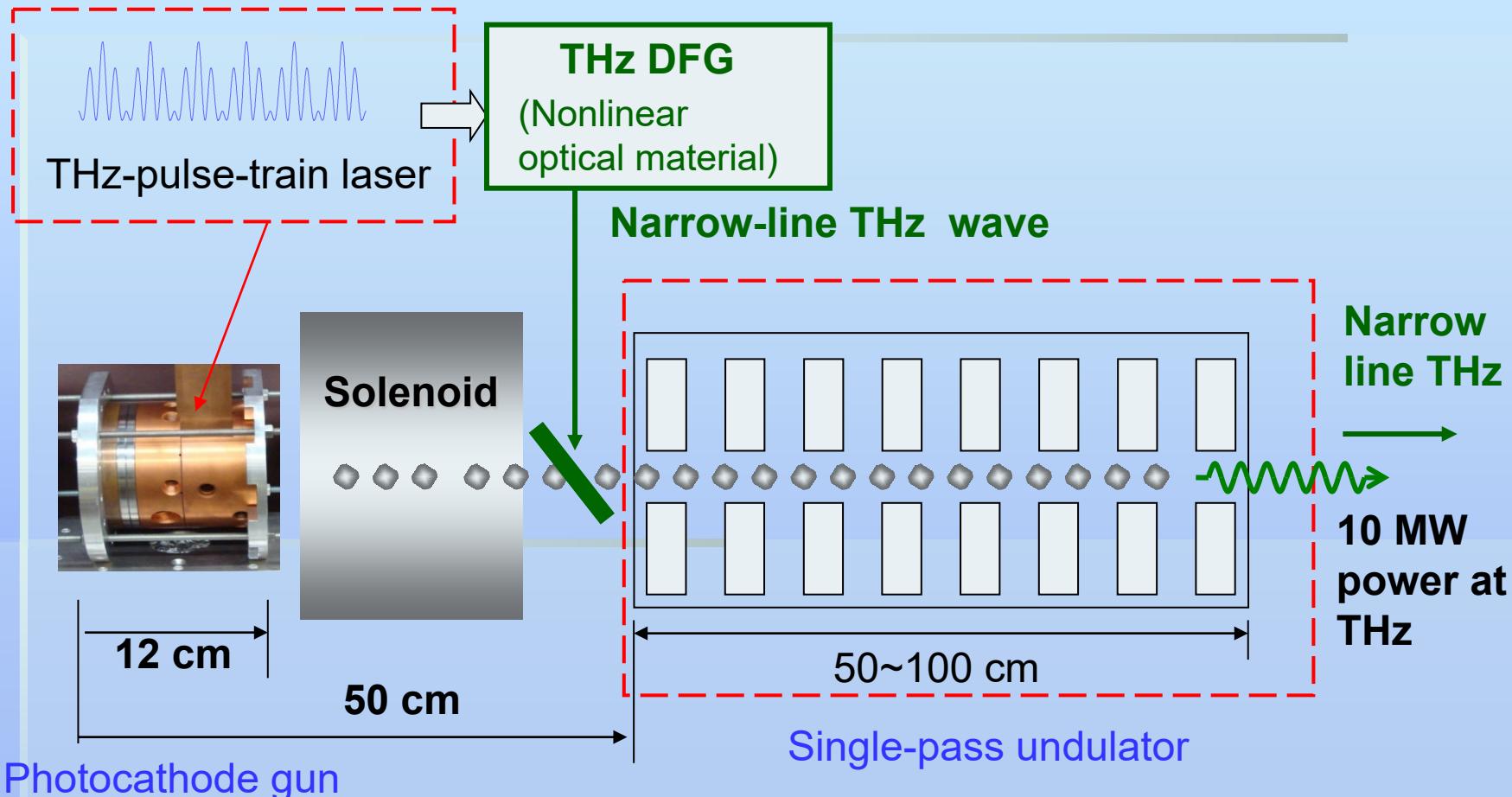
N_b : number of bunched electrons

$$M_p(\omega) = \frac{\sin(N_p \pi \omega / \omega_b)}{N_p \sin(\pi \omega / \omega_b)}$$

Coherent sum of N_p bunches with bunching freq. ω_b

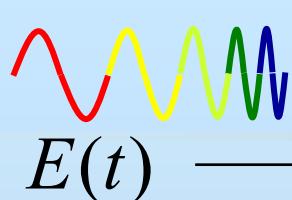


Desktop Narrow-line MW THz Free-electron Laser



Chirped Pulse FEL

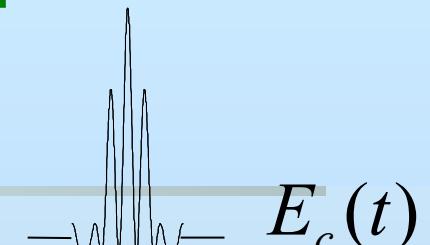
Chirped Pulse Compression → high peak power



Positive group velocity dispersion

Dispersive element
 $e^{j\phi(\omega)}$

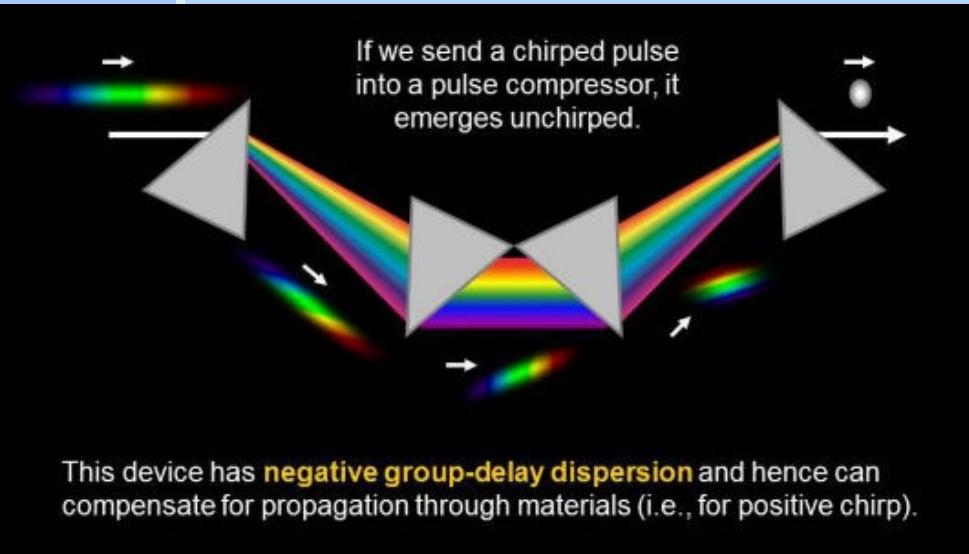
Chirped pulse



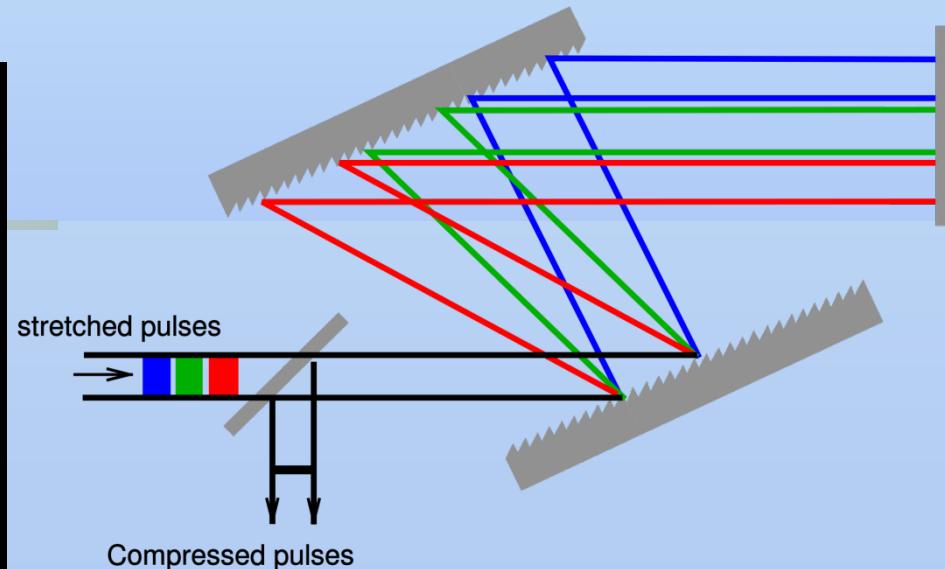
Compressed pulse

$$E(t) = \mathcal{I}^{-1}[E_c(\omega) = E(\omega)e^{j\phi(\omega)}]$$

If we send a chirped pulse into a pulse compressor, it emerges unchirped.



This device has **negative group-delay dispersion** and hence can compensate for propagation through materials (i.e., for positive chirp).



Spectral Energy of undulator radiation (fundamental mode)

$$\left(\frac{d^2W}{d\omega d\Omega} \right) = \boxed{Q^2 \times B^2(\omega)} \frac{N_u^2 \gamma^2}{2\pi} \eta \frac{a_u^2}{(1 + a_u^2 + \gamma^2 \theta^2)^2} \times JJ^2 \left\{ \frac{\sin[N_u \pi (\omega/\omega_r - 1)]}{N_u \pi (\omega/\omega_r - 1)} \right\}^2$$

where $JJ = [J_1(\frac{1}{2} \frac{a_u^2}{1 + a_u^2}) - J_o(\frac{1}{2} \frac{a_u^2}{1 + a_u^2})]$ for a planar undulator

Radiation power for $a_u \sim 1$ is $\boxed{P_Q \sim 0.6 \times Q^2 \times B^2(\omega) \eta f_r^2}$, where f_r is the radiation frequency.

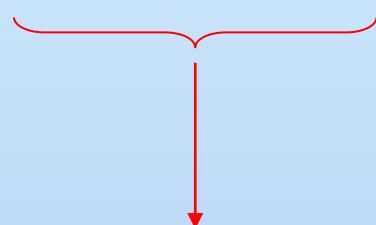
For $Q = 0.3$ nC in 0.15 ps and $B \sim 0.8$ @ $f_r = 1$ THz, radiation power $P_Q \sim 13$ MW, which is 0.15% of the beam power of a 4 MeV beam from a photoinjector.

e⁻¹ energy depletion \Rightarrow chirped radiation



The frequency chirped radiation field

$$E(t_n) = \frac{E_0}{(3r_\tau t_n + 1)^{2/3}} \exp[j2\pi \frac{N_u}{r_\tau} (3r_\tau t_n + 1)^{1/3} + j\phi_0]$$



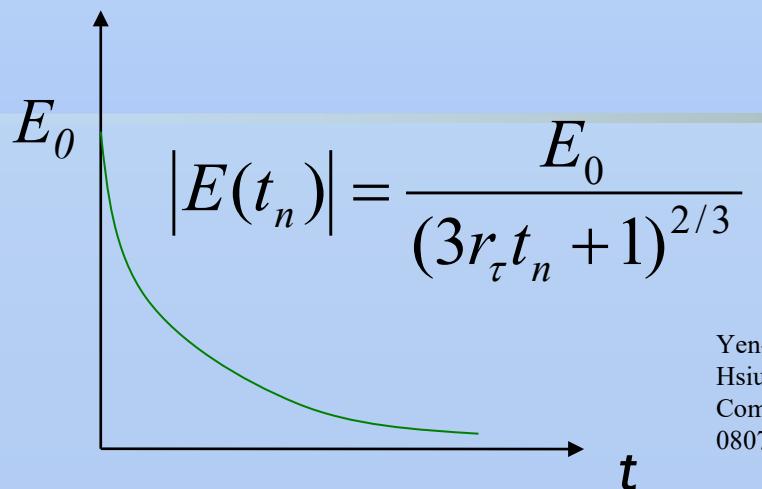
N_u : # of undulator period

$r_\tau = \frac{\text{electron transit time}}{\text{energy depletion time}}$

Amplitude Attenuation

$$r_\tau \gg 1$$

strong e^{-1} energy depletion



Yen-Chieh Huang (黃衍介), Zhen Zhang (張振), Chia-Hsiang Chen (陳家祥), Ming-Hsiung Wu (吳明雄), "Isolated Few-cycle Radiation from Chirped-pulse Compression of Superradiant Free-electron Laser," Physical Review ST AB 18, 080701 (2015). DOI: <http://dx.doi.org/10.1103/PhysRevSTAB.18.080701>

Pulse Elongation

$$E(t_n) = \frac{E_0}{(3r_\tau t_n + 1)^{2/3}} \exp[j2\pi \frac{N_u}{r_\tau} (3r_\tau t_n + 1)^{1/3} + j\phi_0]$$

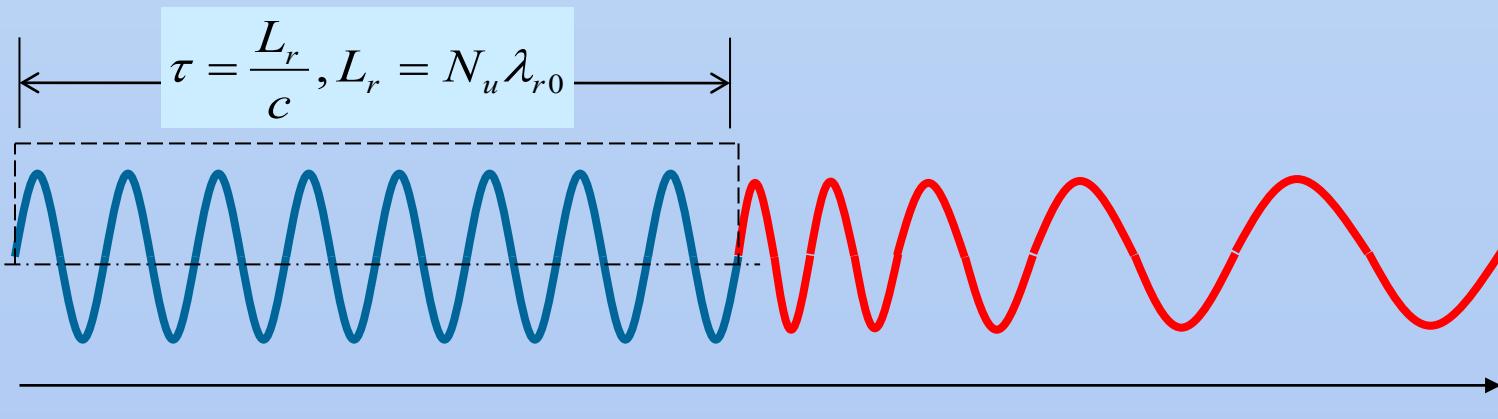
$t_n = f_{r0} \times (t - \tau_u) / N_u$ observation time/unperturbed slippage time

$$0 \leq t_n \leq [1 + r_\tau + \underbrace{\frac{1}{3} r_\tau^2}_3]$$

$$N_u \lambda_{r0}/c = N_u/f_{r0}$$

Radiation pulse increased from e^- energy depletion

w/o e^- energy depletion with e^- energy depletion



Frequency Chirp

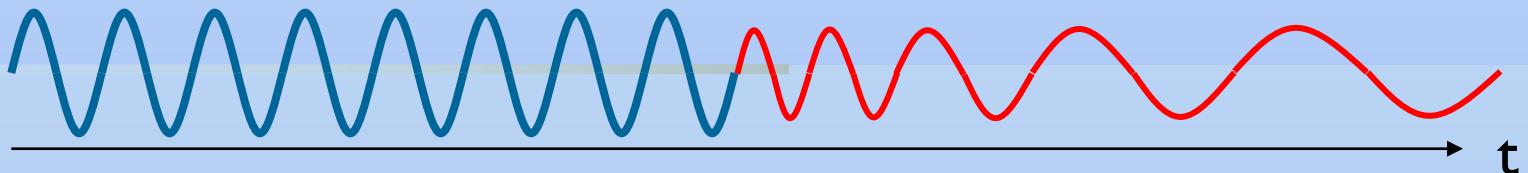
$$E(t_n) = \frac{E_0}{(3r_\tau t_n + 1)^{2/3}} \exp[j2\pi \underbrace{\frac{N_u}{r_\tau} (3r_\tau t_n + 1)^{1/3}}_{\phi(t)} + j\phi_0]$$

Radiation phase: defining freq. chirp

(1) Initially ($r_\tau t_n \ll 1$)

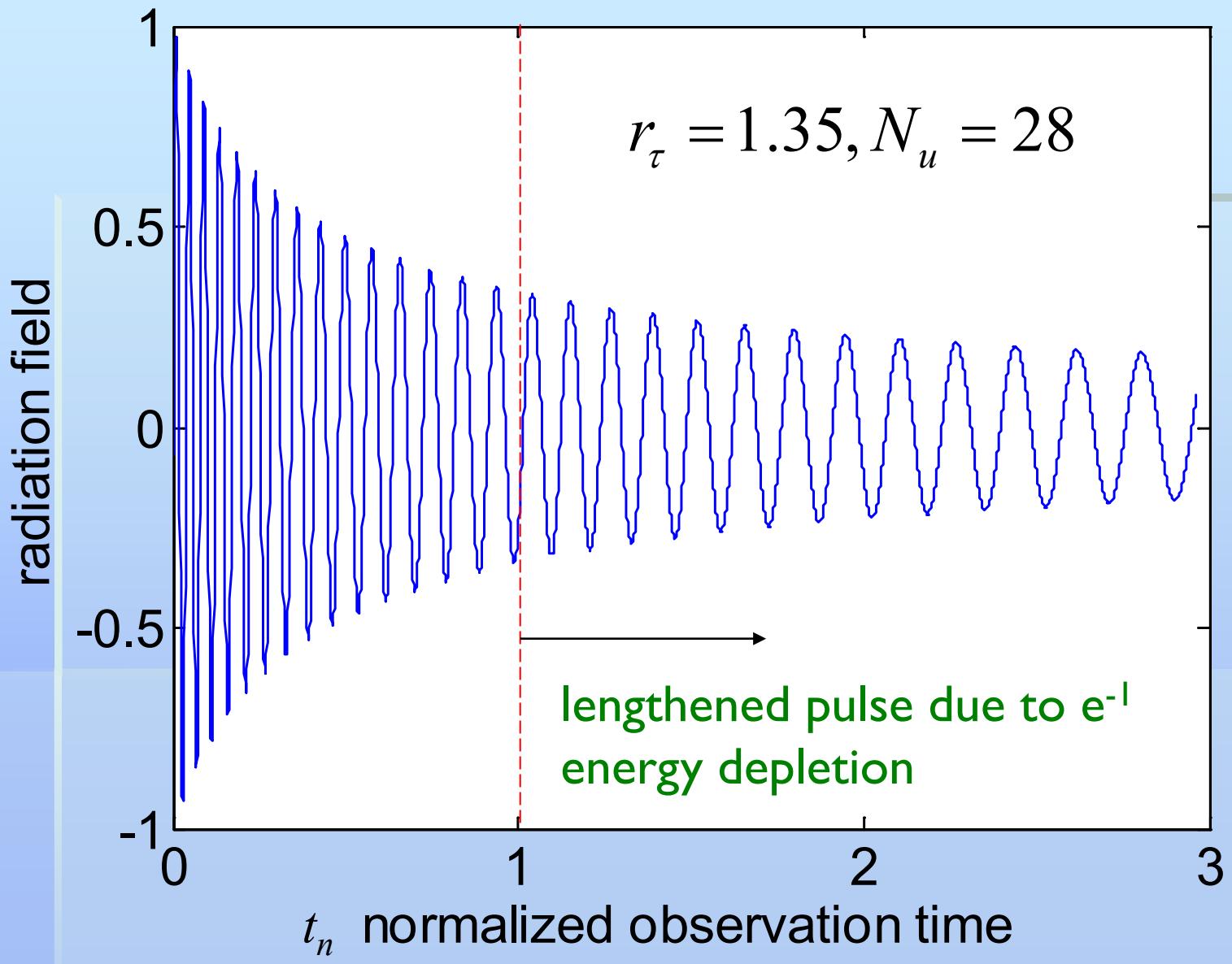
$$\phi(t_n) = 2\pi \frac{N_u}{r_\tau} (3r_\tau t_n + 1)^{1/3} \sim 2\pi N_u t_n = 2\pi N_u \frac{t}{N_u \lambda_{r0}/c} = \omega_{r0} t$$

Radiation frequency is that w/o e^- energy depletion



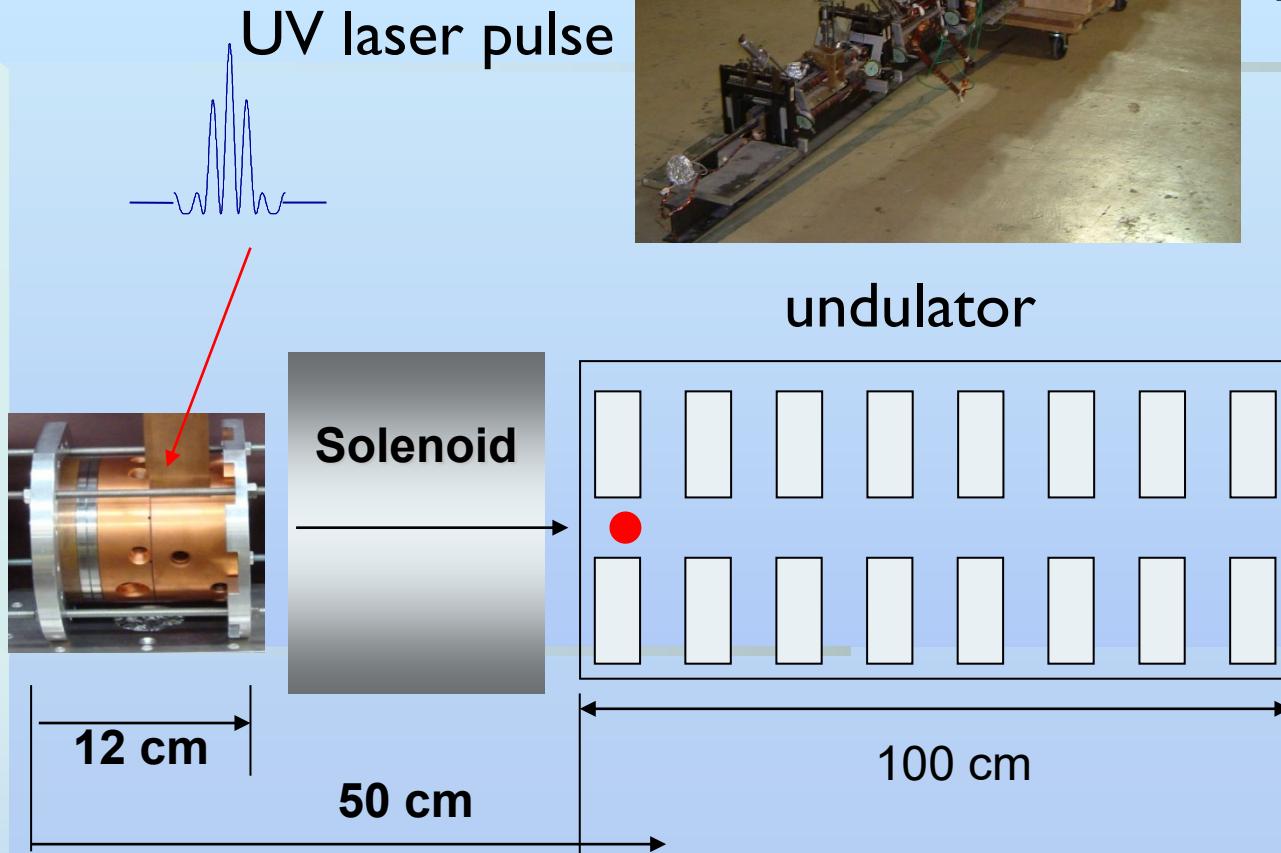
(2) For $r_\tau t_n \gg 1$,

Instantaneous frequency $f_r(t) = \frac{1}{2\pi} \frac{d\phi(t_n)}{dt} \sim \frac{N_u}{(3r_\tau t_n)^{2/3}} \xrightarrow{t_n \rightarrow \infty} 0$



Design Example

TRW undulator



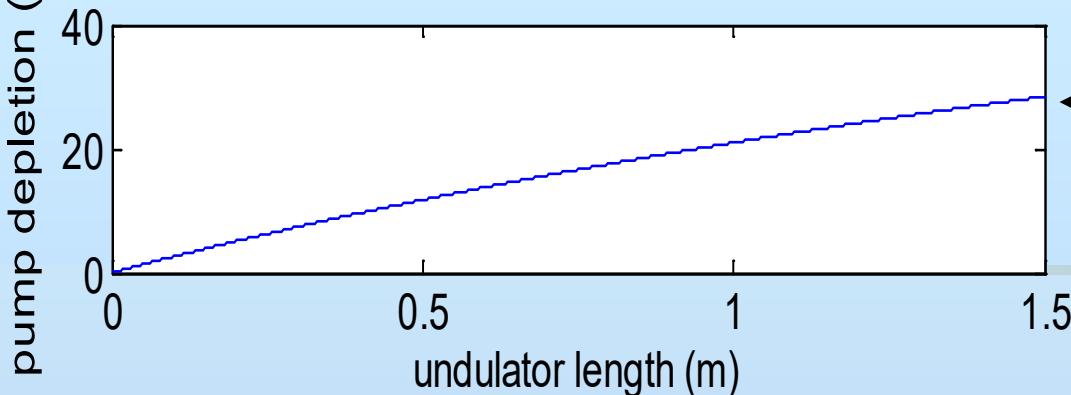
$$\lambda_u = 3.56 \text{ cm}$$

$$a_u = 0.68$$

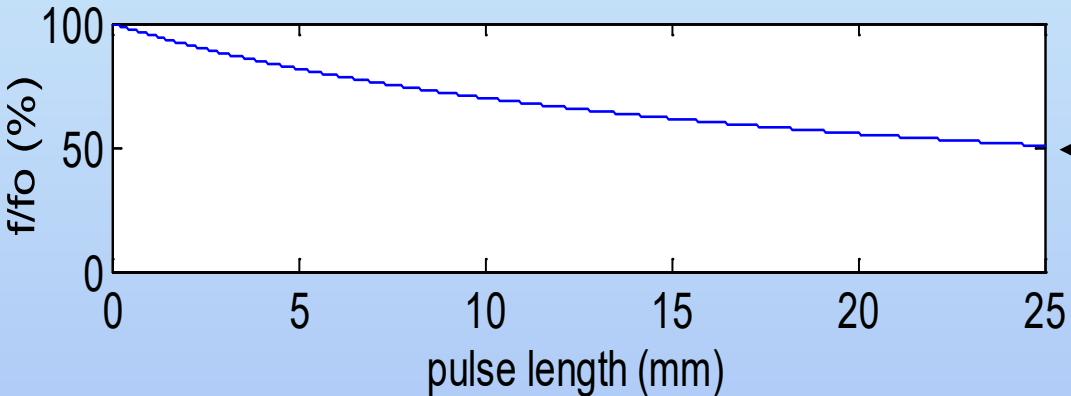
4 MeV Photocathode gun

*Photocathode gun – courtesy of CX Tang of Beijing Tsinghua U and W. K. Lau of NSRRC

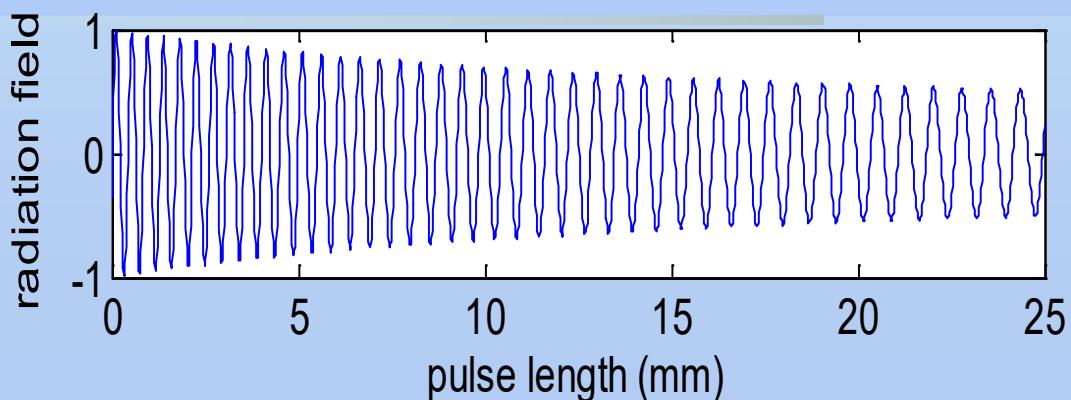
Bunch Charge = 200 pC, initial f_{r0} = 1.1 THz



30% pump depletion



50% frequency chirp



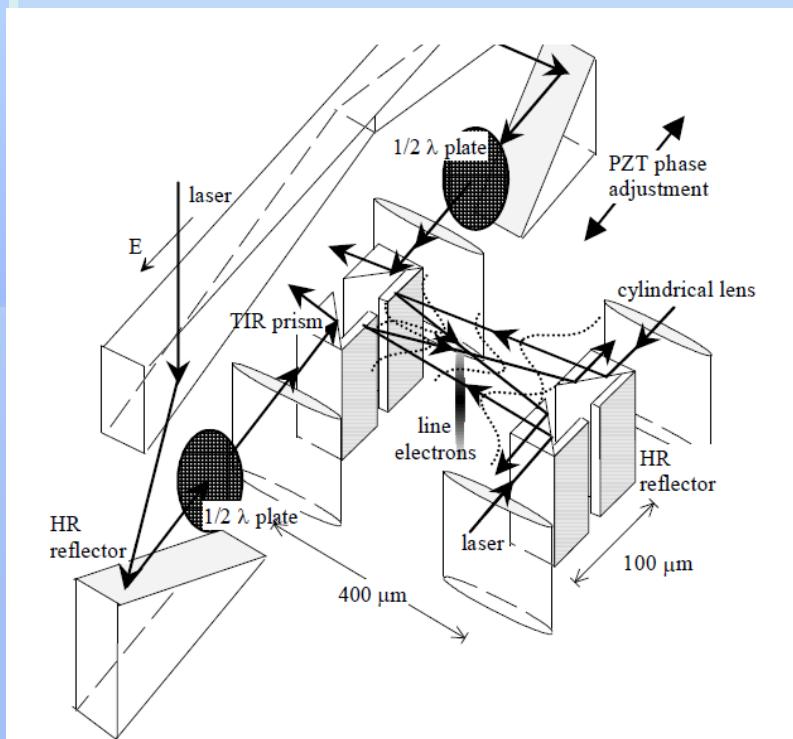
Get your imagination wild....

Dielectric laser accelerator (DLA)

Dielectric Laser Accelerator

1. Solid state → stable
2. Dielectric damage field and thus high acceleration gradient (up to 1-10 GeV/m)
3. Fabrication compatible to semiconductor lithographic patterning technique

$$E_{\parallel} \approx \frac{-j}{k} \nabla_{\perp} E_{\perp} \sim 1 \text{ GV/m}$$



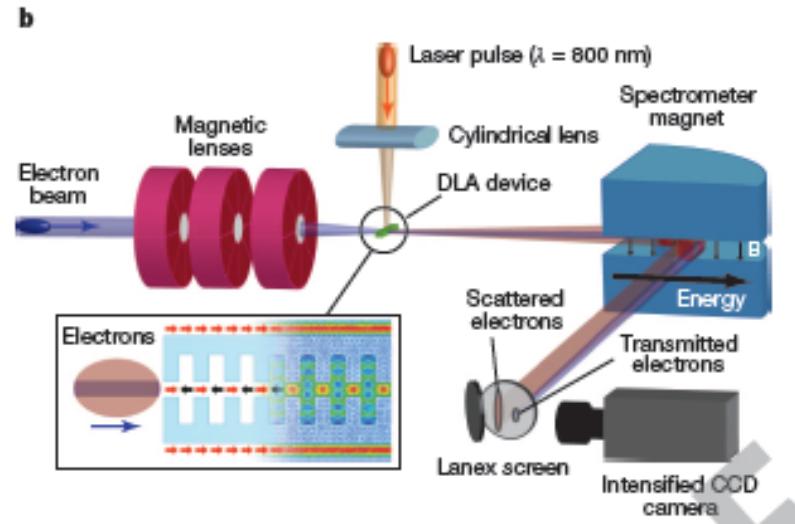
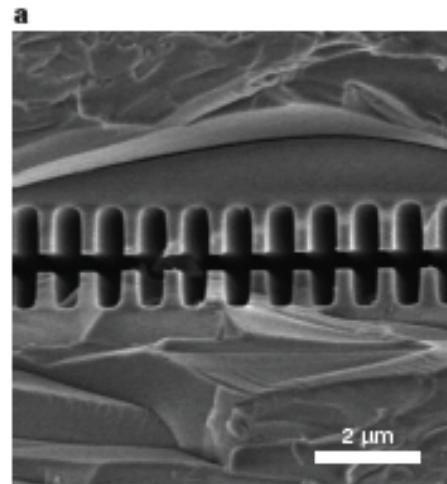
- Y.C. Huang, R.L. Byer, D. Zheng, and W. Tulloch, "A Proposed Structure for GeV per Meter Crossed-laser-beam Electron Linear Accelerator," *Appl. Phys. Lett.* **5**, (1996) 10.
- Y.C. Huang and R.L. Byer, "A Proposed High-gradient, Laser-driven Linear Acceleration using Cylindrical Laser Focusing," *Appl. Phys. Lett.* **69** (15) (1996).

Demonstration of electron acceleration in a laser-driven dielectric microstructure

E. A. Peralta¹, K. Soong¹, R. J. England², E. R. Colby², Z. Wu², B. Montazeri³, C. McGuinness¹, J. McNeur⁴, K. J. Leedle³, D. Walz², E. B. Sozer⁴, B. Cowan⁵, B. Schwartz⁵, G. Travish⁴ & R. L. Byer¹

RESEARCH LETTER

300 MeV/m



Dielectric Laser Undulator

$(\lambda_u \gg \lambda_{\text{laser}}$ to operate with large γ)

$$\lambda_u = \frac{\lambda}{|1/\beta_e - 1/\beta_p|}$$

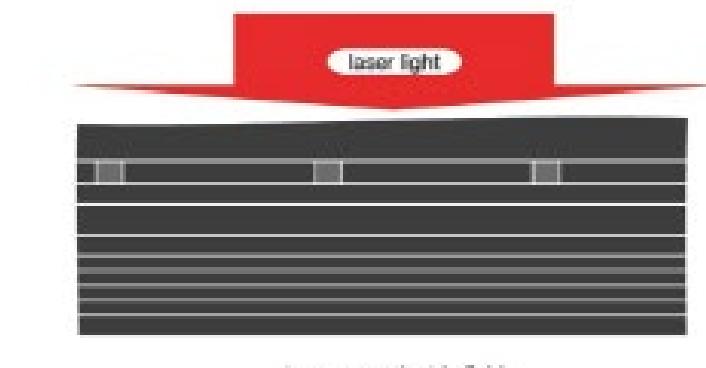
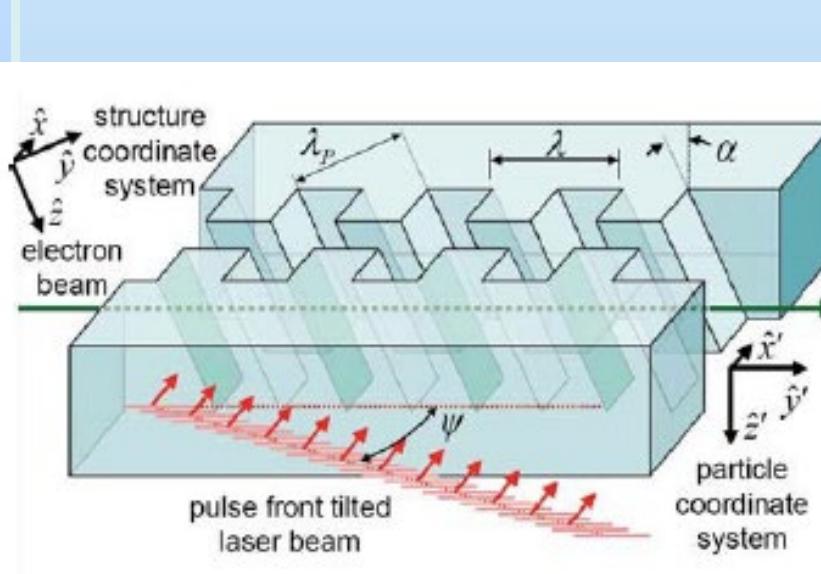
$\beta_e = v_e / c$: Electron velocity

$\beta_p = v_p / c$: Laser phase velocity

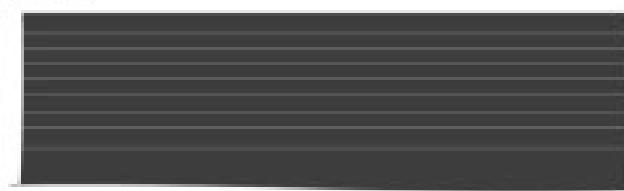
$$\left(\lambda_r = \lambda_u \frac{1 + a_u^2}{2\gamma^2} \right)$$

$$\lambda_u = 1 \sim 10^3 \mu\text{m}, B_u \sim \frac{E_{\text{laser}}}{c} \sim 3.3 \text{ T}$$

for $E_{\text{laser}} \sim 1 \text{ GV/m}$

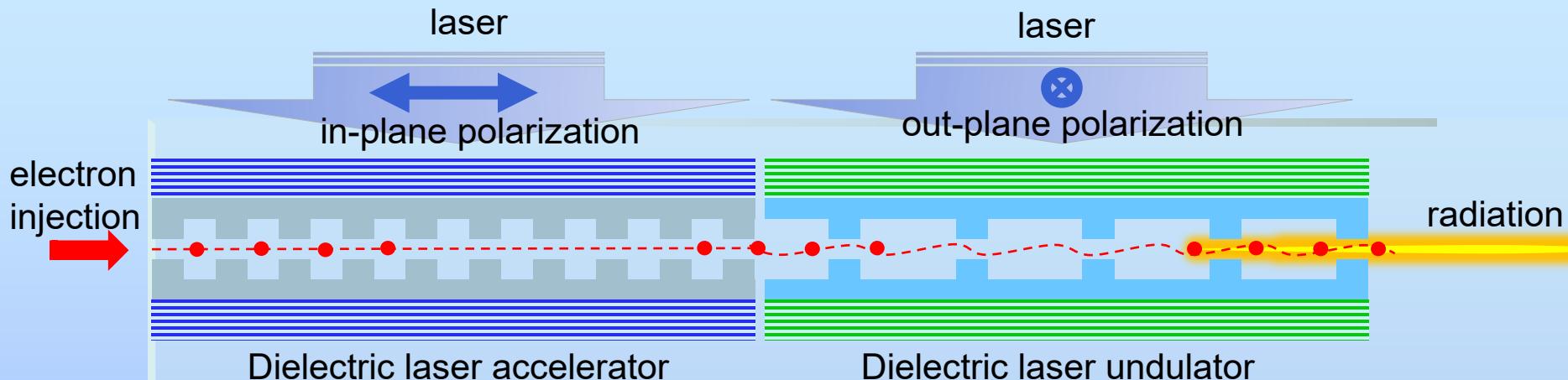


T. Plettner, R. L. Byer, Phys. Rev. ST Accel. Beams **11**, 030704 (2008).

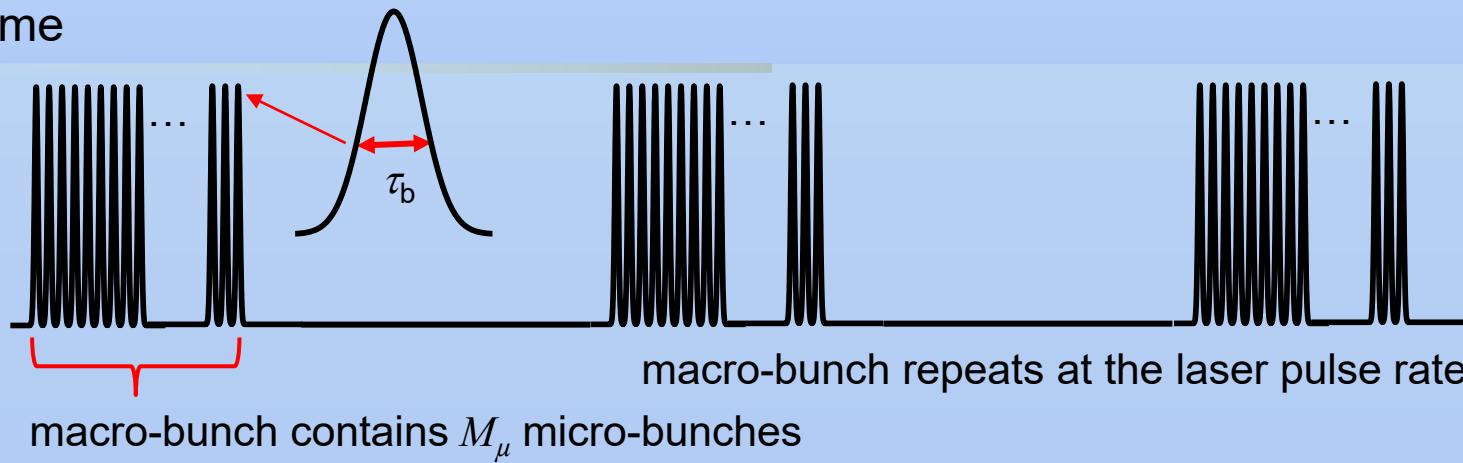


G. Travish and R. B. Yoder, Proceedings SPIE 8079, 23 (2011).

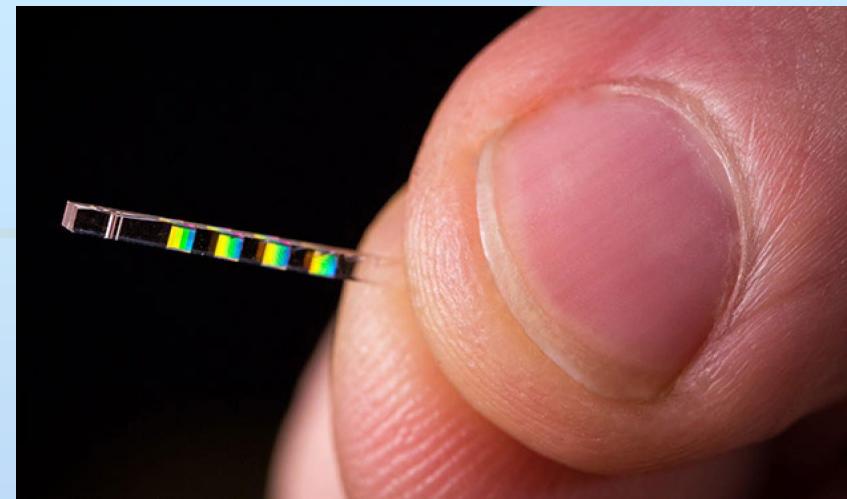
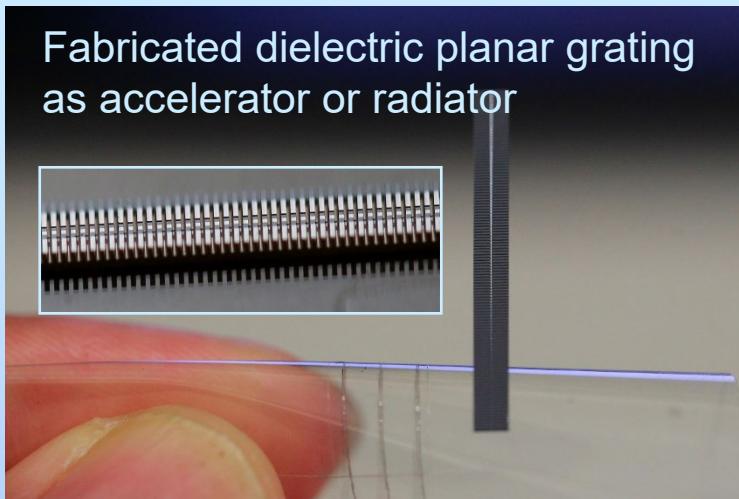
Laser-driven Dielectric Accelerator and Undulator



rms micro-bunch width = $\tau_b \sim$ nanometer → excellent
for coherent undulator radiation (CUR) in the soft-x ray
regime



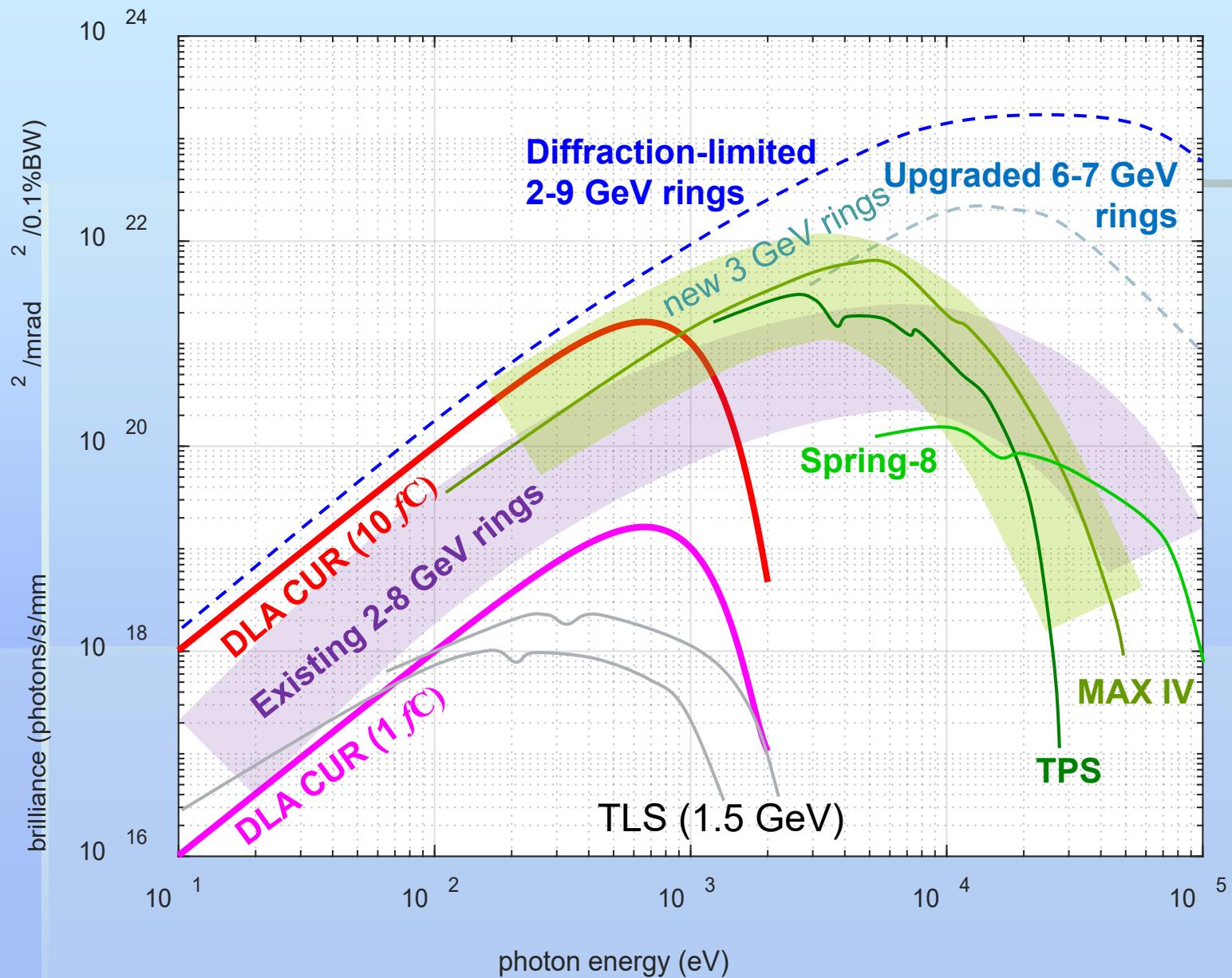
How is DLA-driven CUR compared with monster synchrotron?



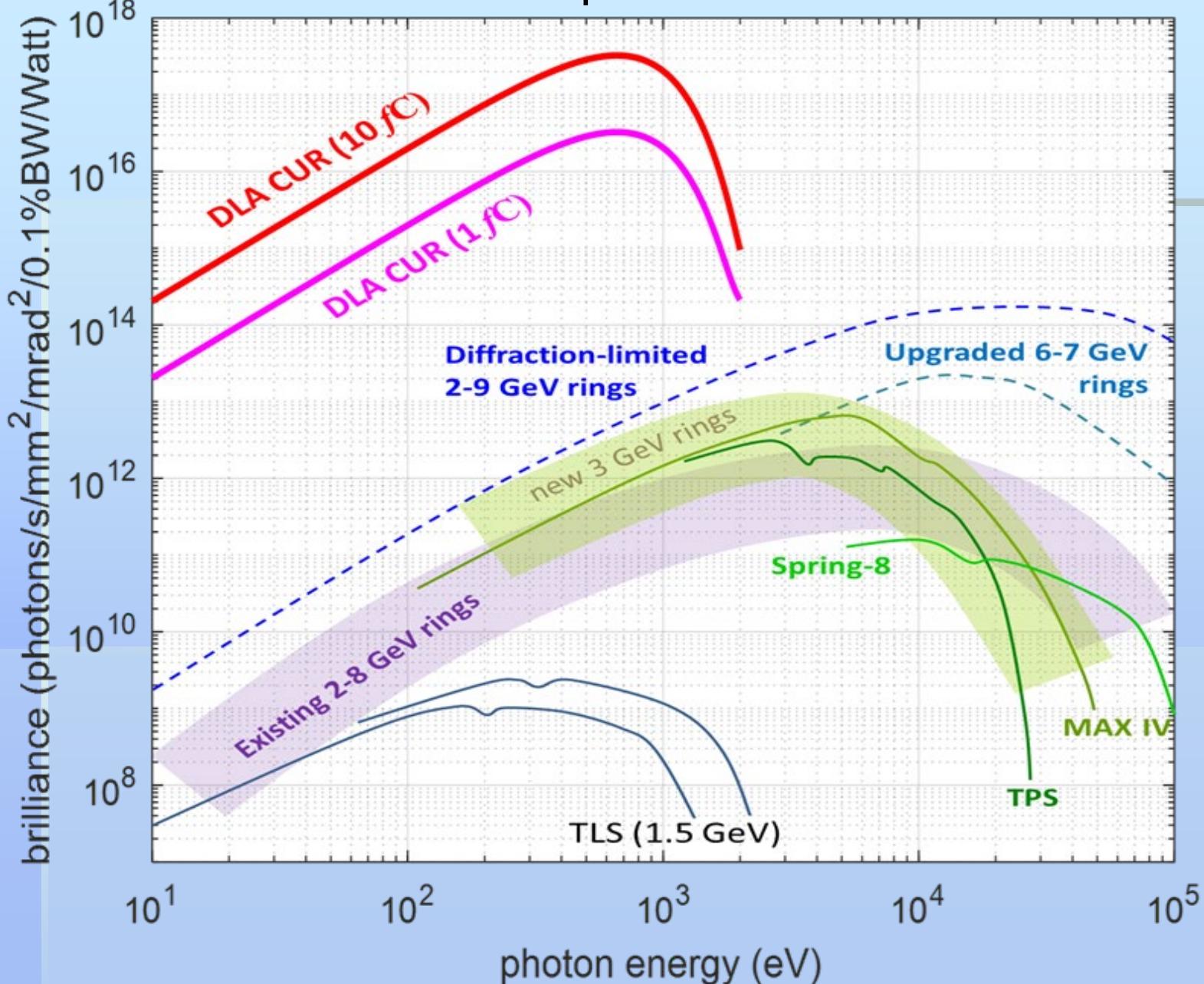
Design Parameters for DLA-driven CUR

item	unit	quantity	remark
Driving laser wavelength	μm	1	100,000 th of the 10-cm S-band RF wavelength
Bunch length	atto-sec/nm	1/0.3 nm	100,000 th of the ~100-fs RF bunch (scaled with wavelength)
Bunch Charge	fC	1, 10	6,250 and 62,500 electrons/bunch
Bunch rate	GHz	1	100 optical cycles in a ~300-fs pulse repeating at 10 MHz
Beam energy	GeV	~0.5	~ half meter long DLA with 1 GV/m gradient
Undulator period	mm	1	to radiate at $\lambda \sim 1$ nm from ~ 0.5 GeV beam
Undulator parameter	NA	0.22	3.3-T peak undulator field under laser damage to dielectric undulator
Number of undulator periods	NA	1000	radiation bandwidth ~0.1%

Brilliance



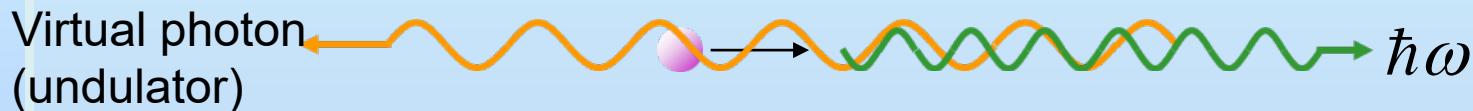
Brilliance per Beam Power



Quantum FEL

Pro: quantum noise added to startup power P_{ω} , usually small, could assist FEL buildup.

Con: (1) 1 photon from 1 electron \rightarrow low efficiency
(2) Electron recoil induced energy spread \ll FEL gain bandwidth



To stay in the gain bandwidth

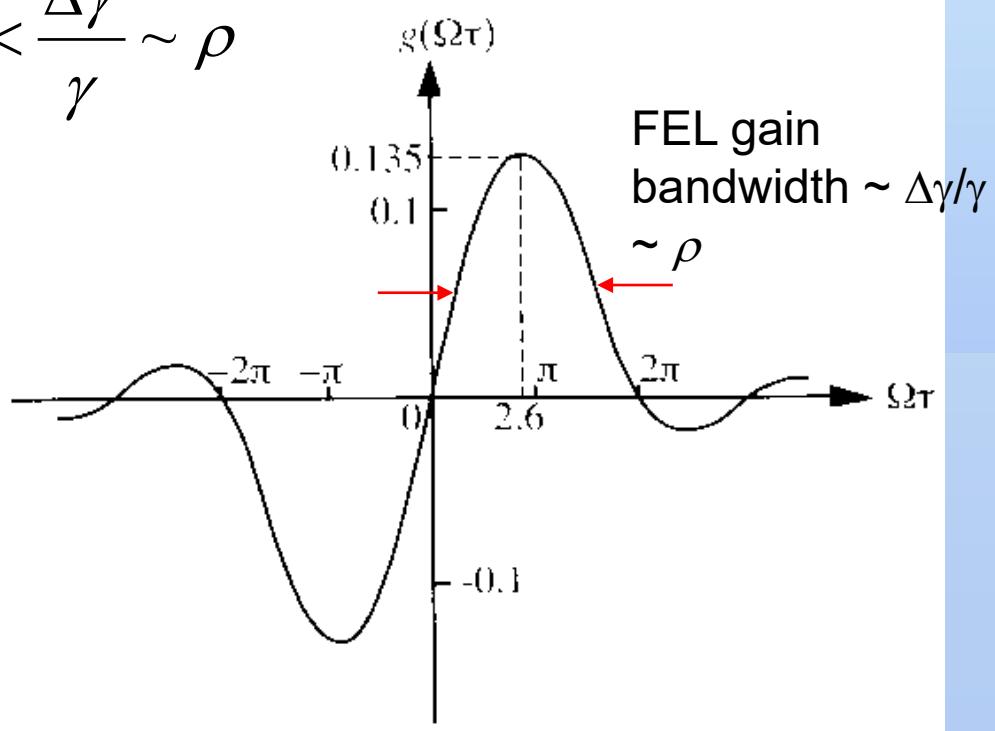
$$\frac{\hbar\omega}{\gamma mc^2} \ll \frac{\Delta\gamma}{\gamma} \sim \rho$$

Define quantum ρ parameter

$$\bar{\rho} = \rho(\gamma mc^2 / \hbar\omega)$$

Classic regime $\bar{\rho} \gg 1$
(γ large enough)

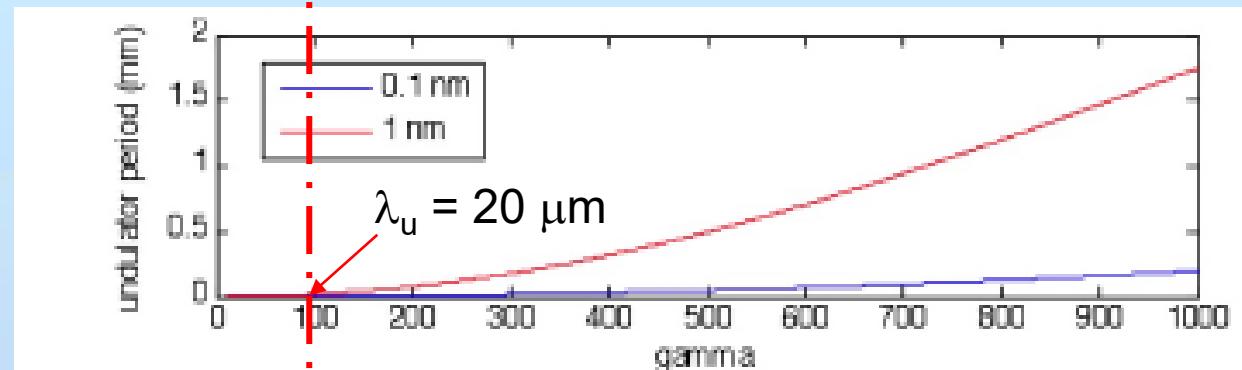
Quantum regime $\bar{\rho} \sim < 1$



DLA-driven soft-x-ray SASE FEL (laser undulator $B_u \sim 3$ T, $\lambda_r = 1$ nm)

Assume rms beam radius = 100 nm

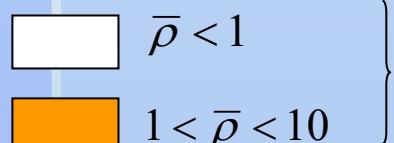
$$\lambda_r = \lambda_u \frac{1 + a_u^2}{2\gamma^2}$$



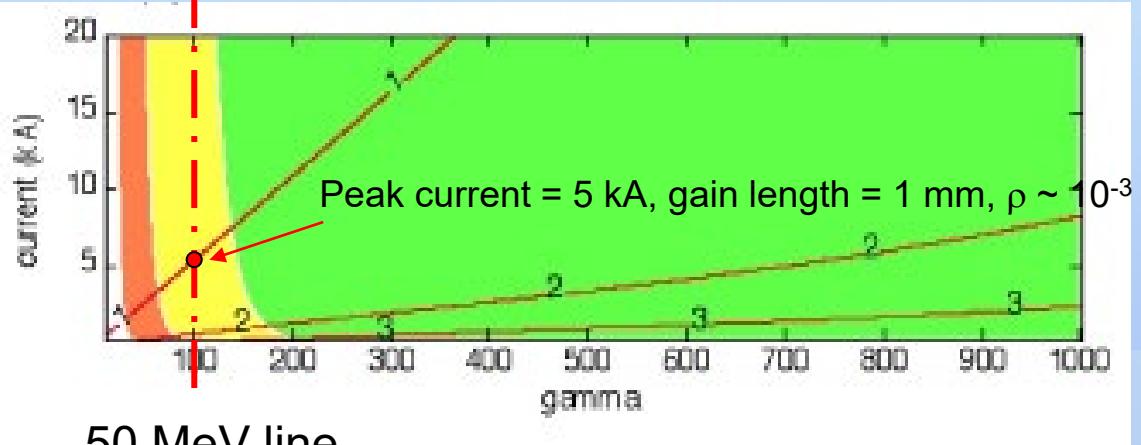
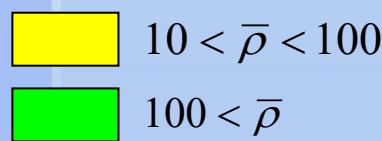
$$\bar{\rho} = \rho(\gamma mc / \hbar k)$$

$$L_g = \lambda_u / 4\pi\sqrt{3}\rho$$

Straight lines are
gain-length
contours in mm



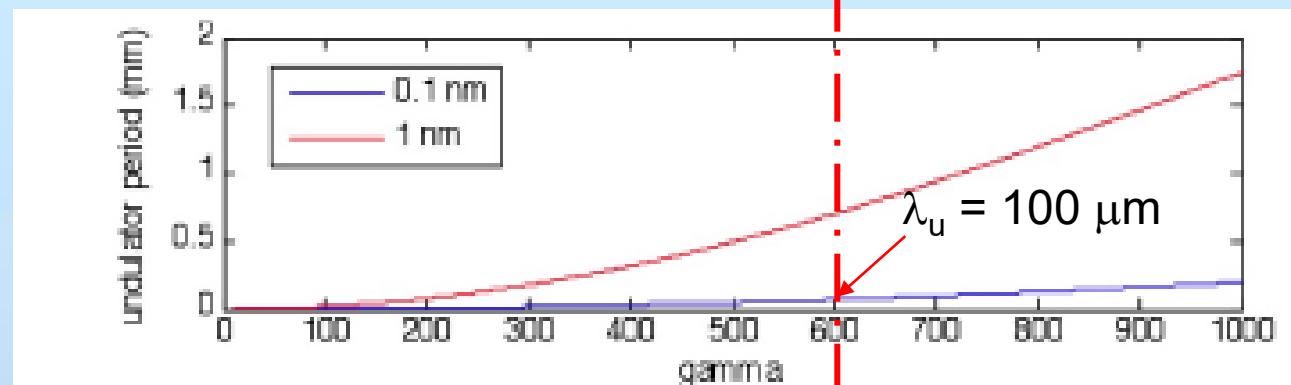
Quantum regime



DLA-driven hard-x-ray SASE FEL (laser undulator $B_u \sim 3$ T, $\lambda_r = 1$ Å)

Assume rms beam radius = 100 nm

$$\lambda_r = \lambda_u \frac{1 + a_u^2}{2\gamma^2}$$

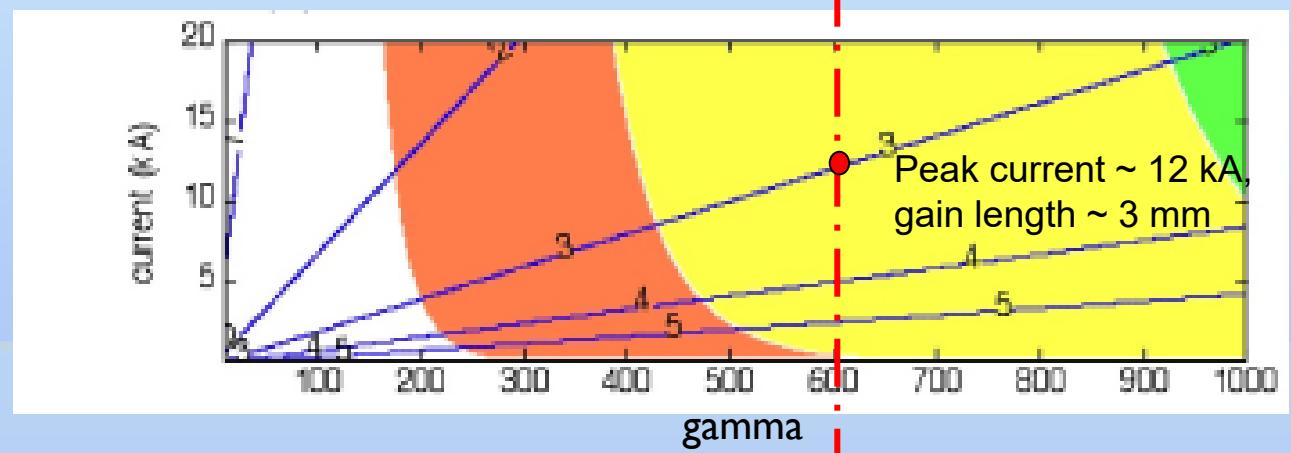


$$\bar{\rho} = \rho(\gamma mc / \hbar k)$$

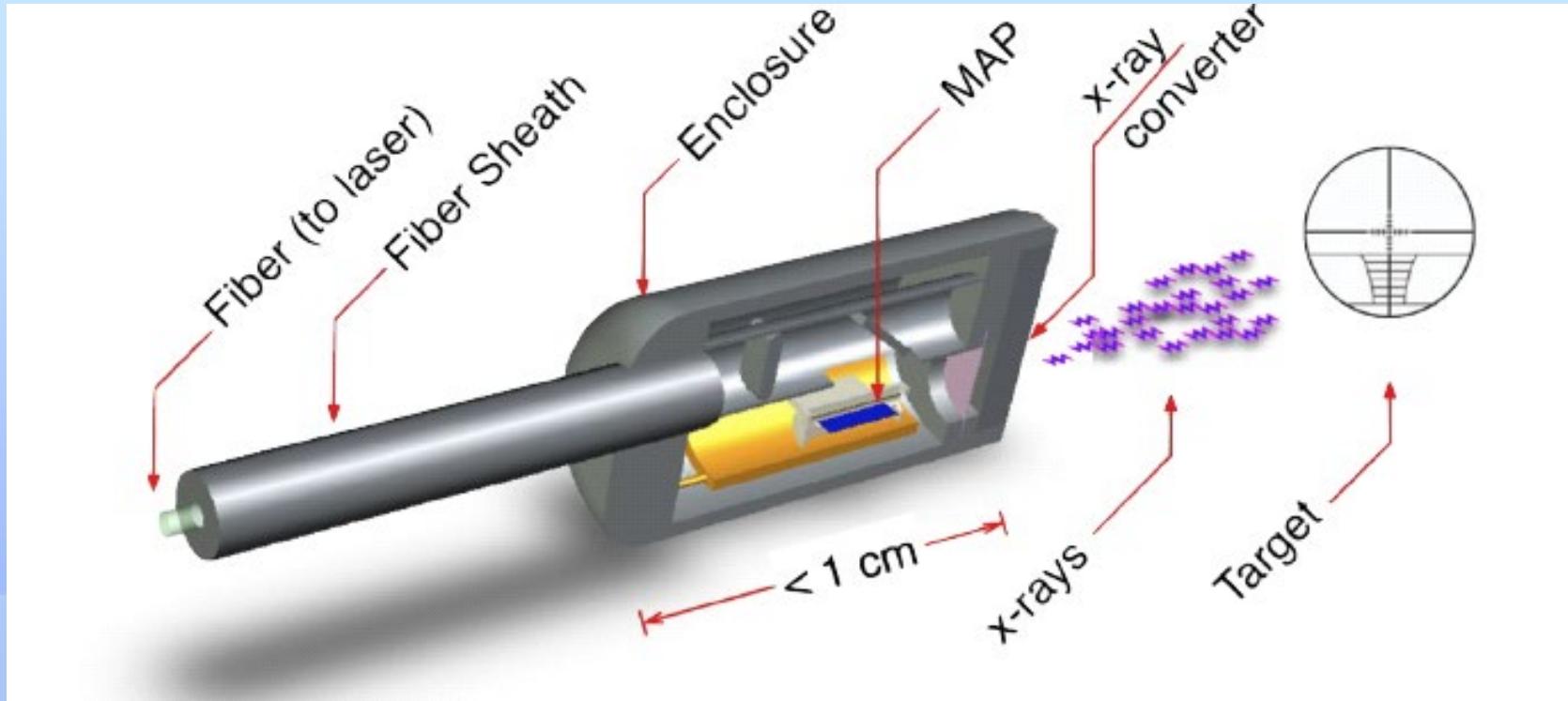
$$L_g = \lambda_u / 4\pi\sqrt{3}\rho$$

Straight lines are gain-length contours in mm

- | | |
|---|---|
| $\bar{\rho} < 1$
 $1 < \bar{\rho} < 10$
 $10 < \bar{\rho} < 100$
 $100 < \bar{\rho}$ | } |
| Quantum regime | |



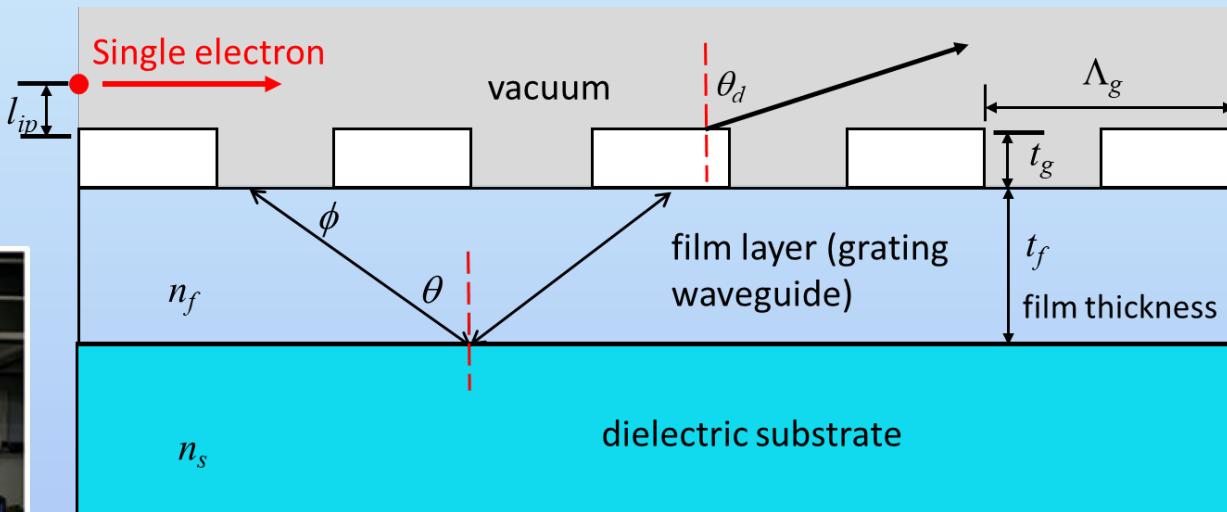
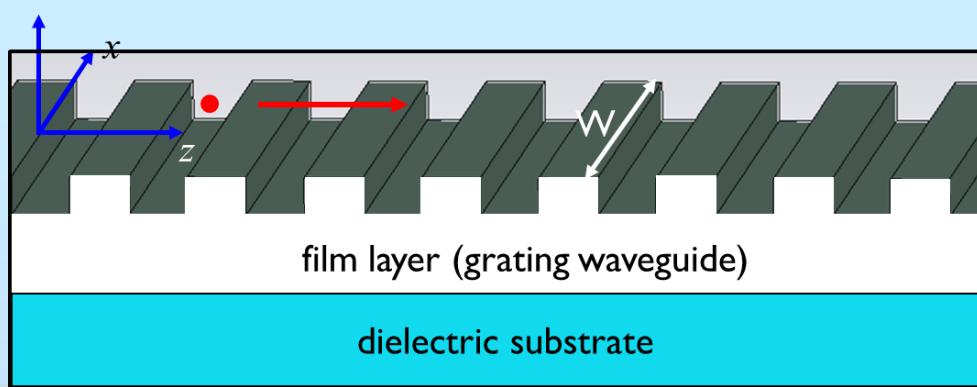
Artist's rendering of a hand-held XFEL!!



Travish, G., and R.B. Yoder, 2011, "Laser-powered dielectric-structures for the production of high-brightness electron and x-ray beams", in *Laser Acceleration of Electrons, Protons, and Ions; and Medical Applications of Laser-Generated Secondary Sources of Radiation and Particles*, Prague, Czech Republic, edited by K. W. D. Ledingham *et al*, (SPIE, Bellingham, WA, 2011), Vol. 8079 of *Proceedings of SPIE*, p. 80790K

Photonic-chip FEL

Yen-Chieh Huang, Luo-Hao Peng, Hossein Shirvani, Wen-Chi Chen, Karthickraj Muthuramalingam, Wei-Chih Wang, and Andrzej Szczepkowicz, "Single-electron Nano-chip Free-electron Laser," APL Photonics 7, 096101 (2022). (editor featured article and cover story of the journal).

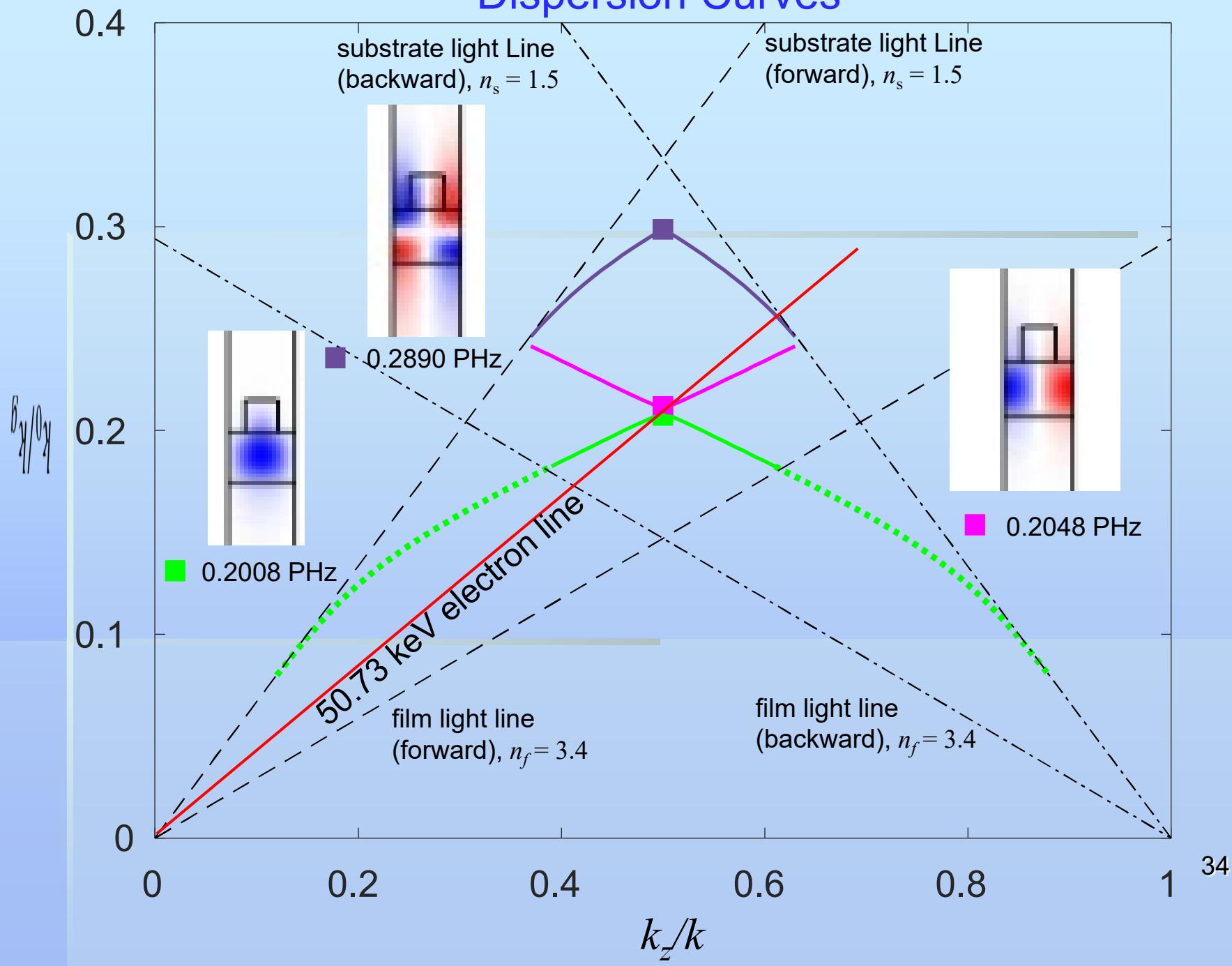


120 keV TEM in
HOPE Lab/NTHU

TABLE I. The first-order design parameters for a $1.5\text{-}\mu\text{m}$ nano-chip FEL with a silicon ($n_f = 3.4$) grating waveguide on a glass substrate ($n_s = 1.5$).

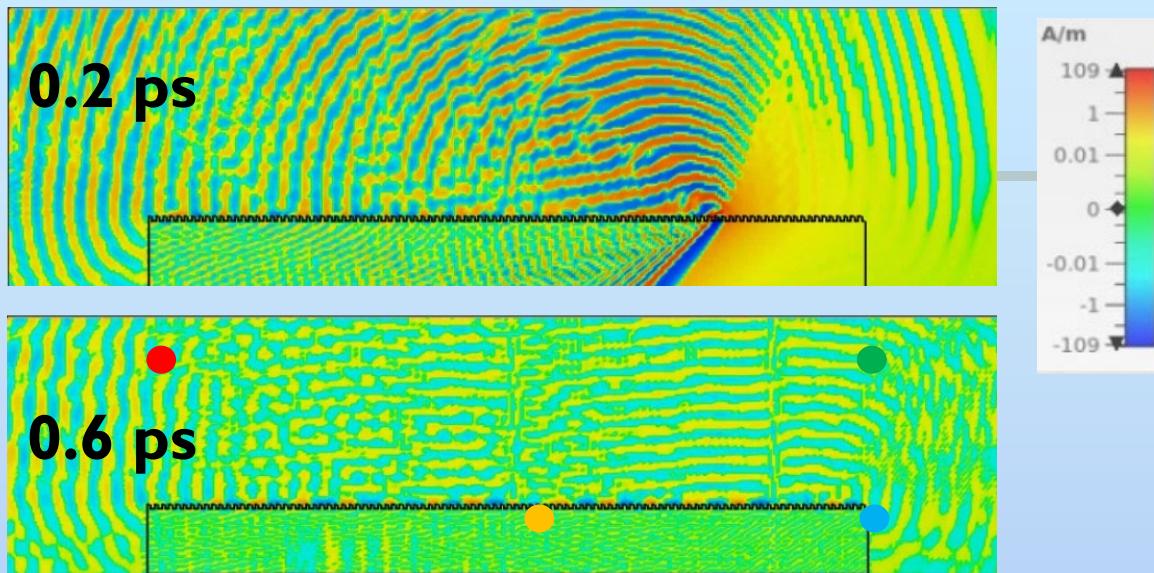
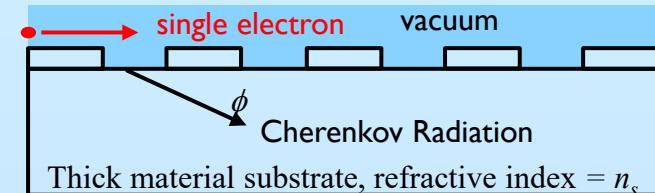
Design wavelength (μm)	Electron energy (keV)	Grating period Λ_g (nm)	Grating depth t_g (nm)	Film thickness t_f (nm)	Impact parameter l_{ip} (nm)
1.5	50	310	160	240	100

Dispersion Curves

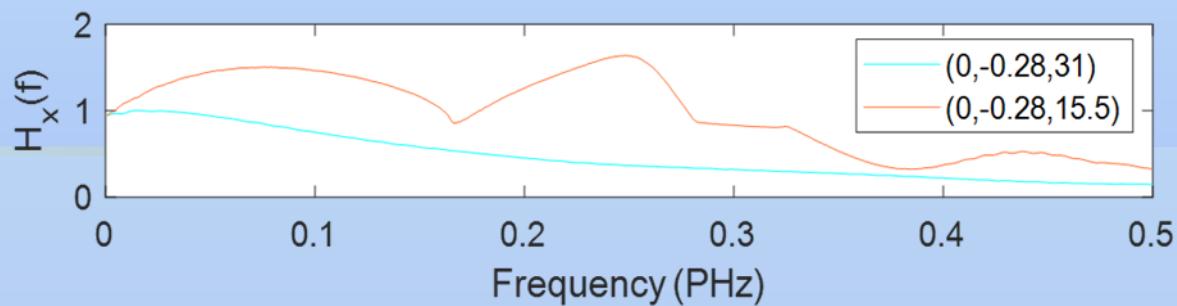


Radiation W/O Waveguide

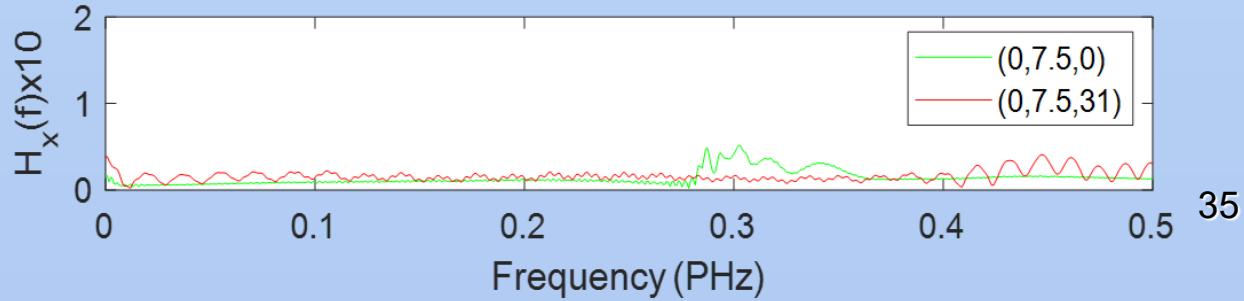
CST simulation



Internal
radiation
(broadband)

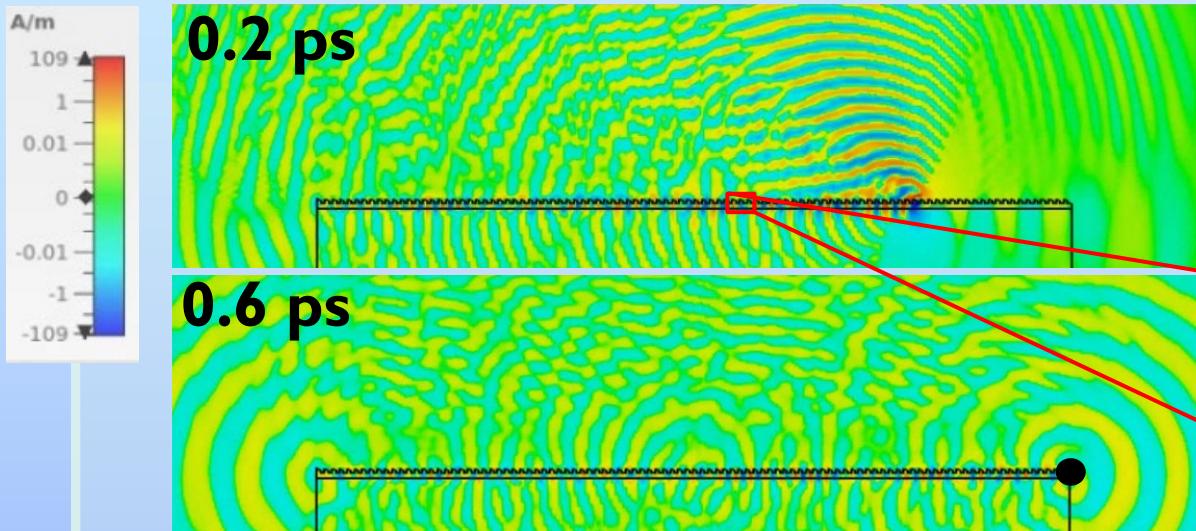
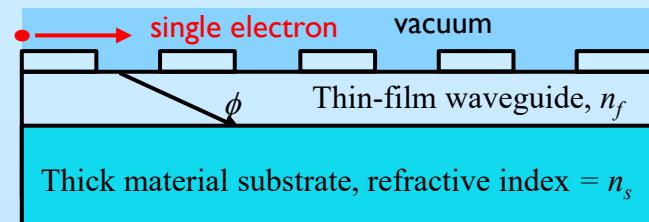


External
radiation
(weak &
broadband)

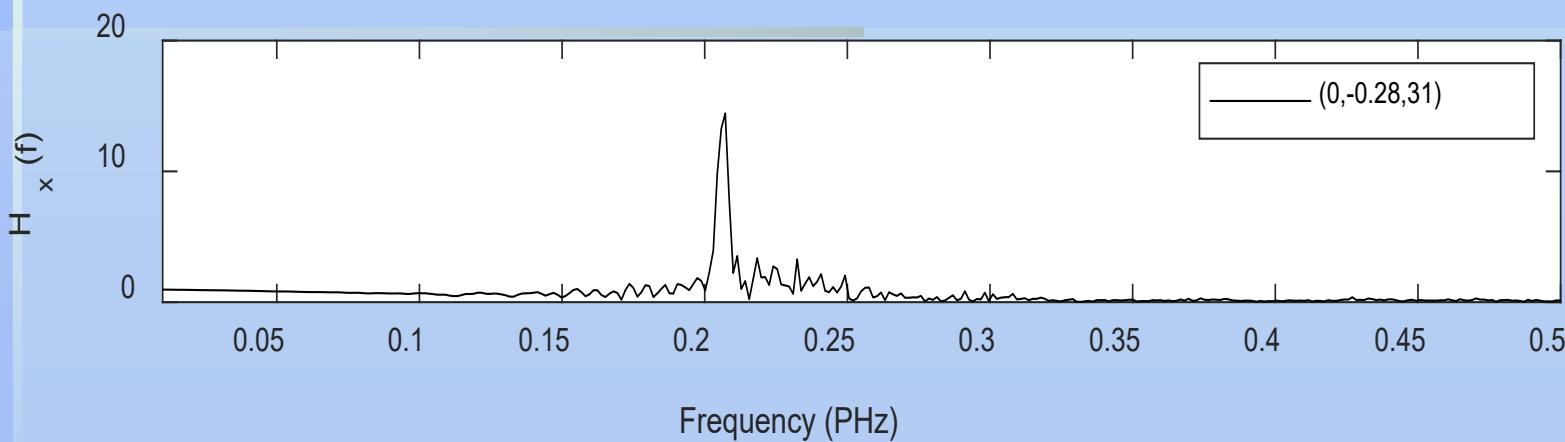


Grating-waveguide FEL driven by 1 electron – single-electron FEL

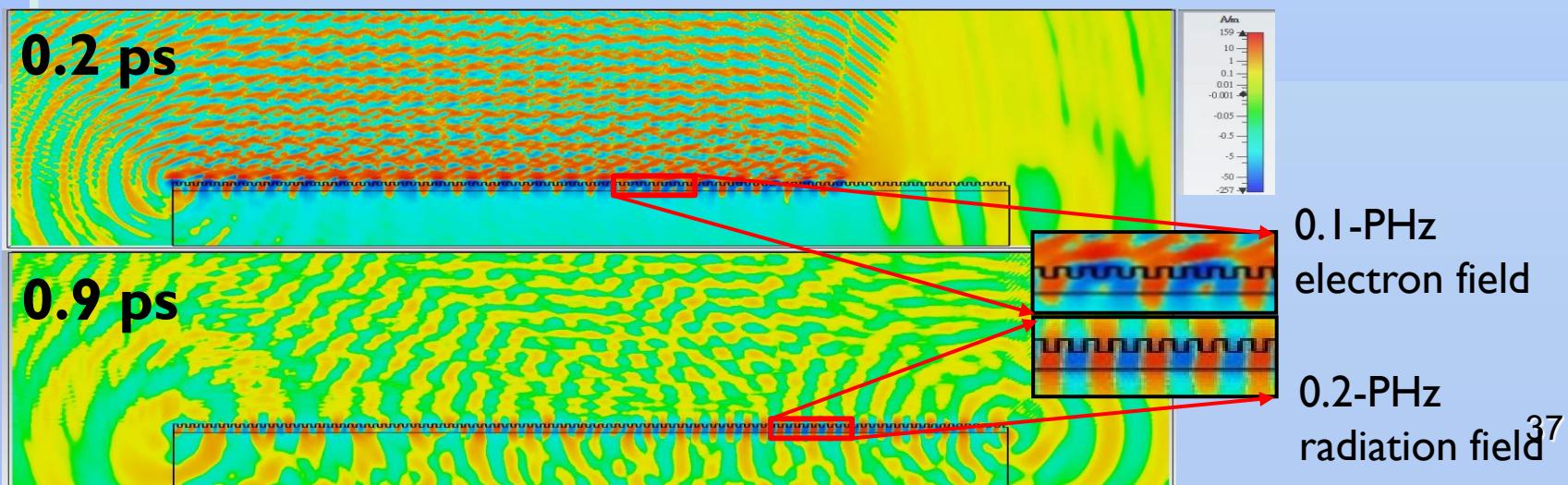
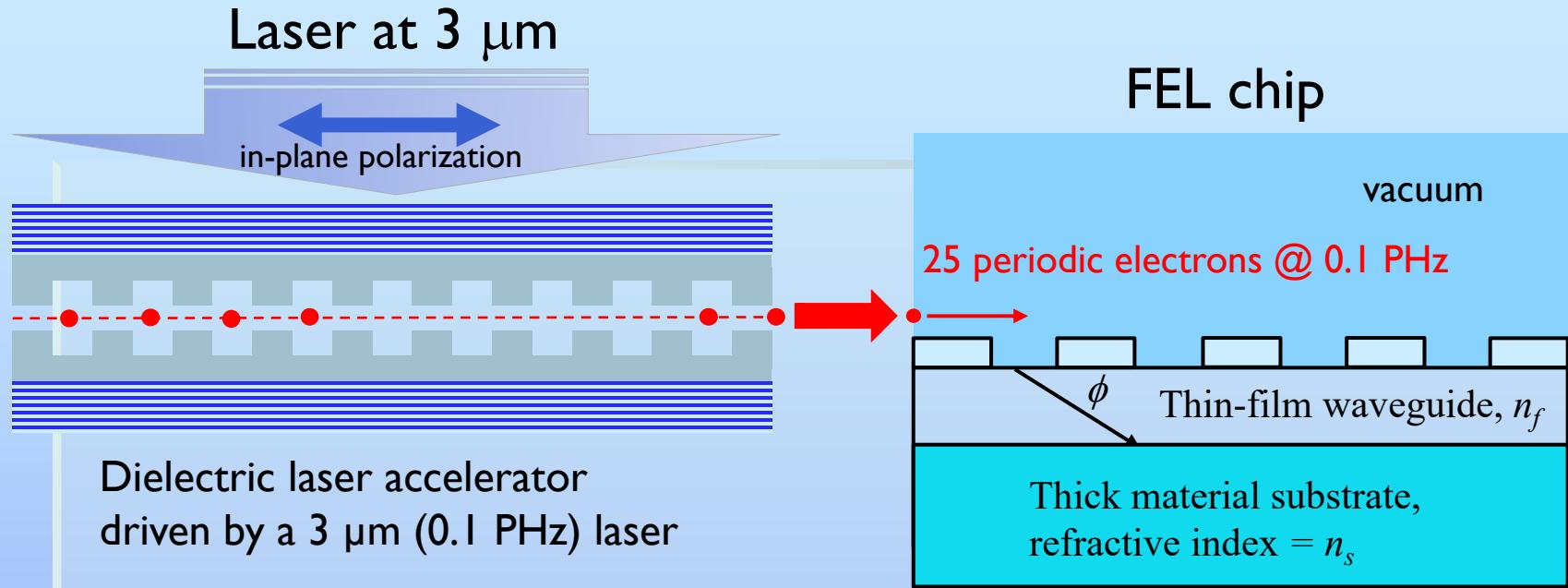
CST simulation



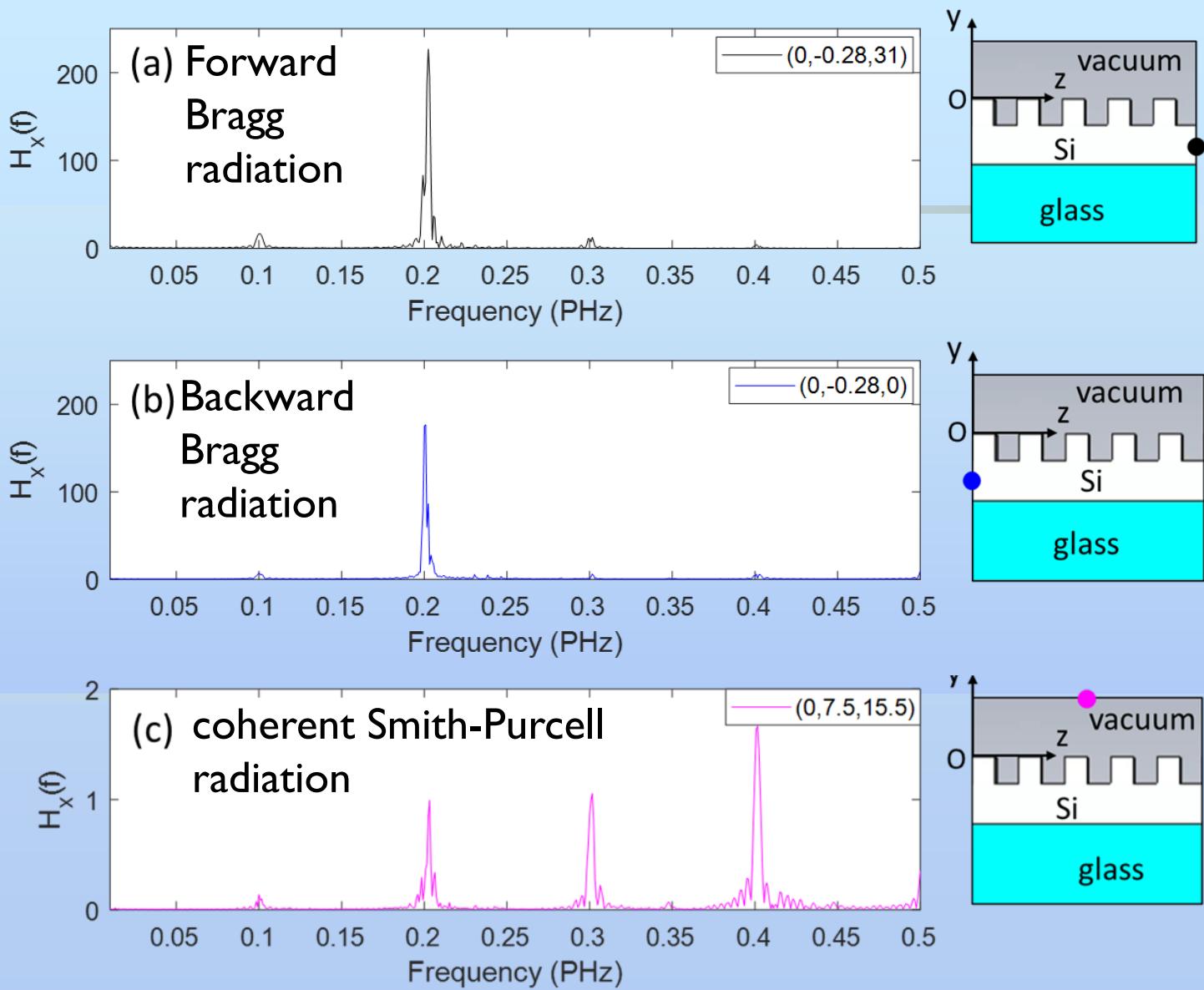
$$\Lambda_g = \frac{\lambda_z}{2}$$



Grating-waveguide FEL driven by Periodic Single Electrons

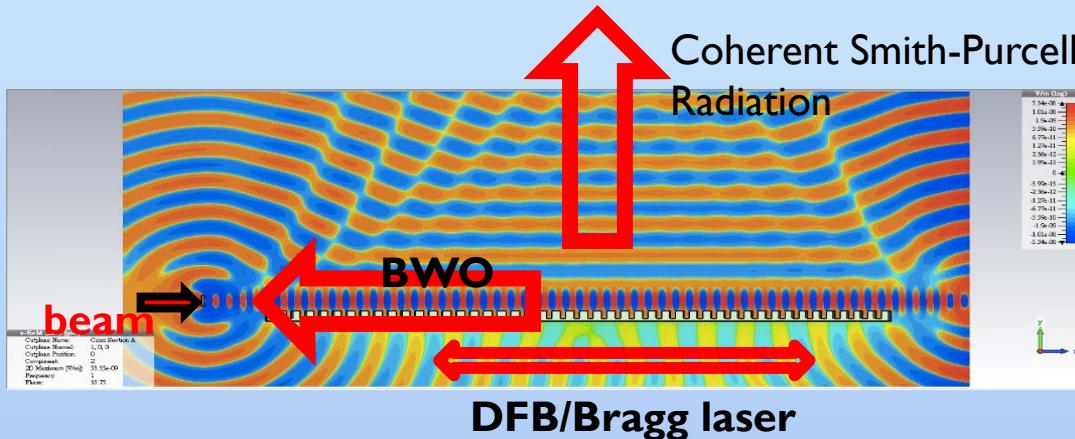


Harmonic Radiation Spectrum

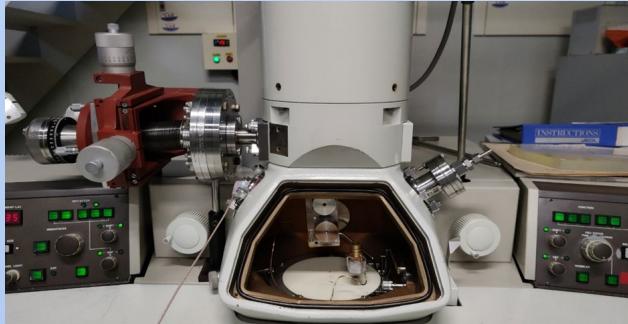


Conclusions for Photonic-chip FEL

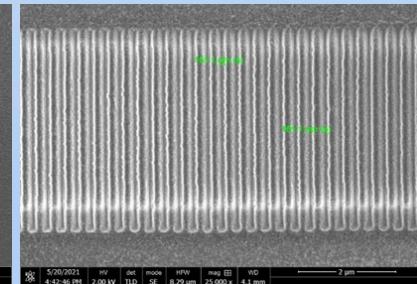
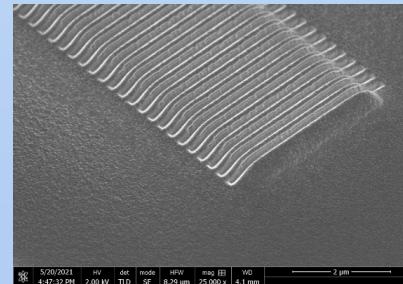
1. Dielectric laser accelerator and photonic FEL can be integrated into a chip-size structure via microfabrication techniques.
2. Single-electron FEL built upon a dielectric-grating waveguide is numerically demonstrated at 0.2 PHz and its harmonics.



3. Experimental tests are on-going by using a TEM beam.



TEM experimental chamber



Fabricated structure on Si (courtesy of Prof. Wei-Chih Wang of NTHU)