

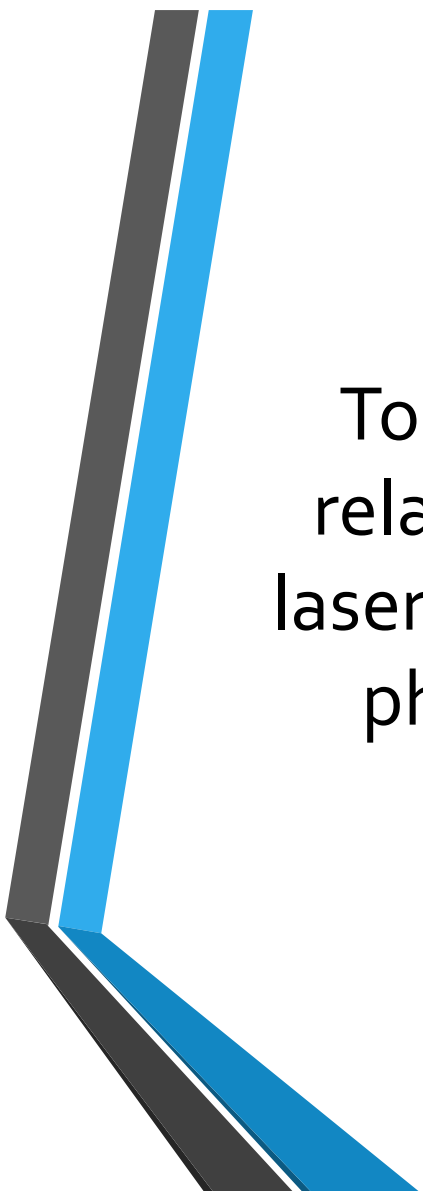
Laser-driven Plasma Accelerator and Application

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05.07.2023





Topics in relativistic laser-plasma physics

Introduction of plasma physics

Laser wakefield accelerator (LWFA)

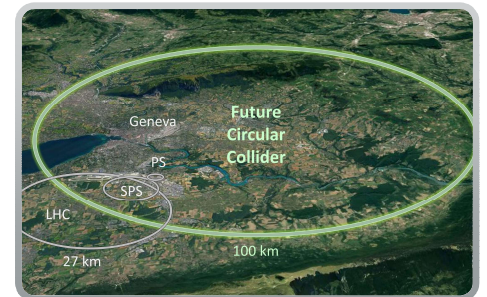
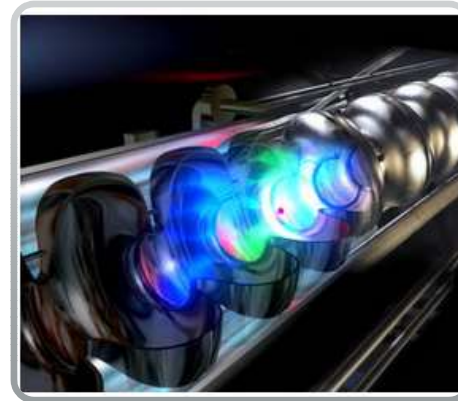
Relativistic optics

Application

- X-ray imaging
- Free electron laser

Conventional accelerators

- Accelerating fields of 10^5 MV/m due to material breakdown
- Colossus infrastructure
- CERN: \$8 billion to construct and \$1 billion pro year to maintain

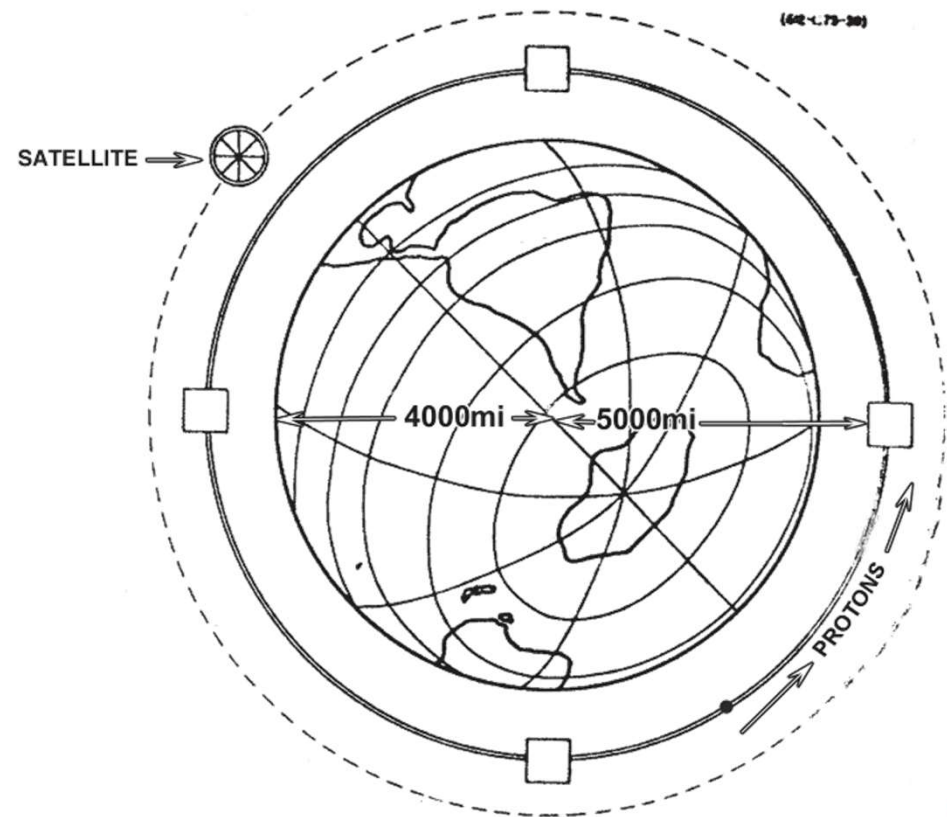


Limitations of Classical RF accelerators: 1 TeV electron



Fermi's Ultimate Accelerator

- To get PeV proton
- Preliminary design...8000 km, 20,000 gauss, 1600 km above the surface of earth
- "What we can learn impossible to guess...main element surprise...some things look for but see others...Look for multiple production...antinucleons...strange particles...puzzle of long lifetimes...large angular momentum?...double formation? (now called associated production) At present more probable..."
- Cost (in 1954): 170 B\$ (LHC: 8 B\$ in 2010)
- Estimated completion date: 1994



Orear, Jay. Enrico Fermi - The Master Scientist. 2004



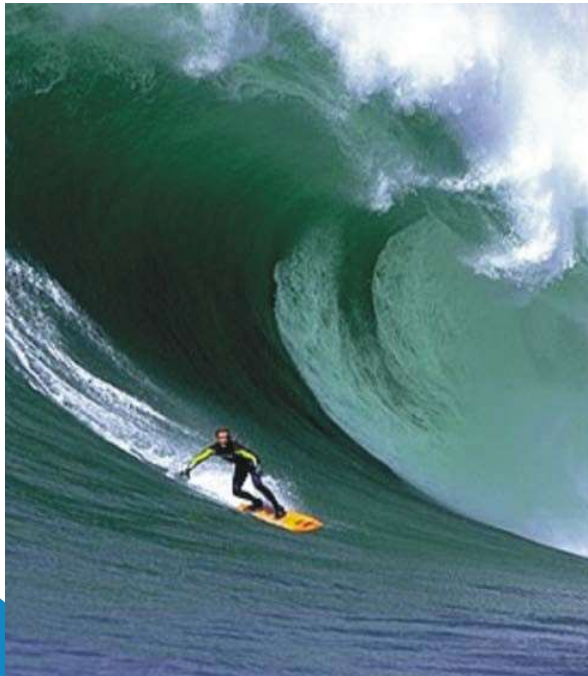
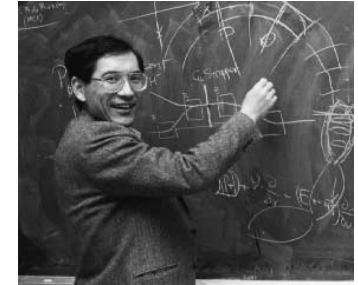
Laser Electron Accelerator

T. Tajima and J. M. Dawson

Department of Physics, University of California, Los Angeles, California 90024

(Received 9 March 1979)

An intense electromagnetic pulse can create a weak of plasma oscillations through the action of the nonlinear ponderomotive force. Electrons trapped in the wake can be accelerated to high energy. Existing glass lasers of power density 10^{18} W/cm^2 shone on plasmas of densities 10^{18} cm^{-3} can yield gigaelectronvolts of electron energy per centimeter of acceleration distance. This acceleration mechanism is demonstrated through computer simulation. Applications to accelerators and pulsers are examined.

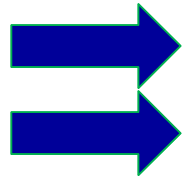




Classical RF accelerators v.s. plasma

Maximal accelerating fields due to breakdown:

$$E_{\text{max}} = 20 \text{ MV/m}$$



10-30km long accelerators to generate TeV
expensive

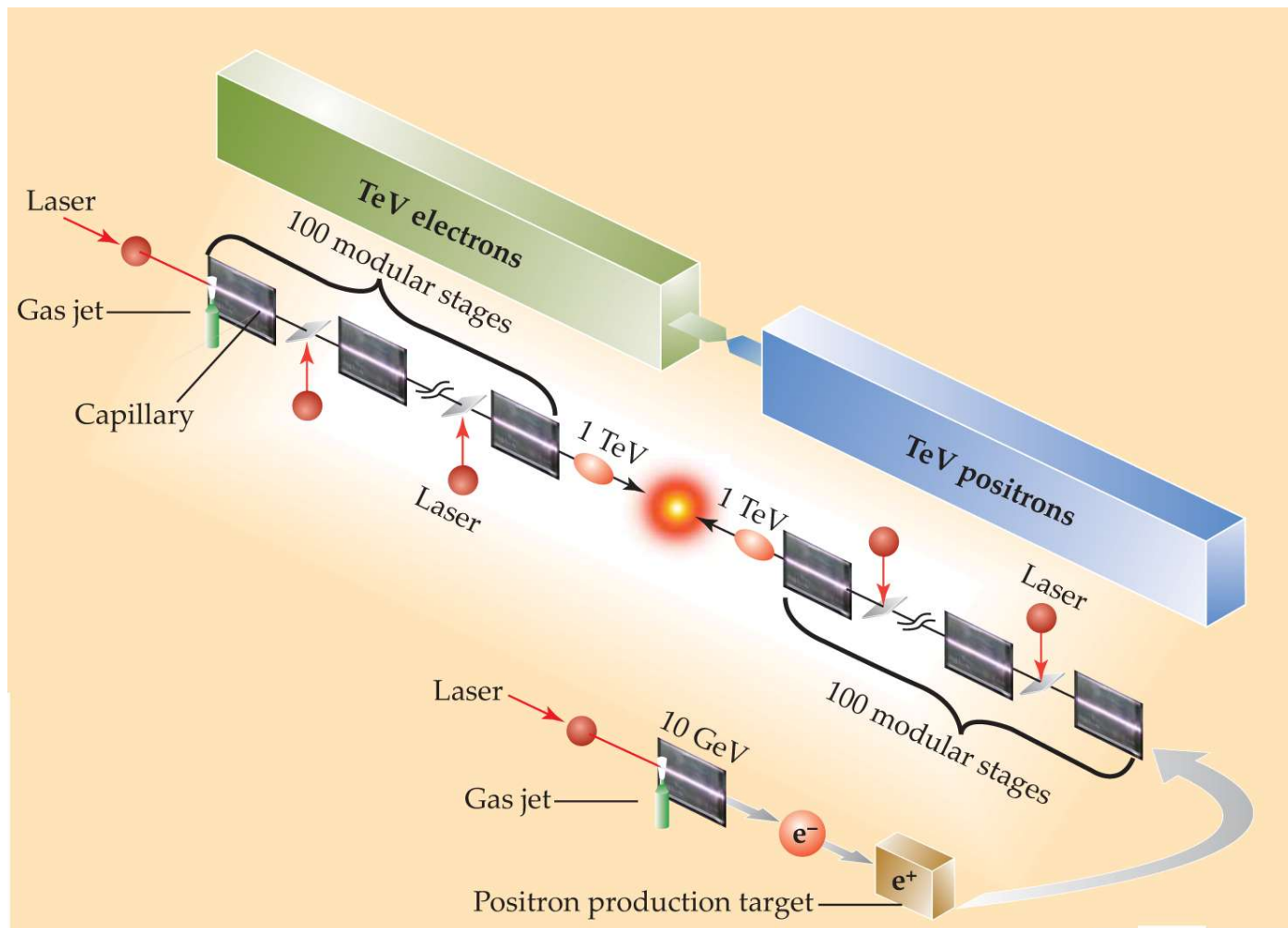
In plasmas there is no breakdown ! much higher
accelerating fields in

plasma density oscillations – plasma waves



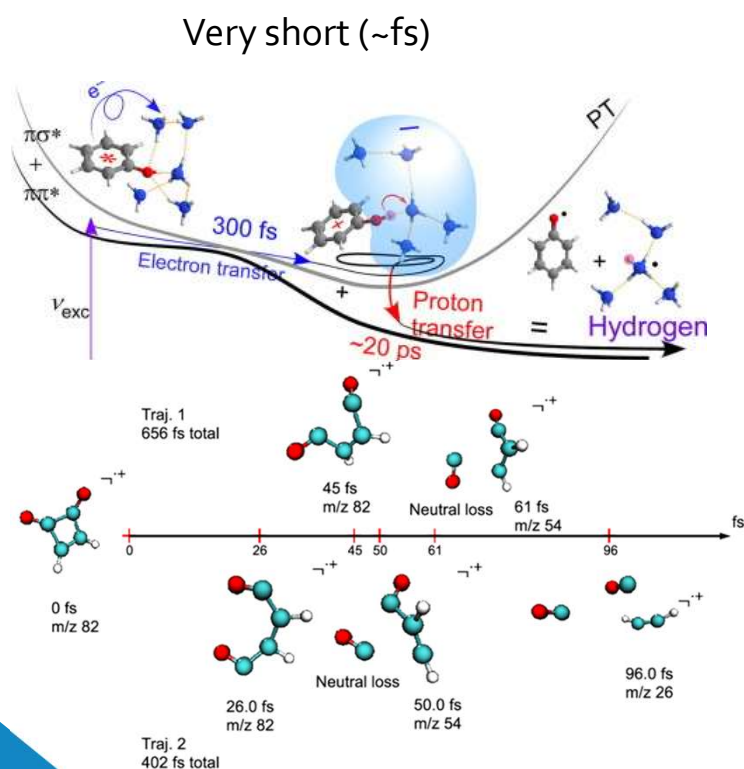
$E = 100 \text{ GV/m} - 1 \text{ TV/m}$

$10^3 - 10^4$ x higher fields
than in RF acceler

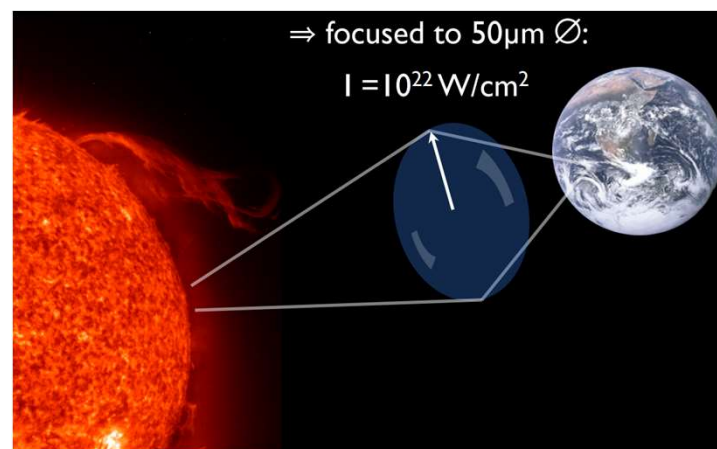


Physics Today **62**, 3, 44
(2009); <https://doi.org/10.1063/1.3099645>

What kind of laser are we talking about?



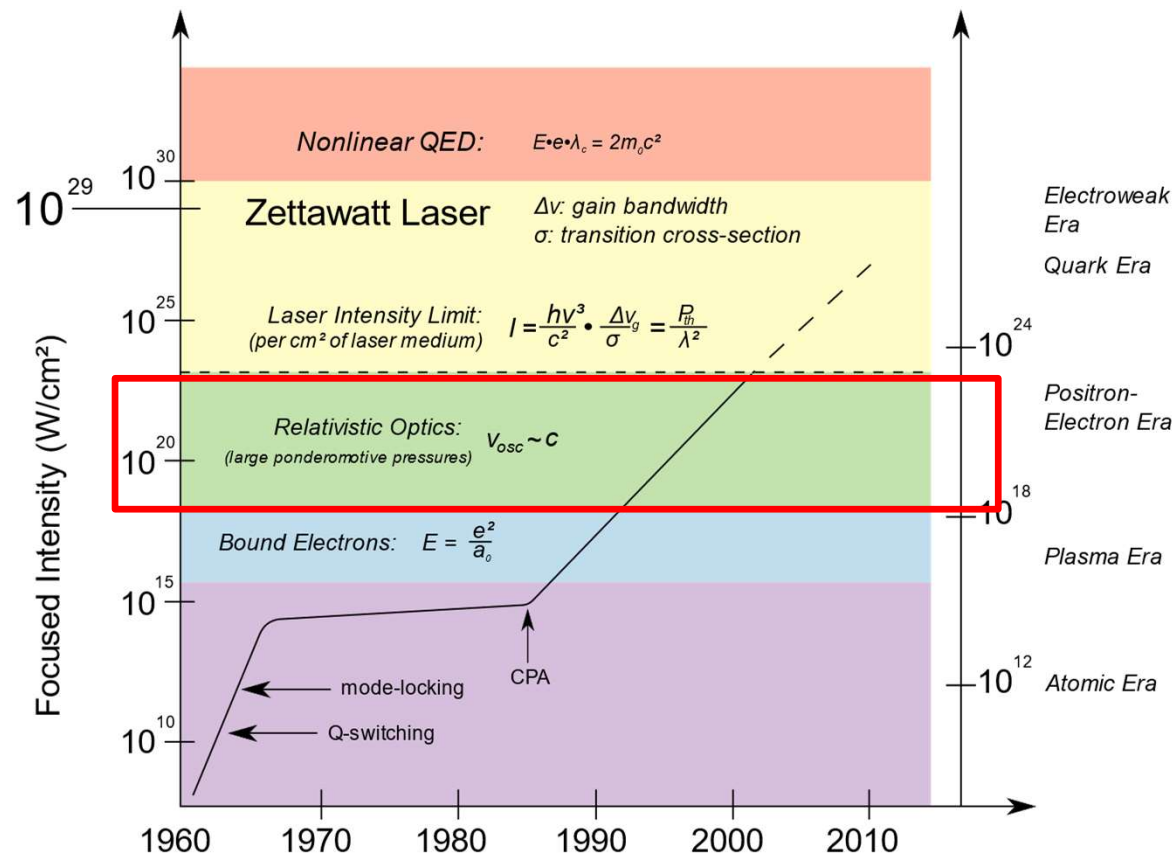
Very strong



Total power of sun on earth: $1.73 \times 10^{17} \text{ W}$

$$\frac{1.73 \times 10^{17} \text{ W}}{\pi \times (25 \mu\text{m})^2} \sim 8.8 \times 10^{21} \frac{\text{W}}{\text{cm}^2}$$

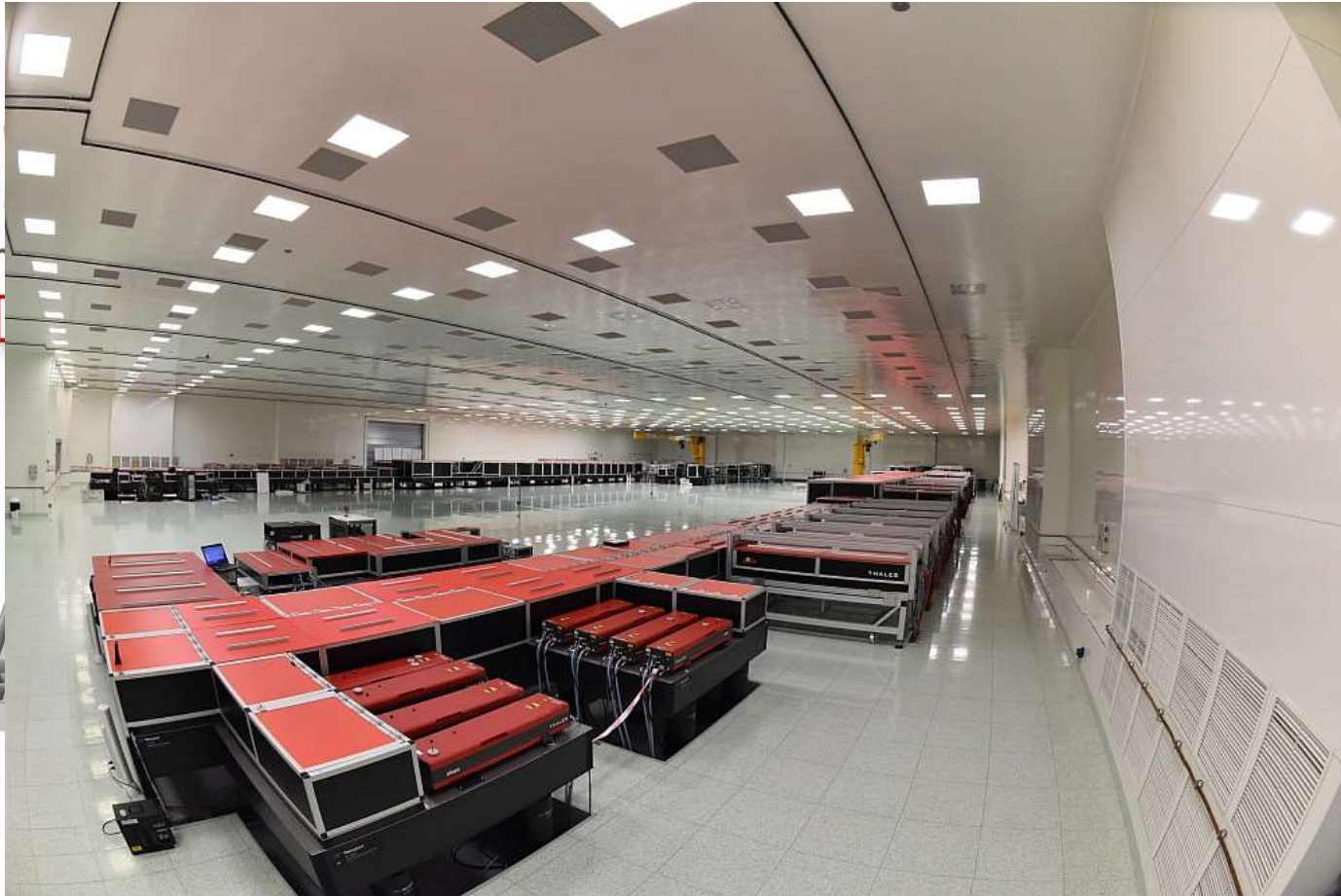
Introduction: What happens to the matter when the E field intensity increases?



NCU 100TW Laser System

beam line	first	second	third
pulse energy	3.3 J	450 mJ	200 mJ
central wavelength	810 nm	805 nm	870–920 nm
bandwidth (FWHM)	35 nm	34 nm	35 nm
duration (FWHM)	30 fs	34 fs	38 fs
peak power	110 TW	13 TW	5 TW
energy fluctuation	1.1%	1.8%	2.6%
focusability (M^2)	1.2	1.1	1.3
enclosed energy	77%	81%	72%
pointing fluctuation	4.5 μrad	4.8 μrad	4.7 μrad
temporal contrast at -100 ps	4×10^{-10}	2×10^{-9}	2×10^{-9}

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Vulcan OPCPA

**All Optical Parametric Amplification allows
for Ultra-broad Bandwidth**

Vulcan OPcpa PETawatt Laser

	Vulcan Long Pulse	Vulcan Short Pulse	VOPPEL
Pulse Length	3 ns	500 fs	< 30 fs
Spectral Bandwidth	-	5nm	160 nm
Final Beam Size	96 mm	600 mm	200 mm
Energy on Target	250 J	500 J	30 J
Final Beam Shape	Round	Round	Round
Rep. Rate	20 min	20 min	20 min*

*To be decreased to 5 min air-cooled amplifier

What is plasma?

- Composition: ionized gas, ion and free electron
- Plasma is electrically conducting
- Weak coupling between pairs of particles, but strong collective interactions:

- Debye shielding
- electron plasma oscillations

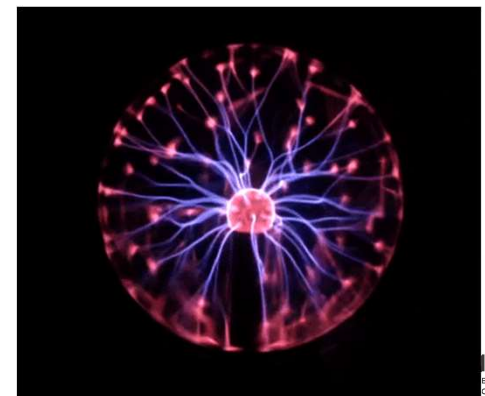
$$\lambda_{Ds} \equiv \left(\frac{\epsilon_0 K T_e}{n e^2} \right)^{1/2} \quad \omega_p \equiv \left(\frac{n e^2}{\epsilon_0 m} \right)^{1/2}$$

- Hierarchy of length scales, expressed in terms of fundamental plasma parameter

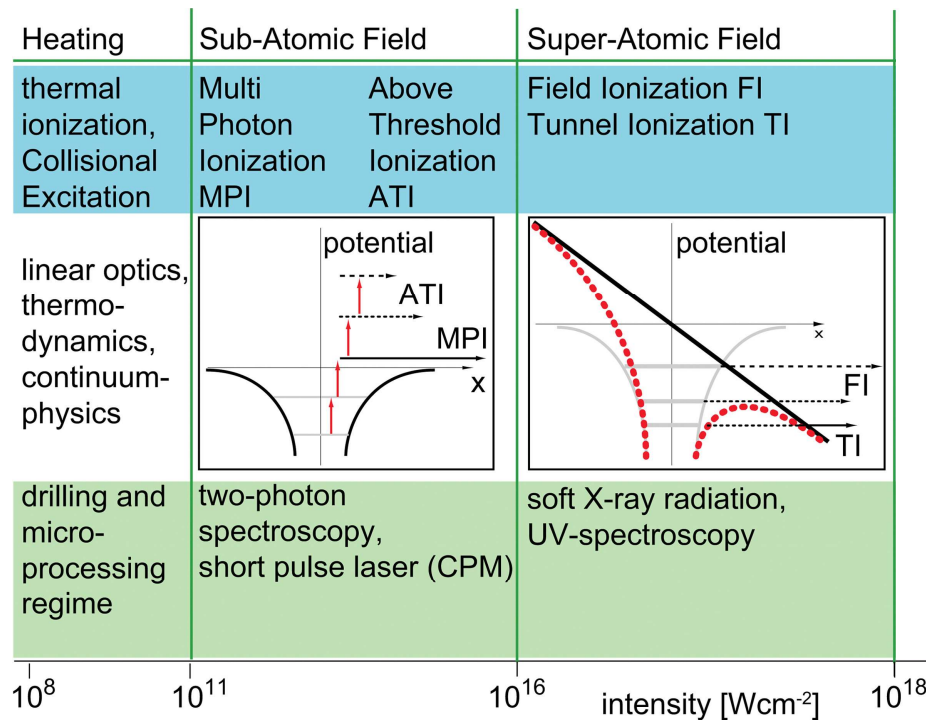
$$\Lambda \equiv \frac{4\pi}{3} \lambda_D^3 n = \# \text{ of particles in a Debye sphere}$$

- Dispersive for electromagnetic wave

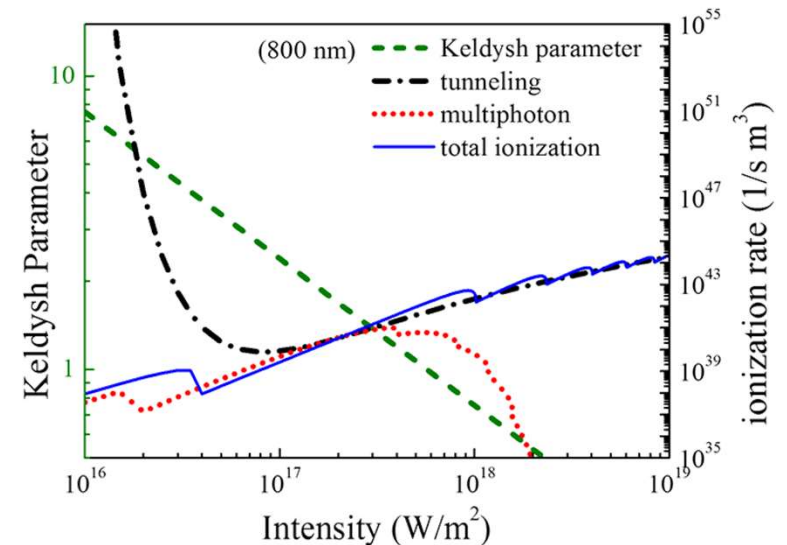
$$\text{Critical density } n_{crit}: \omega_p = \omega_0$$



Ionization by laser



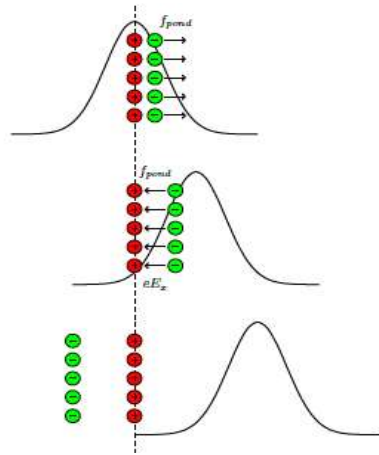
Journal of Laser Applications **25**, 012006
(2013)
DOI: 10.5772/65637



Keldysh parameter:

$$\gamma = \frac{\sqrt{2mE_{ion}}}{eE_L/\omega}$$

Generation of Wakefield



Laser energy → Plasma Wave
 Plasma Wave → Electrons

Plasmas can support large
 Electrostatic fields of 100 GV/m for
 $n_e = 10^{18} \text{ cm}^{-3}$

Quiver motion:

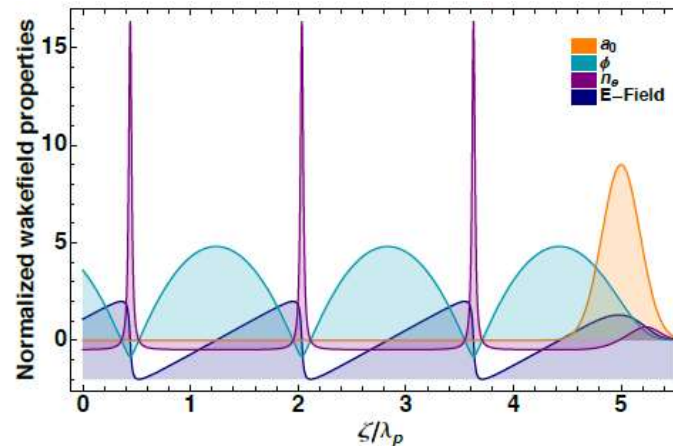
$$a_0 = \frac{eE}{m\omega c} = 1 \Leftrightarrow I = 10^{18} \text{ W cm}^{-2}$$

Ponderomotive force:

$$\langle F^{(2)}(r, t) \rangle = -\frac{1}{4} \frac{e^2}{m\omega^2} \nabla E(r, t)^2$$

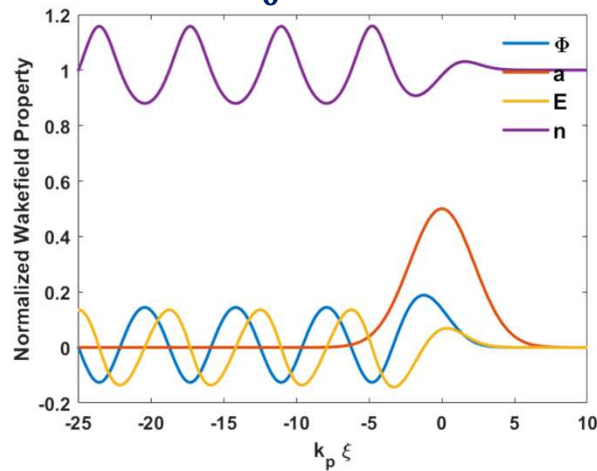
Resonant Excitation:

$$c\tau_{\text{pulse}} \approx \frac{\lambda_p}{2} = \frac{\pi c}{\omega_p} \sim \frac{1.7 \times 10^{10}}{\sqrt{n_e}}$$

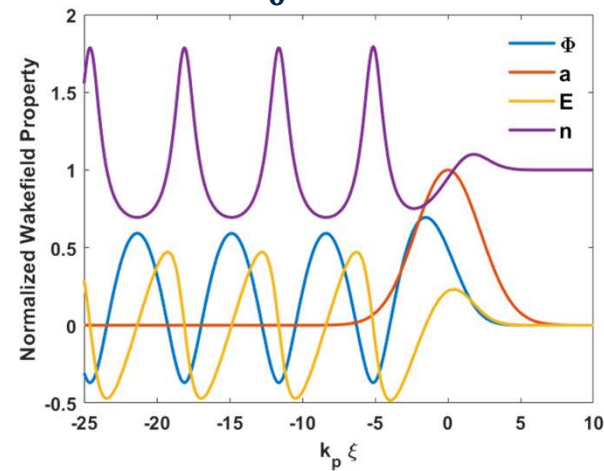


1D description, linear and non-linear plasma waves

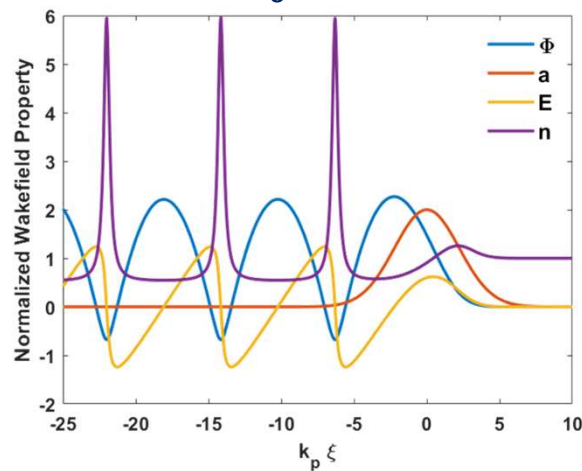
$a_0 = 0.5$



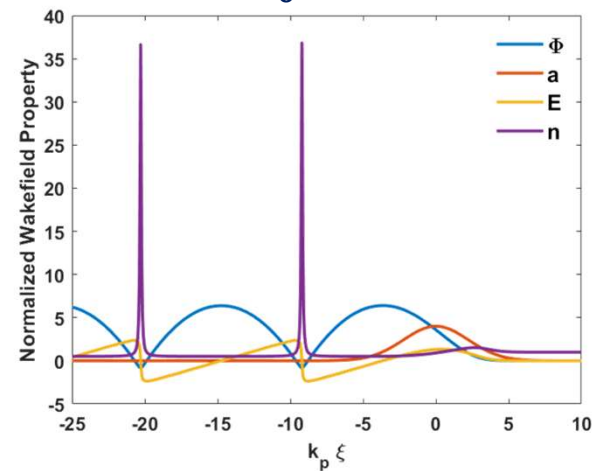
$a_0 = 1$



$a_0 = 2$



$a_0 = 4$





Strength of Wakefield

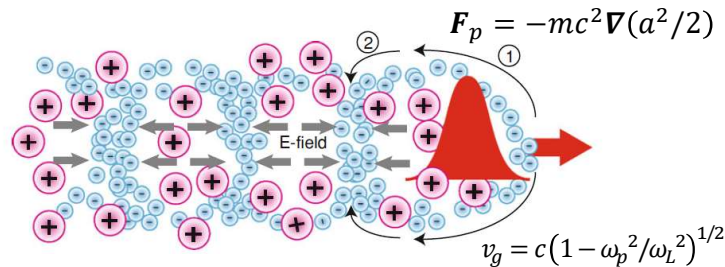
Single electron \Rightarrow plasma ($n_{\text{crit}} = 10^{21} \text{ cm}^{-3}$ @800nm)

- In plasma, laser interaction generates additional
 - E -fields (due to the separation of electrons from ions)
 - B -fields (due to laser-driven electron currents)
- They are quasi-stationary and of the same order as laser fields:
 - $E_L \approx 3 \times 10^{12} \text{ V/m} \cdot a_0$
 - $B_L \approx 10^8 \text{ Gauss} \cdot a_0$
- Plasma is governed by **collective oscillatory electron motion**

T. Tajima & J.M. Dawson,
Phys. Rev. Letter 43, 267 (1979)

Wakefield excitation

Laser field (vector potential a)



1-D linear approximation $a^2 \ll 1$ $\xi = z - ct$

$$\left(\frac{\partial^2}{\partial \xi^2} + k_p^2 \right) \frac{\delta n}{n_0} = \nabla^2 \frac{a^2(\xi)}{2}$$

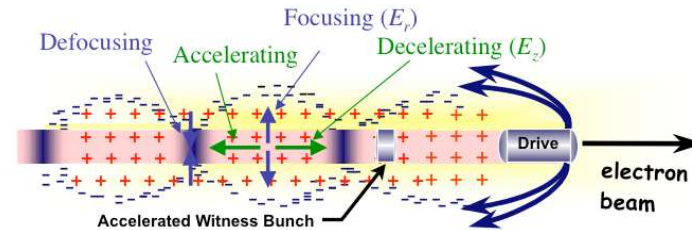
Plasma wave forced density oscillation Ponderomotive Force

1. The ponderomotive force (gradient of laser intensity) pushes electrons away from axis and generates a charge separation between ions and electrons

2. The restoring force initiates a local density oscillation with frequency $\omega_p = (4\pi e^2 n_0 / m_e)^{1/2}$

vector potential $a_0 = \frac{eA}{mc} = 0.854 \sqrt{I_0 [10^{18} \text{ W/cm}^2] \cdot \lambda [\mu\text{m}]}$
 wave number $k_p = \omega_p / c$

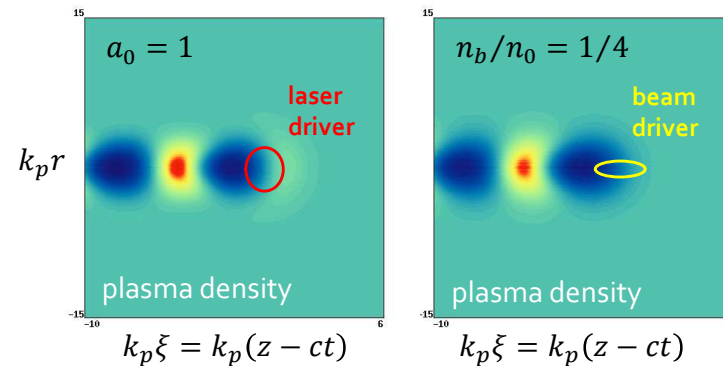
Particle beam field



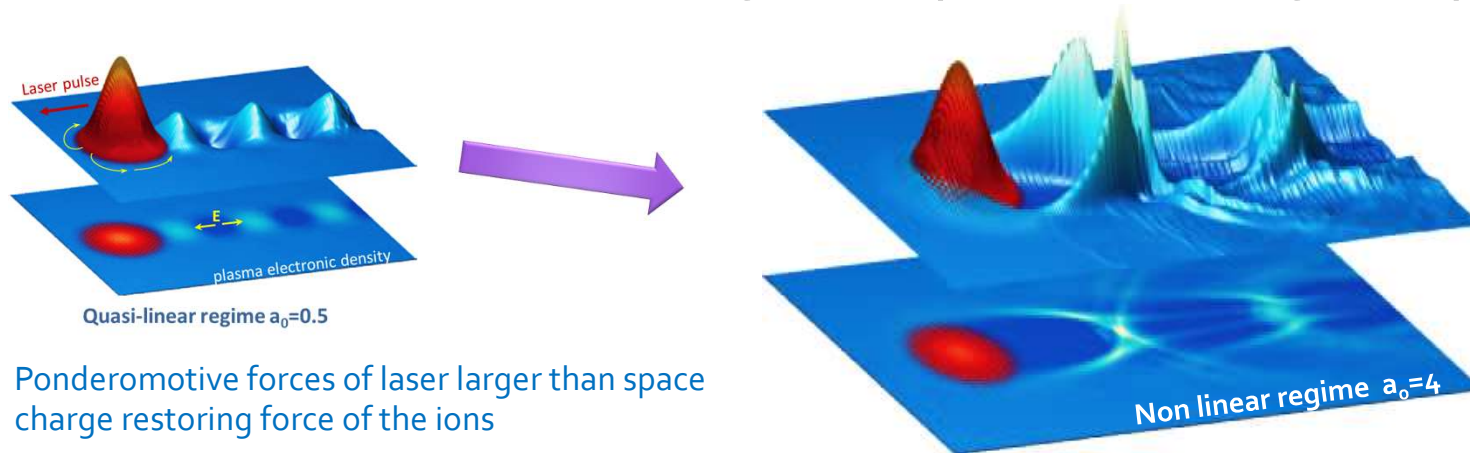
1-D linear approximation $n_b/n_0 \ll 1$

$$\left(\frac{\partial^2}{\partial \xi^2} + k_p^2 \right) \frac{\delta n}{n_0} = -k_p^2 \frac{n_b}{n_0}$$

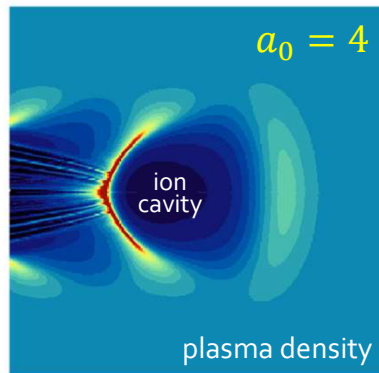
Space charge force



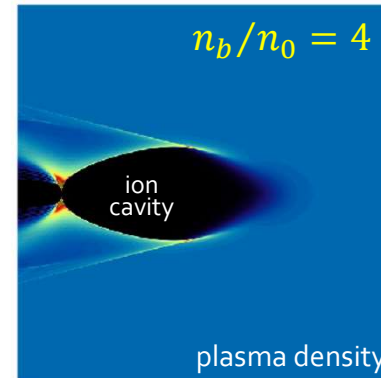
3D non-linear regime (bubble regime)



- Ponderomotive forces of laser larger than space charge restoring force of the ions
 \Rightarrow All electrons of the plasma are expelled
 \Rightarrow ionic cavity \Rightarrow linear fields

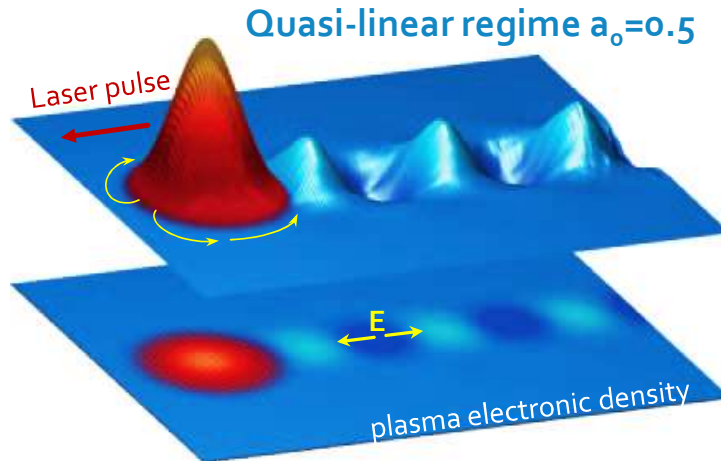


*driver =
laser*



*driver =
e-beam*

Characteristic scales of LWFA



➤ Plasma wavelength

$$\lambda_p \approx 33 \mu\text{m} / \sqrt{n_0 (10^{18} \text{cm}^{-3})}$$

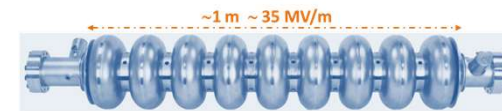
- for $\sim 10^{18} \text{cm}^{-3} \rightarrow \lambda_p \sim 30 \mu\text{m}$
- LWFA produce ultrashort bunches

➤ Accelerating field

assumption: in linear regime, all electrons oscillate at ω_p

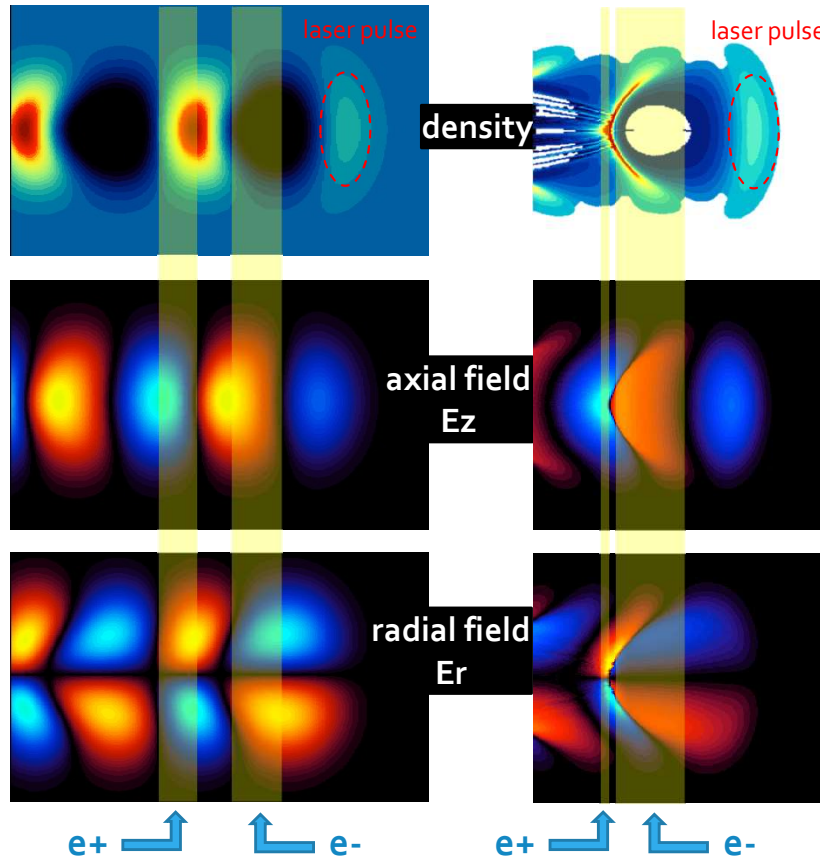
$$E_0 = m_0 c \omega_p / e \approx 96 \text{ GV/m} \cdot \sqrt{n_0 (10^{18} \text{cm}^{-3})}$$

- for $\sim 10^{18} \text{cm}^{-3} \rightarrow E_0 \sim 100 \text{ GV/m}$
- Accelerating gradients several orders of magnitude larger than conventional RF cavities



Useable phase range (accelerating & focusing)

Quasi linear regime $a_0 \sim 1$ Non linear regime $a_0 \geq 4$



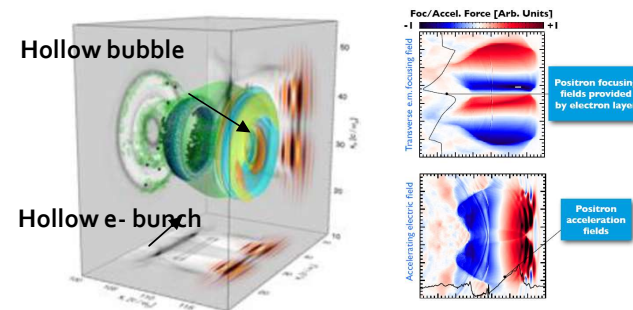
❑ **Quasi linear regime**
quasi-symmetric ranges of useable phases

❑ **Bubble regime**
very asymmetric regions

- focusing for e^-
- defocusing for e^+

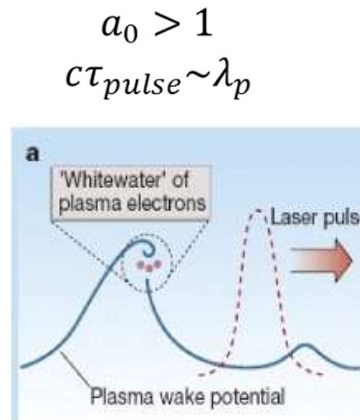
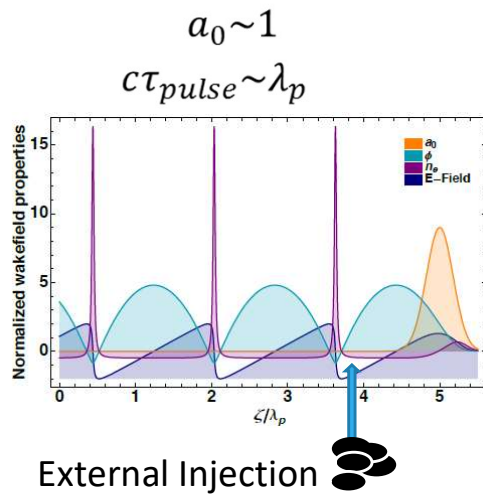
❑ **Ideas for e^+ focusing**

- Combine multiple laser modes
- Use of orbital angular momentum lasers (OAM) to drive doughnut wakefields



J. Vieira and J.T. Mendonça, PRL 112, 215001 (2014)

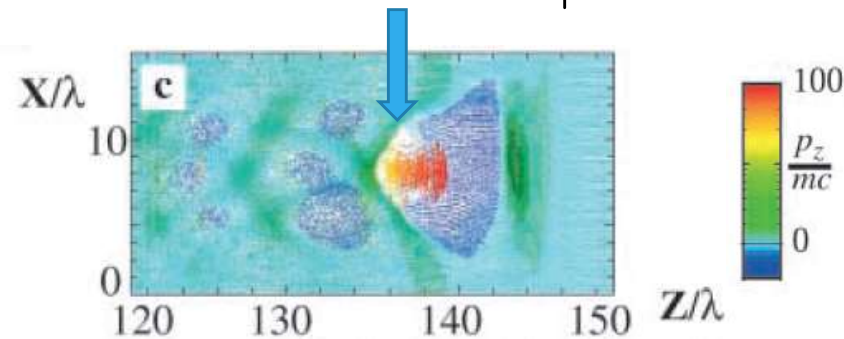
Injection in Wakefield



Wavebreaking
Self-injection



Wave breaks at first
oscillation. No plasma wave



Bubble acceleration principle

Characteristics:

- High efficiency (~20%)
- Quasi-monoenergetic electron spectrum ($\Delta E/E \sim$ as low as 0.2 %)
- Low normalized emittance (few mm•mrad)
- Large accelerating fields (100GV/m – 1TV/m)
- Very short acceleration distance (100 μ m – 1mm)

Requirements:

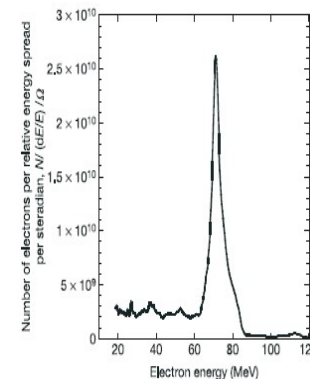
- Relativistic laser intensities 10^{18} - 10^{19} W/cm²



C.G.R. Geddes et al.,
Nature 431, 538, (2004)

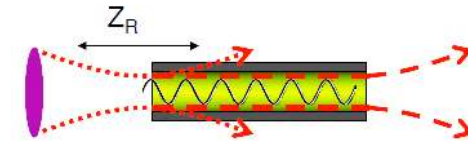
S.P.D. Mangles et al.,
Nature 431, 535, (2004)

J. Faure et al.,
Nature 431, 541, (2004)



Energy gain limitations

- ❑ Laser **Diffraction** ~ Rayleigh range
 - Controlled by relativistic self-guiding, pre-formed plasma channel, capillary guiding...



- ❑ Beam – plasma wave **Dephasing** ($v_p < v_{e-}$)
 - Controlled by density tapering

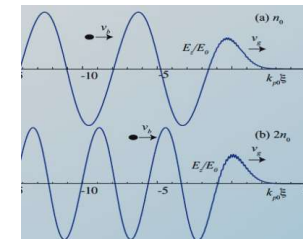
$$L_{max} \propto n_0^{-3/2}$$

$$n_0 \nearrow \Rightarrow \lambda_p \searrow$$

- ❑ Laser energy **Depletion**

$$L_{deplete} \propto \lambda_p^3 / \lambda_L^2 \propto n_0^{-3/2}$$

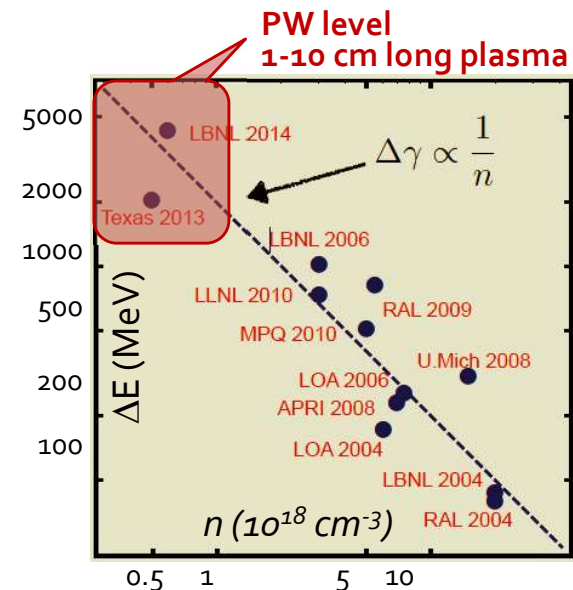
- Laser energy deposition into wave excitation



- ❖ Accelerating Gradient $G \sim E_0 = mc\omega_p/e \propto \sqrt{n_0}$
- ❖ Energy Gain $W = G \times L_{acc} \propto 1/n_0$
- ❖ Laser peak power $P_{laser} \propto 1/n_0$



To increase the energy gain in a plasma module
Decrease the density and increase the laser power



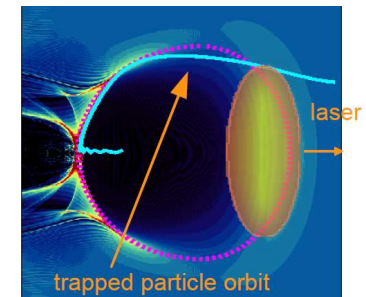
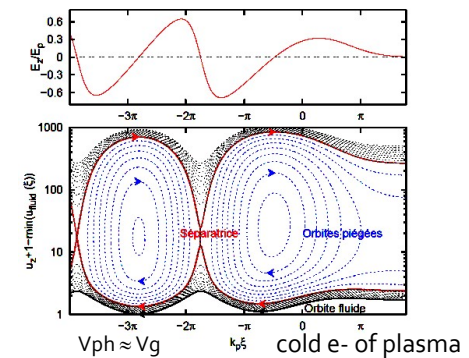
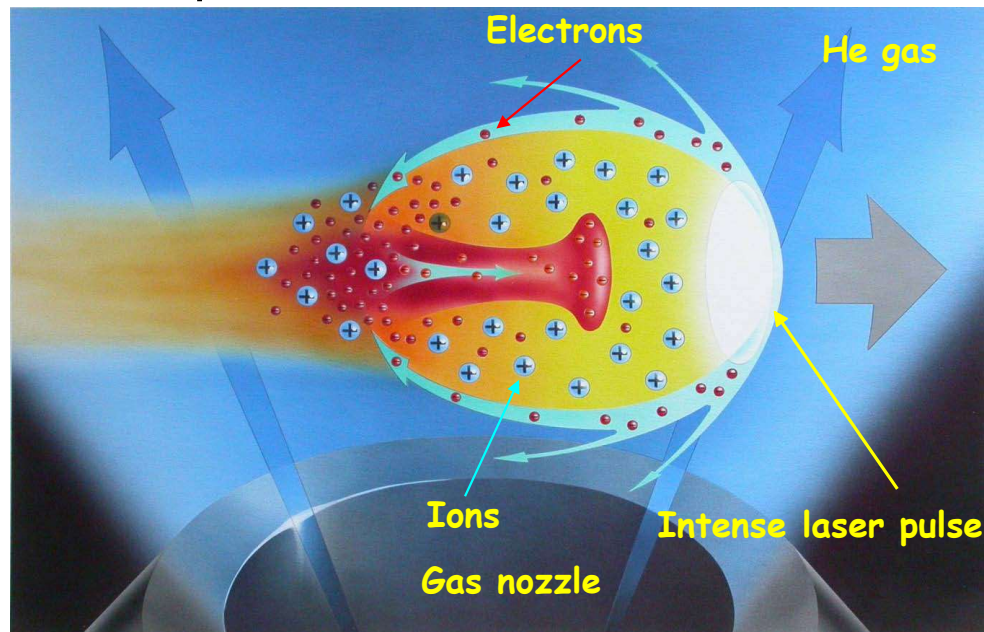


Injection mechanisms

- **Injector is of prime importance**
 - Determines the performances of the overall accelerator : charge, energy spread, emittance
- **Better to decouple the injection mechanism from the acceleration mechanism**
 - Individual adjustment of parameters, stability, control
- **External injection**
 - Fine definition of beams delivered by conventional photo-injectors
 - Issues: to achieve an ultra-short bunch $< \lambda_p/4 \sim 10\mu\text{m}$ (30 fs)
possible solution : longer bunch injection, further self-compressed and accelerated in the plasma wakefield, but limited to low charges (1-2 pC)
 - Synchronization between laser and injected beam
- **Internal injection**
 - Various mechanisms more or less complex to implement
 - Issues: to achieve high charge $> 100 \text{ pC}$, and low energy spread
 \Rightarrow the trapping of e^- by the plasma wave should be highly localized

Injection mechanisms

1. Self-injection (so-called bubble or blow out regime): needs strong wakefield to trap the cold plasma e-



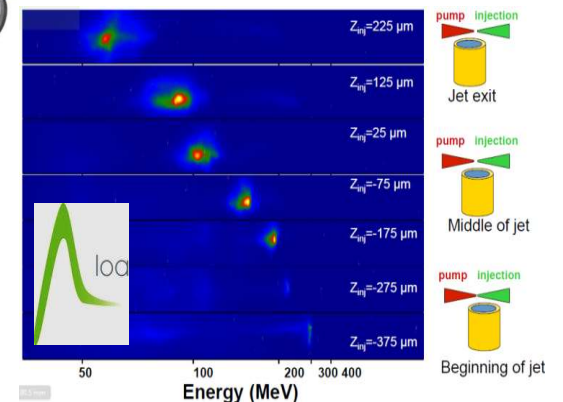
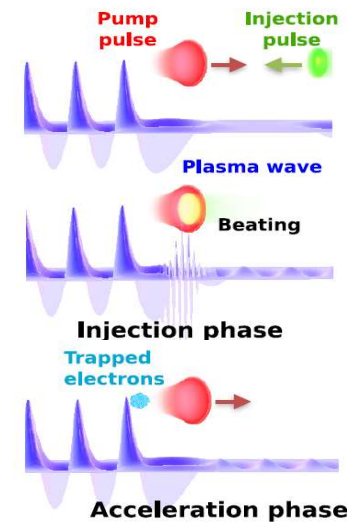
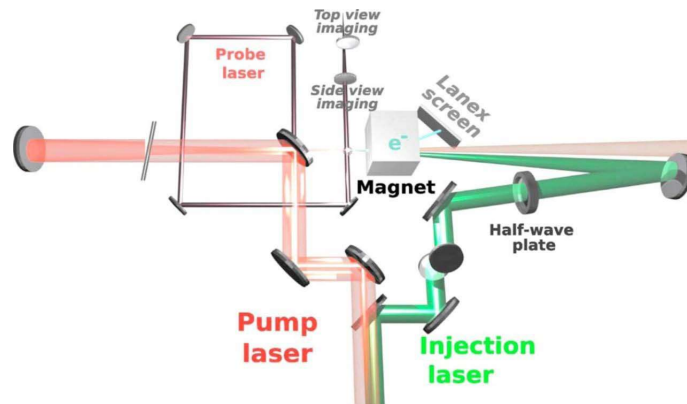
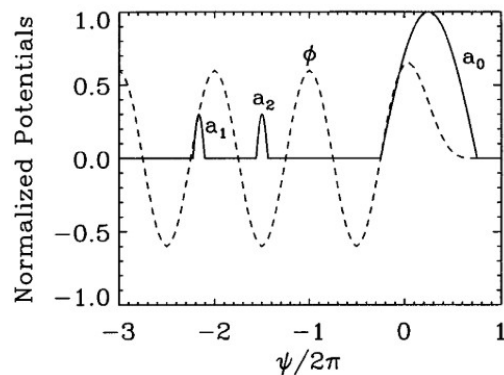
- On-axis or off-axis injection
 - Non-linear regime
 - Self-guiding
- ⇒ Uneasy to control

S. Bulanov et al., PRE 58, R5257 (1998)
C.G.R. Geddes et al., PRL 100, 215004 (2008)

Injection mechanisms

2. Counterpropagating Pulses

- Excite plasma wave below the self-injection threshold
- Counter-propagating injection pulse: to generate a beating with main pulse → triggers the injection



Faure et al, Nature 444 (2006)

VOLUME 79, NUMBER 14 PHYSICAL REVIEW LETTERS 6 OCTOBER 1997

Electron Injection into Plasma Wake Fields by Colliding Laser Pulses

E. Esarey,¹ R. F. Hubbard,¹ W. P. Leemans,² A. Ting,¹ and P. Sprangle¹

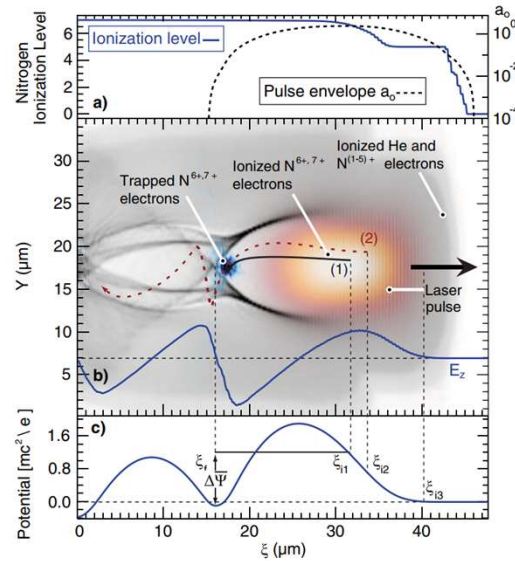
¹Beam Physics Branch, Plasma Physics Division, Naval Research Laboratory, Washington, D.C. 20375-5346

²Ernest Orlando Lawrence Berkeley National Laboratory, University of California at Berkeley, Berkeley, California 94720
(Received 6 February 1997)

Injection mechanisms

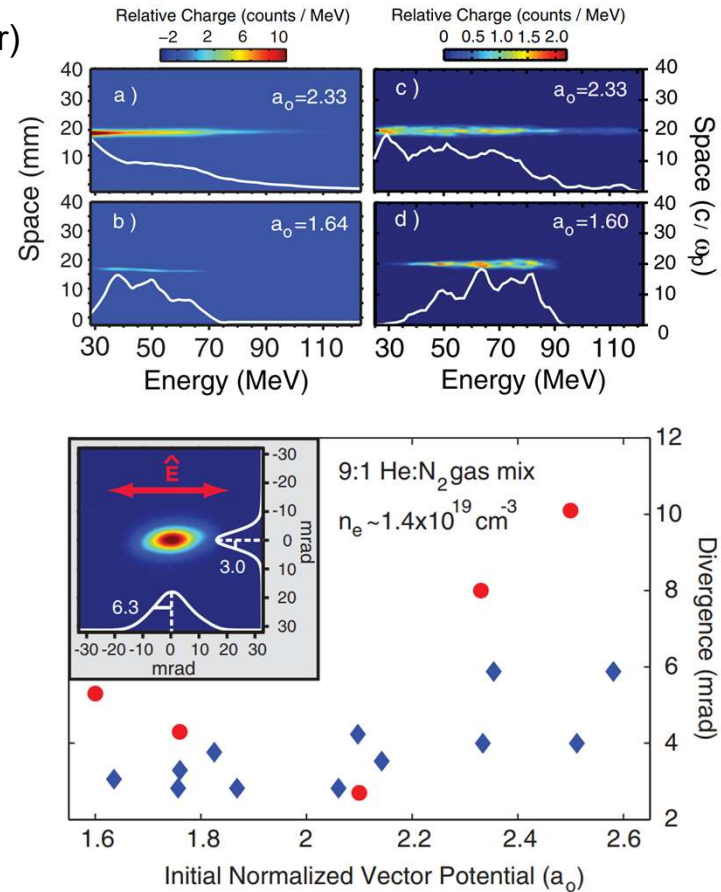
3. Ionization-induced injection

- Ionization of inner shells of high Z atom (N,Kr,Ar) at the peak intensity of the laser pulse
- e- injected at the proper phase for trapping and acceleration to high energies
- Potential for high charge > 100 pC but high E_spread



Injection and Trapping of Tunnel-Ionized Electrons into Laser-Produced Wakes

A. Pak, K. A. Marsh, S. F. Martins, W. Lu, W. B. Mori, and C. Joshi
Phys. Rev. Lett. **104**, 025003 – Published 15 January 2010



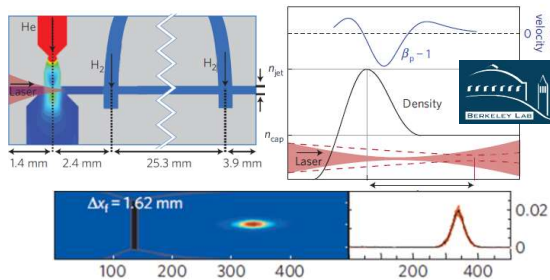
Injection mechanisms

4. Downramp injection

- **soft gradient** $L_{\text{grad}} \gg \lambda_p$

→ slows down the plasma wave

A.J. Gonsalves et al, Nature Physics (2011)



$$\frac{v_p}{c} = \left(1 + \frac{\zeta}{k_p} \frac{dk_p}{dz} \right)^{-1}$$

$$dk_p/dz = (k_p/2n_e)(dn_e/dz)$$

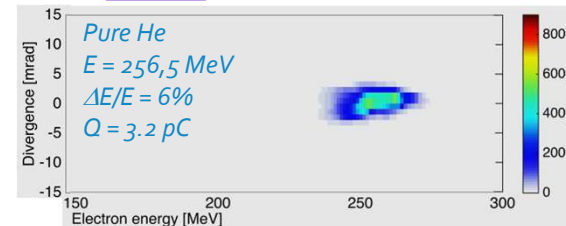
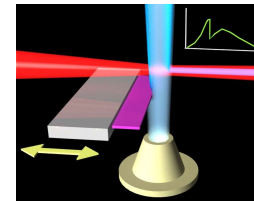
- **sharp density ramp** $L_{\text{grad}} \leq \lambda_p$

→ Increase of the bubble size

places e- at the right phase

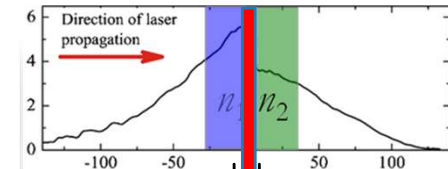
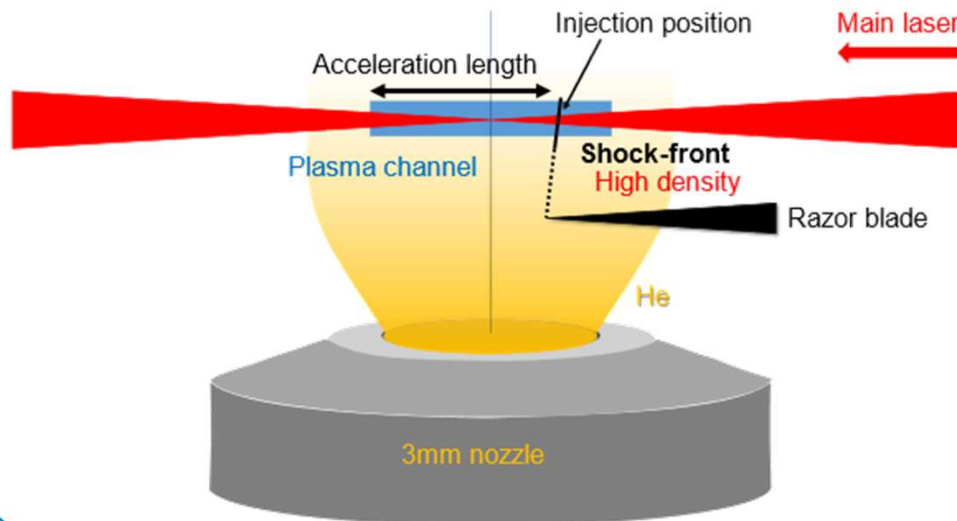
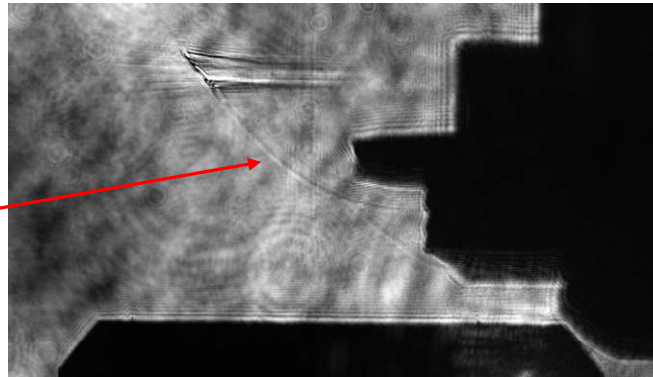
→ Localize injection → reduce E spread

movable blade

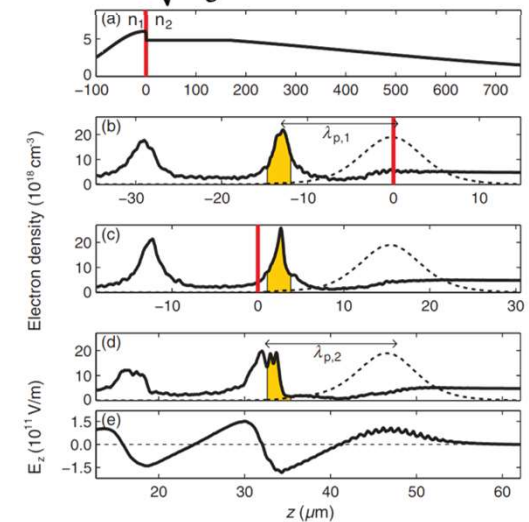


Principle of shock-front injection

Shock wave

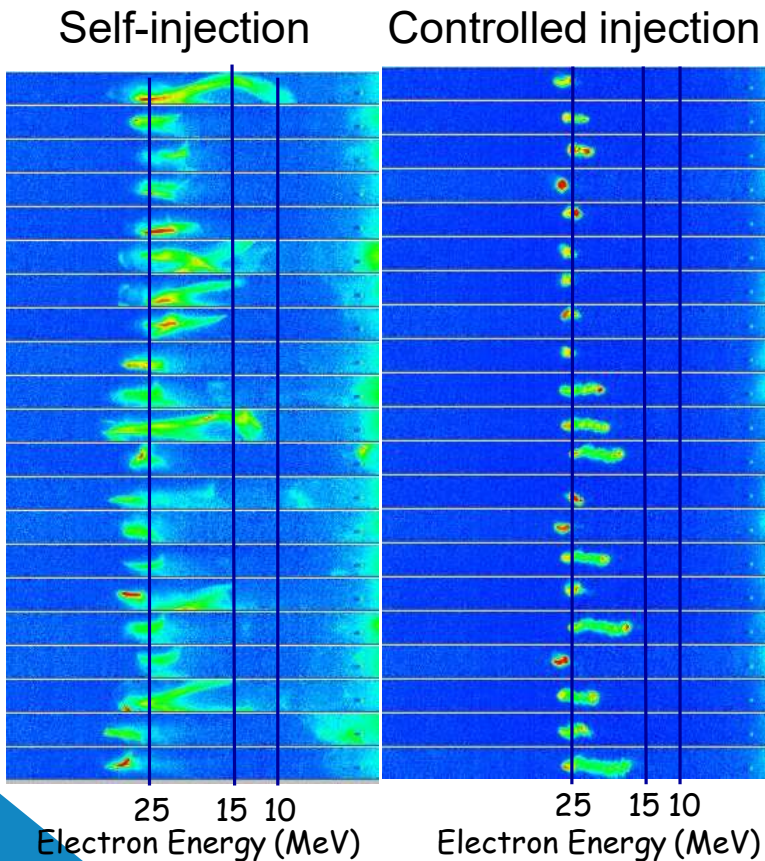


$$\lambda_p \propto \frac{1}{\sqrt{n_e}} \Rightarrow \lambda_{p,1} < \lambda_{p,2}$$



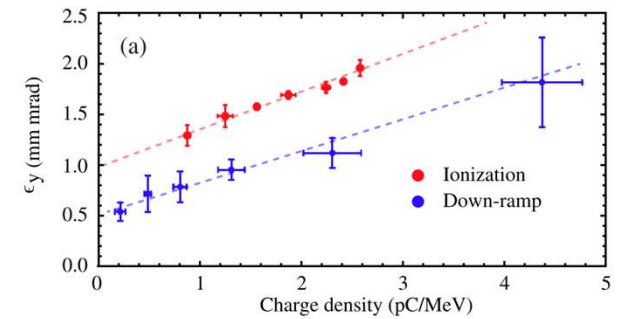
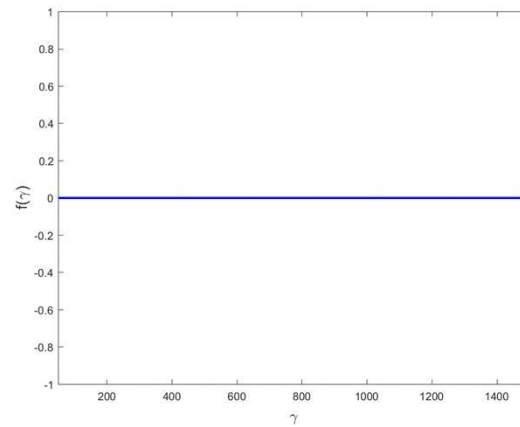
Phys. Rev. Lett. 110, 185006 (2013)
Phys. Rev. ST-Acc and Beams 13, 091201 (2010)

Improved electron spectra



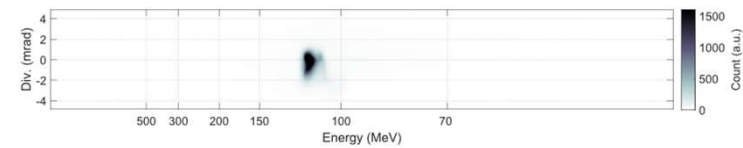
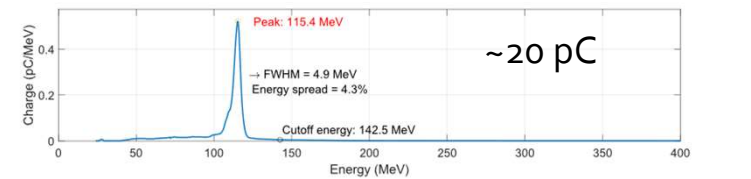
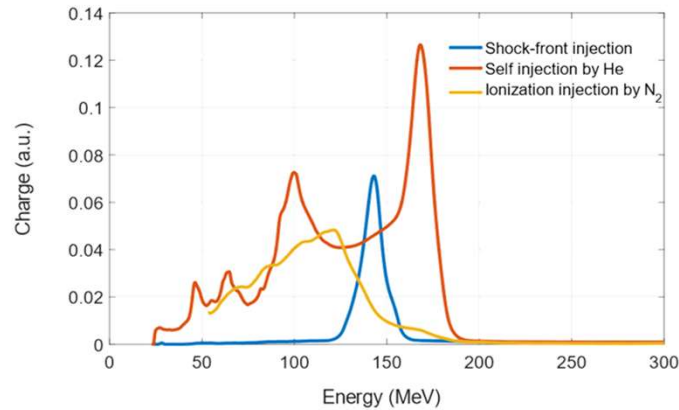
Comparing the two good runs with injection by transverse wave-breaking and by sharp density transition

	Self-injection		Sharp density transition	
Peak energy	25.1	8.0 MeV	27.6	3.9 MeV
Mean energy spread	8.5 % (RMS)		2.6 % (RMS)	
Peak charge	3.8	3.8 pC	2.9	1.9 pC
Divergence	11.0	2.9 mrad	11.0	1.7 mrad
Pointing	3.8 mrad (RMS)		3.8 mrad (RMS)	

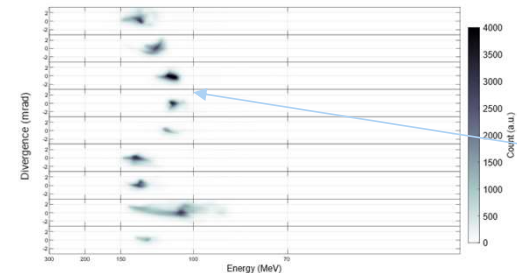


Properties of Shock-front Injection

Comparison of injection mechanism by NCU 100TW laser system



Consecutive shots



Due to PID oscillation

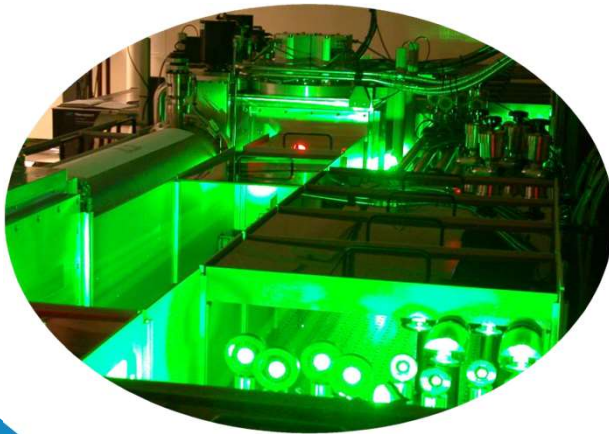
Injection method	Electron density (cm ⁻³)
Ionization injection (N ₂)	2.0×10^{18} (neutral)
Self-injection (He)	$8.5 \times 10^{18} - 1.0 \times 10^{19}$
Shock-front injection (He)	3.7×10^{18}

GeV: channeling over cm-scale

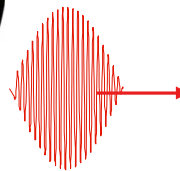
- Increasing beam energy requires increased dephasing length and power:

$$\text{Capillary } \Delta W[\text{GeV}] \sim I[\text{W}/\text{cm}^2] / n[\text{cm}^{-3}]$$

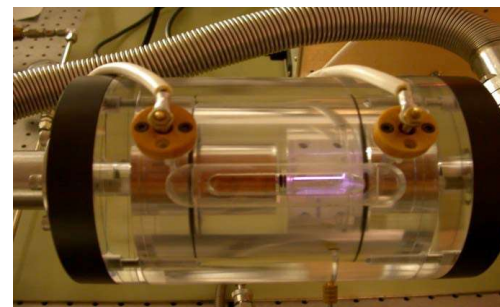
- Scalings indicate cm-scale channel at $\sim 10^{18} \text{ cm}^{-3}$ and $\sim 50 \text{ TW}$ laser for GeV
- Laser heated plasma channel formation is inefficient at low density
- Use capillary plasma channels for cm-scale, low density plasma channels



Laser: 40-100 TW,
40 fs 10 Hz



Plasma channel technology: Capillary



3 cm

1 GeV

e⁻ beam

0.5 GeV Beam Generation

225 μm diameter and 33 mm length capillary

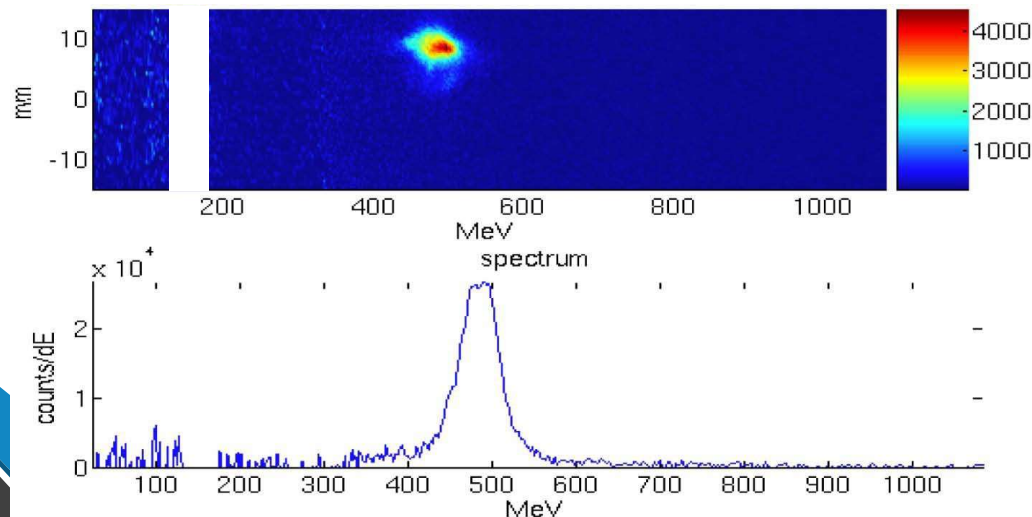
Density: $3.2\text{--}3.8 \times 10^{18}/\text{cm}^3$

Laser: $950(\pm 15\%) \text{ mJ/pulse}$ (compression scan)

Injection threshold: $a_0 \sim 0.65$ ($\sim 9\text{ TW}$, 105 fs)

Less injection at higher power

- Relativistic effects
- Self-modulation



Stable operation
500 MeV Mono-energetic beams:

$a_0 \sim 0.75$ (11 TW, 75 fs)

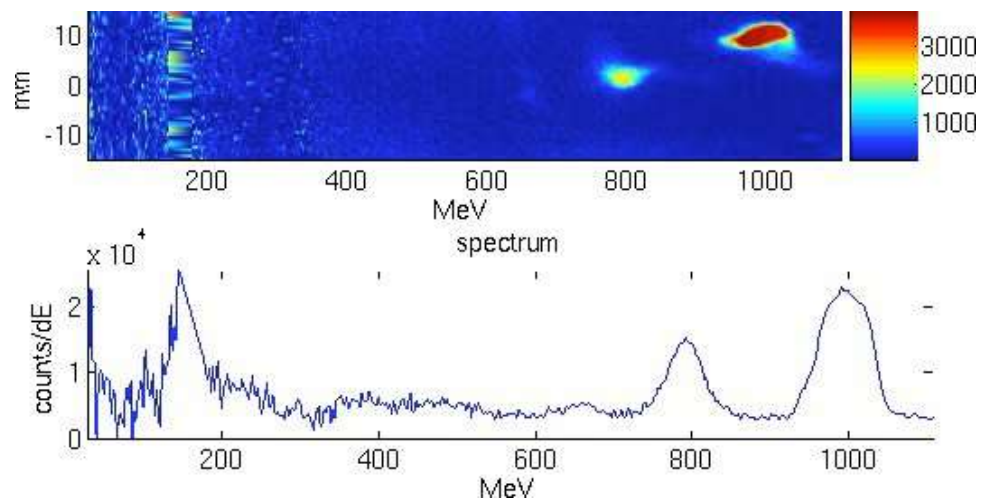
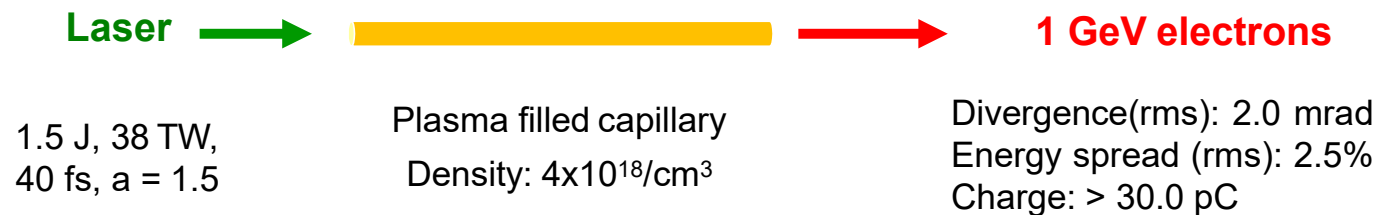
Peak energy: 490 MeV
Divergence(rms): 1.6 mrad
Energy spread (rms): 5.6%
Resolution: 1.1%
Charge: $\sim 50 \text{ pC}$

LETTERS

GeV electron beams from a centimetre-scale accelerator

Published online: 24 September 2006;
doi:10.1038/nphys418 G

W. P. LEEMANS^{1*}, B. NAGLER¹, A. J. GONSALVES², Cs. TÓTH¹, K. NAKAMURA^{1,3}, C. G. R. GEDDES¹,
E. ESAREY^{1*}, C. B. SCHROEDER¹ AND S. M. HOOKER²



1.0 GeV Beam Generation

312 μm diameter and 33 mm length capillary

Laser: 1500($\pm 15\%$) mJ/pulse

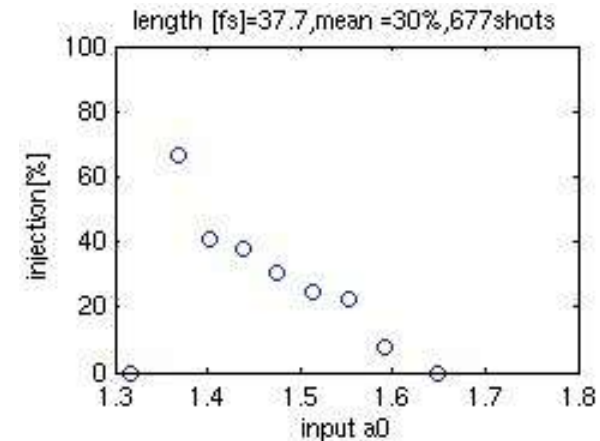
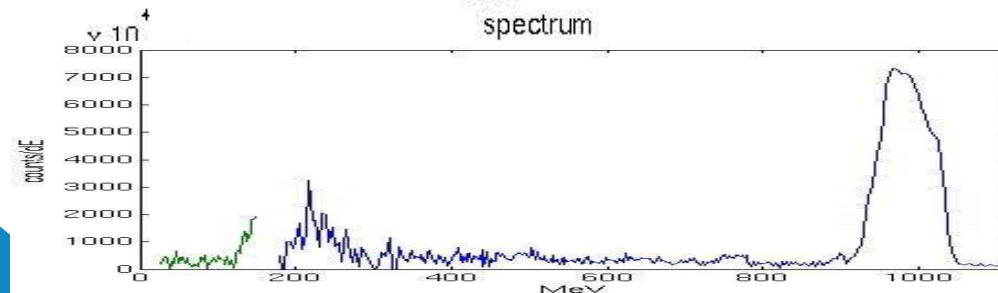
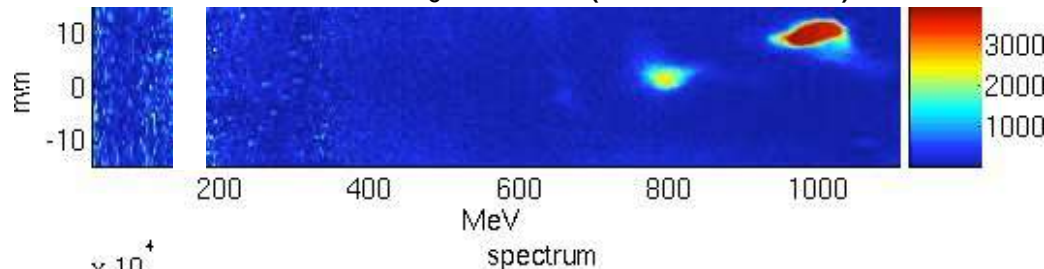
Density: $4 \times 10^{18}/\text{cm}^3$

Injection threshold: $a_0 \sim 1.35$ ($\sim 35\text{TW}$, 38fs)

Less injection at higher power

Relativistic effect, self-modulation

1 GeV beam: $a_0 \sim 1.46$ (40 TW, 37 fs)



Peak energy: 1000 MeV
Divergence(rms): 2.0 mrad
Energy spread (rms): 2.5%
Resolution: 2.4%
Charge: > 30.0 pC

**Less stable
operation**

Laser power fluctuation, discharge timing, pointing stability

Multi-GeV Beam Generation

PRL 111, 165002 (2013)

PHYSICAL REVIEW LETTERS

week ending
18 OCTOBER 2013

Enhancement of Electron Energy to the Multi-GeV Regime by a Dual-Stage Laser-Wakefield Accelerator Pumped by Petawatt Laser Pulses

Hyung Taek Kim,^{1,2} Ki Hong Pae,¹ Hyuk Jin Cha,¹ I Jong Kim,^{1,2} Tae Jun Yu,^{1,2} Jae Hee Sung,^{1,2}
Seong Ku Lee,^{1,2} Tae Moon Jeong,^{1,2,*} and Jongmin Lee^{1,*}

¹Advanced Photonics Research Institute, GIST, Gwangju 500-712, Korea

²Center for Relativistic Laser Science, Institute for Basic Science (IBS), Gwangju 500-712, Korea

(Received 17 July 2013; published 15 October 2013)

PHYSICS OF PLASMAS 22, 056703 (2015)

Generation and pointing stabilization of multi-GeV electron beams from a laser plasma accelerator driven in a pre-formed plasma waveguide^{a)}

A. J. Gonsalves,¹ K. Nakamura,¹ J. Daniels,¹ H.-S. Mao,¹ C. Benedetti,¹ C. B. Schroeder,¹
Cs. Tóth,¹ J. van Tilborg,¹ D. E. Mittelberger,^{1,2} S. S. Bulanov,^{1,2} J.-L. Vay,¹
C. G. R. Geddes,¹ E. Esarey,¹ and W. P. Leemans^{1,2,b) c)}

¹Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA

²Department of Physics, University of California, Berkeley, California 94720, USA

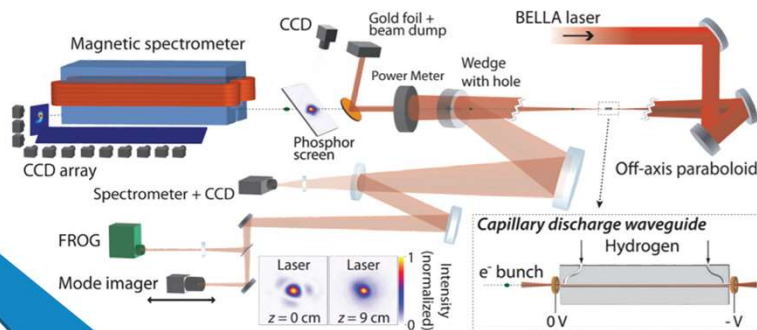
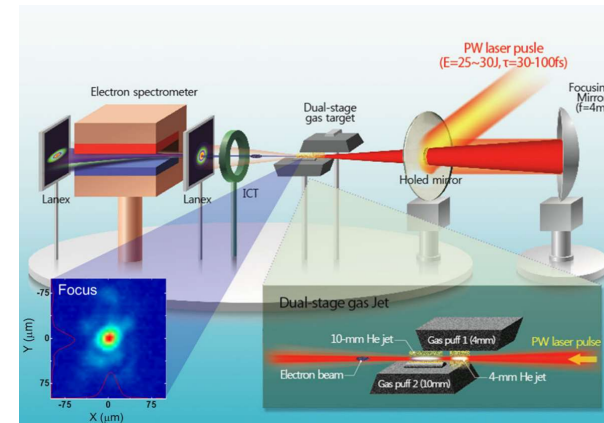


FIG. 1. Schematic of the experimental setup showing the target (inset) and diagnostics of the laser and electron beam. Typical laser spatial profiles with input laser pulse energy 16.6 J are shown at focus ($z=0$) and at the exit of the capillary ($z=9$ cm) for density $8 \times 10^{17} \text{ cm}^{-3}$. The width of each image is $500 \mu\text{m}$. The Airy ring observed in the image of the laser at focus is both due to the input mode and the hole in the first wedge.



Petawatt Laser Guiding and Electron Beam Acceleration to 8 GeV in a Laser-Heated Capillary Discharge Waveguide

A. J. Gonsalves, K. Nakamura, J. Daniels, C. Benedetti, C. Pieronek, T. C. H. de Raadt, S. Steinke, J. H. Bin, S. S. Bulanov, J. van Tilborg, C. G. R. Geddes, C. B. Schroeder, Cs. Tóth, E. Esarey, K. Swanson, L. Fan-Chiang, G. Bagdasarov, N. Bobrova, V. Gasilov, G. Korn, P. Satorov, and W. P. Leemans
Phys. Rev. Lett. 122, 084801 – Published 25 February 2019

0.85 PW

Propagation of non-relativistic laser light in plasma

- Index of refraction: $\omega/k = c/n_R$

$$\omega^2 = \omega_p^2 + c^2 k^2$$

$$n_R = \sqrt{1 - \frac{\omega_p^2}{\omega^2}} = \sqrt{1 - \frac{N_e}{N_c}}$$

$$\omega_p = \sqrt{\frac{4\pi N_e e^2}{m_e}}$$

- Critical density:

$$N_c = \frac{m_e \omega^2}{4\pi e^2}$$

$$N_c = 1.1 \times 10^{21} \text{ cm}^{-3} / \lambda_{\mu m}^2$$

- Criteria of overdense ($n_e > N_c$) and underdense ($n_e < N_c$) plasmas
- Index of refraction in plasma is smaller than 1 !

$$n_R < 1$$

- No problem with relativity!
- Group velocity ($\frac{\partial \omega}{\partial k}$):

$$v_g = c n_R$$

Relativistic non-linear optics

$$\omega^2 = \omega_p^2 + c^2 k^2$$

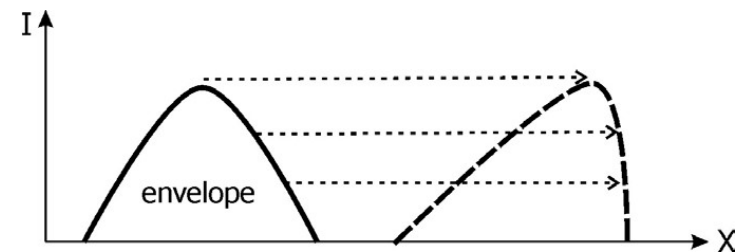
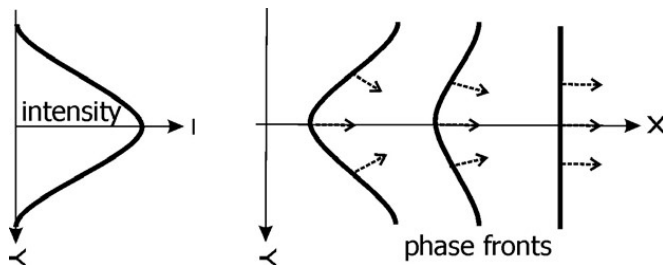
$$\omega_p^2 = 4\pi e^2 n_e / m \langle \gamma \rangle$$

$$\gamma = 1 / \sqrt{1 - \frac{v^2}{c^2}}$$

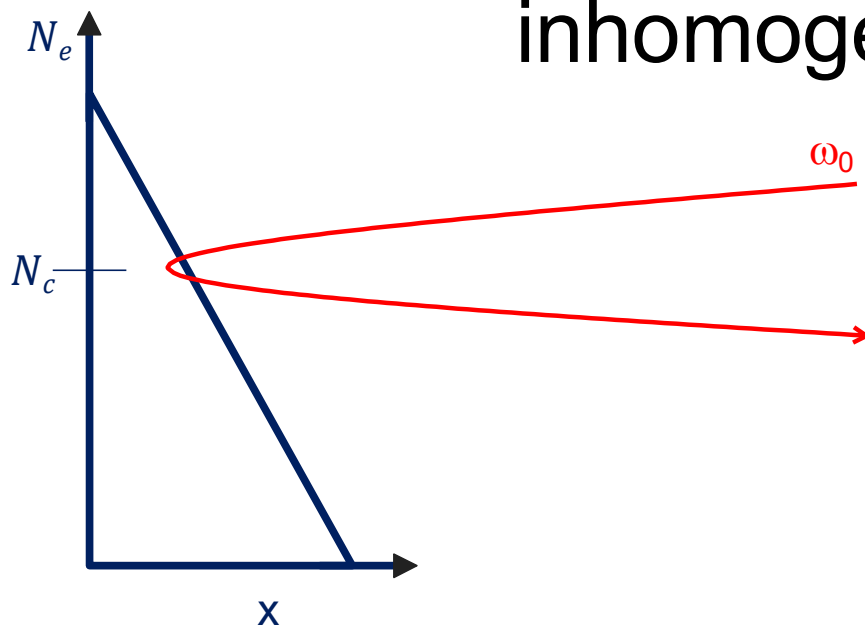
$$n_R = \sqrt{1 - \frac{\omega_p^2}{\omega^2}}$$

Self-focusing: $v_{ph} = c/n_R$

Profile steepening: $v_g = c n_R$



Propagation of non-relativistic laser light in inhomogeneous plasma



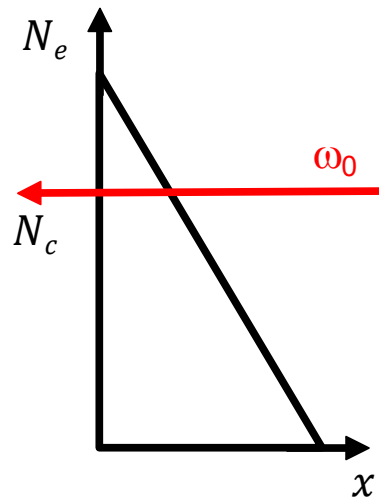
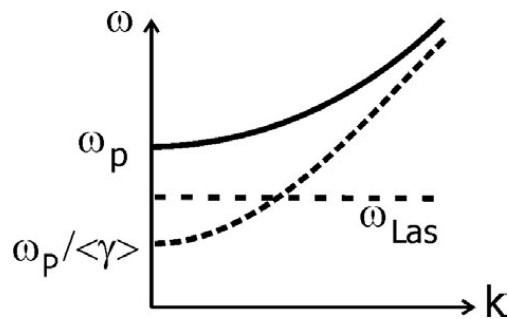
$$\vec{A}(r, t) = \text{Re}\{\vec{A}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}\}$$

$$\omega/k = c/n_R$$

- Reflection from the critical density (by perpendicular incidence)
- No significant penetration into the overdense region (just evanescent wave)
- Reflection from lower densities ($N_c \cos^2 \alpha$) by oblique incidence (α)

Relativistic non-linear optics

Induced transparency:



$$\omega^2 = \omega_p^2 + c^2 k^2$$

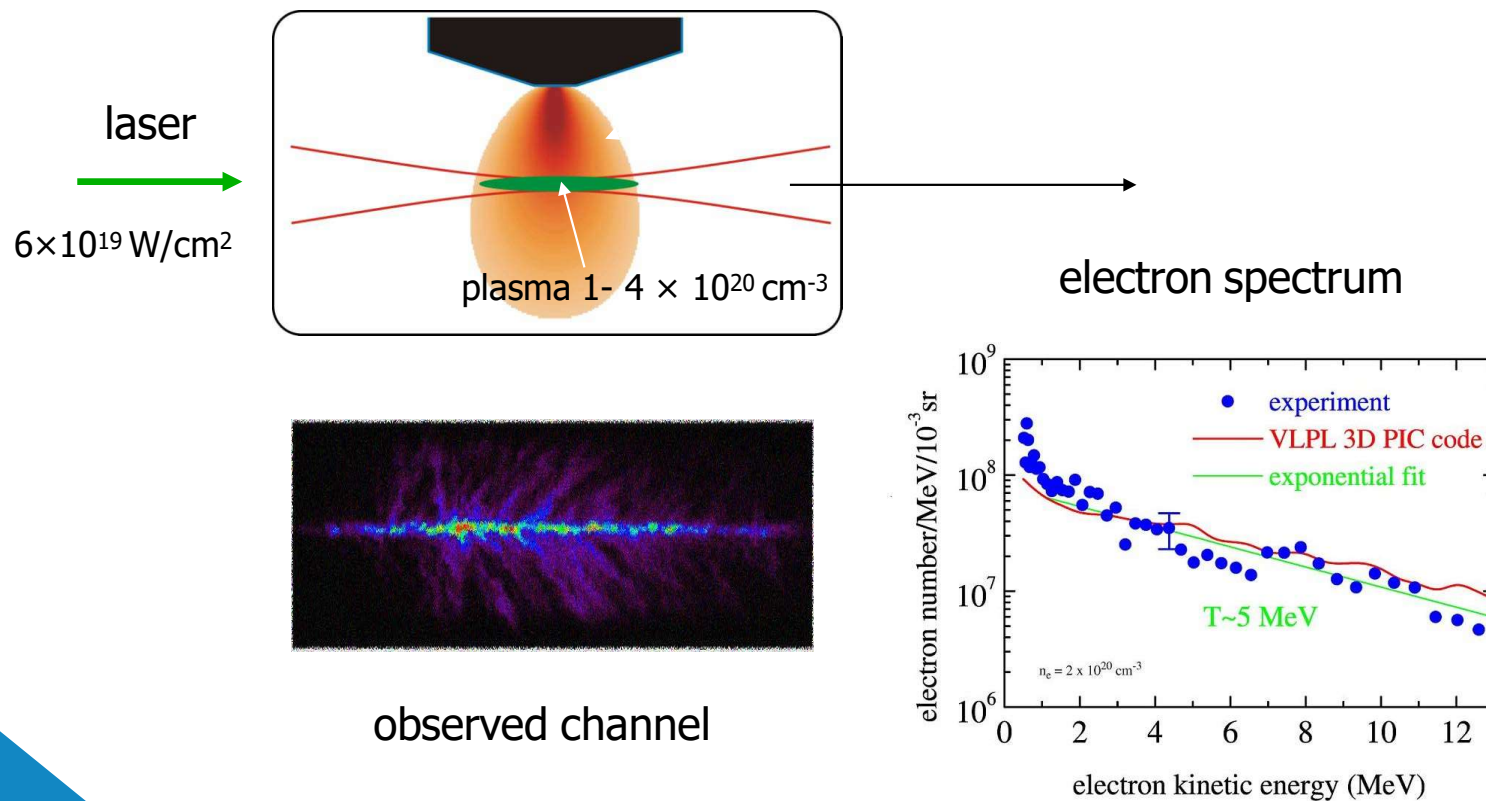
$$\omega_p^2 = 4\pi e^2 n_e / m \langle \gamma \rangle$$

$$\gamma = 1 / \sqrt{1 - \frac{v^2}{c^2}}$$

$$n_R = \sqrt{1 - \frac{\omega_p^2}{\omega^2}}$$

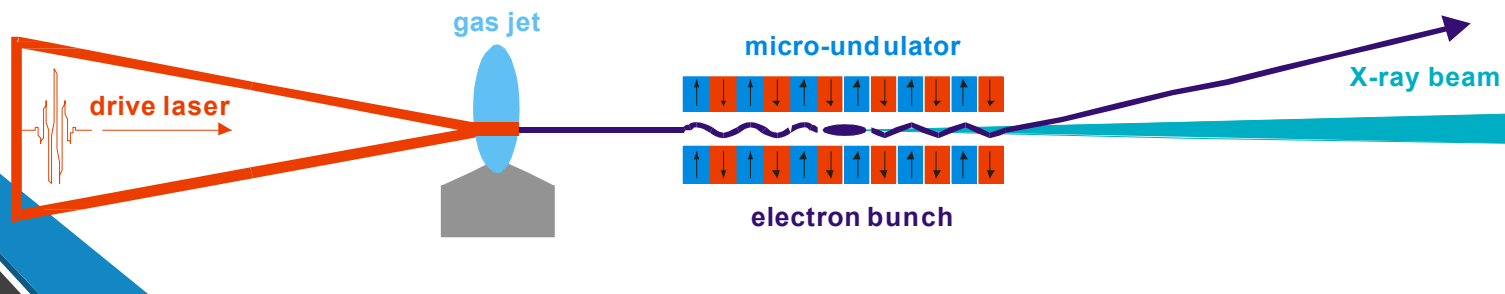
Plasma channels and electron beams observed

C. Gahn et al. PRL 83, 4772 (1999)



Application: Laser-driven X-ray sources

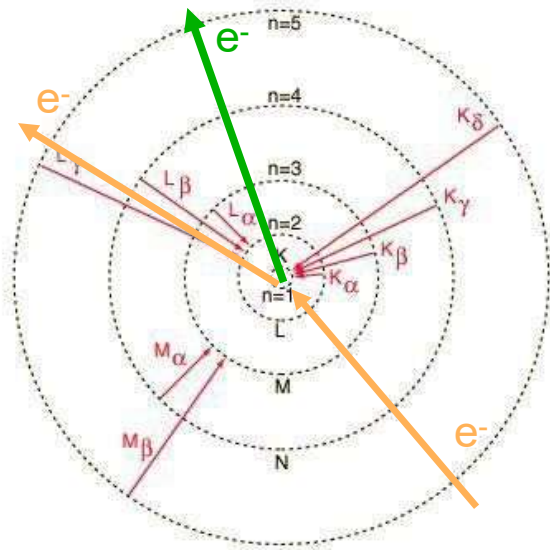
- K_{α} line radiation from laser produced plasma
- Bremsstrahlung from accelerated particles
- Transverse betatron oscillation of electrons in the „Bubble“
- Undulator, Table Top - Free Electron Laser



K_α line radiation and Bremsstrahlung

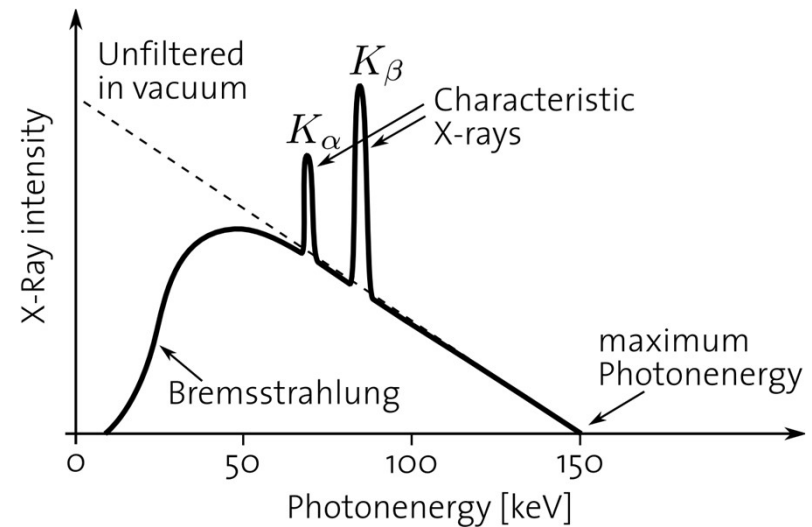
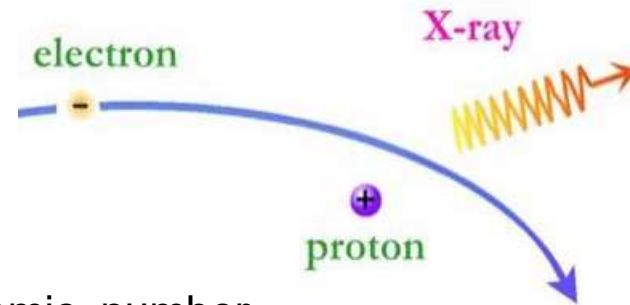
K_α radiation

- Line emission \rightarrow monochromatic
- Incoherent
- Duration can be few 100 fs
- 1-10 keV photon energy



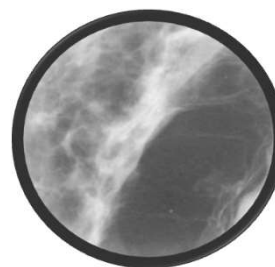
Bremsstrahlung

- Broad spectrum
- Incoherent
- Intensive
- $\sim Z^2$ Z : atomic number

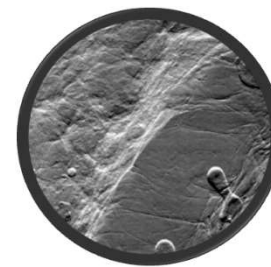


Imaging

- Application of bright X-Rays:
 - Small angle X-ray scattering
 - Phase Contrast Imaging
 - Ultrafast science
 - Biomolecular Imaging
- Requirements:
 - High energy (10-160 keV)
 - Spatial coherent X-Ray beams
 - Small emittance



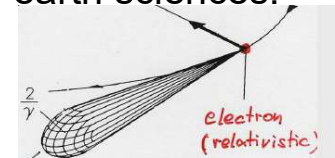
X-ray absorption
radiography image



X-ray phase
contrast image

Synchrotron radiation light source

- **Synchrotron radiation** emitted from accelerated charged particles can produce very intense radiation at X-ray frequencies
- The last decades, vast increase in the use of synchrony radiation **for photon science**. Some uses: material sciences; life sciences; earth sciences.

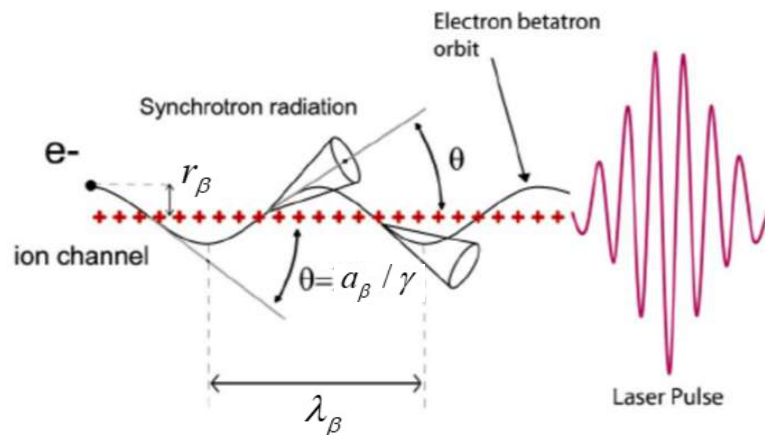


Radiation from ultra-relativistic electrons: forward direction.

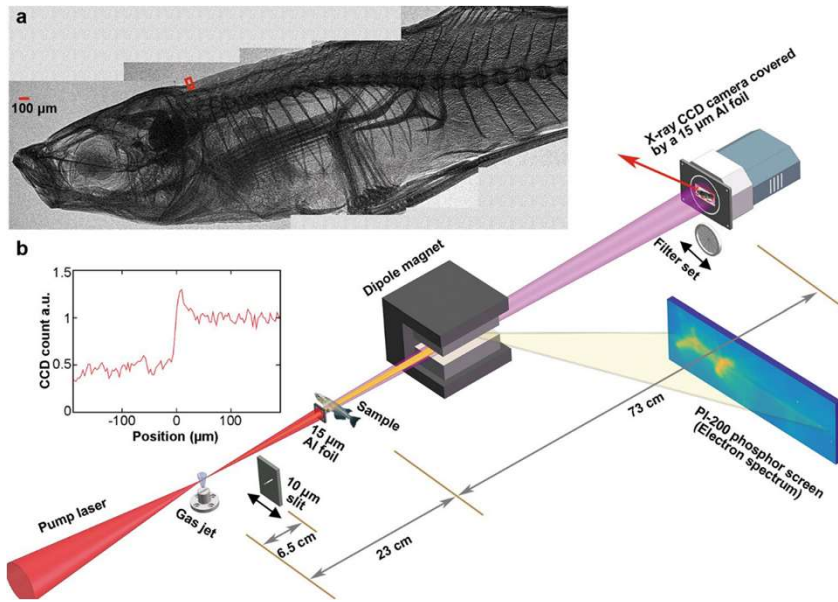


Introduction of Betatron Radiation

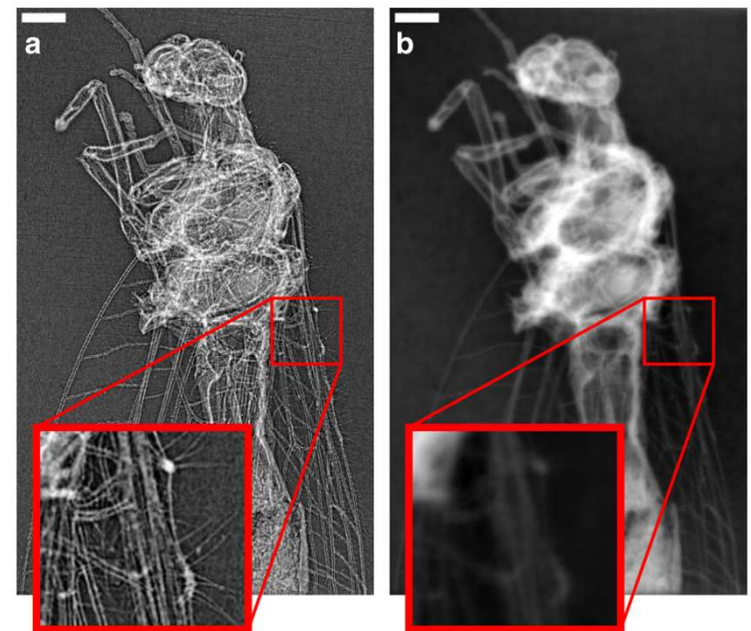
SCALING LAWS



- Betatron frequency: $\omega_\beta = \omega_p / \sqrt{2\gamma}$
- Transverse momentum: $a_\beta \propto \sqrt{\gamma n_e} r_\beta$
- Divergence: $\vartheta = a_\beta / \gamma$
- Critical photon energy: $E_c \propto \gamma^2 n_e r_\beta$
- Efficiency: $N_{\text{phot/cycle}} = \alpha a_\beta$
- Wavelength:
$$\lambda_h = \frac{\lambda_\beta}{h 2 \gamma_e^2} \left(1 + \frac{a_\beta^2}{2} + (\gamma_e \varphi)^2 \right) = \frac{\sqrt{3} \pi c}{h \omega_p \gamma_e^{3/2}} \left(1 + \frac{a_\beta^2}{2} + (\gamma_e \varphi)^2 \right)$$



[Scientific Reports](#) volume 9, Article number: 7796 (2019)



[Nature Communications](#) volume 6, Article number: 7568 (2015)

Free Electron Laser

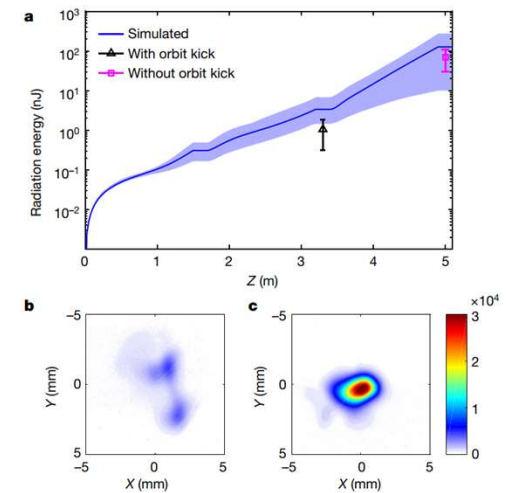
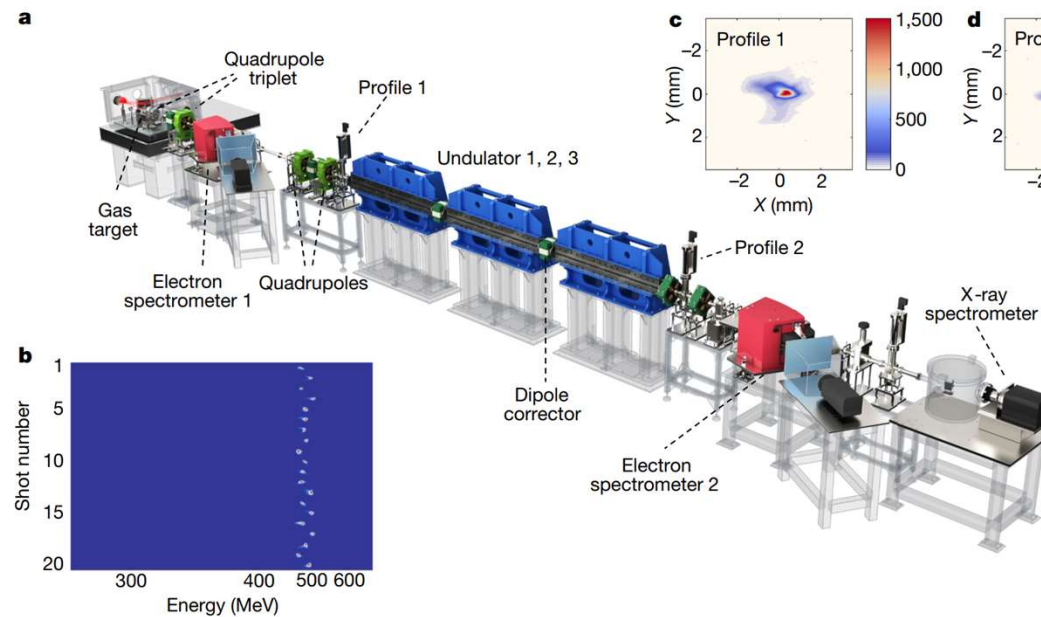


Fig. 3 | Undulator radiation measurement at 27 nm. **a**, Measured radiation energy with (black) and without (magenta) the orbit kick and the simulated energy along the undulator. Error bars represent the r.m.s. statistical uncertainty in the measured energy averaged over 20 shots. **b**, **c**, Corresponding transverse-beam patterns of the radiation measured with (b) and without (c) the orbit kick. The scale bar is normalized.

Nature volume 595, pages 516–520 (2021)