Lecture 2 - Superradiance FEL

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1. superradiance

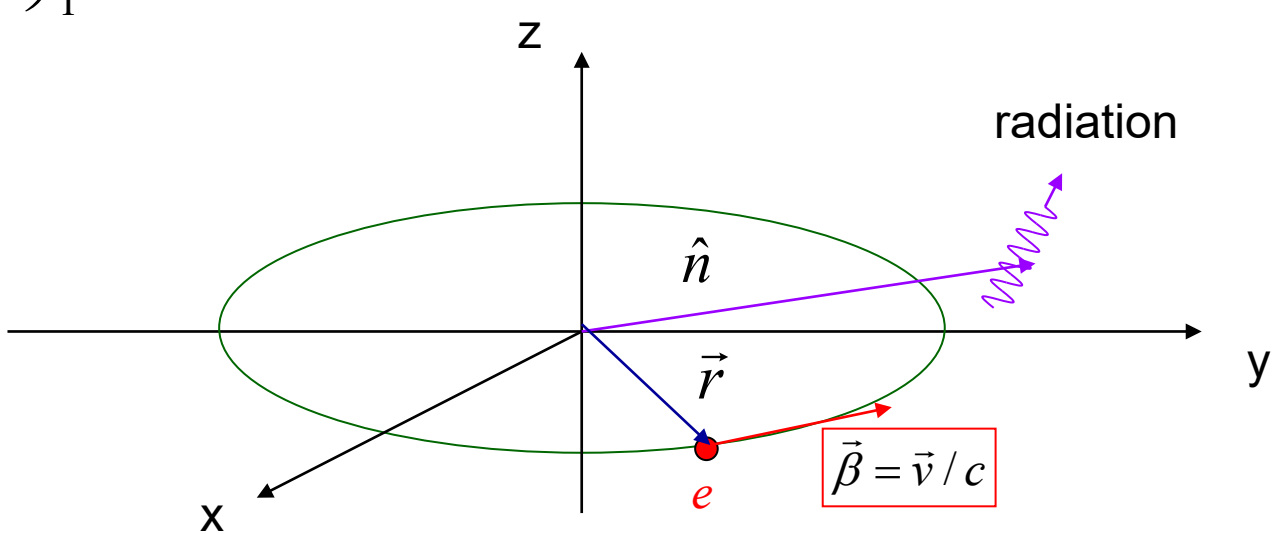
Radiation Spectral Energy of a Single Electron

Radiation spectral energy per solid angle $\left(\dfrac{d^2W}{d\omega d\Omega}\right)_1$

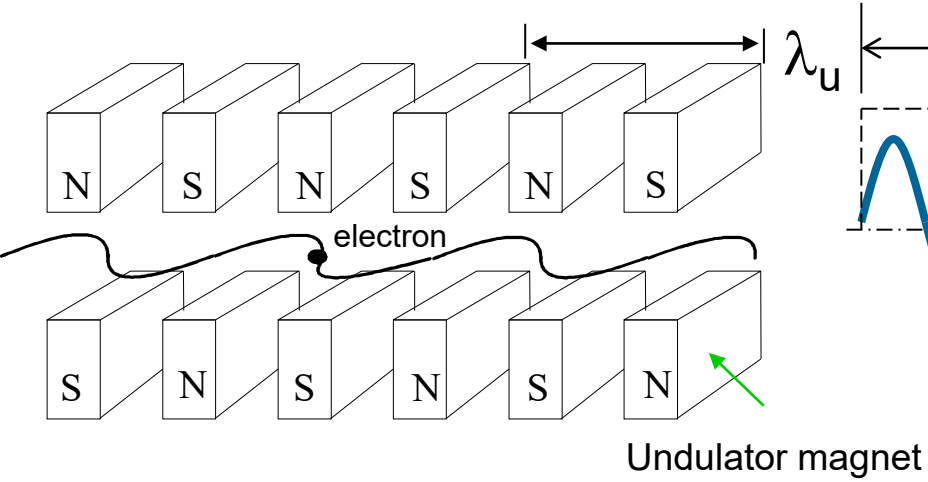
W : radiation energy
ω: radiation frequency
μ_0: vacuum permeability
e: electron charge
t: time variable
Ω: solid angle

Single-electron Radiation spectral energy

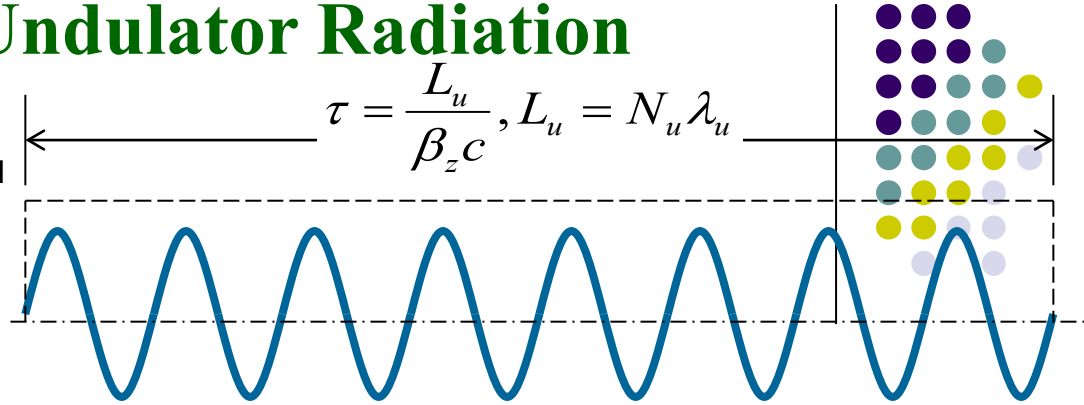
$$\left(\frac{dW}{d\omega}\right)_1 = \frac{\mu_0 c \omega^2 \boxed{e^2}}{16\pi^3} \int \left| \int_{-\infty}^{\infty} (e^{j\omega(t-\hat{n}\cdot\vec{r}/c)} \hat{n} \times \vec{\beta}) dt \right|^2 d\Omega$$



Singe-electron Undulator Radiation



$$\tau = \frac{L_u}{\beta_z c}, L_u = N_u \lambda_u$$



$$e^{i\omega_r t} \times \mathrm{rect}[\frac{t}{L_u/(\beta_z c)}]$$

Lemma

Rectangular function: $\mathrm{rect}[t] = \begin{cases} 1 & \text{for } |t| \le 1/2 \\ 0 & \text{otherwise} \end{cases}$

N_w: number of wiggler periods
L_w: wiggler length
τ: radiation time
ω_r: resonant radiation frequency

Fourier Transform $\{\mathrm{rect}[t]\} = \frac{\sin(\omega/2)}{\omega/2} = \mathrm{sinc}(f)$

So,

Fourier Transform $\{e^{i\omega_r t} \times \mathrm{rect}[\frac{t}{L_u/(\beta_z c)}]\}$

$$\propto \frac{\sin[N_w(\omega/\omega_r - 1)]}{N_w(\omega/\omega_r - 1)}$$

Synchronous radiation frequency

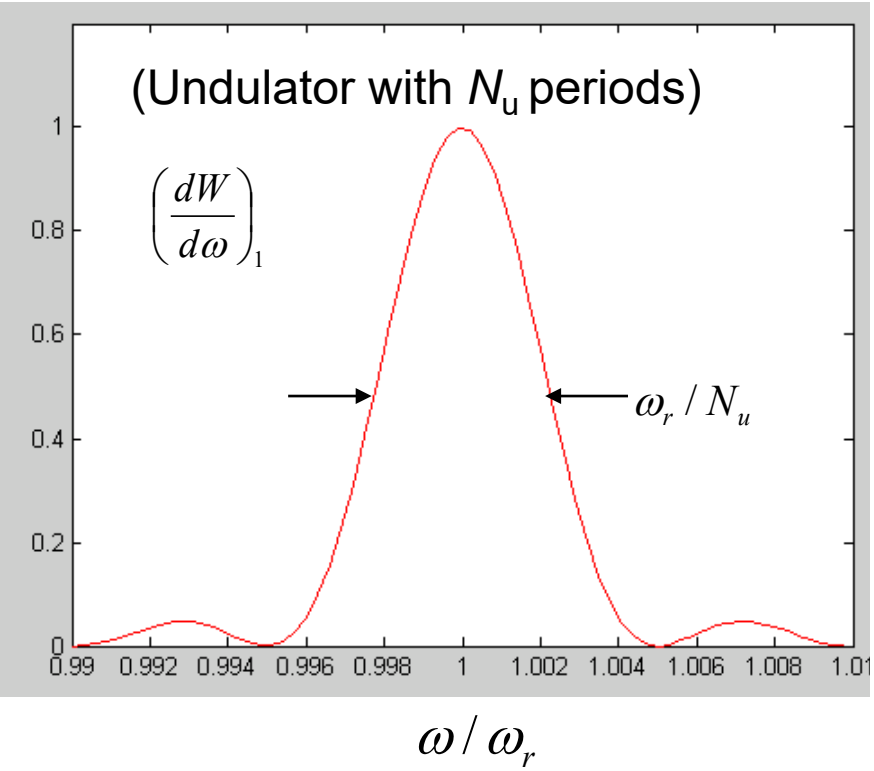
$$\omega_r = 2\pi c \left[\frac{1+a_u^2}{2\gamma^2}\lambda_u\right]^{-1}$$

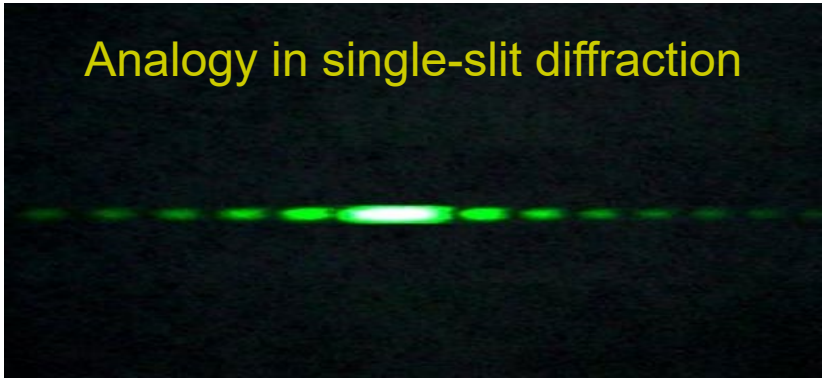
Undulator parameter

$$a_u = 0.093 B_{rms}(\text{kgauss}) \times \lambda_u(\text{cm})$$

Spectral energy $\left(\frac{dW}{d\omega}\right)_1 \propto \{\mathrm{sinc}[N_w(\omega/\omega_r - 1)/\pi]\}^2$

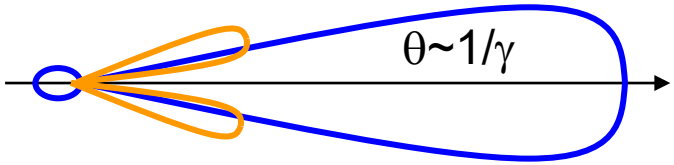
Spectral-energy Lineshape Function





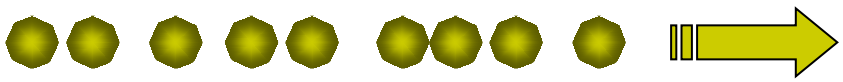
For a helical undulator

$$\left(\frac{d^2W}{d\omega d\Omega}\right)_1 = \frac{e^2 N_u^2 \gamma^2}{2\pi} \eta \frac{a_u^2}{(1 + a_u^2 + \gamma^2\theta^2)^2} \left\{\frac{\sin[N_u \pi(\omega/\omega_r - 1)]}{N_u \pi(\omega/\omega_r - 1)}\right\}^2$$



Solid angle $\Delta\Omega \sim \pi/\gamma^2$ η: Wave impedance

Radiation from many electrons



$$\left(\frac{dW}{d\omega}\right)_N = \left(\frac{dW}{d\omega}\right)_1 \left|\sum_{i=1}^{N} e^{-j\phi_i}\right|^2$$

Assume no energy exchange between electrons

$$\left(\frac{dW}{d\omega}\right)_1$$ Spectral energy radiated by a single particle

$\phi_i(t,r)$ Radiation phase of ith particle

N: number of electrons

$$\left|\sum_{i=1}^{N} e^{-j\phi_i}\right|^2 = \begin{cases} \sim N & \text{for random phase } \phi_i \\ N^2 & \text{for a constant phase } \phi_i = \phi_0 \end{cases}$$

$\longleftarrow$ **superradance**

Bunching Factor

$$|M| = \left| \sum_{i=1}^{N} e^{-j\phi_i} \right| / N \qquad 0 \le |M| \le 1$$

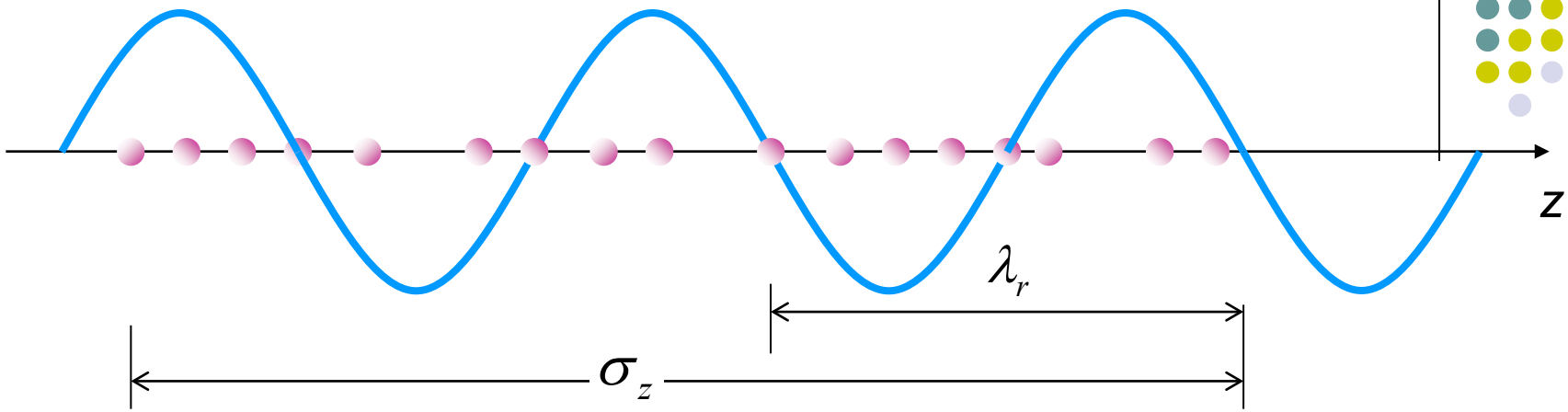
In the limit of continuous distribution with a temporal distribution function *f(t),* where $\int_{-\infty}^{\infty} f(t)dt = 1$, the bunching factor becomes

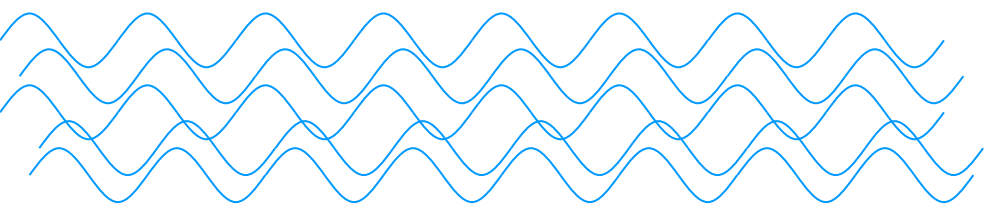
$$|M(\omega)| \rightarrow \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt \equiv$$ Fourier Transform of $f(t)$

$$|M(\omega_n)| = 1$$ means perfect bunching at frequency ω_n, yielding the maximum enhancement factor of *N* for radiation

Incoherent Radiation

$\sigma_z >> \lambda_r, r_i$ is random



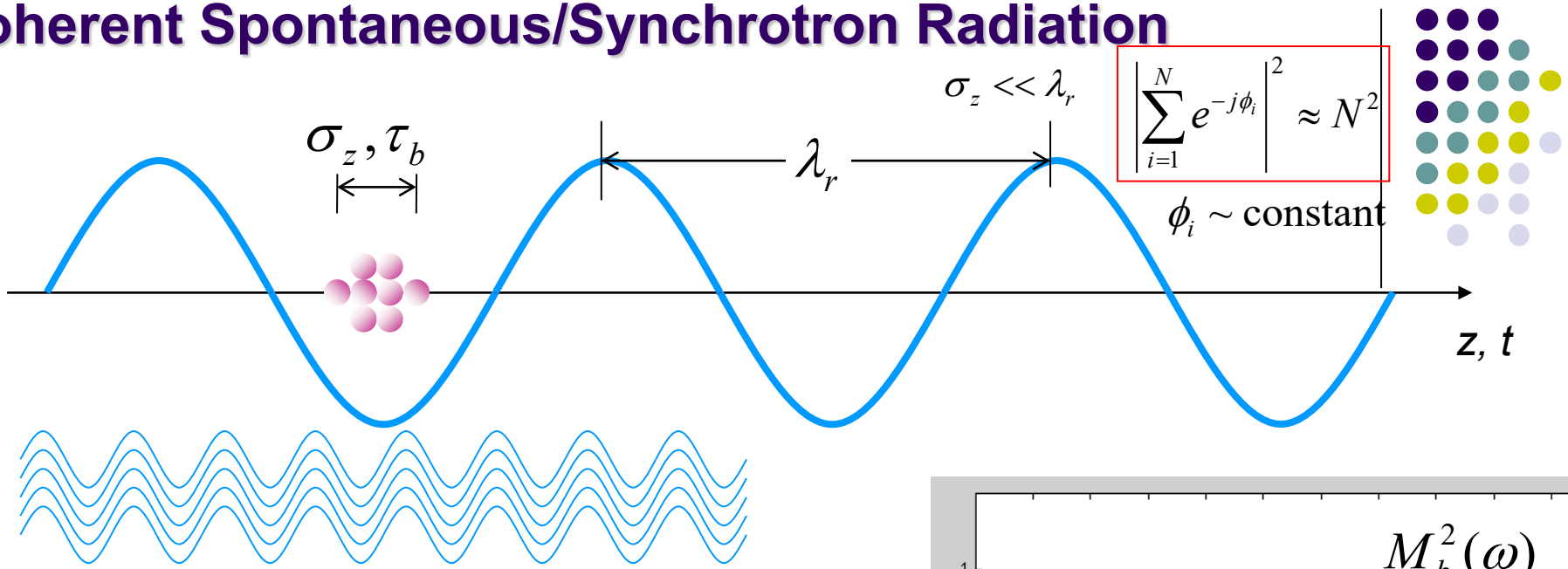


$$\left| \sum_{i=1}^{N} e^{-j\phi_i} \right|^2 \approx N$$

Total Spectral Energy $\left(\frac{dW}{d\omega}\right)_{inc,N} = \boxed{N}\left(\frac{dW}{d\omega}\right)_1 \propto N$ *N*: number of electrons

$\left(\frac{dW}{d\omega}\right)_1$ Radiation spectral energy of a single electron

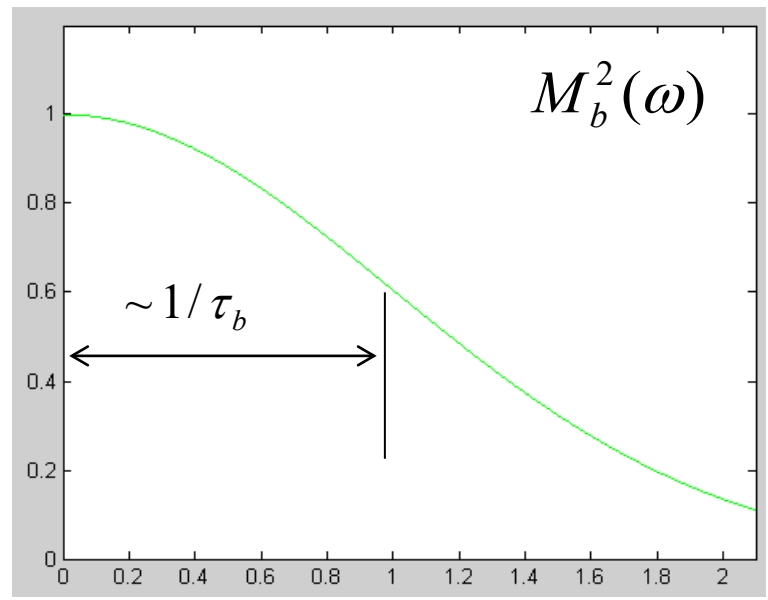
Coherent Spontaneous/Synchrotron Radiation



Spectral Energy $\left(\frac{dW}{d\omega}\right)_{SR,N_b} = N_b^2 \left(\frac{dW}{d\omega}\right)_1 M_b^2(\omega)$

$M_b(\omega)$:Fourier transform of the bunch shape

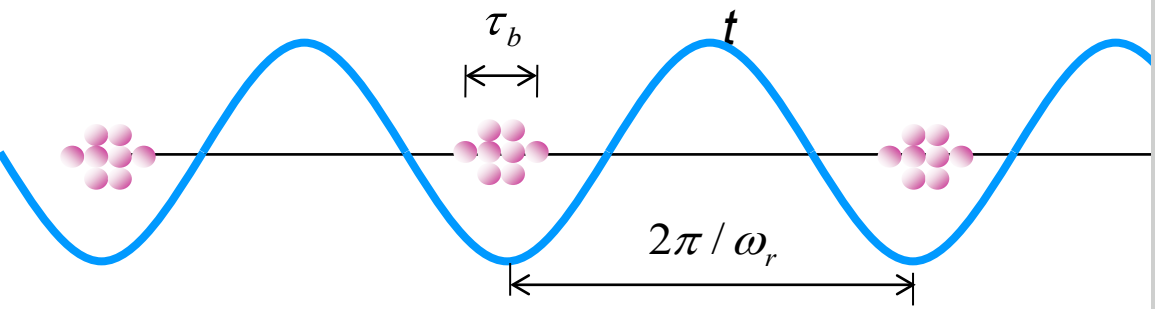
N_b: number of electrons in the bunch

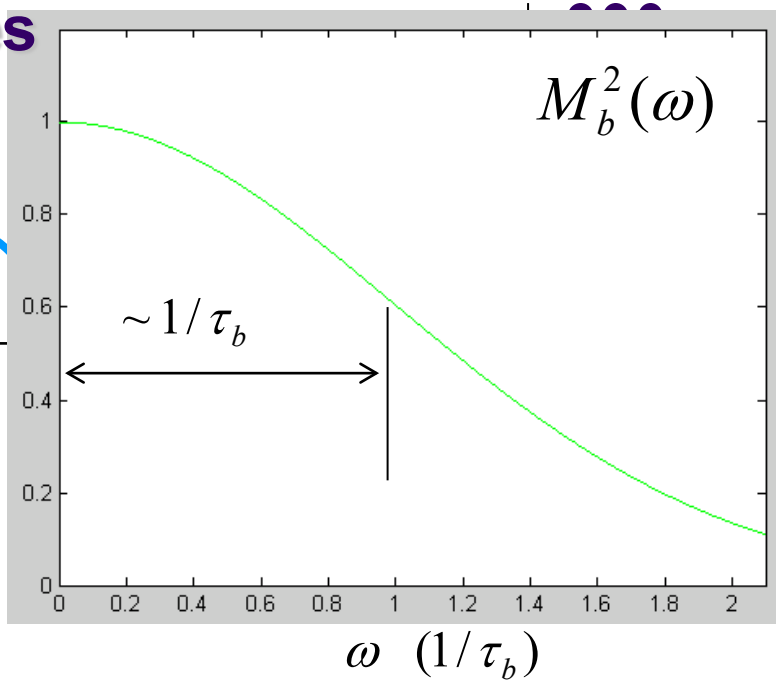


* For 1 nC in 10 ps, $N_b = 6.25 \times 10^9$!

For Gaussian bunch $f(t) = \frac{\exp\left(-t^2/\tau_b^2\right)}{\sqrt{\pi}\tau_b} \Rightarrow M_b(\omega) = \exp\left(-\frac{\omega^2\tau_b^2}{4}\right)$ $\omega\tau_b$

Superradiance from Periodic Bunches
(comb-pulse superradiance)





Spectral Energy

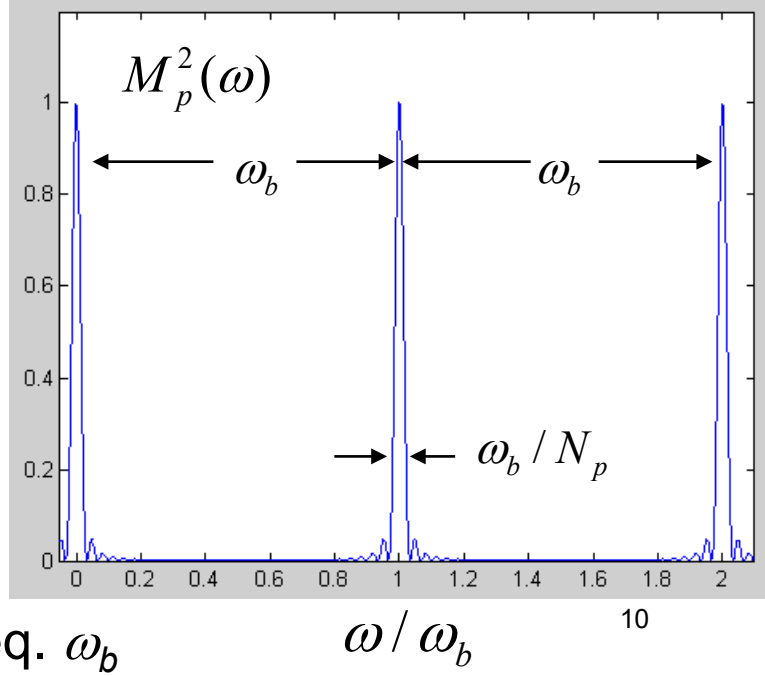
$$\left(\frac{dW}{d\omega}\right)_{SR} = \left(\frac{dW}{d\omega}\right)_1 N_b^{\,2} M_b^2(\omega) \times N_p^2 M_p^2(\omega)$$

$M_b(\omega)$:Fourier transform of the bunch shape

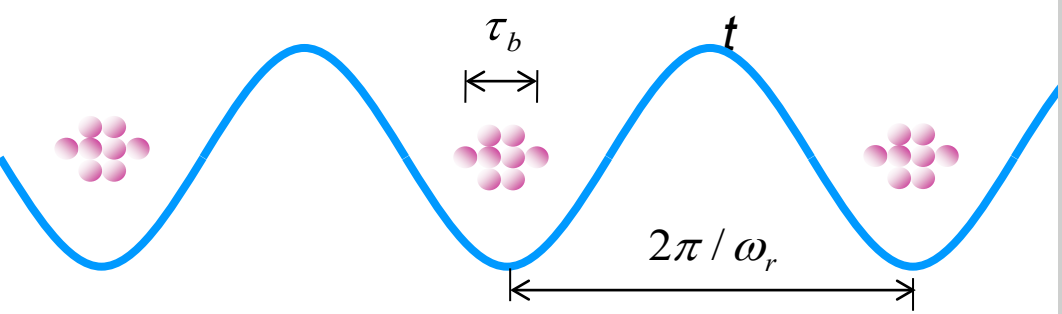
N_b: number of bunched electrons

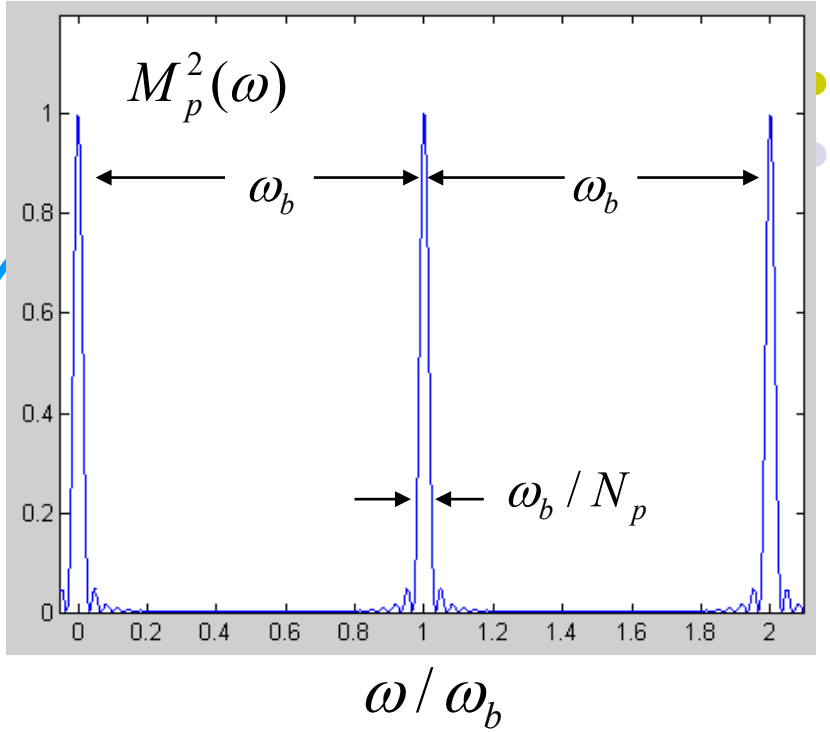
$$M_p(\omega) = \frac{\sin(N_p \pi\omega/\omega_b)}{N_p \sin(\pi\omega/\omega_b)}$$

Coherent sum of N_p bunches with bunching freq. ω_b

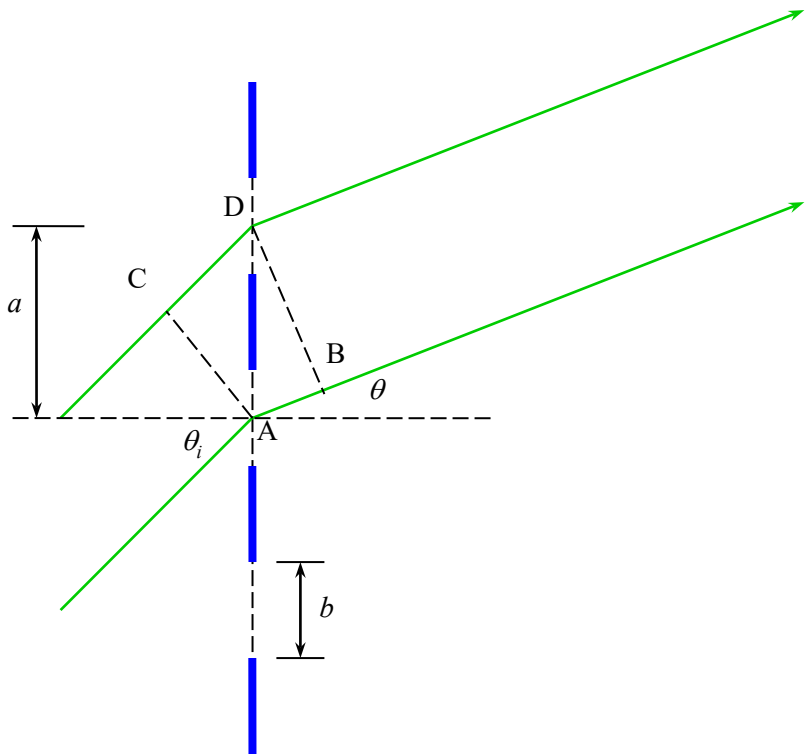


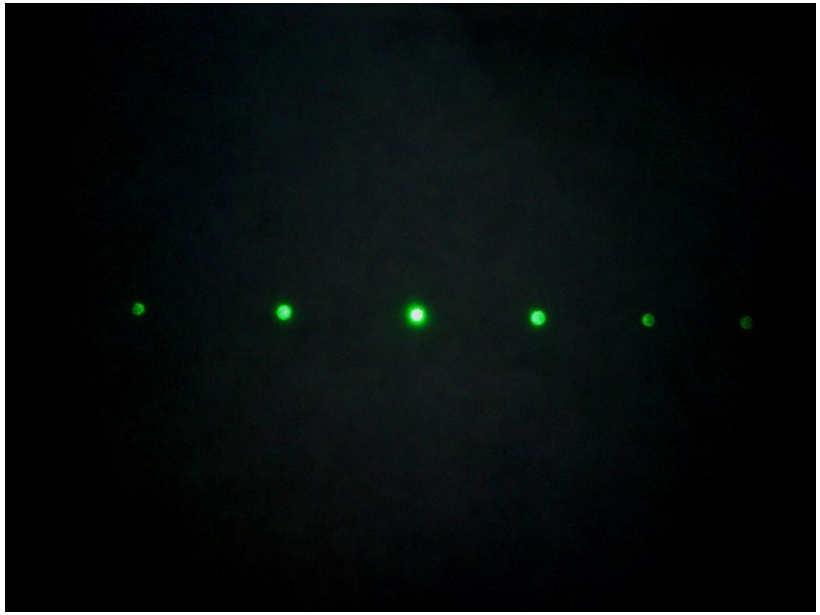
Multi-bunch radiation





Analogy in grating diffraction





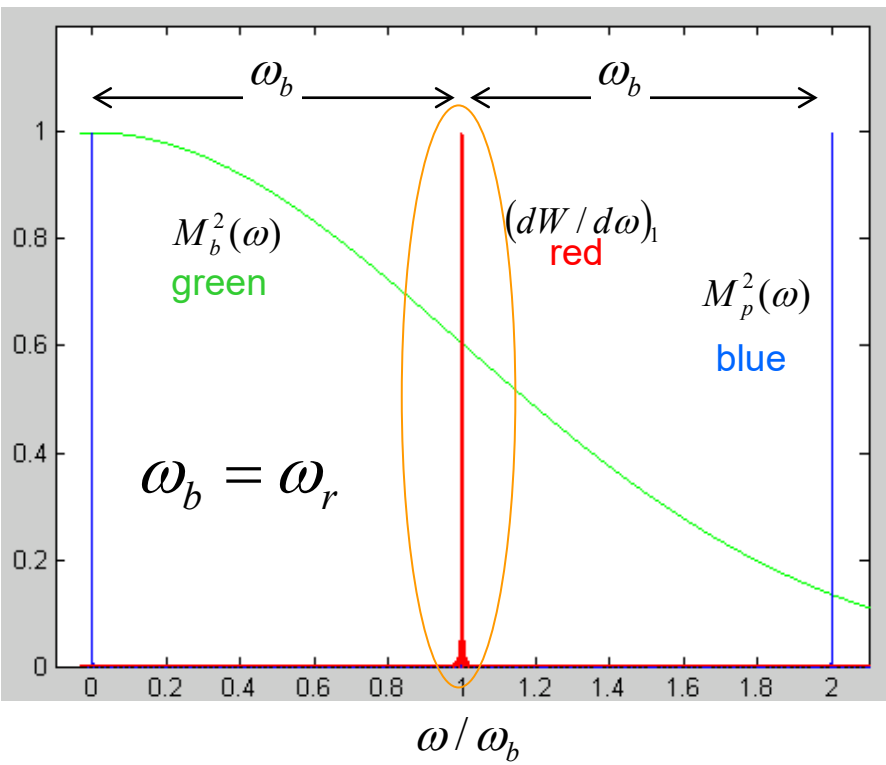
Spectral narrowing from comb-pulse superradiance

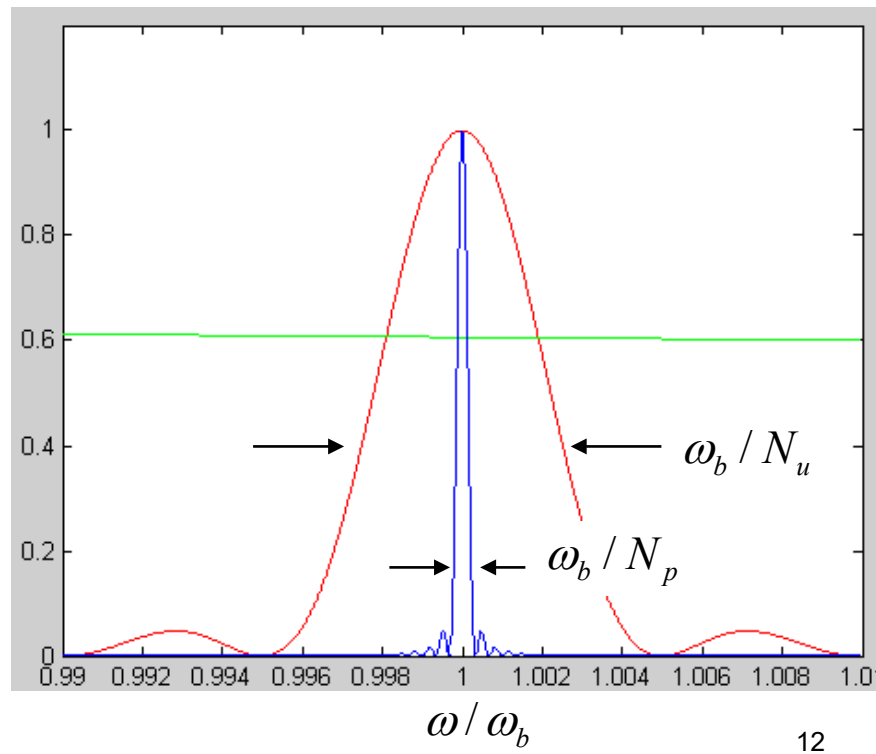
Spectral Energy $\left(dW/d\omega\right)_{SR,N_p\times N_b} = (N_p N_b)^2 \left(dW/d\omega\right)_1 M_b^2(\omega) M_p^2(\omega)$

Assume undulator radiation $\Rightarrow$ $\left(dW/d\omega\right)_1 \propto \mathrm{sinc}^2[N_u(\omega/\omega_r - 1)]$

$M_p(\omega) = \sin(N_p \pi\omega/\omega_b)/\sin(\pi\omega/\omega_b)/N_p$ for bunching freq.= ω_b

$\left|M_b(\omega)M_p(\omega)\right|$ is the overall *bunching factor*

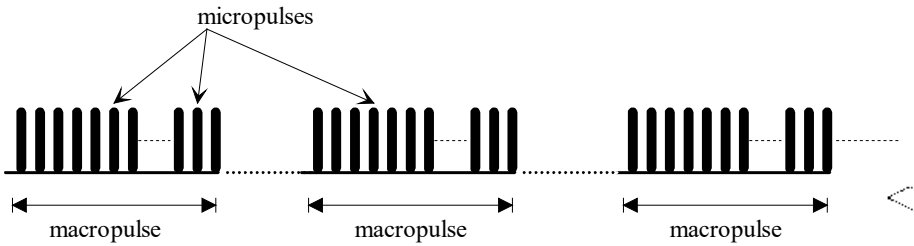


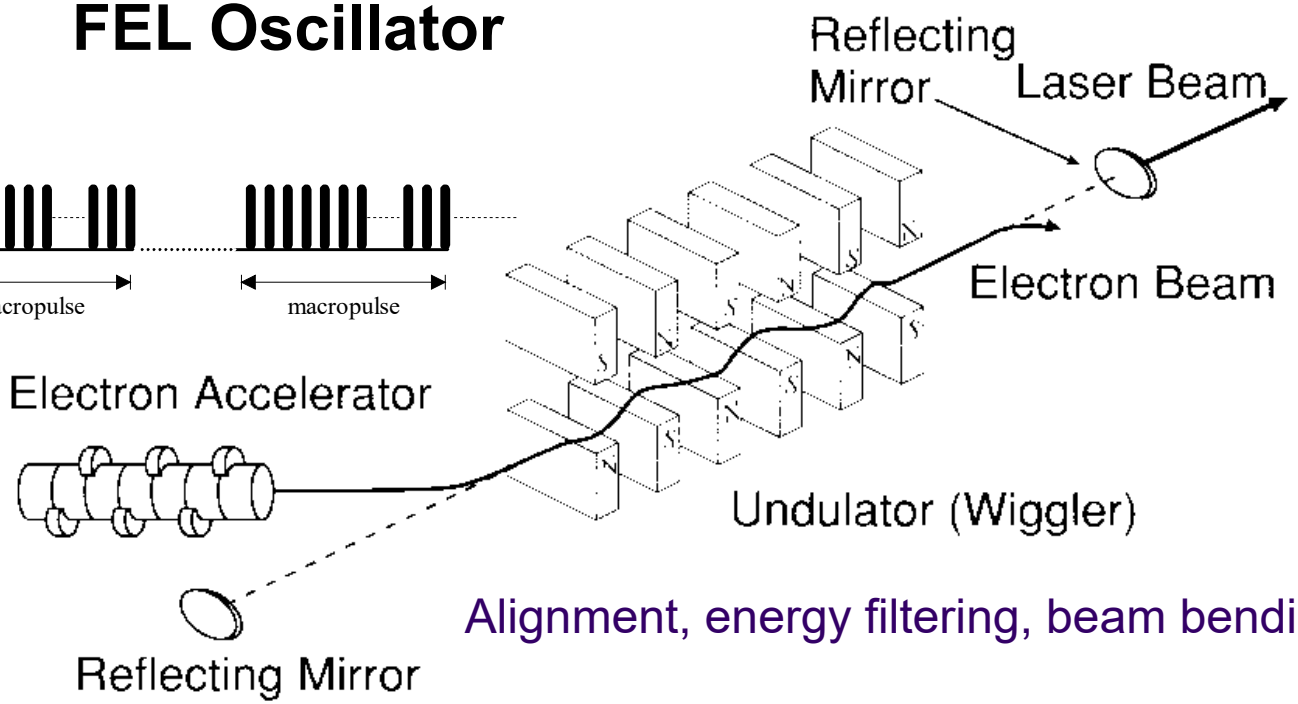


Tight bunch, large bunching factor, periodic bunching are desirable

2. Ordinary FEL
vs.
Superradiance FEL

FEL Oscillator

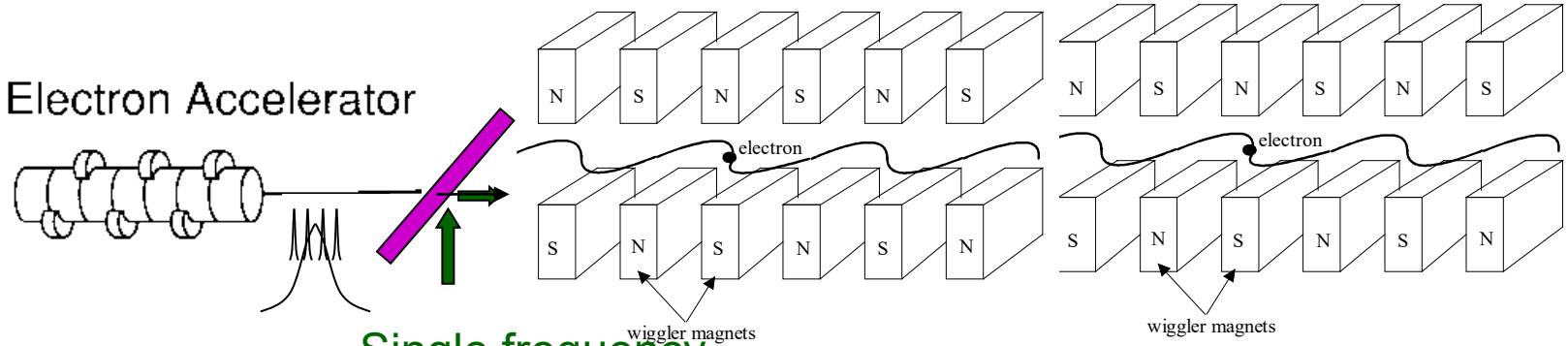




Alignment, energy filtering, beam bending…

Single-pass FEL

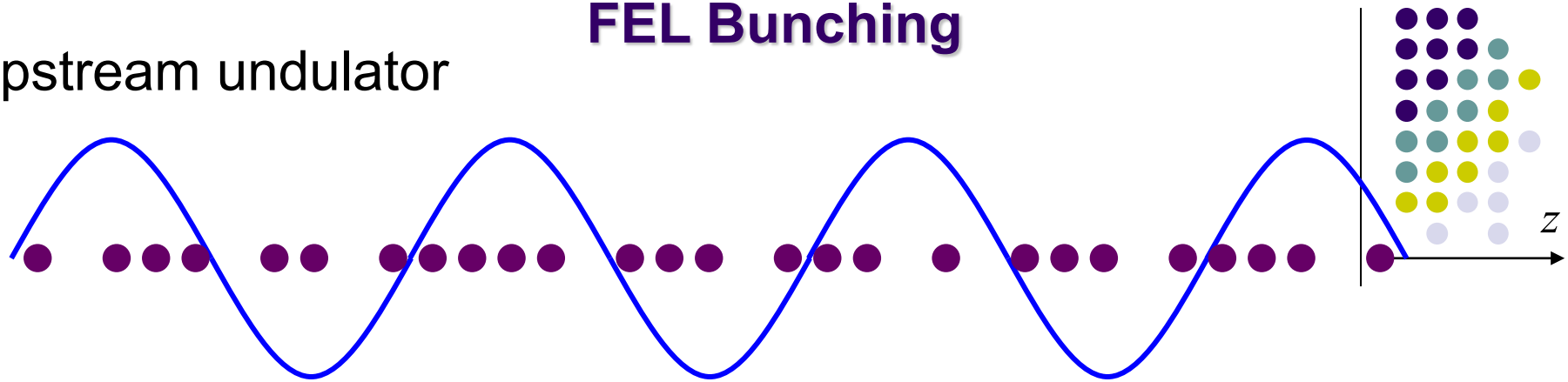
Long undulator for a SASE FEL



Single frequency
Seeding laser

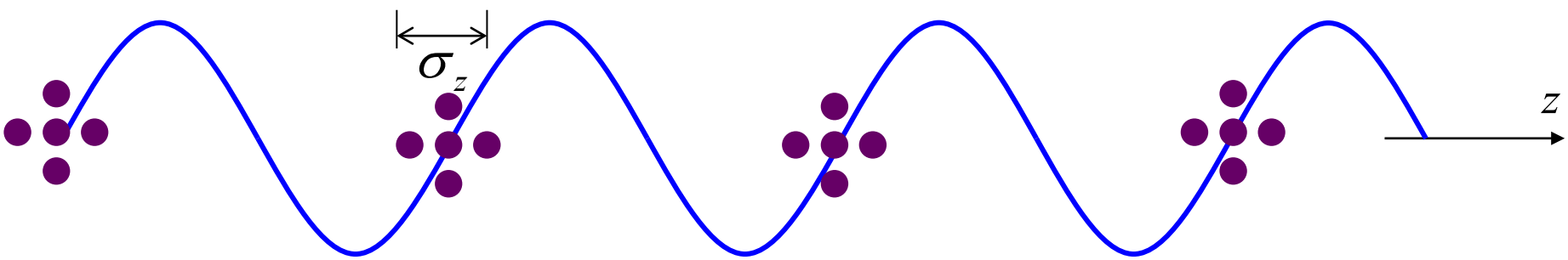
FEL Bunching

upstream undulator



As an electron beam propagates down an FEL, electrons gain energy modulation ⇒ spatial modulation and are bunched to emit coherent radiation.

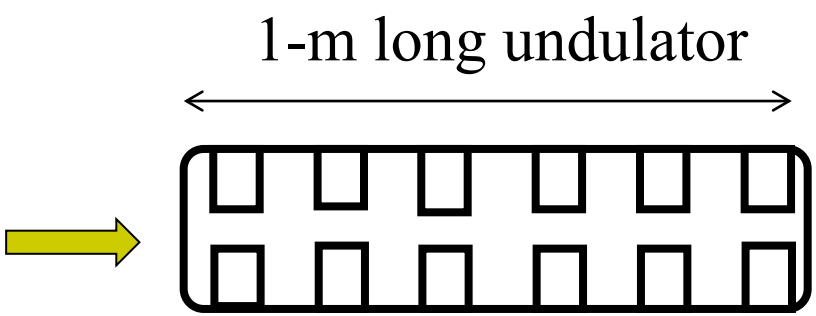
Down-stream undulator ~ superradiance FEL



Radiation Spectral Energy $$\left(\frac{dW}{d\omega}\right)_N = \left(\frac{dW}{d\omega}\right)_1 \left|\sum_{i=1}^{N} e^{-j\phi_i}\right|^2$$

Coherent Radiation $\sigma_z << \lambda$, Power $\propto N^2$ $N \sim 10^{8\text{-}10}$!

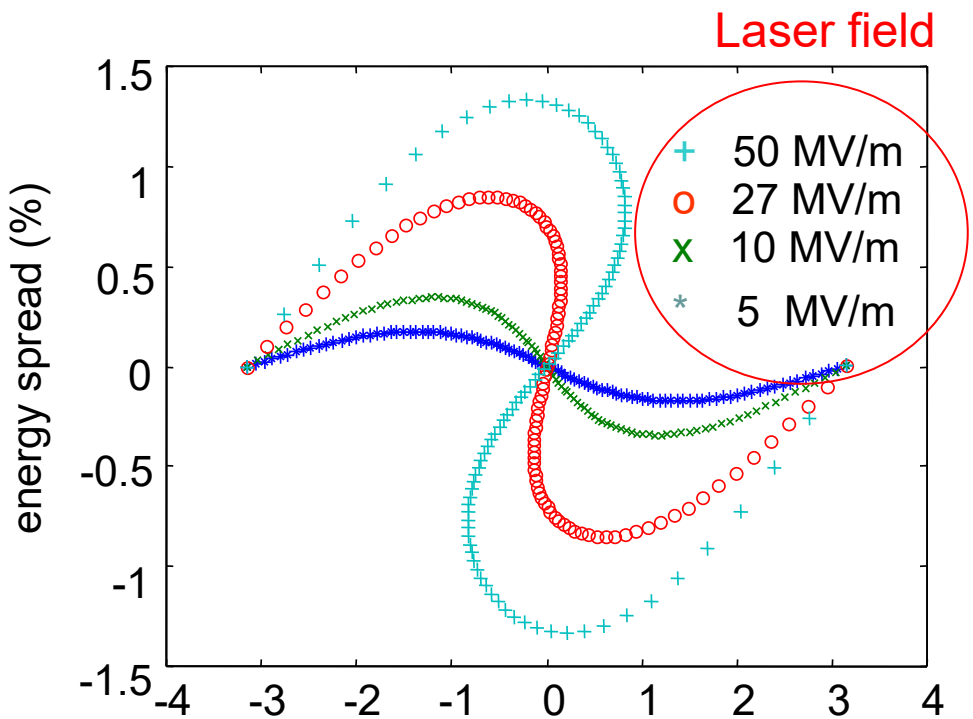
Energy ⇒ Spatial Modulation in a Undulator



$B_u = 2.9\,\mathrm{kG}, \qquad \lambda_u = 3.56\,\mathrm{cm}$

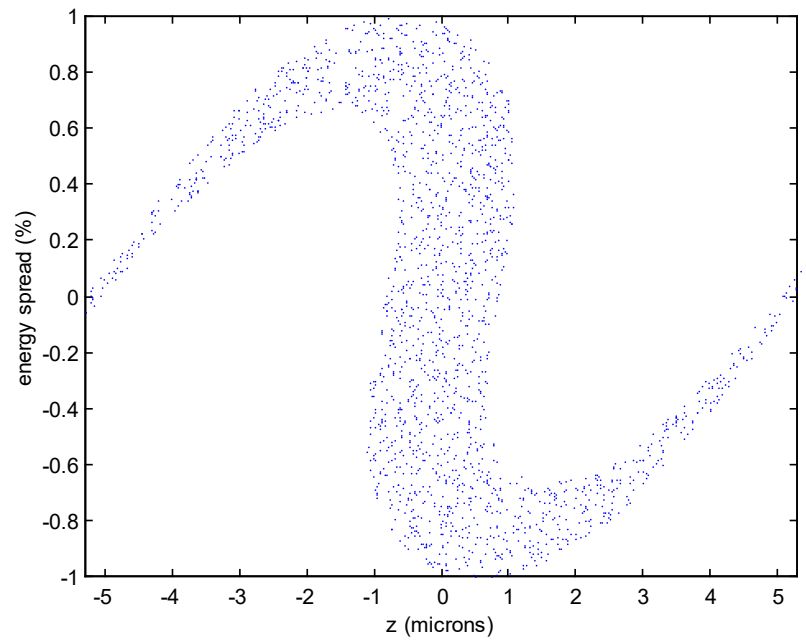
Phase matching wavelength = 10 μm

Laser @ λ = 10 μm



Pondermotive phase

$$\theta = [k - (k + k_u)\beta_z]L / \beta_z$$



After the end of the 1-m wiggler for 27MV/m laser field

SASE FEL

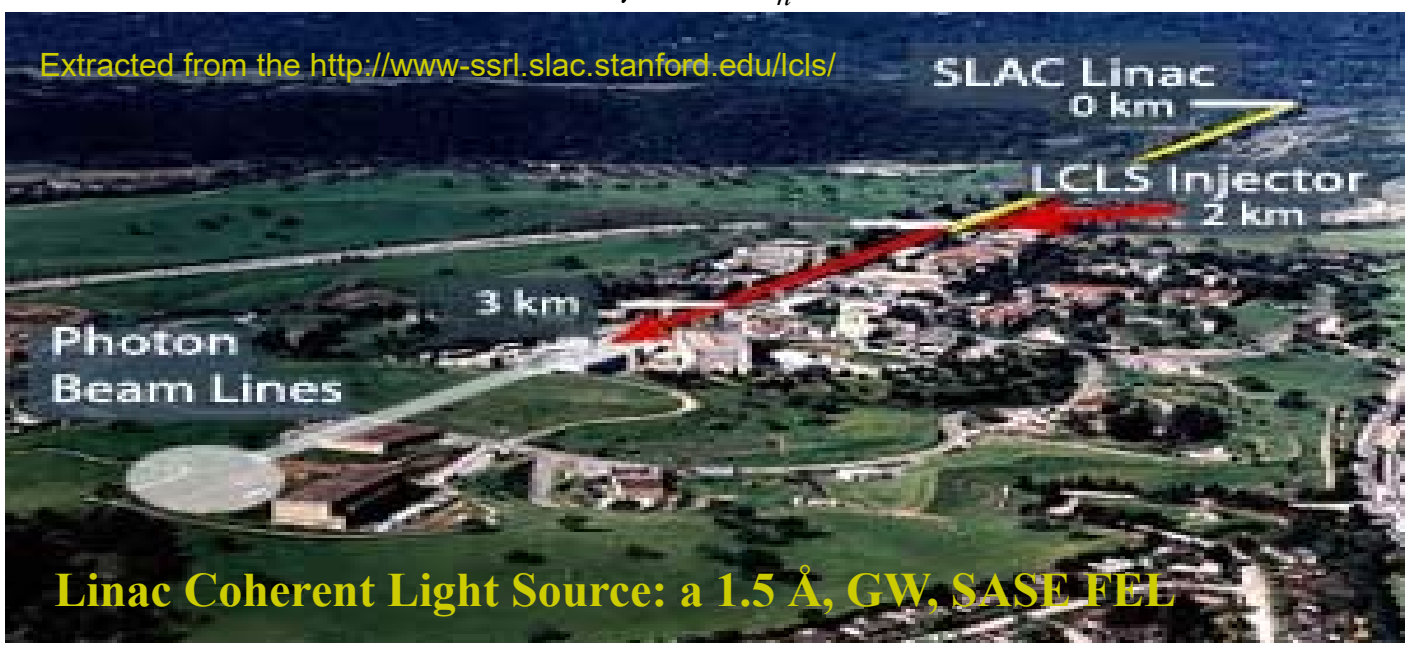
Self-amplified Spontaneous Emission (SASE)

As electrons propagate down a long wiggler, electrons are bunched, emit coherent radiation, and amplify the radiation .

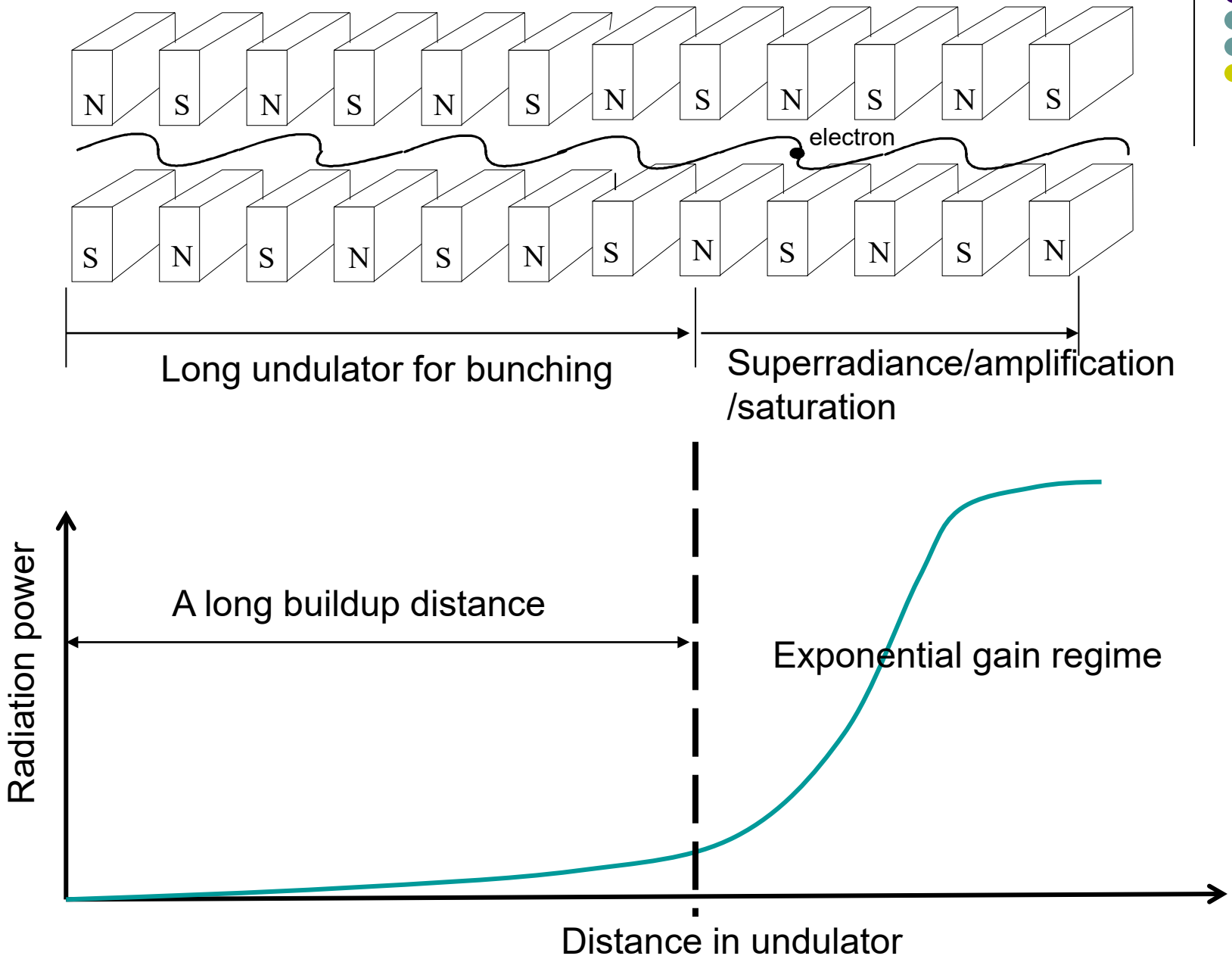
1-D Model

In the high-gain regime, the SASE FEL gain $G = \frac{P(z)}{P_{in}} = \frac{1}{9}\exp(2z/L_g)$

$L_g = \lambda_u / 2\sqrt{3}\pi\rho$: gain length, $\rho \approx \frac{0.88}{\gamma}\frac{B_u \lambda_u^{4/3} I^{1/3}}{\varepsilon_n^{1/3}}$: fundamental FEL parameter



SAFE FEL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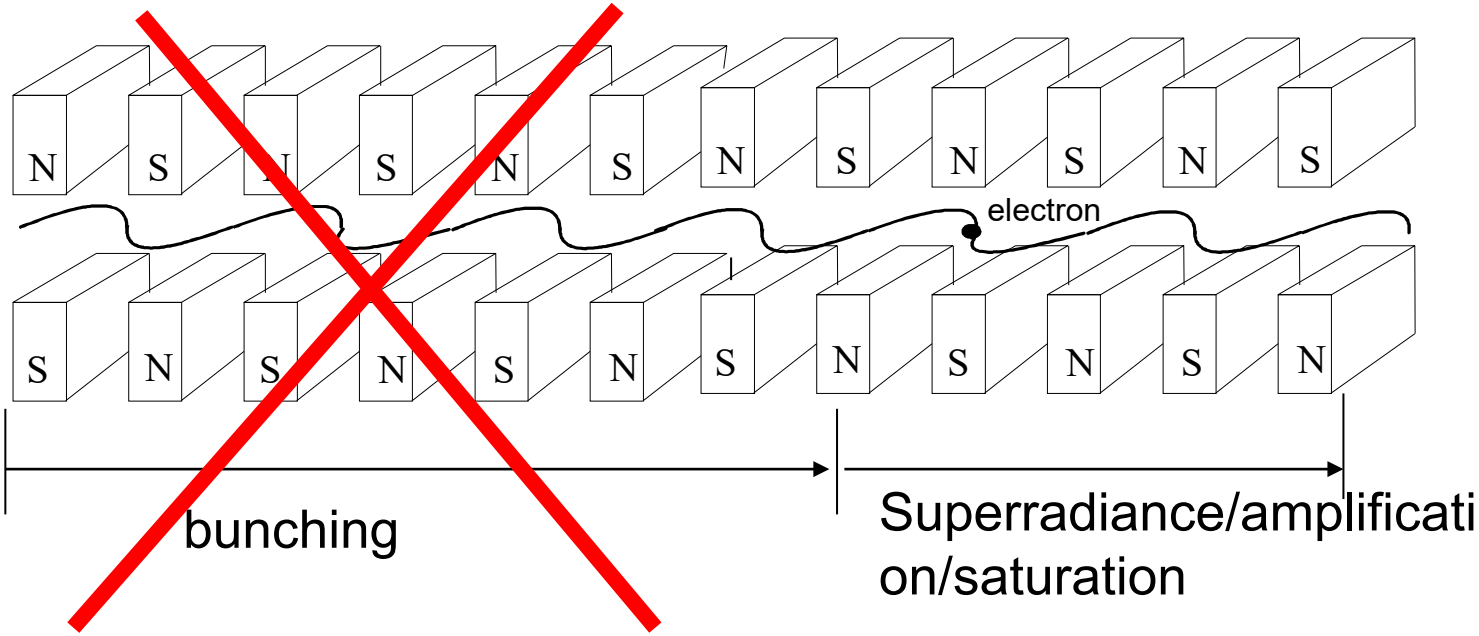
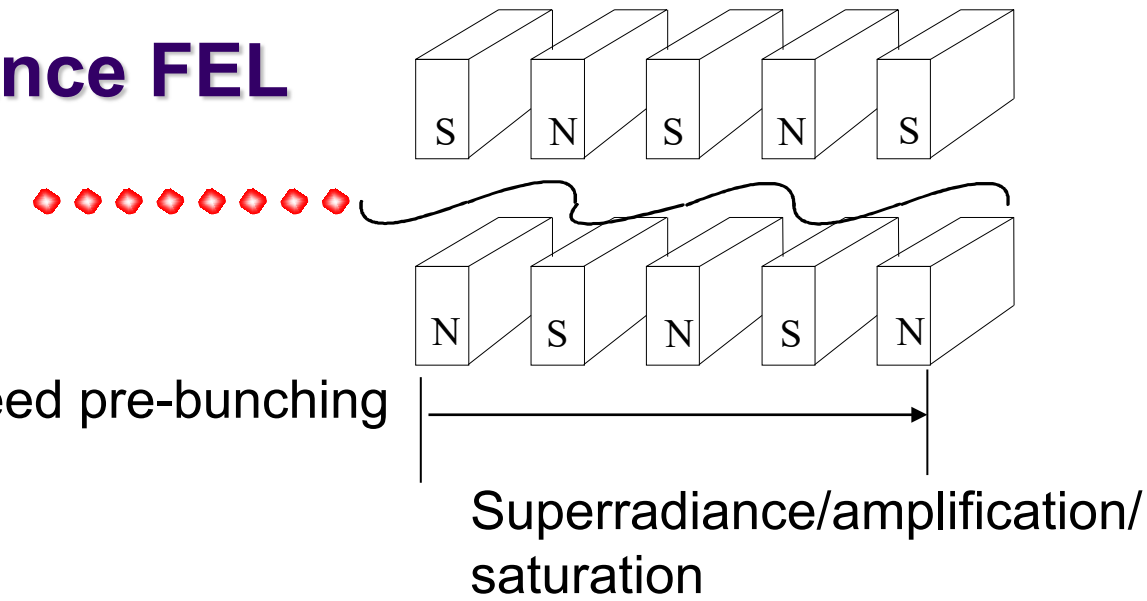
Long undulator for bunching
Superradiance/amplification
/saturation
Radiation power
A long buildup distance
Exponential gain regime
Distance in undulator

Conventional SAFE FEL



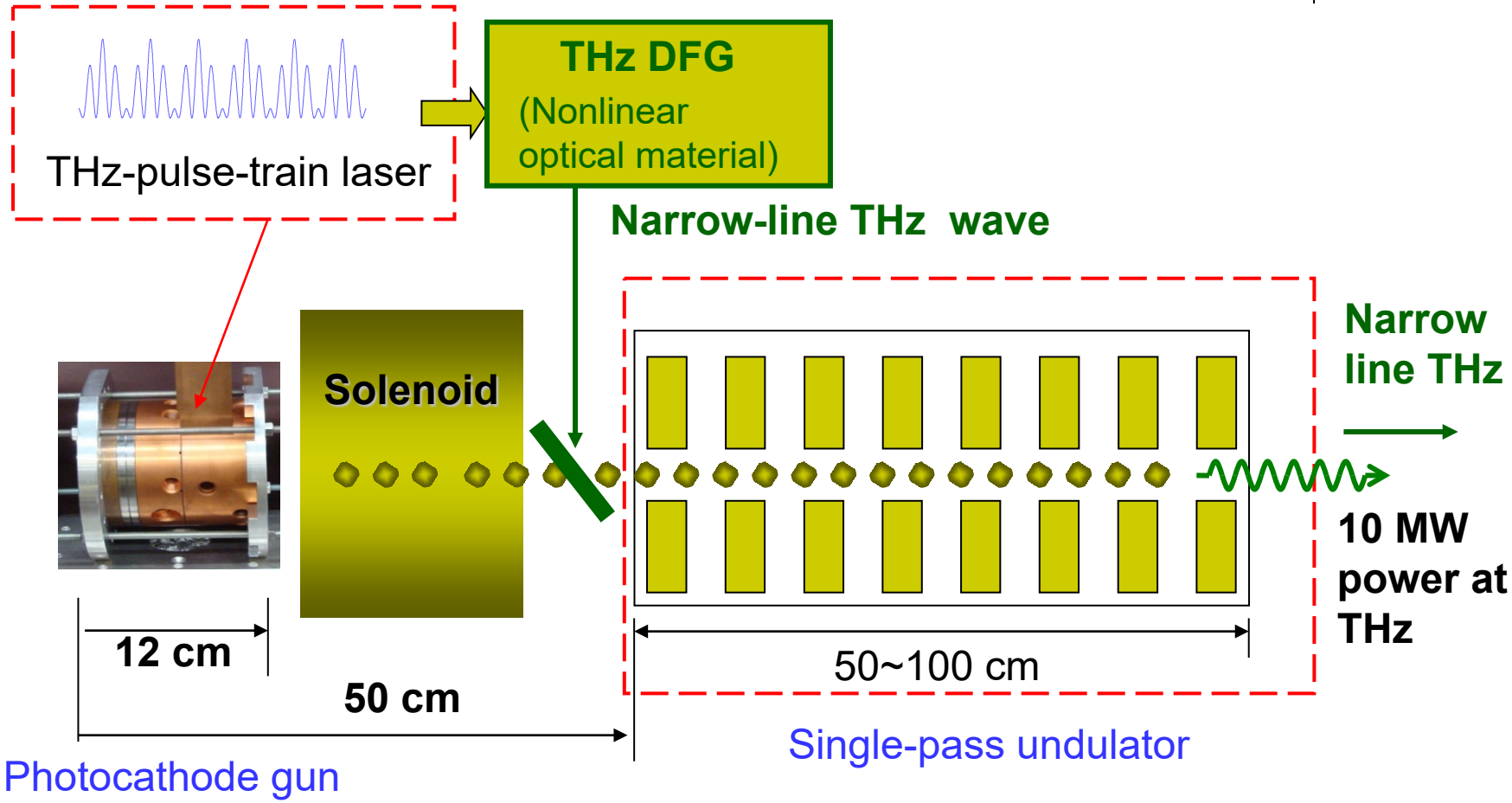
Superradiance FEL



Much shorter, need pre-bunching

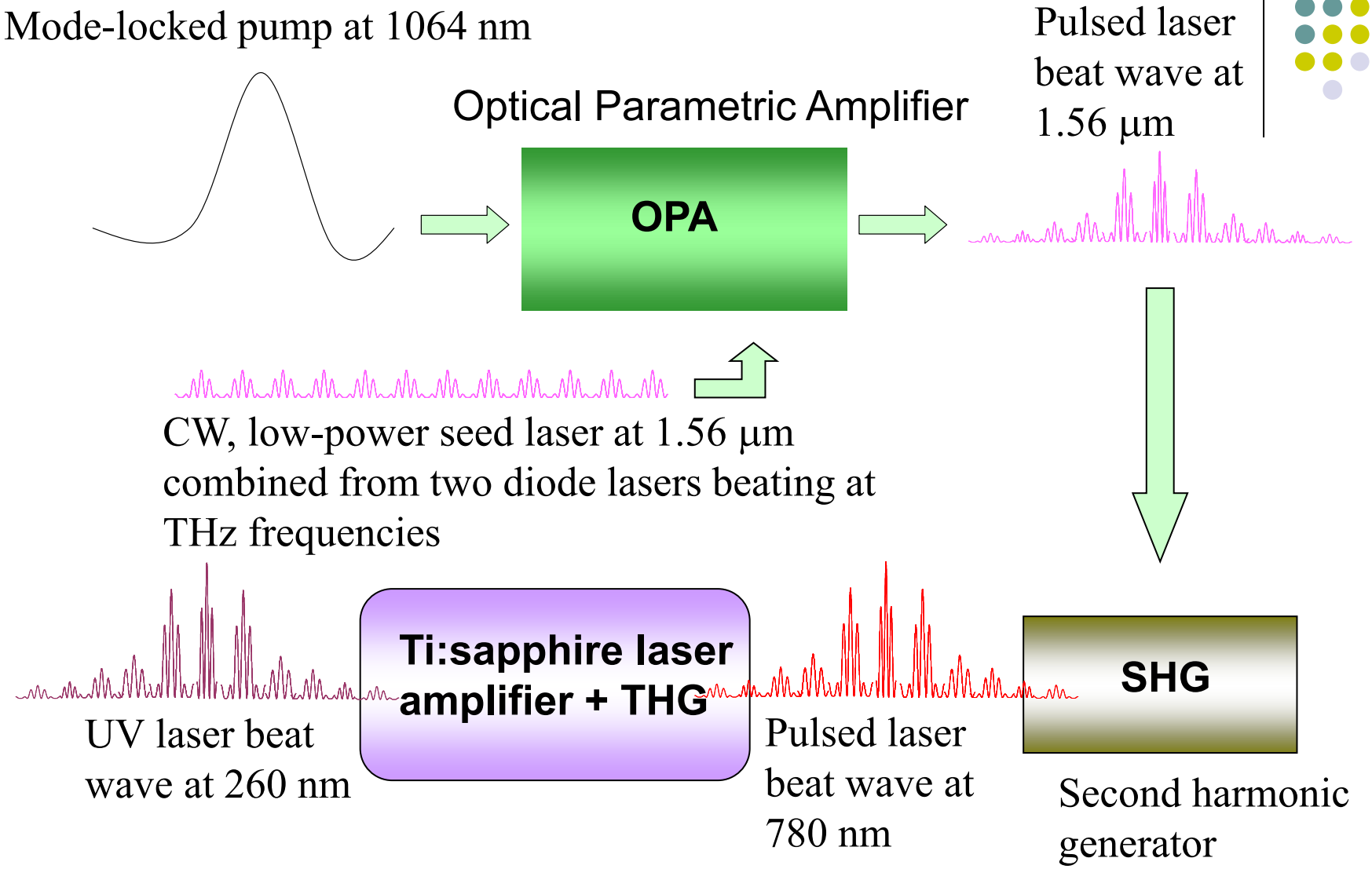
3. A Design Example of a Superradiance FEL

Desktop Narrow-line MW THz Free-electron Laser



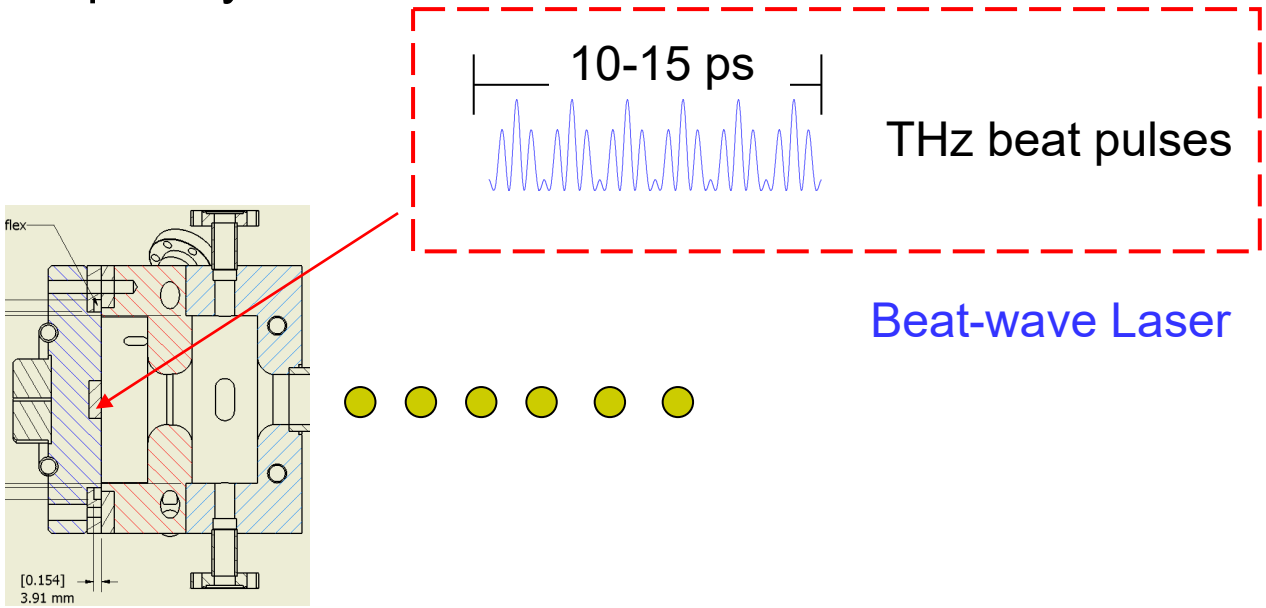
Yen-Chieh Huang, "Desktop MW Superradiant Free-electron Laser at THz Frequencies," Applied Physics Letters, **96**, 231503 (2010)

THz-pulse-train Laser System

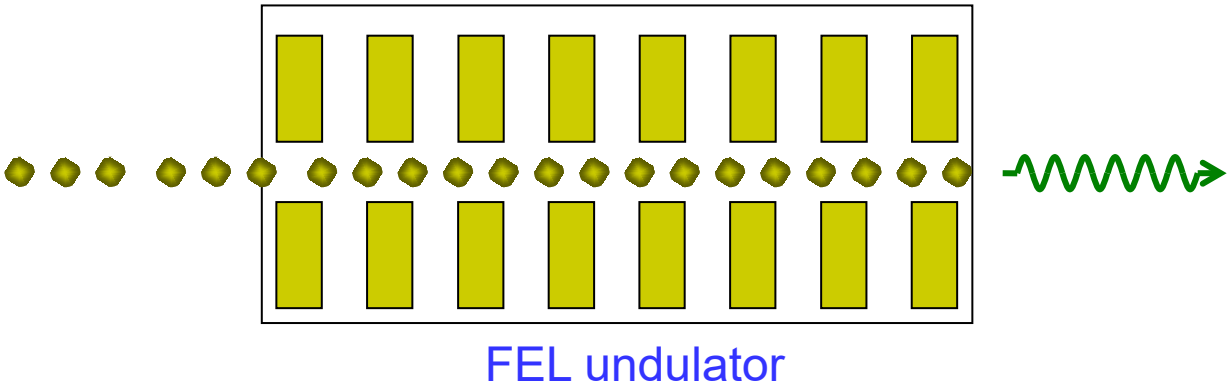


Simulation Tools

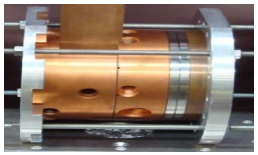
1. Injector Simulation: ASTRA – A space-charge tracking code developed by DESY



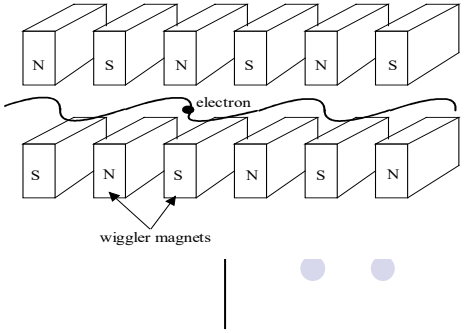
2. FEL Simulation: GENESIS – a SASE FEL code developed by UCLA/DESY

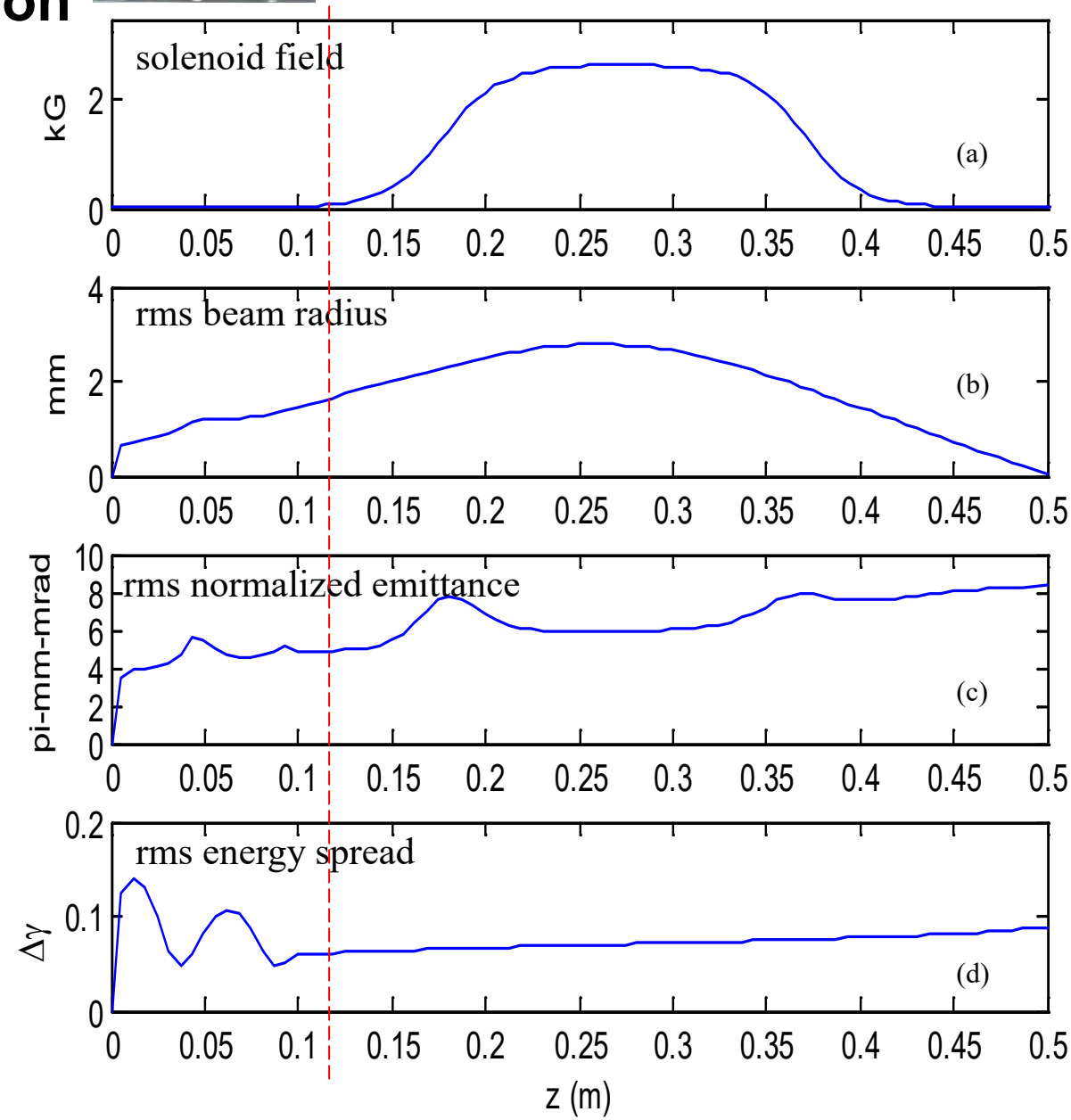


ASTRA Simulation

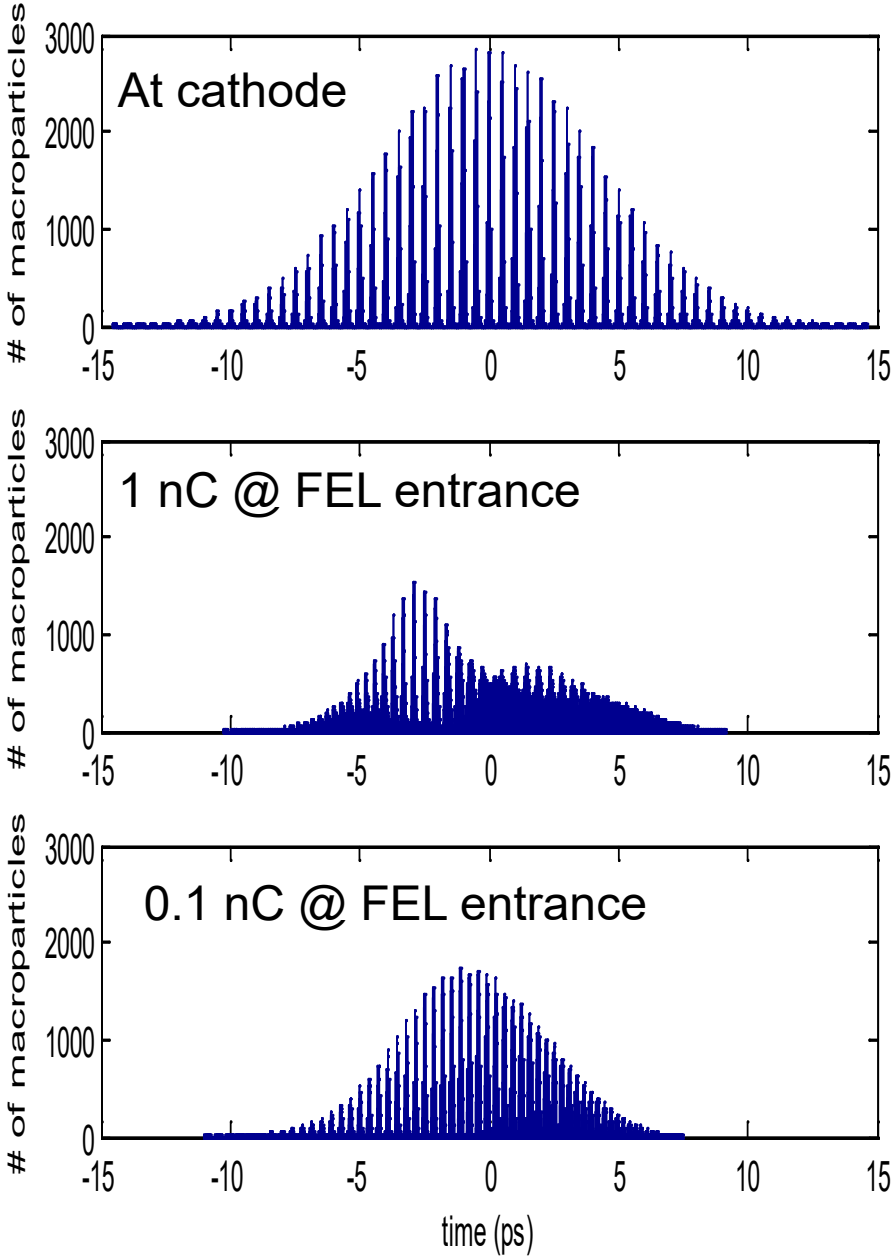


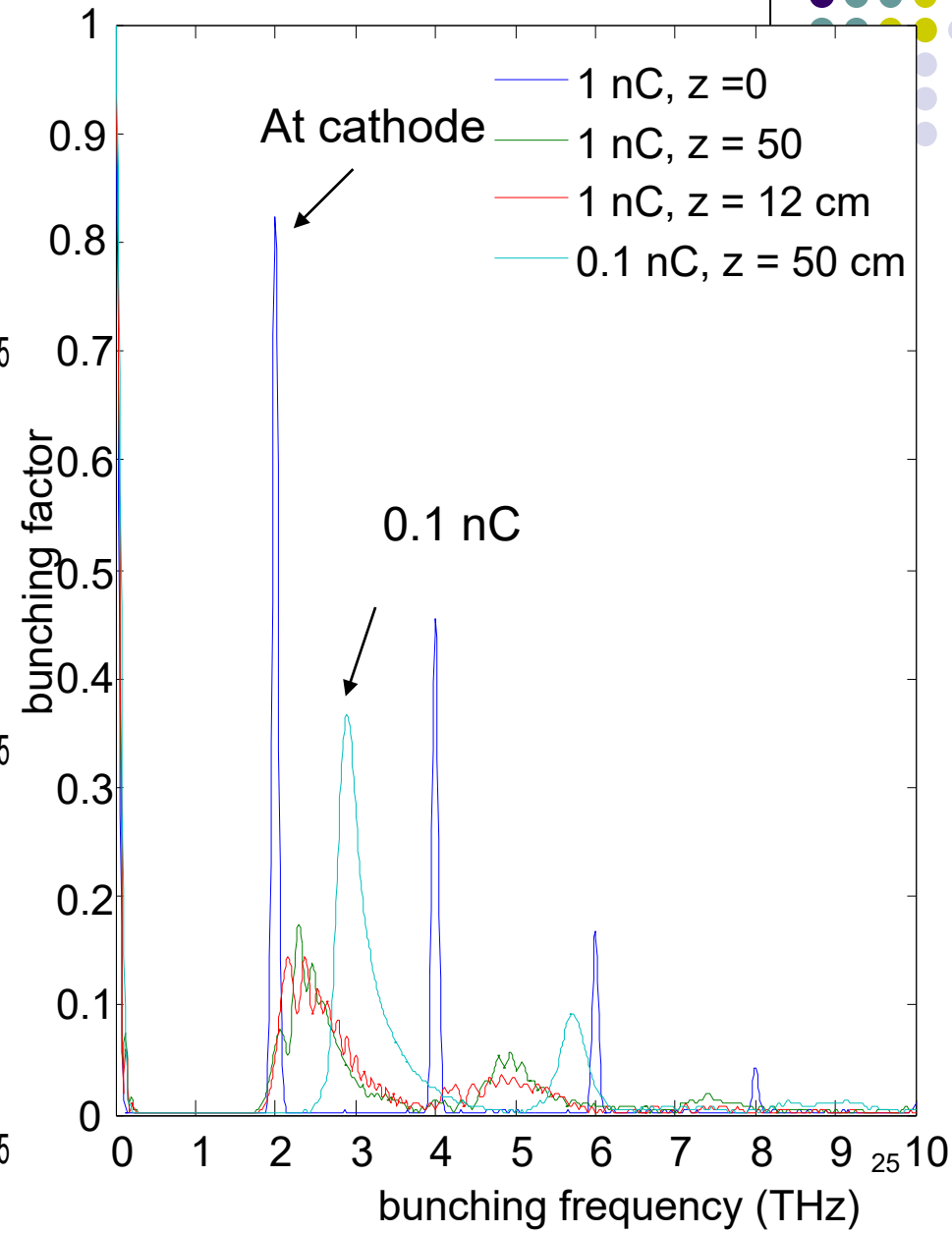
Solenoid





120 MV/m Acceleration Gradient, comb-f Beat Wave





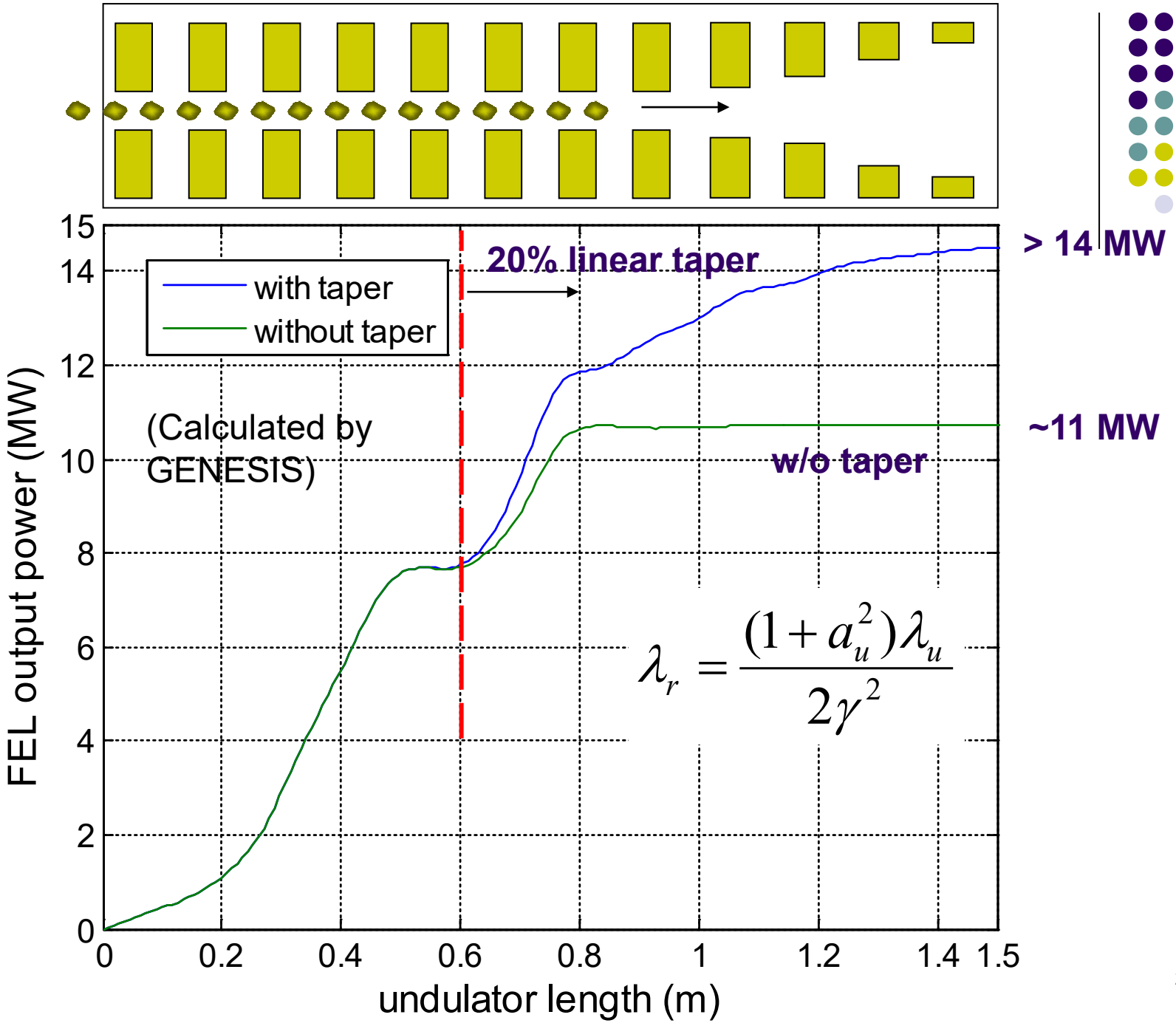
Undulator Parameters

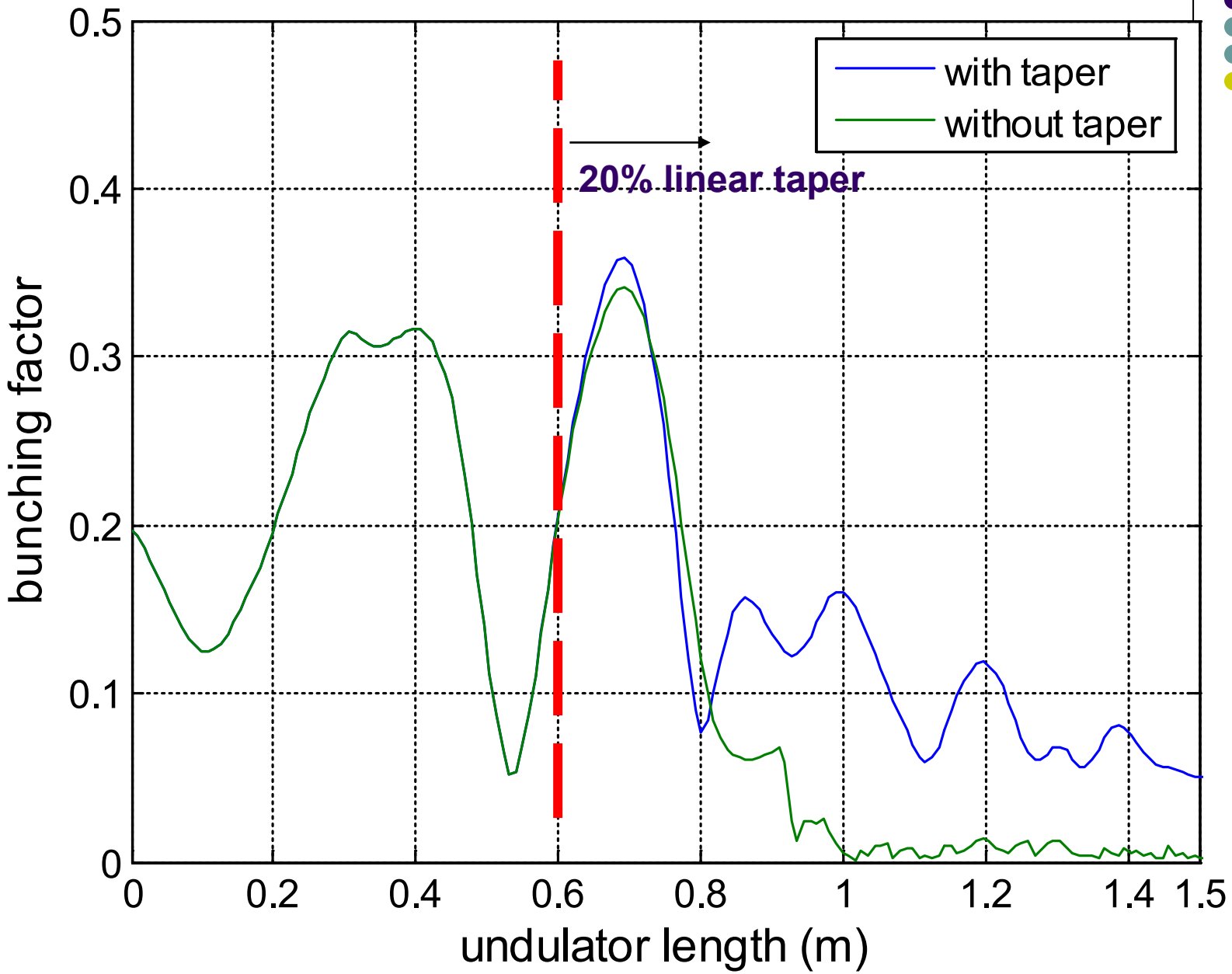
Type: helical
Period: 1.8 cm
Undulator parameter = 0.98

Beam Parameters (from ASTRA code)

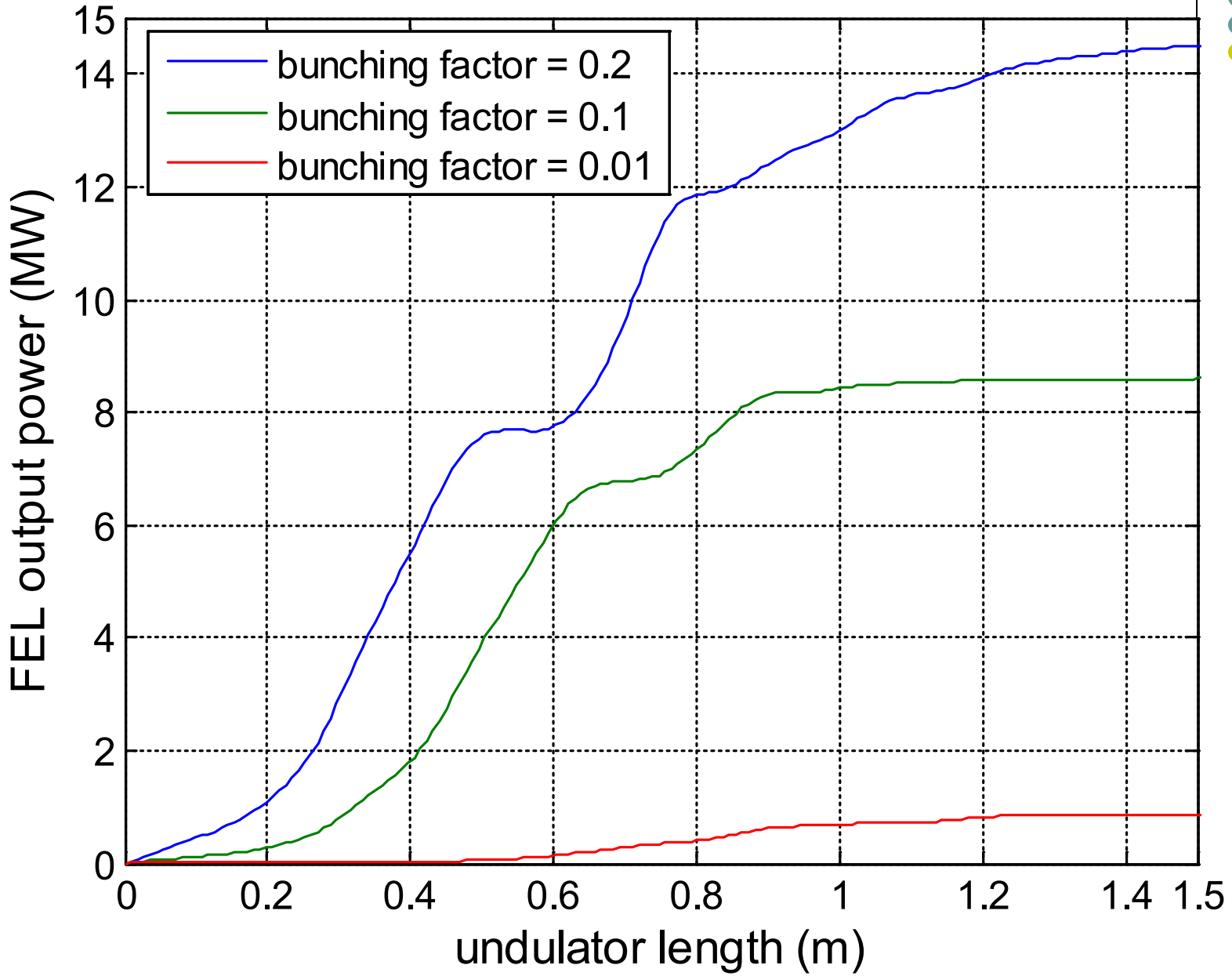
Beam parameters at Cathode						
peak gradient	Charges	rms beam radius (mm)	Macro-bunch length (σ_M)	micro-pulse length (σ_μ)	micro-pulse rate	Bunching factor @ 2 THz
120 MV/m	1 nC	0.6 (radial distribution)	4.25 ps (10-ps FWHM)	50 fs	2 THz	0.85

Beam Parameters at FEL Entrance				
rms beam energy (γ)	rms energy spread $\Delta\gamma$	rms beam radius (mm)	rms emittance (10^{-6} π-m)	bunching factor @ 2.4 THz
11.9	7.4×10^{-2} (0.61%)	6.3×10^{-2}	6.88	0.21


with taper
without taper
20% linear taper
(Calculated by GENESIS)
w/o taper
> 14 MW
~11 MW
$\lambda_r = \frac{(1+a_u^2)\lambda_u}{2\gamma^2}$
FEL output power (MW)
undulator length (m)
0
2
4
6
8
10
12
14
15
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.5

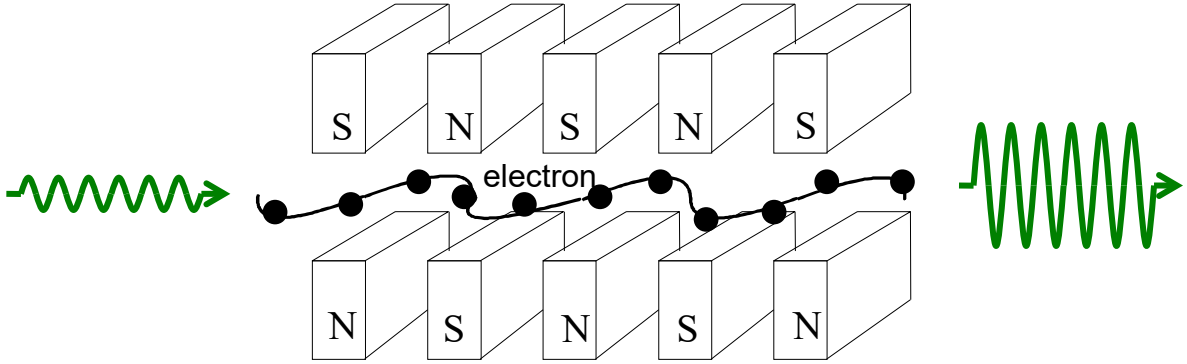

with taper
without taper
20% linear taper
bunching factor
undulator length (m)
0
0.1
0.2
0.3
0.4
0.5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.5

How critical is the prebunching?

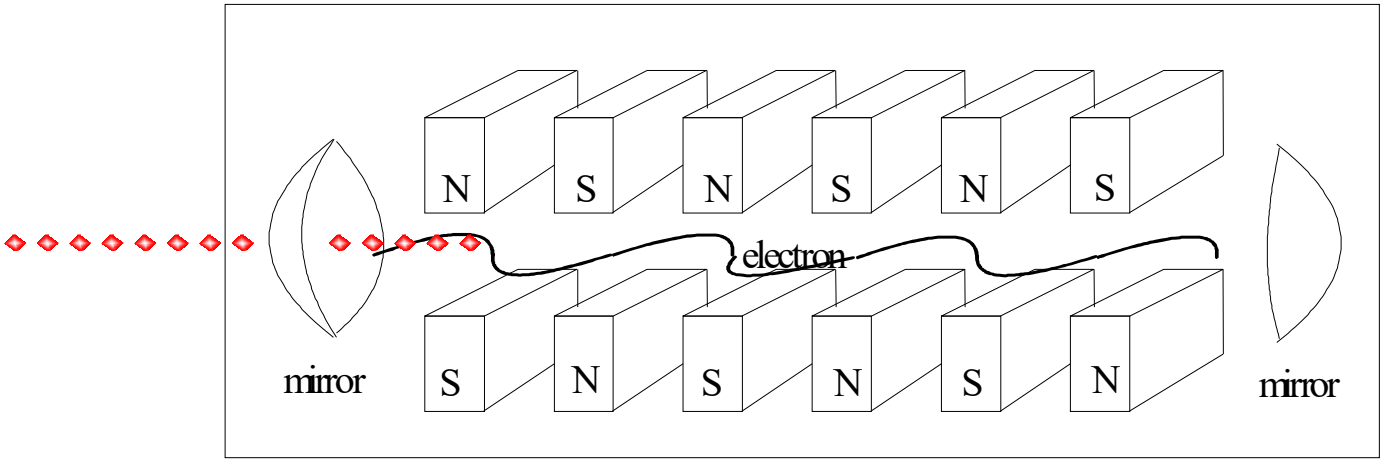


Variants of Superradiance FEL

1. Superradiance FEL amplifier: narrow-line, fully coherent radiation



2. Superradiance FEL Oscillator?

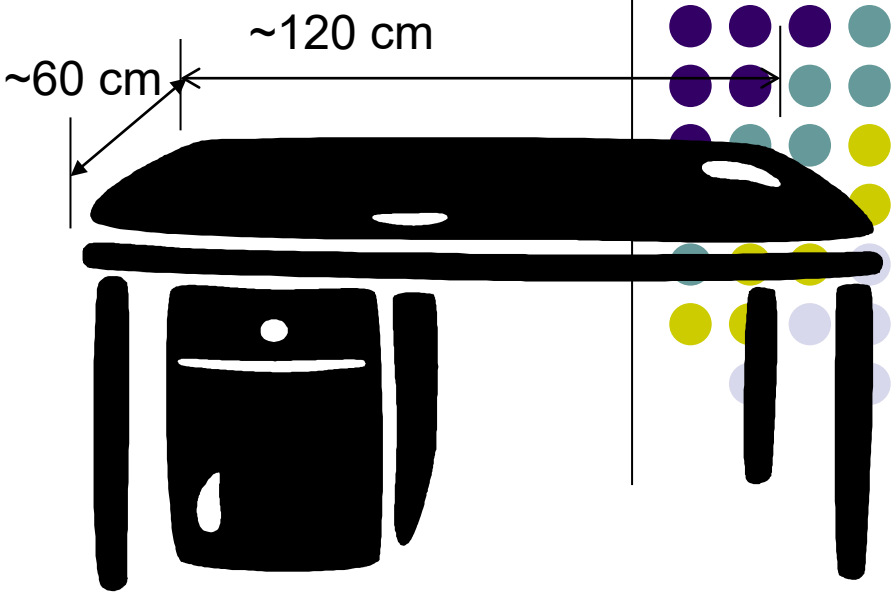


New physics yet to be discovered

“Desktop” MW Superradiant Free-electron Laser at THz Frequencies

High-energy OPtics & Electronics (HOPE) Laboratory
National Tsinghua University, Hsinchu, Taiwan

NTHU HOPE Lab
(established in Feb. 2008)



Concept Review

1. A superradiance FEL is driven by well pre-bunched electrons

2. Radiation enters the coherent regime right away with a pre-bunched electron beam.

3. A superradiance FEL skips the slow bunching process in a long undulator, permitting an ultra-compact size.

4. Comb-pulse superradiance narrows down the radiation linewidth.

5. A tapered undulator is recommended for a highly efficiency and ultracompact superradiance FEL.

6. The physics of a superradiance FEL oscillator is yet to be investigated.