電子加速器物理基礎介紹

Fundamentals of Electron Particle Accelerator Physics

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OUTLINE

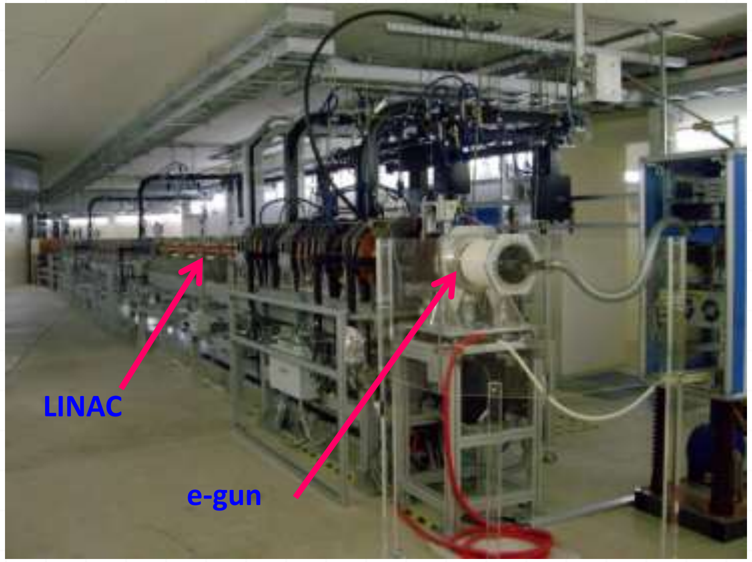
- Overview of charged particle accelerator
- Some basic concepts
- Beam transportation and bunch compressor
- Characteristics of synchrotron radiation
- Storage ring and FEL light sources

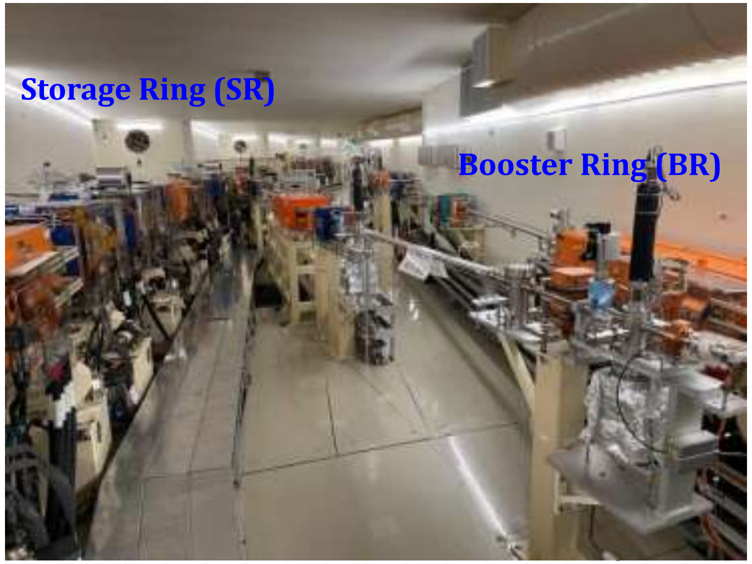
CHARGED PARTICLE ACCELERATOR

- A charged **particle accelerator** is a machine that uses electromagnetic fields to propel charged particles to very high speeds and energies, and to contain them in well-defined beams.

- **Proton** → particle nuclear physics *ex. 6.5 TeV, 13 km LHC @ CERN*
  → particle therapy、radio-isotope production、ion implanters

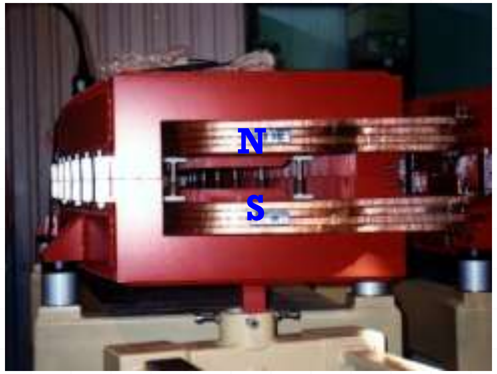
- **Electron** → SR light sources *ex. 3GeV, 518.4 m TPS、1.5 GeV, 120 m TLS*



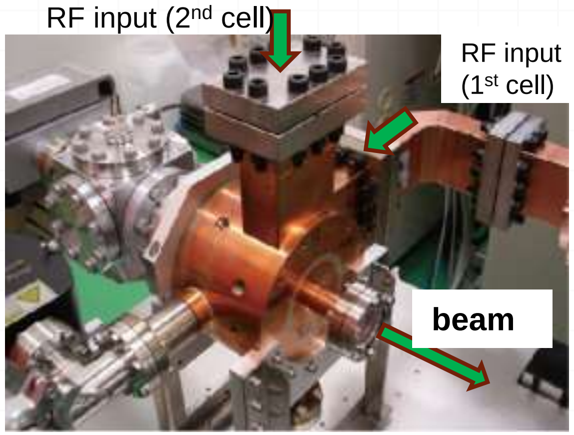


[ref] https://en.wikipedia.org/wiki/Particle_accelerator
https://www.shi.co.jp/industrial/en/product/medical/proton-therapy/proton-therapy-system.html

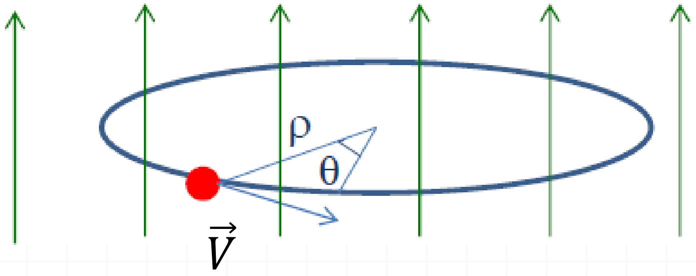
MOTION OF CHARGED PARTICLES



$$\vec{F} = \frac{d\vec{P}}{dt} = q(\vec{E} + \vec{v} \times \vec{B})$$

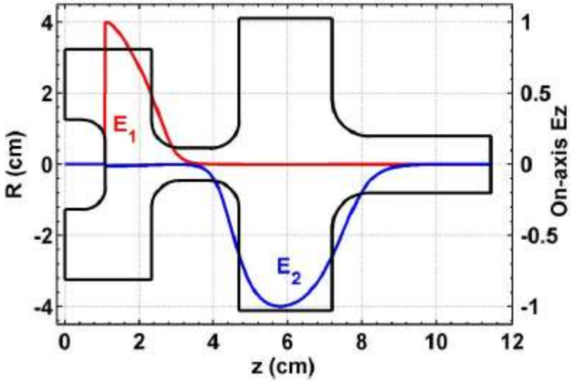


Uniform constant magnetic field **B**



For a relativistic charged particle (V ~ c)

F_B (B = 1 T) ~ F_E (E = 300 MV/m)



Magnetic fields for deflection (bending and focusing).
B ~ 1T

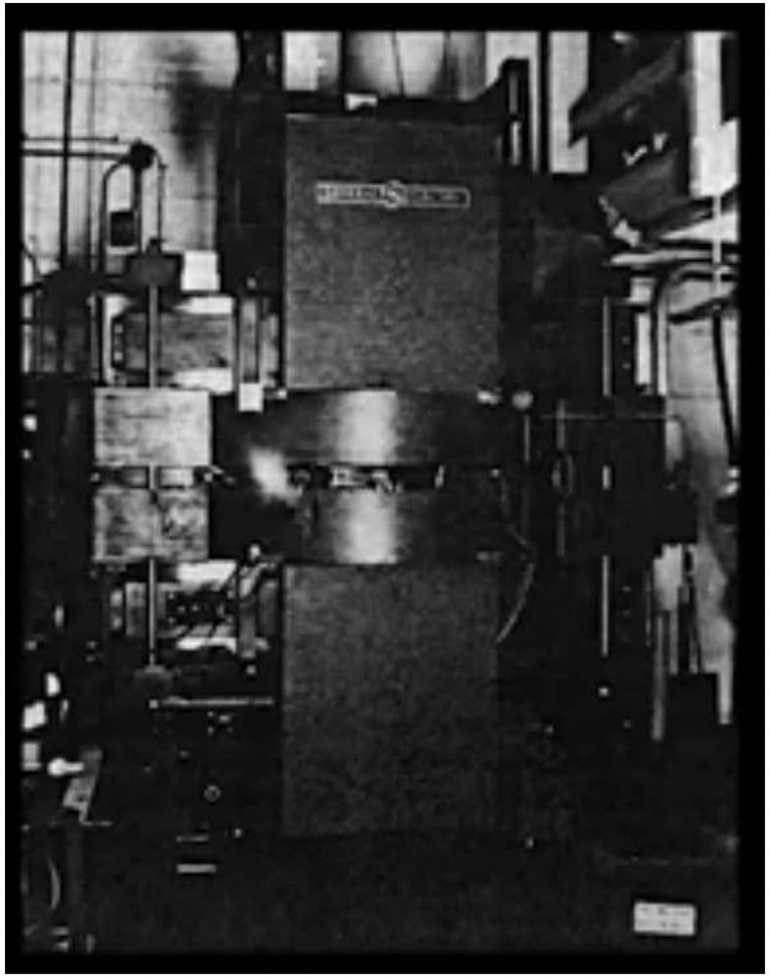
$$\overrightarrow{F_B} = q\vec{v} \times \vec{B} = m\frac{v^2}{\rho}$$

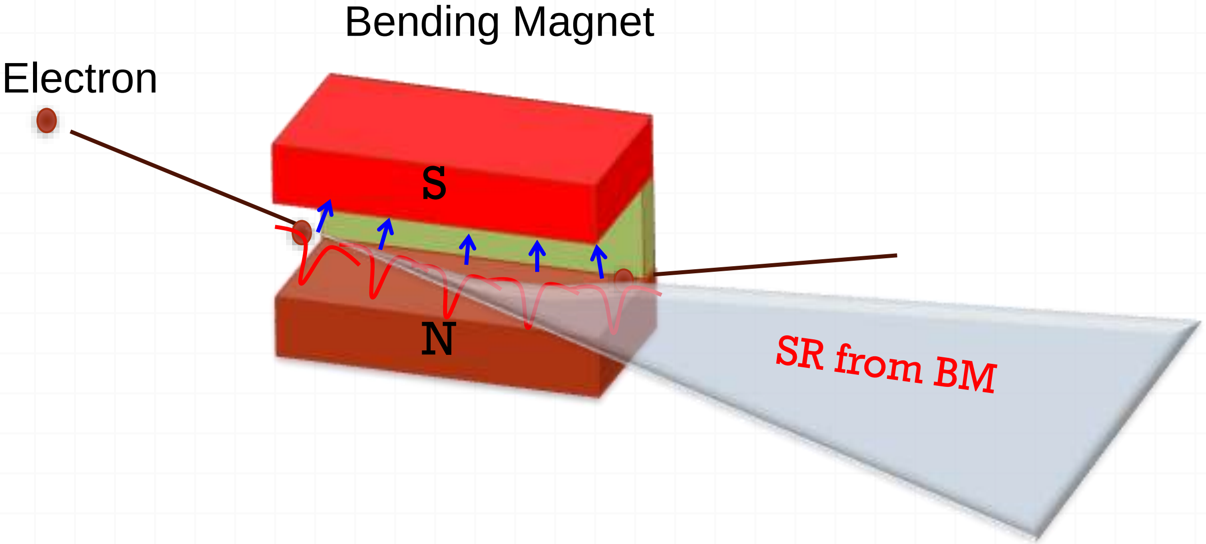
Electric fields for acceleration (RF cavity).
E ~ 10 MV/m

$$\overrightarrow{F_E} = q\vec{E}$$

$$\overrightarrow{F_E} \bullet \overrightarrow{dS} = \Delta\ Energy\ Gain$$

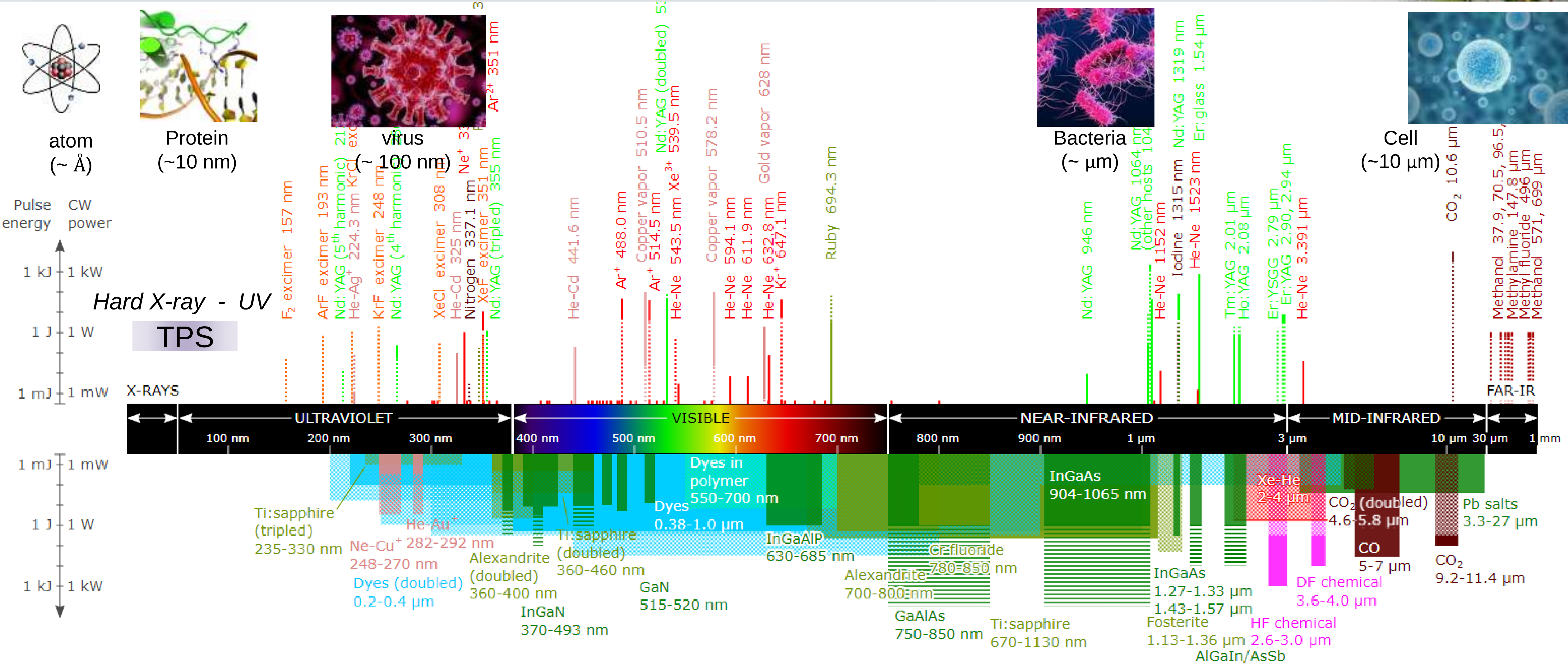
SYNCHROTRON RADIATION





- ➢ First observed in 1947 by General Electric Research Laboratory in Schenectady, New York
- ➢ Basic element: (relativistic electron、bending magnet)

LIGHT TO EXPLORE THE EXTREME RESOLUTION



- Wavelength of commercially laser: ~ 200 nm – 100 μm (No laser available at the extreme short wavelength region)
- To explore the science at extreme tiny resolution, SR is a solution to intense VUV、EUV and X-ray source

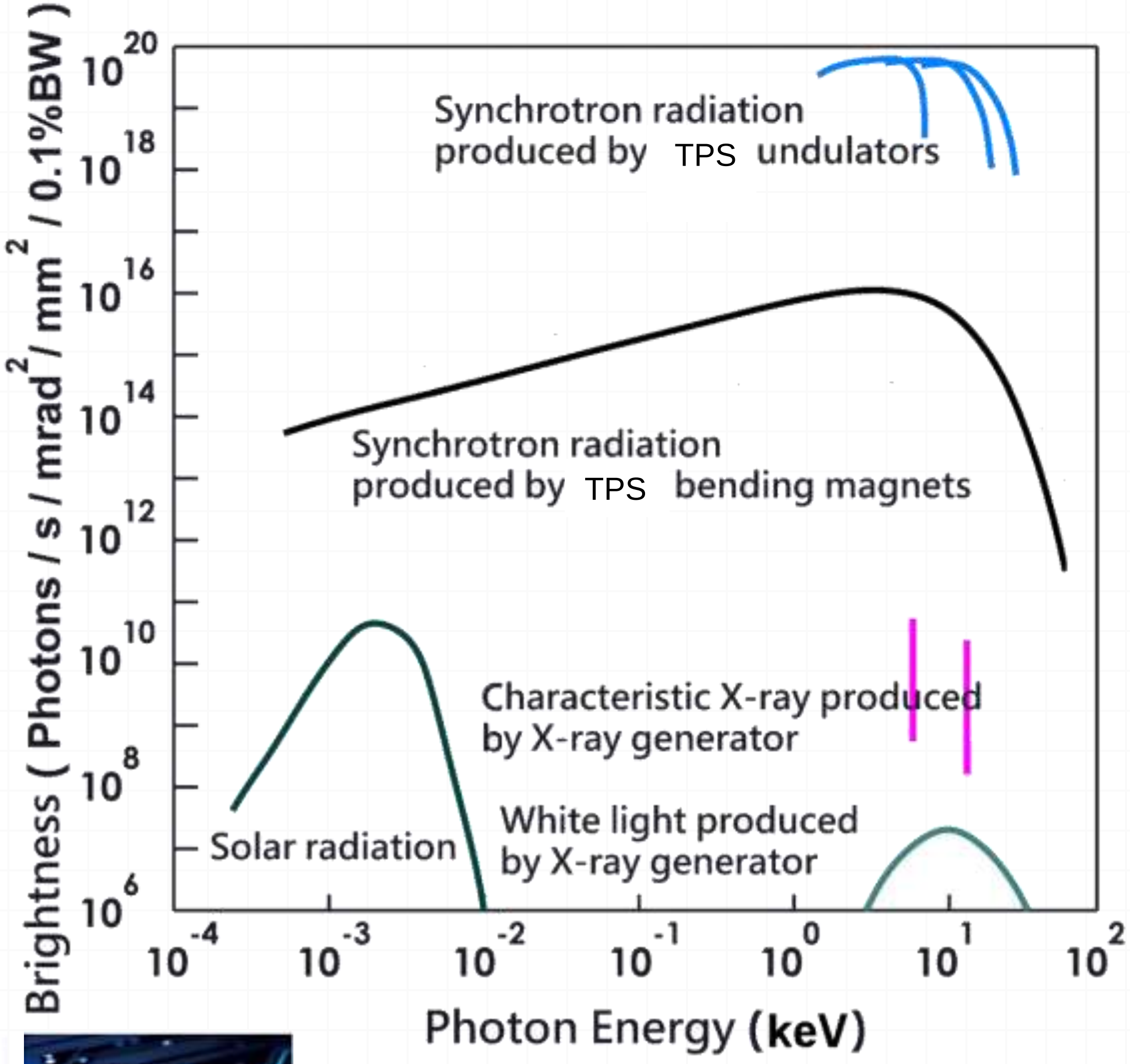
[ref] https://en.wikipedia.org/wiki/List_of_laser_types ;
Lecture of "Foundation of FEL", Takashi Tanaka, RIKEN SPring-8 Center, ISBA 2022.

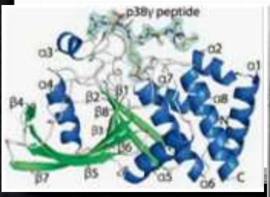
PROPERTIES OF SYNCHROTRON RADIATION LIGHT

- ✓ **High brightness** (>> 10^6 x Solar radiation)
- ✓ **Tunable wavelength** (extend to hard-X-ray)
- ✓ **Excellent collimation** (~ $1/\gamma$)
- ✓ **small spot size and divergence**
- ✓ **Full polarization**
- ✓ **Pulsed time structure** (several ps with spacing of ns)

$$Brightness = \frac{flux}{(2\pi)^2\Sigma_x\Sigma_{x'}\Sigma_y\Sigma_{y'}} \left[\frac{photons}{sec-mm^2-mrad^2-0.1\%B.W.}\right]$$

$$spectral\ flux = \frac{N_{photon}}{\Delta T \cdot \Delta\omega/\omega}$$





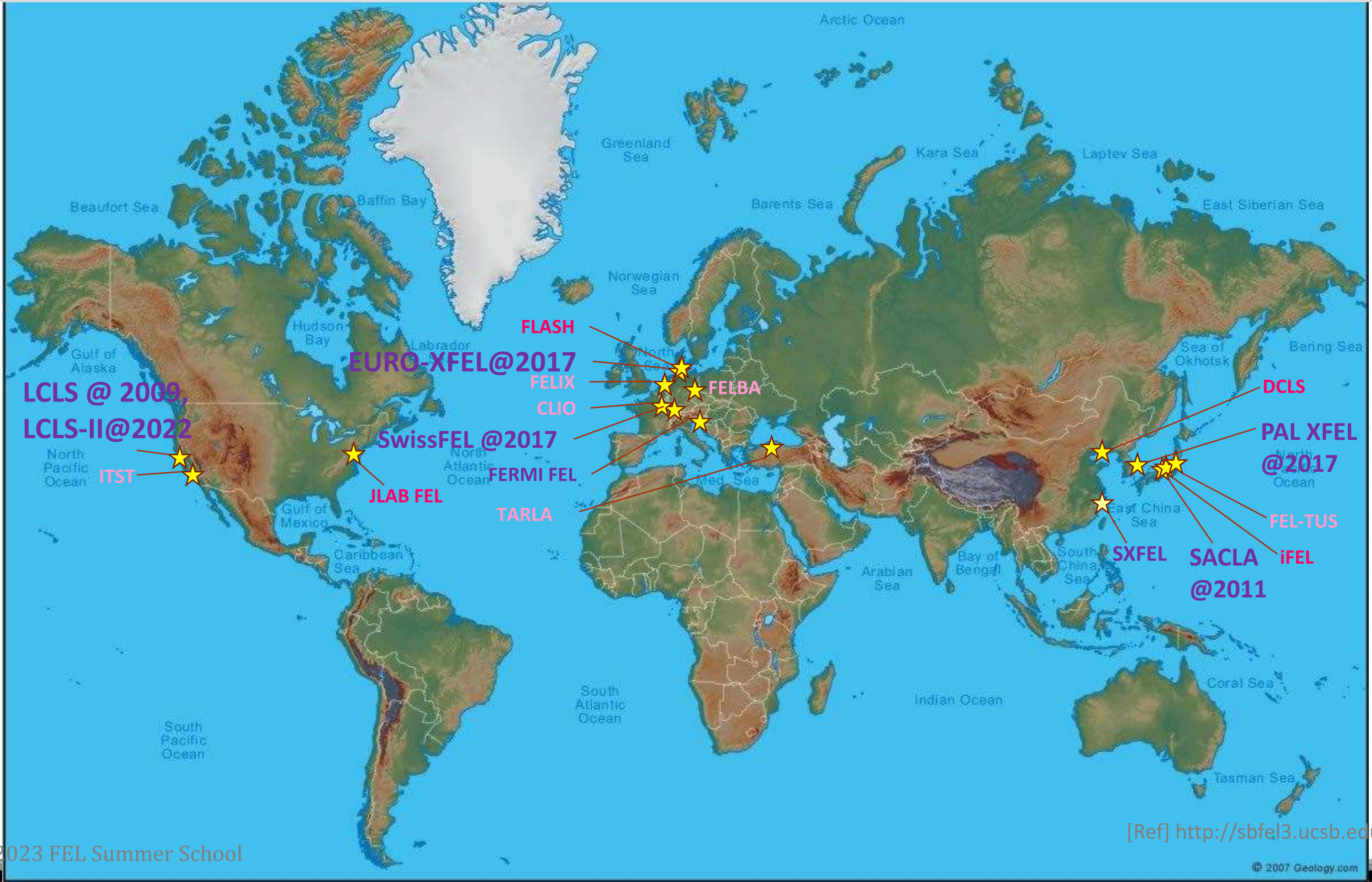




WORLD'S SYNCHROTRON RING FACILITIES

[Ref]: https://www.aps.anl.gov/spring-8-encyclopedia-of-synchrotron-radiation-facilities%E2%80%932nd-edition
Encyclopedia of the Synchrotron Radiation Facilities

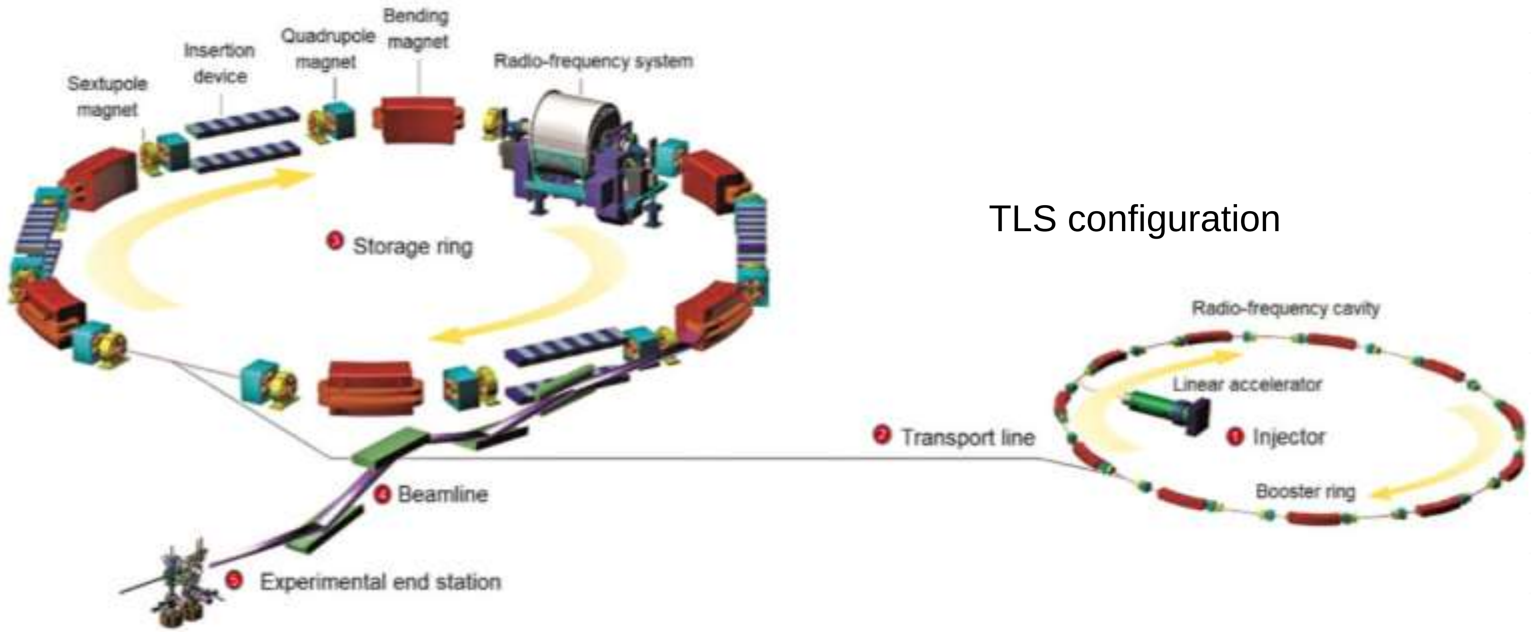
WORLD'S FREE ELECTRON LASER FACILITIES



[Ref] http://sbfel3.ucsb.edu/www/vl_fel.html

OVERVIEW OF SR AND LINAC-FEL FACILITIES

SR
(STORAGE RING)

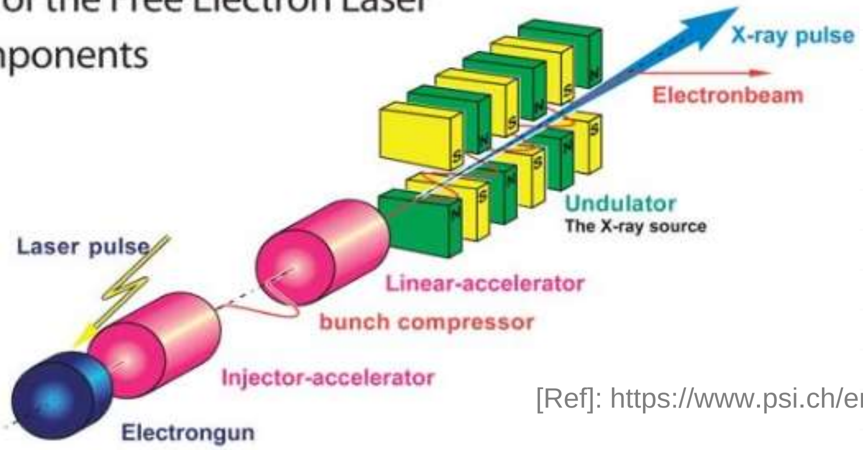


TLS configuration

LINAC FEL
(LINEAR ACCELERATOR FREE ELECTRON LASER)

Schematic design of the Free Electron Laser with different components

1) Electrongun
2) Injector
3) Accelerator
4) Undulator



[Ref]: https://www.psi.ch/en/swissfel/how-it-works

SOME BASIC CONCEPTS

RELATIVISTIC CHARGE PARTICLES

Speed of light

$$c = 2.99792458 \times 10^{8}\ \text{m/sec}$$

Lorentz factor

$$\gamma = \frac{E}{E_0} = \frac{1}{\sqrt{1-\beta_\gamma^2}},$$

$$\beta_\gamma = \frac{v}{c}$$

Total energy 、Rest energy、kinetic energy、

$$E = E_0 + E_k = m c^2 = \gamma m_0 c^2,$$

$$E_0 = m_0 c^2,$$

$$E_k = E - E_0$$

$$E_{0,electron} = 0.511\ MeV$$
$$E_{0,proton} = 938.08\ MeV$$

Momentum

$$p = \gamma m_0 \mathbf{v} = \beta_\gamma \frac{E}{c},$$

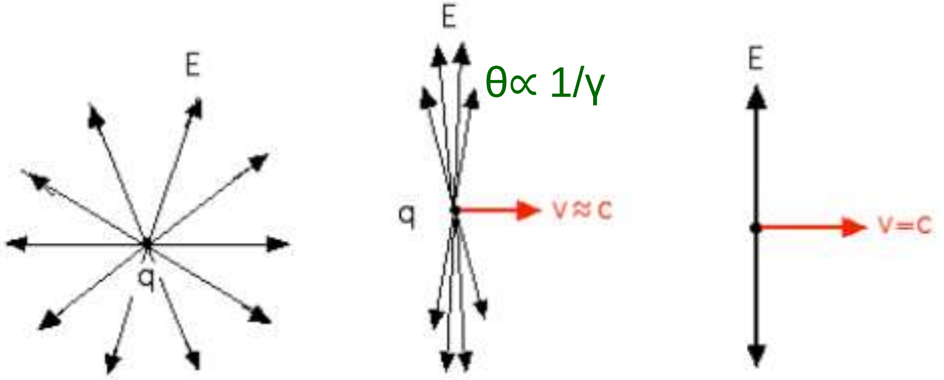
Velocity of a moving electron particle

E_k	$\beta_\gamma = v/c$	γ
100 keV	0.5482204	1.1956
150 MeV	0.99999424	294.5420
3 GeV	0.9999999855	5871.8414

Ultra-relativistic particles (V>0.99c)

- Move ~ 7.4 turns around the earth in 1 sec
- Move TPS machine ~580k turns in 1 sec (~ 1.75 μs/turn)

RELATIVISTIC EM FIELD & SPACE CHARGE FORCE



(a) stationary charge (b) a charge moving relativistically (c) when v=c

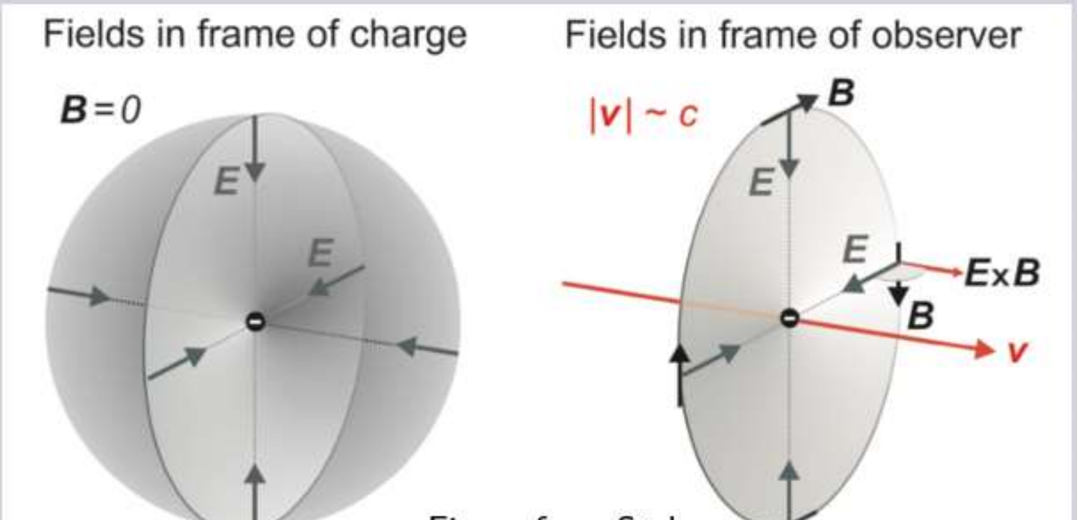
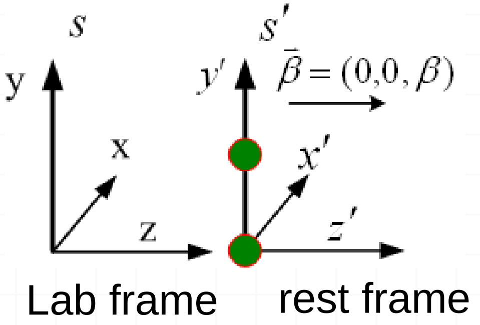


Figure from Stohr, Siegmann, "Magnetism"

The electric and magnetic field under Lorentz transformation between ***s*** and ***s'*** frame takes the following form

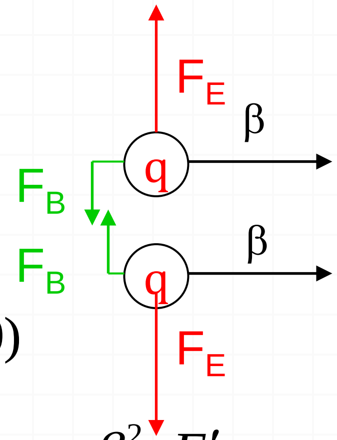


$$E_z = E_z' \qquad B_z = B_z'$$

$$E_x = \gamma(E_x' + c\beta B_y') \qquad B_x = \gamma(B_x' - \frac{1}{c}\beta E_y')$$

$$E_y = \gamma(E_y' - c\beta B_x') \qquad B_y = \gamma(B_y' + \frac{1}{c}\beta E_x')$$

The Lorentz force between two charged particles parallel moving at a distance of d is



$$F_E = qE_y = \gamma q E_y' \qquad \vec{E}' = (0, E_y', 0) \qquad \vec{B}' = (0,0,0)$$

$$F_B = qc(\vec{\beta}\times\vec{B})_y = qc(\beta_z B_x - \beta_x B_z) = qc\beta B_x = -\gamma\beta^2 qE_y'$$

$$\boxed{F = F_E + F_B = \gamma(1-\beta^2)qE_y' = \frac{1}{\gamma}qE_y'} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

- *The larger the beam energy, the weaker the space charge force*

- 6-D phase space of beam

$$(x, x', y, y', t, \delta)$$

- Transverse phase space (horizontal and vertical)

$$(x, x');\ (y, y')$$

$$x' = v_x / v_z; \quad y' = v_y / v_z$$

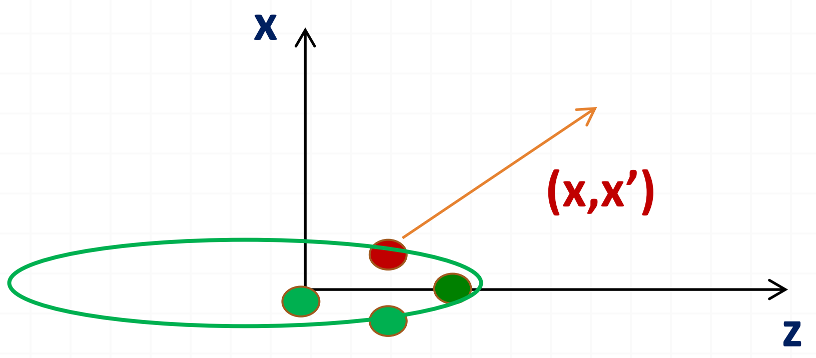
- Longitudinal phase space

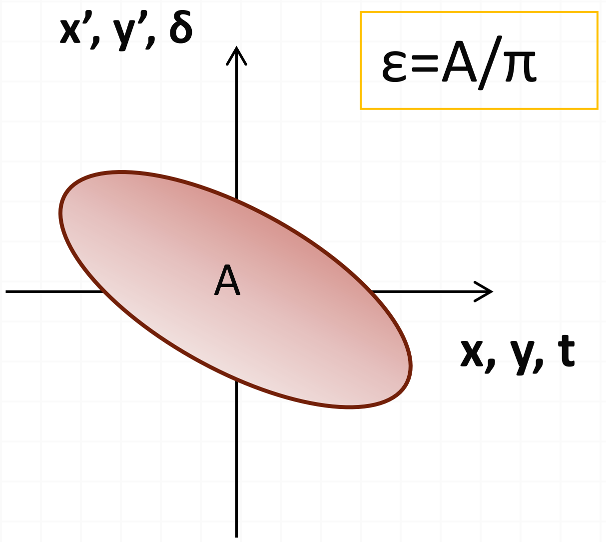
$$(t, \delta);\ \delta = \frac{\Delta p}{p} = \left(\frac{1}{\beta^2}\right)\frac{\Delta E}{E}$$

- Beam emittance

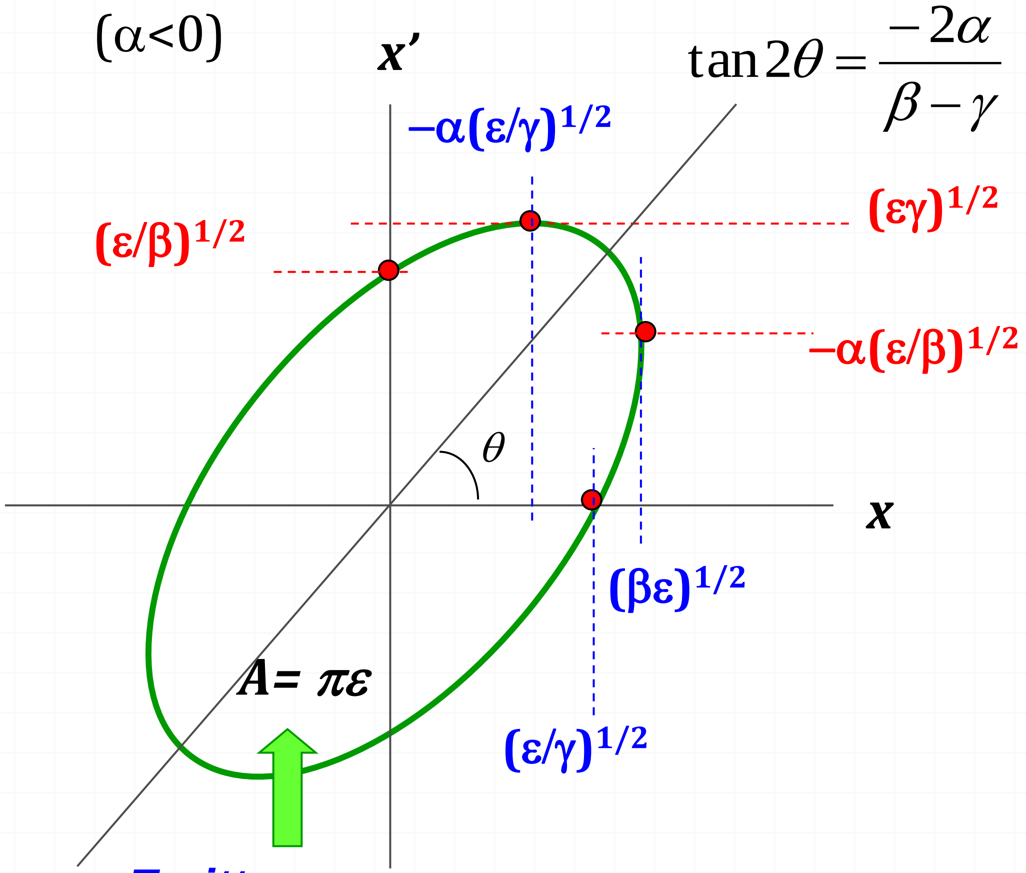
$$\varepsilon = A/\pi,$$

$$\varepsilon_x = \sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle x\,x'\rangle^2}.$$





COURANT-SNYDER PARAMETERS



Emittance ***(an invariant of betatron motion)***

- **Liouville's theorem**: the population density of particles in the phase space keeps the same constant under the conservative force of system (no energy difference)

$$x''(s)+k(s)x(s)=0,$$

$$x(s)=\sqrt{\varepsilon}\sqrt{\beta}\cos(\phi(s)-\phi_0),$$

$$\frac{1}{2}\left(\beta\beta''-\frac{1}{2}\beta'^2\right)-\beta^2\phi'^2+\beta^2k=0,$$

$$\beta\phi'=\text{constant}.$$

$$\phi(s)-\phi_0=\int_0^s\frac{dS}{\beta(s)}.$$

$$\beta''+2k\beta-2\gamma=0,\qquad \gamma x^2+2\alpha xx'+\beta x'^2=\varepsilon.$$

Courant-Snyder parameters are defined as,

$$\beta(s)$$

$$\alpha(s)=-\frac{\beta'(s)}{2}$$

$$\gamma(s)=\frac{1+\alpha^2(s)}{\beta(s)}$$

the beam size

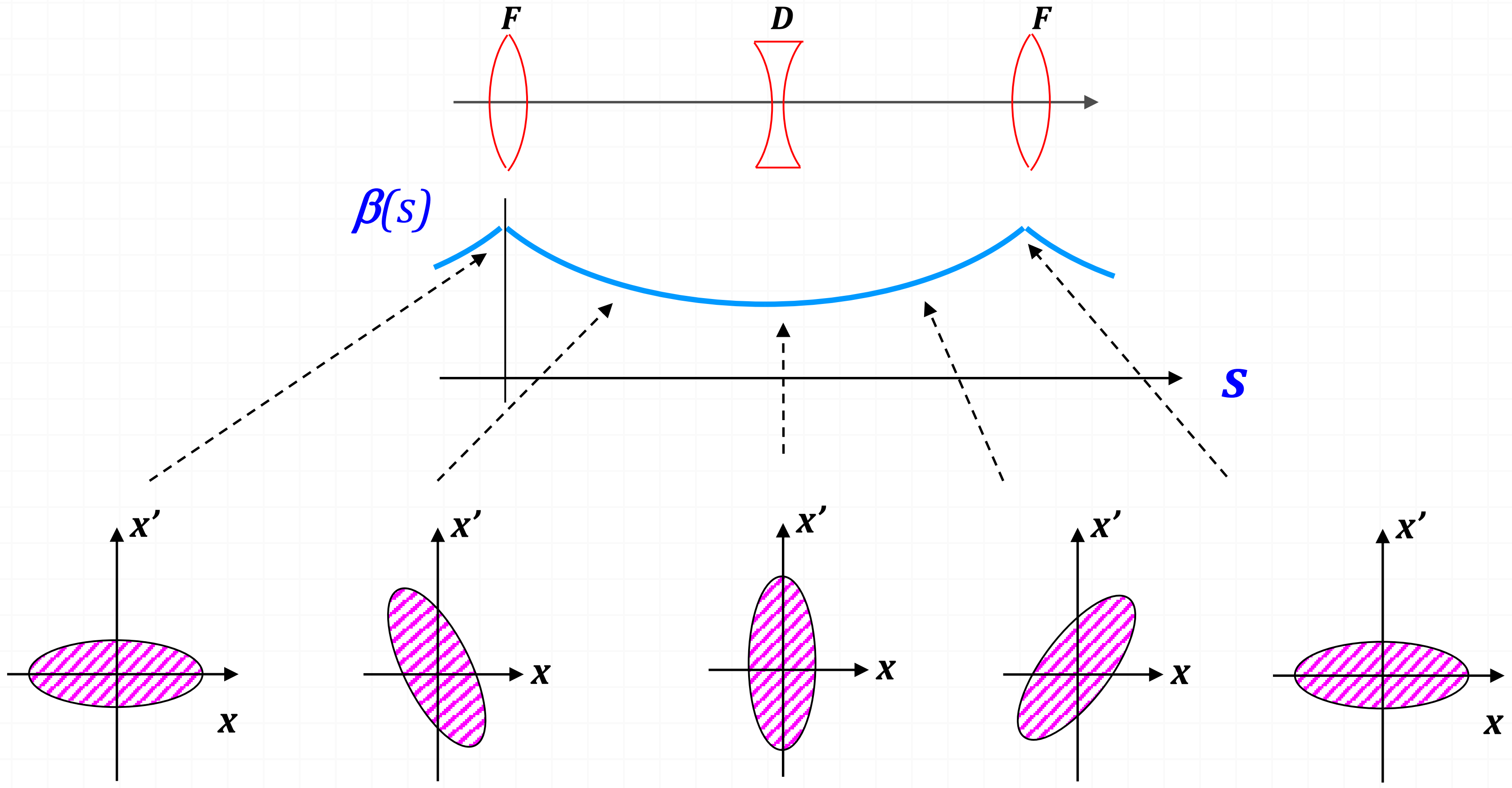
$$\sigma(s)=\sqrt{\varepsilon\cdot\beta(s)}$$

the slope of beam envelope evolution

the beam divergence

$$\sigma'(s)=\sqrt{\varepsilon\cdot\gamma(s)}$$

COURANT-SNYDER PARAMETERS



σ_{xmax}
$\alpha = 0$

Converging beam,
$\alpha > 0$

Beam waist
σ_{xmin}, $\alpha = 0$

Diverging beam,
$\alpha < 0$

σ_{xmax}
$\alpha = 0$

EMITTANCE OF RING AND LINAC (1/2)

Equilibrium emittance of ring

- Balance between the radiation damping and quantum excitation
- ε_0 depends on beam energy and lattice configuration

$$\varepsilon_0 = C_q \Upsilon^2 \frac{<H/|\rho|^3>}{j_x <1/|\rho|^2>}$$

Quantum excitation related terms

radiation damping related terms

$$H = \gamma_x \eta_x^2 + 2\alpha_x \eta_x \eta'_x + \beta_x \eta'^2_x \propto \rho\theta^3$$

Normalized emittance of linac

- For the linear injector case that the beam energy is increasing along the beamline,
- the geometric emittance is damping during energy ramping, the normalized beam emittance ε_n is preserved as a constant

geometric emittance

$$\varepsilon = \frac{\varepsilon_n}{\beta\,\gamma} \propto \frac{1}{E},$$

Normalized emittance

$$\varepsilon_n = \beta\gamma\varepsilon,$$

more information for the concept of normalized emittance:
“Acceleration and Normalized Emittance”, Prof. Steven M. Lund, USPAS 2018

EMITTANCE OF RING AND LINAC (2/2)

Equilibrium emittance of ring

$$\varepsilon_0 \propto E^2\theta^3 \qquad \varepsilon_0 \propto \frac{E^2}{(\text{Circumference})^3}$$

- the smaller bending angle
  (the concept of MBA lattice)

$$\sigma_x(s) = \sqrt{\varepsilon_x \cdot \beta_x(s)} \propto E$$

$$\sigma_x'(s) = \sqrt{\varepsilon_x \cdot \gamma_x(s)} \propto E$$

$$\sigma_\gamma/\gamma(s) \propto E$$

- *Lower beam energy is beneficial to the beam emittance*
- *Beam energy is mainly decided by the desired radiation photon beam energy*
- *As a result of advancements in magnet technology, the beam energy of recently designed machines is decreasing in order to reduce energy consumption while simultaneously improving beam emittance. (Spring8 → Spring8II: 8GeV→6GeV)*

Normalized emittance of linac

- The minimum normalized emittance of the whole injector is limited at the stage of beam generation.

$$\varepsilon_n = \sqrt{(\varepsilon^{th})^2 + (\varepsilon^{rf})^2 + (\varepsilon^{sc})^2 + \cdots}$$

- higher beam energy helps to reduce the geometric emittance, hence also the transverse electron beam size and divergence

$$\sigma_x(s) = \sqrt{\frac{\varepsilon_n}{\beta\gamma} \cdot \beta(s)} \propto \sqrt{\frac{1}{E}}$$

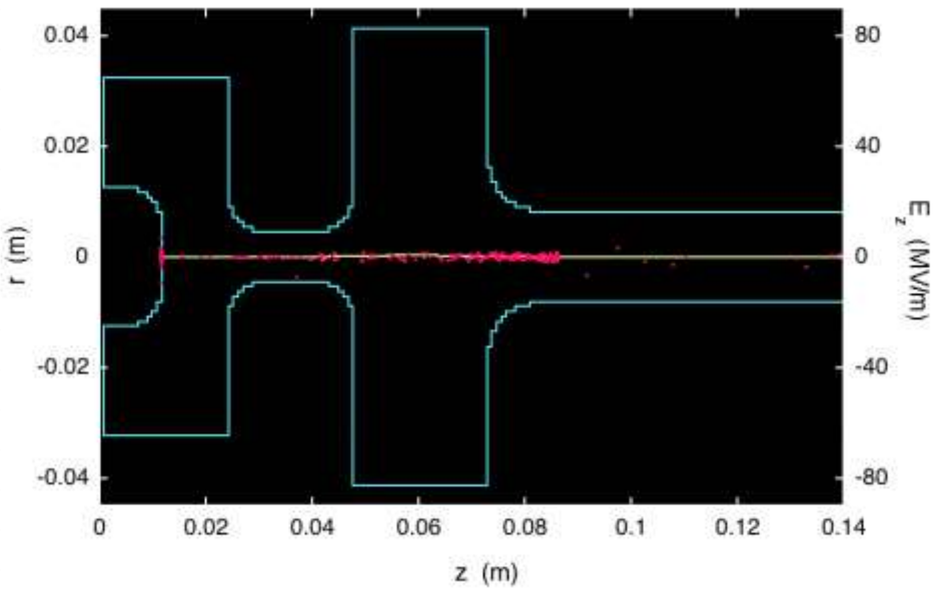
$$\sigma_x'(s) = \sqrt{\frac{\varepsilon_n}{\beta\gamma} \cdot \gamma(s)} \propto \sqrt{\frac{1}{E}}$$

$$\sigma_\gamma/\gamma(s) \propto \frac{1}{E}$$

- *damping of ε with acceleration with conserved ε_n improves beam quality*

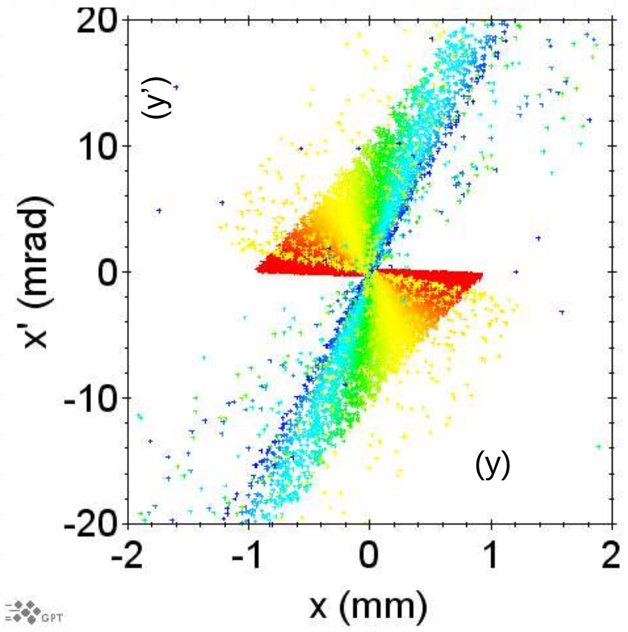
GENERATED ELECTRON DISTRIBUTION OF THERMIONIC RF GUN

Simulated electron dynamics in a thermionic RF gun

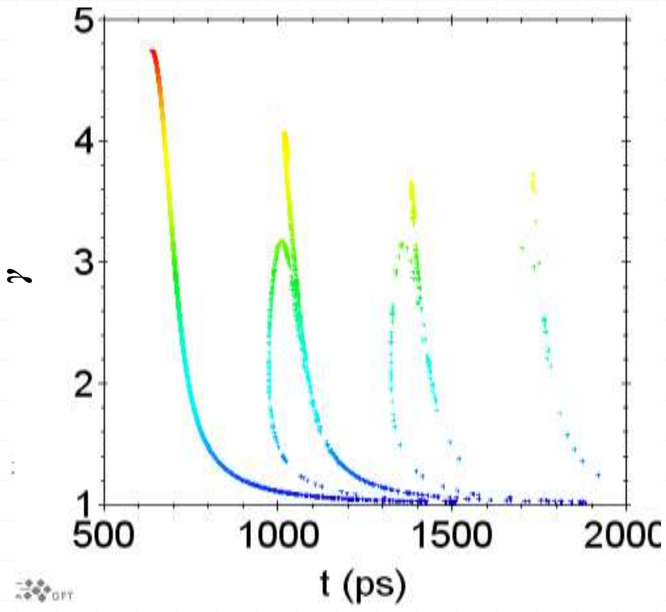


Particle distribution of generated electrons at the thermionic rf gun exit

- transverse phase space -



- longitudinal phase space -



ELECTRON BEAM PARAMETERS OF OPERATING FACILITIES

SR

FACILITY NAME	Size and Location	Energy	Pulse length	Energy spread	Equilibrium emittance	Rep. rate
TPS 2016	C=518.4 m TAIWAN	3 GeV	22 ps	8.86×10^{-4}	1.6 nm-rad	~ 500 MHz (M-mode) ~580 kHz (S-mode)
MAX-IV 2017	C=528 m Sweden	3 GeV	400 ps	7.7×10^{-4}	320 pm-rad	~ 100 MHz (M-mode)
EBS 2020	C=844 m France	6 GeV	23 ps	9.3×10^{-4}	132 pm-rad	~ 350 MHz (M-mode)

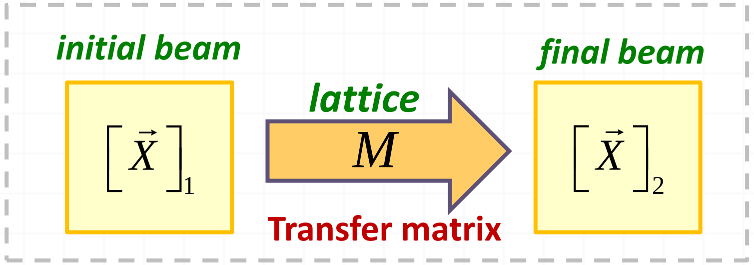
LINAC FEL

FACILITY NAME	Size and Location	Energy	Pulse length	Energy spread	ε_{nx}	Geometric emittance	Rep. rate
SACLA, 2011	0.72 km RIKEN, JAPAN	8.5 GeV	10 fs	10^{-4}	0.7 mm-mrad	42 pm-rad	30-60 Hz
PAL-XFEL 2017	1.1 km Pohang, Korea	10 GeV	10 – 100 fs	----	0.5 mm-mrad	26 pm-rad	60 Hz
LCLS-II, 2022	3.5 km SLAC, USA	4 GeV	50 fs	$< 10^{-3}$	0.45 mm-mrad	57 pm-rad	0.62 MHz

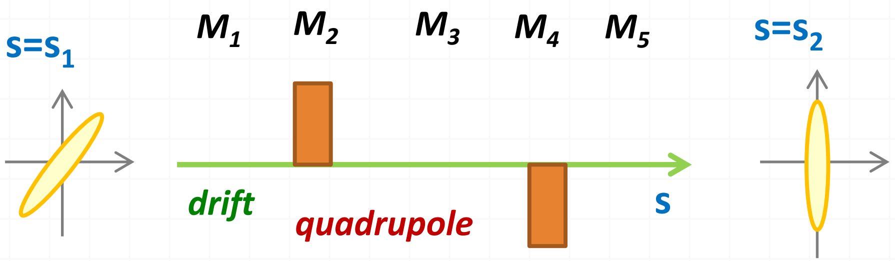
BEAM TRANSPORTATION & BUNCH COMPRESSOR

- the particle motion in a 6-D phase space can be expressed by the transfer matrix

$$\left[\vec{X}\right]_2 = \mathrm{M}\left[\vec{X}\right]_1, \qquad \vec{X} = \left(x, x', y, y', z, \delta\right), \quad \delta = \Delta E / E = \Delta\gamma / \gamma$$



(Example of beam transport line)



$$\boxed{M = M_5 M_4 M_3 M_2 M_1}$$

$$\begin{bmatrix} x(s_2) \\ x'(s_2) \\ y(s_2) \\ y'(s_2) \\ z(s_2) \\ \delta(s_2) \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} & R_{16} \\ R_{21} & R_{22} & R_{23} & R_{24} & R_{25} & R_{26} \\ R_{31} & R_{32} & R_{33} & R_{34} & R_{35} & R_{36} \\ R_{41} & R_{42} & R_{43} & R_{44} & R_{45} & R_{46} \\ R_{51} & R_{52} & R_{53} & R_{54} & R_{55} & R_{56} \\ R_{61} & R_{62} & R_{63} & R_{64} & R_{65} & R_{66} \end{bmatrix} \begin{bmatrix} x(s_1) \\ x'(s_1) \\ y(s_1) \\ y'(s_1) \\ z(s_1) \\ \delta(s_1) \end{bmatrix},$$

where

$$R_{11}\left(x|x_0\right),$$

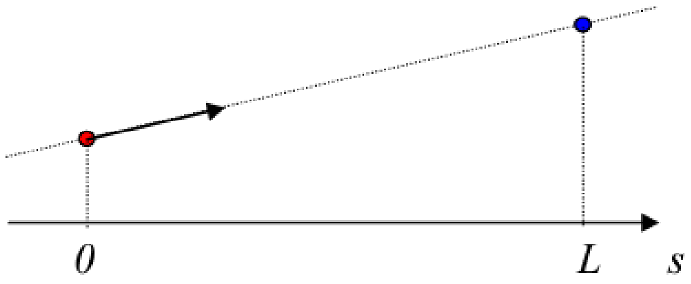
$$R_{12}\left(x|x'_0\right),\ldots$$

$$R_{26}\left(x'|\delta_0\right),$$

$$R_{56}\left(z|\delta_0\right), T_{566}\left(z|\delta_0^2\right), U_{5666}\left(z|\delta_0^3\right),\ldots$$

[Ref]: Handbook of Accelerator Physics and Engineering, Alexander Wu Chao et al., World Scientific.

(1) Drift space



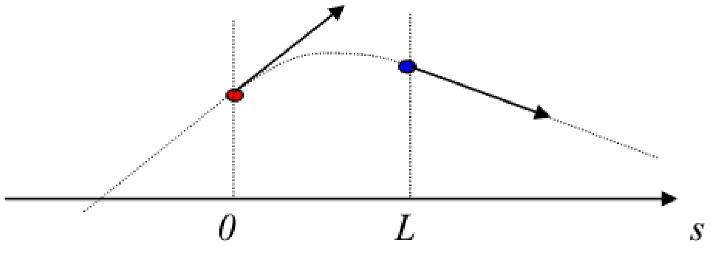
$$x_2 = x_1 + x_{p1} \cdot L$$

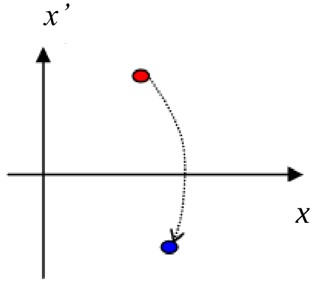
$$x_{p2} = x_{p1}$$

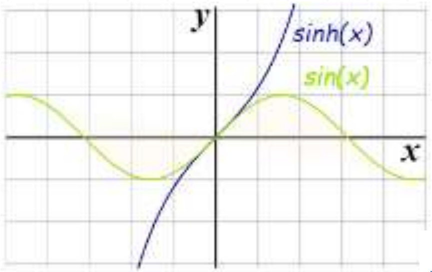
$$R_{drift} = \begin{bmatrix} 1 & L & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & L & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

- The slope remains constant
- Position varies linearly as distance

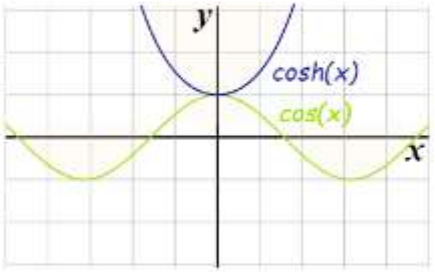
(2) Quadrupole magnet







$$\sinh(x) = \frac{e^{x} - e^{-x}}{2}$$



$$\cosh(x) = \frac{e^{x} + e^{-x}}{2}$$

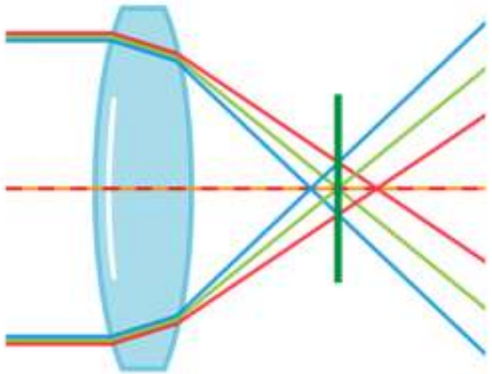
$$R_{quad} = \begin{bmatrix} \cos kL & \frac{\sin kL}{k} & 0 & 0 & 0 & 0 \\ -k\sin kL & \cos kL & 0 & 0 & 0 & 0 \\ 0 & 0 & \cosh kL & \frac{\sinh kL}{k} & 0 & 0 \\ 0 & 0 & k\sinh kL & \cosh kL & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$k = \sqrt{\frac{g}{B\rho}} \qquad B\rho[T-m] = \frac{p}{q} = 3.33\beta_{\gamma}\mathrm{E}\ [\mathrm{GeV}]$$

- Horizontal focusing while vertical defocusing
- Momentum dependent focusing effects $k = k(\delta)$

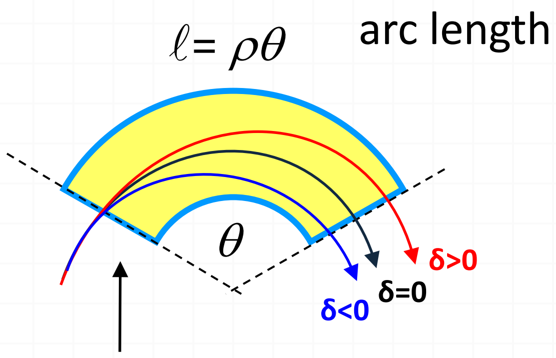
→ Different focal length for off-momentum particle

→ chromatic aberration



NSRRC

(3) Sector dipole magnet (hard-edge)



Particle enters and exits normal to the magnet pole faces

$$R_{sector} = \begin{bmatrix} \cos\theta & \rho\sin\theta & 0 & 0 & 0 & \rho(1-\cos\theta) \\ -\frac{1}{\rho}\sin\theta & \cos\theta & 0 & 0 & 0 & \sin\theta \\ 0 & 0 & 1 & \ell & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \sin\theta & \rho(1-\cos\theta) & 0 & 0 & 1 & \rho(\theta-\sin\theta) \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

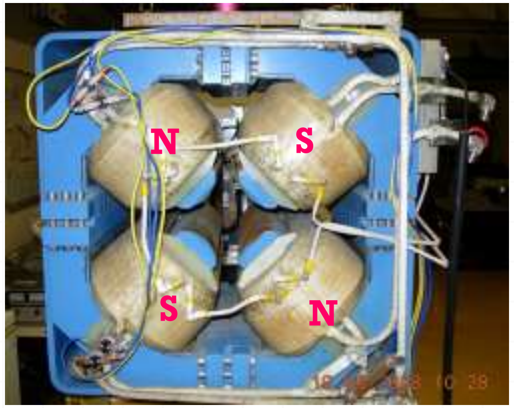
- a natural focusing effects in the horizontal deflection plane ; Vertical direction acts as a pure drift space
- **R16**: momentum dependent horizontal displacement term (larger momentum particle gets less bending)

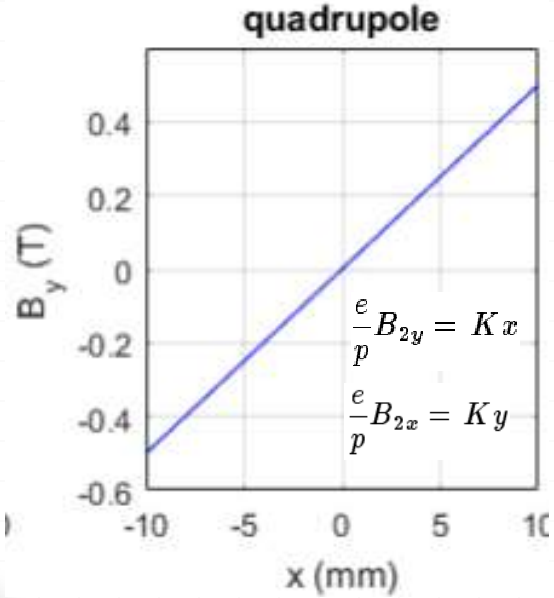
  → dispersion η → cause beam size broadening $\sigma_x = \sqrt{\varepsilon\beta} + \eta\delta$

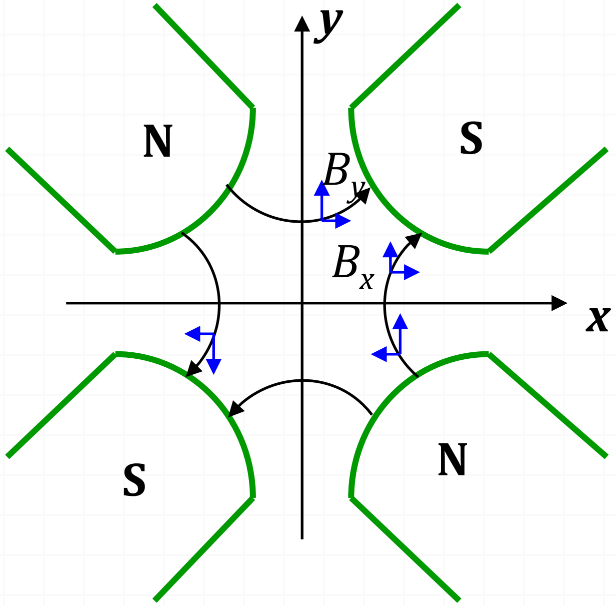
  → **control of R16 for dispersion compensation**
- **R56**: momentum dependent longitudinal displacement term (larger momentum particle has shorter traveling path)

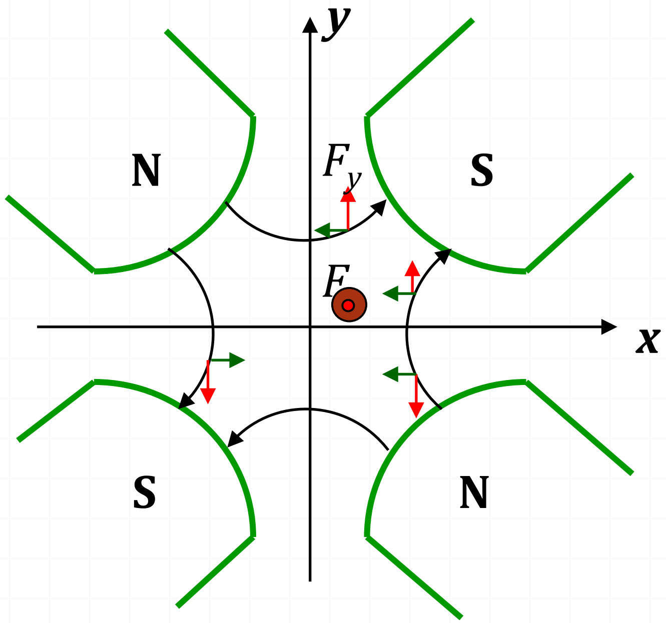
  → will lead to bunch lengthening → **control of R56 for the design of a bunch compressor**

QUADRUPOLE MAGNETS FOR BEAM FOCUSING



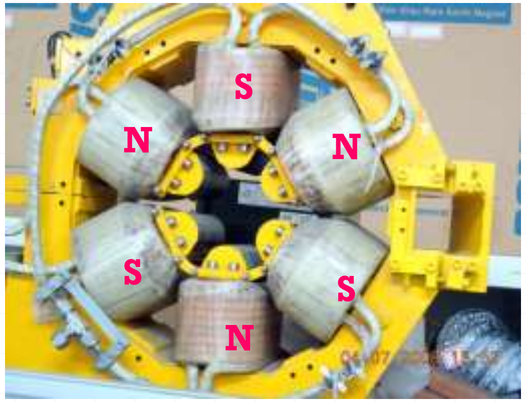


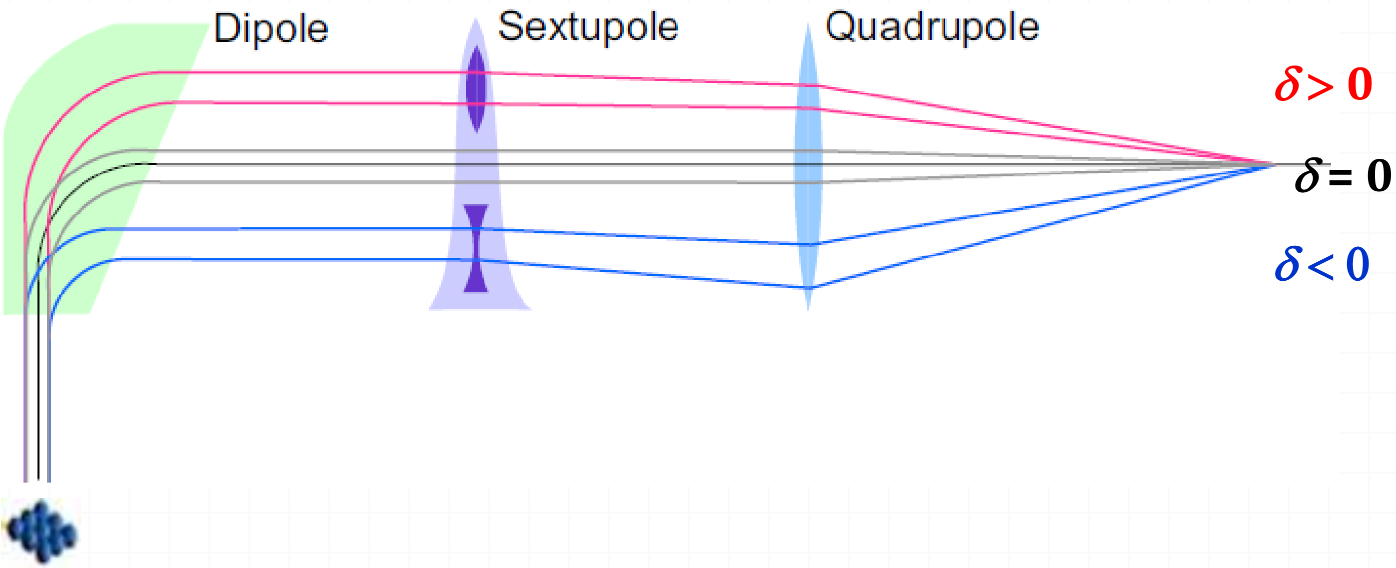


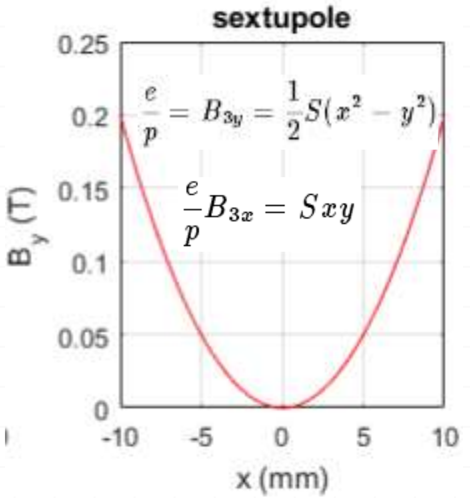


- Horizontal focusing, while vertical defocusing
- Needs at least two quadrupole magnets for beam focusing

SEXTUPOLE MAGNET FOR CHROMATIC CORRECTION







- Sextupole magnet

+x particle, increase focusing strength
-x particle, decrease focusing strength

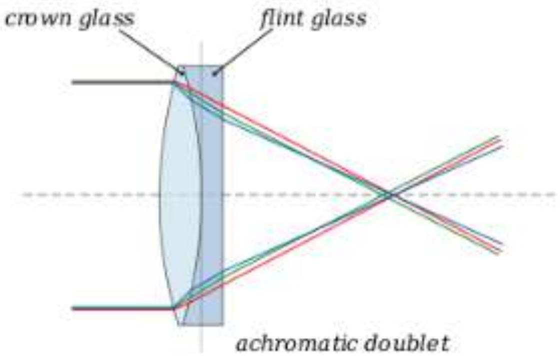
- Chromatic aberration originated from the quadrupole

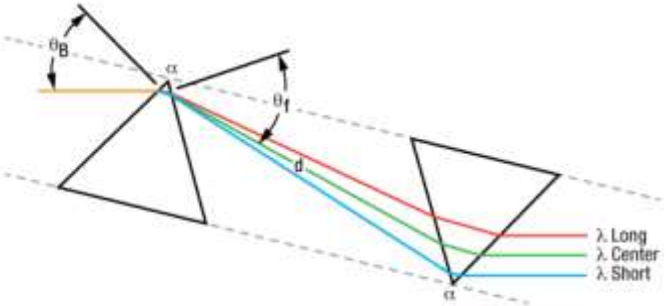
could be corrected by the inclusion of dispersive dipole + sextupole element

p.s. however, the nonlinear sextupole magnet will introduce the geometric aberration

CHARGED PARTICLE BEAM AND PHOTON BEAM TRANSPORTATION

	Charged particle	Photon beam
Deflection	Bending magnet	Mirror、prism
Focusing	Quadruple magnet-s	Lens
Chromatic correction	quadrupole 、 Dipole+ sextupole	Lens complex
Dispersion compensation (or bunch length control)	Dipole pairs	Prism pairs





BUNCH COMPRESSION SCHEME

Magnetic Bunching

- Step 1. RF chirp
- Step 2. dispersive section

- space for two-stage process
- wide range of adopted beam energy

< Issue >

- nonlineartity
- CSR emittance degradation,
- wake field …

Velocity Bunching

- 1 step in accelerating structure:
  acceleration + compression

- compact and simple operation
- suitable for low energy beam

< Issue >

- nonlinearilty
- Space charge effects

CONCEPT OF MAGNETIC BUNCHING

- **Step 1. energy modulation** - rf section
- **Step 2. dispersive region**
  - – bending magnet, chicane (four-dipole system)…

→ **Path difference of energy correlated bean**

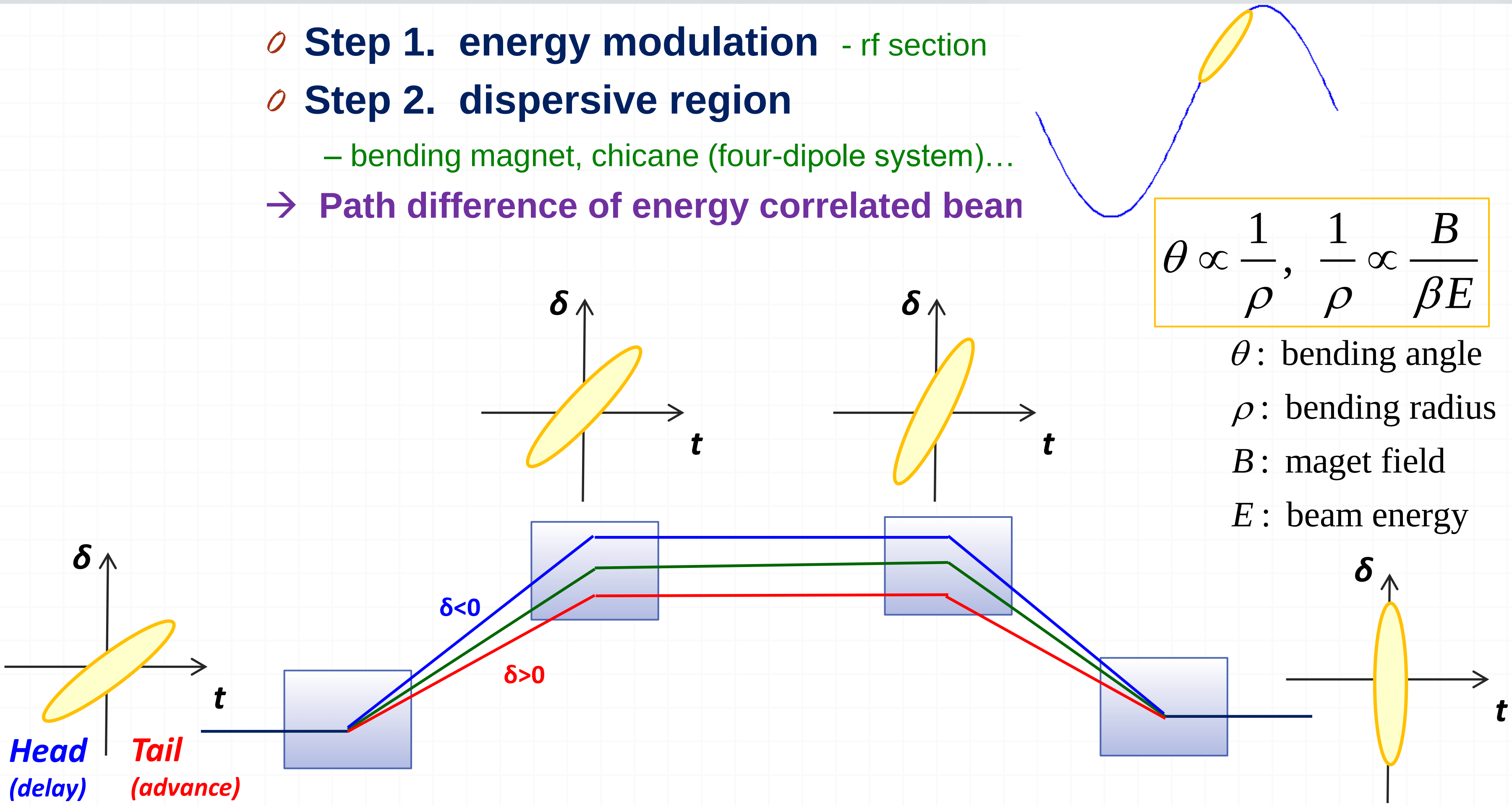
$$\theta \propto \frac{1}{\rho}, \quad \frac{1}{\rho} \propto \frac{B}{\beta E}$$

θ : bending angle

ρ : bending radius

B : maget field

E : beam energy



trajectory of lower energy (larger bending angle, longer path, delay)

trajectory of central energy

trajectory of higher energy (smaller bending angle, shorter path, advance)

LIMITATION OF MAGNETIC BUNCHING (1/2)

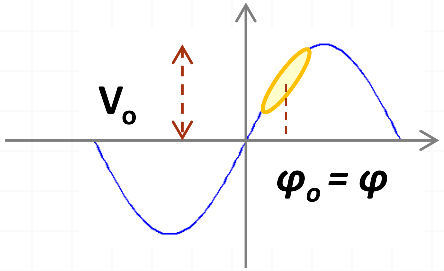
- Energy is modulated by the rf field as

$$E_f(z) = E_i + eV_0\cos(\varphi_0 + 2\pi z/\lambda), \qquad \text{(eq. 1)}$$

- Assumed the injected electron centroid energy is E_i and no chirp for simplicity ($\delta_i = 0$)

→ The correlated energy spread for the electron centroid operated at $\varphi_0 = \varphi$

$$\boxed{\delta(z) = \frac{eV_0\cos(\varphi + 2\pi z/\lambda)}{E_f} = h_1^b z + h_2^b z^2 + h_3^b z^3 + \ldots,} \qquad \text{(eq. 2)}$$



$$\begin{cases} h_1^b = -\dfrac{E_f - E_i}{E_f} k\tan\varphi, \; 1^{\text{st}} \text{ order chirp} \\ h_2^b = -\dfrac{E_f - E_i}{2E_f} k^2, \; 2^{\text{nd}} \text{ order chirp } <0 \\ h_3^b = -\dfrac{E_f - E_i}{6E_f} k^3\tan\varphi, \; 3^{\text{rd}} \text{ order chirp} \end{cases} \qquad \text{(eq. 3)}$$

- For a general dispersive area includes the high order nonlinear term,
- By combining eq.2 and eq.4 → the electron position can be expressed as

$$\boxed{z_f = z_i + R_{56}\delta + T_{566}\delta^2 + U_{5666}\delta^3 + \ldots,} \qquad \text{(eq. 4)}$$

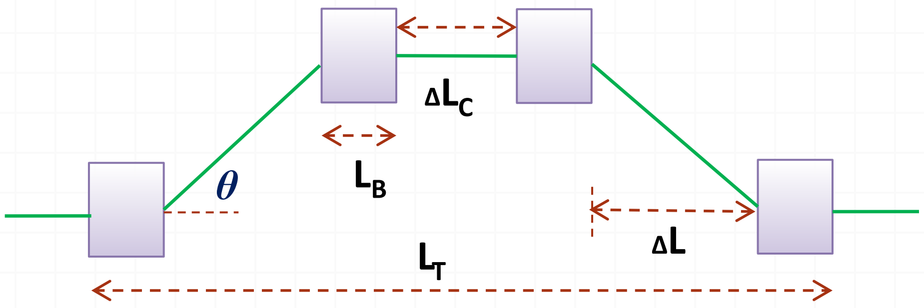
→ Nonlinear term in LPS

$$\boxed{z_f = z_i/C + A_2 z_i^2 + A_3 z_i^3 + \ldots,} \qquad \text{(eq. 5)}$$

$$\boxed{\begin{aligned} &C = 1/\left(1 + h_1^b R_{56}\right), \; \text{linear compression ratio} \\ &A_2 = h_2^b R_{56} + \left(h_1^b\right)^2 T_{566}, \quad A_3 = h_3^b R_{56} + 2h_1^b h_2^b T_{566} + \left(h_1^b\right)^3 U_{5666}. \end{aligned}} \qquad \text{(eq. 6)}$$

LIMITATION OF MAGNETIC BUNCHING (2/2)

- For typical chicane (4 dipole+drift)



$$S = L_T\sqrt{1+\theta^2} \approx L_T\left(1+0.5\theta^2\right)$$

$$\left[\begin{array}{l}\Delta S \propto \theta^2 = \left(\dfrac{\theta_0^2}{1+\delta}\right)^2 = \theta_0^2\left(1-2\delta+3\delta^2-4\delta^3+\ldots\right), \\ \Delta S = \Delta S_0 + R_{56}\delta + T_{566}\delta^2 + U_{5666}\delta^3 + \ldots\end{array}\right.$$

$$\rightarrow T_{566} = -1.5R_{56}, U_{5666} = 2R_{56}$$

$$\text{where } R_{56} \approx -2\theta^2\left(\frac{L_T}{2} - \frac{4}{3}L_B - \frac{\Delta L_c}{2}\right) < 0,$$

$$\text{then } T_{566} > 0, \quad U_{5666} < 0$$

$$z_f = z_i / C + A_2 z_i^2 + \ldots,$$

$$\text{where } A_2 = h_2^b R_{56} + \left(h_1^b\right)^2 T_{566},$$

$\rightarrow A_2 > 0$ for conventional chicane

→ high order nonlinear term in the longitudinal phase space

→ current spike in the compressed bunch (unwanted current structure for FEL)

➢ **Solution:**

introduction of quadrupole and sextupole

$$R_{56} = \int \frac{R_{16}}{\rho} dS,$$

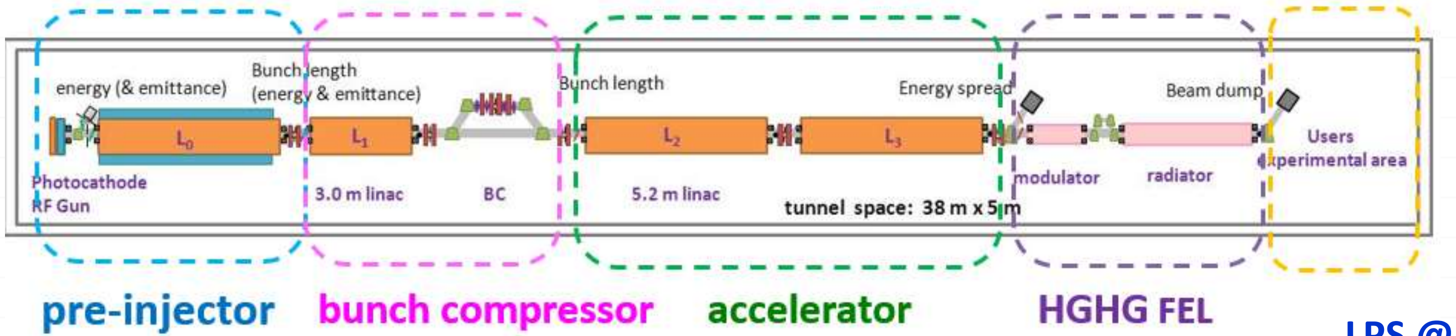
$$T_{566} = \int \frac{T_{166}}{\rho} + \frac{1}{2}R_{26}^2 + \frac{1}{2}\left(\frac{R_{16}}{\rho}\right)^2 dS,$$

→ **compressor with adjustable R_{56} and T_{566}**

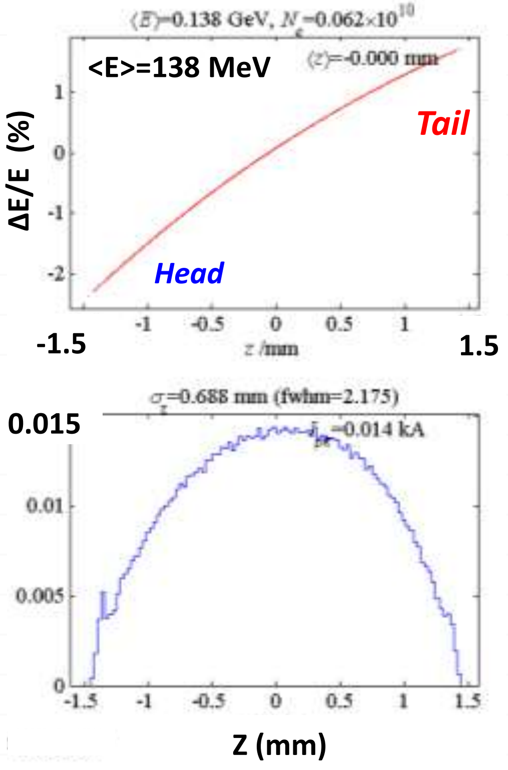
$$\text{when } \boxed{T_{566} = \frac{-h_2^b R_{56}}{\left(h_1^b\right)^2}} \rightarrow A_2 = 0$$

→ smooth current distribution of compressed bunch is possible

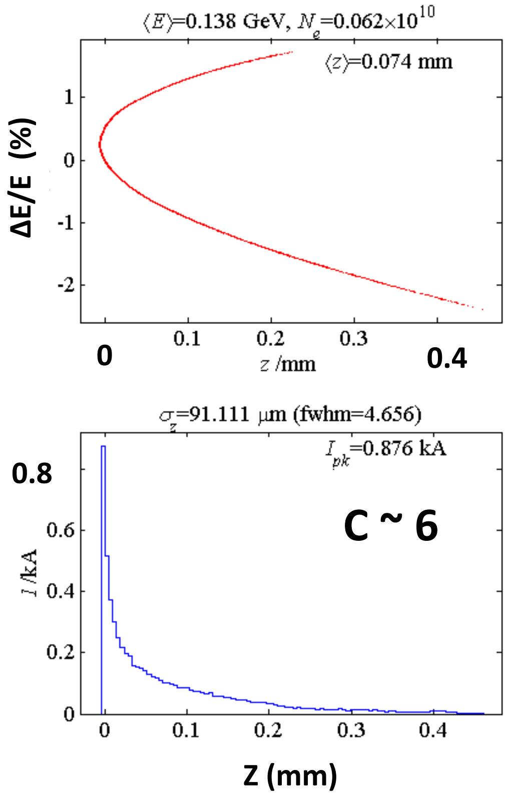
MAGNETIC BUNCHING COMPRESSION



- LPS @ L1 exit -

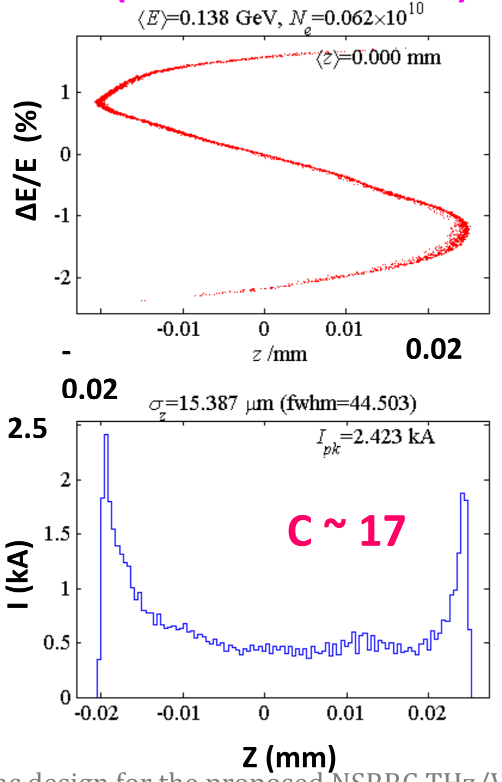


- LPS @ BC exit-



LPS @ BC exit

(with QS correction)

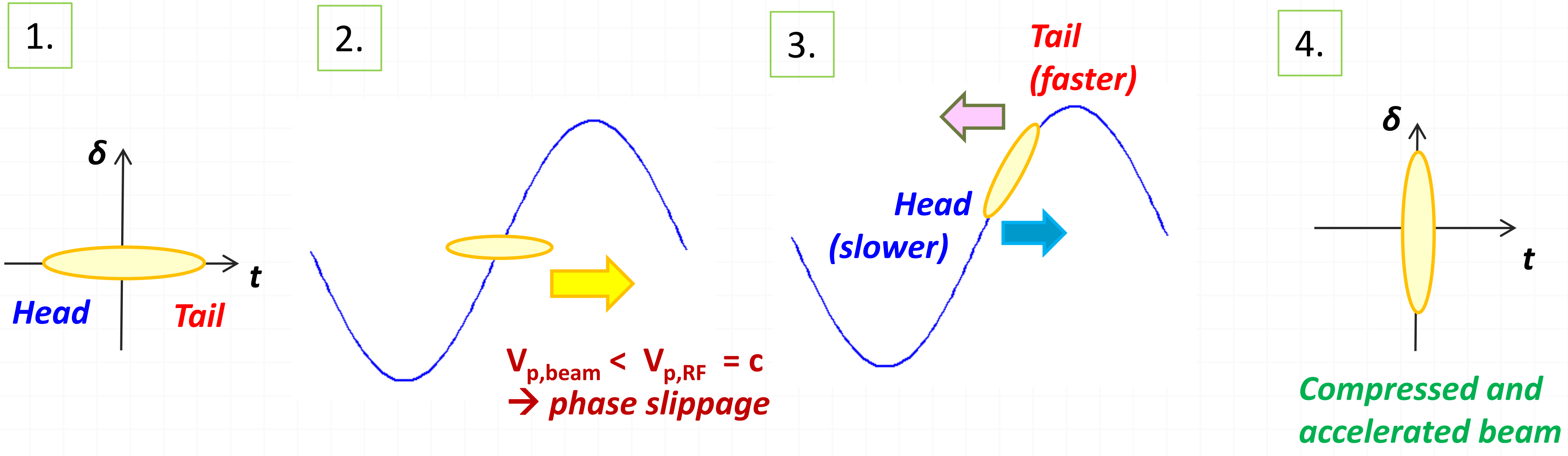


[ref] Linac design for the proposed NSRRC THz/VUV FEL facility, IPAC2014.

CONCEPT OF VELOCITY BUNCHING

1. Injected the beam near the rf zero crossing phase
2. Phase slippage due to velocity difference of electron beam and rf field
3. Rotation of longitudinal phase space
4. Compression and acceleration simultaneously in the accelerating structure

- Velocity difference of energy modulated beam → velocity bunching



$$V = \beta c = c\sqrt{1-\frac{1}{\gamma^2}}$$

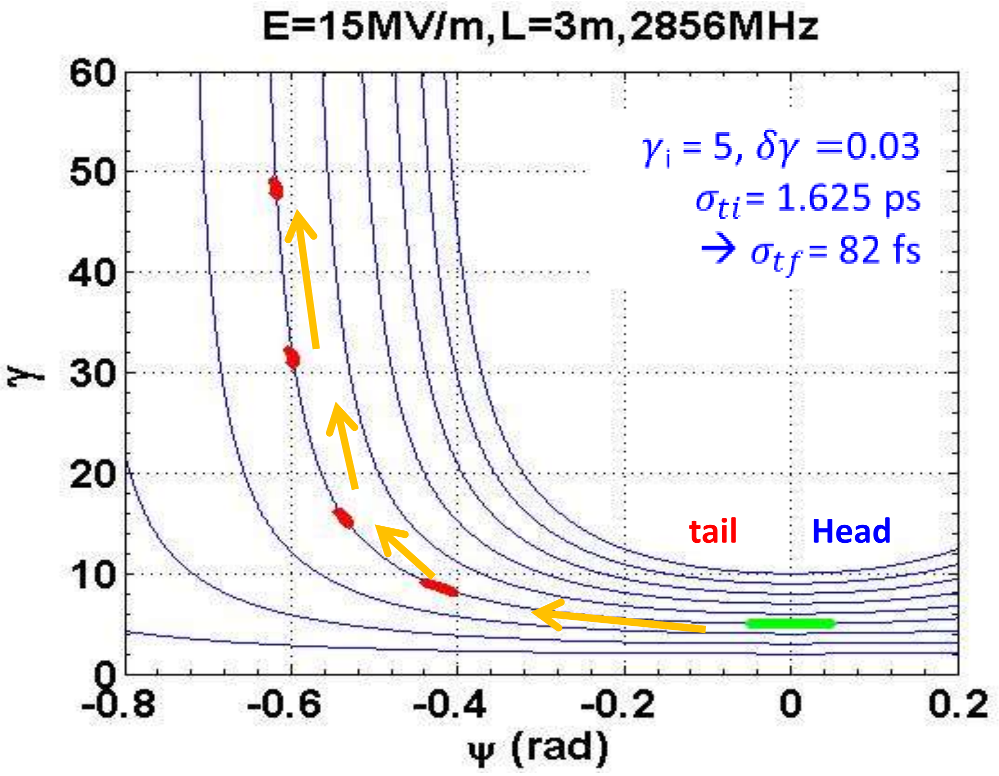
- For high energy beam (γ>>1, β~1), particle velocity is saturated to c.
- This scheme works only for low energy particle (β<1).

Velocity Bunching

→ the deformation of longitudinal phase space during phase slippage

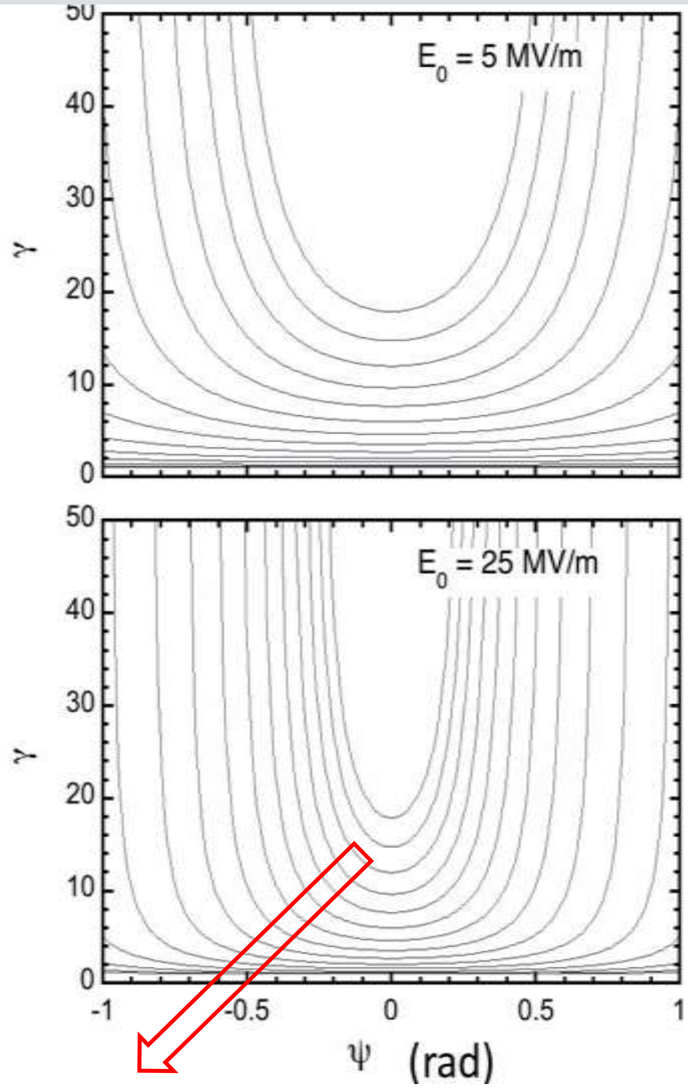
→ bunch is compressed and accelerated simultaneously through the accelerating structure

$$\begin{cases} \dfrac{d\gamma}{dz} = -\alpha k \sin(\psi) = -\dfrac{\partial H}{\partial \psi}, \\ \dfrac{d\psi}{dz} = k\left(1 - \dfrac{\gamma}{\sqrt{\gamma^2 - 1}}\right) = \dfrac{\partial H}{\partial \gamma}, \\ H = \left(\gamma - \sqrt{\gamma^2 - 1}\right)k - \alpha k \cos(\psi). \end{cases}$$



✓ [note] The minimum compressed bunch length will not be zero even for the ideal injected beam → the limitation of rf nonlinearity

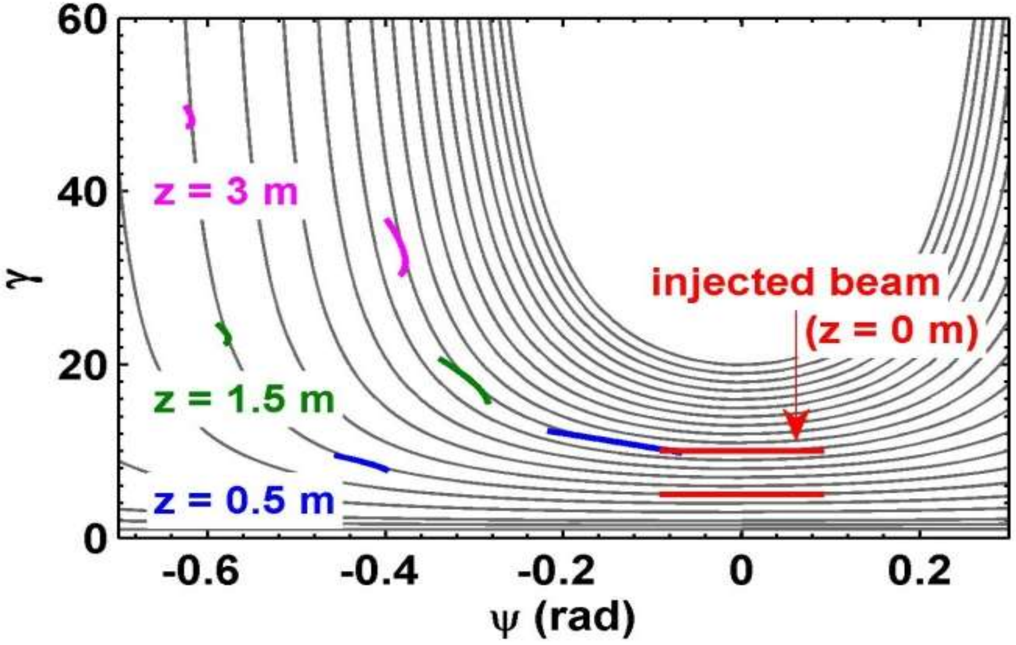
LIMITATION OF VELOCITY BUNCHING



nonlinearity of the equi-potential line at higher injection energy is stronger

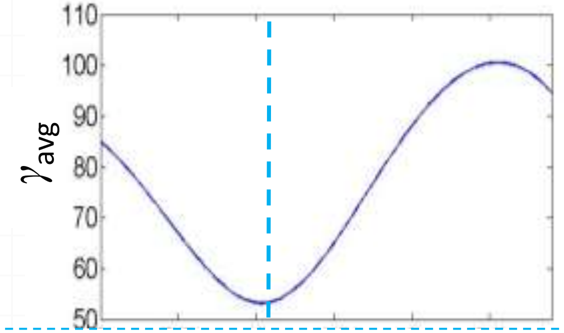
- RF nonlinearity leads to asymmetric distribution of longitudinal phase space
- → the limitation of compression in bunch length
- Possible solution to mitigate this effect
  - → injection of lower energy beam (higher compression)

(1) γ_i=10, 3 ps → γ_f=33, 335 fs, C=9
(2) γ_i=5, 3 ps → γ_f=49, 135 fs , C =22

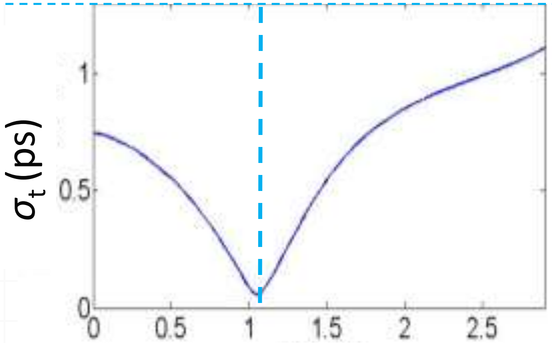


BUNCH COMPRESSION VIA VELOCITY BUNCHING IN LINAC

- Relation of rf phase v.s. beam energy and bunch length at the linac exit-

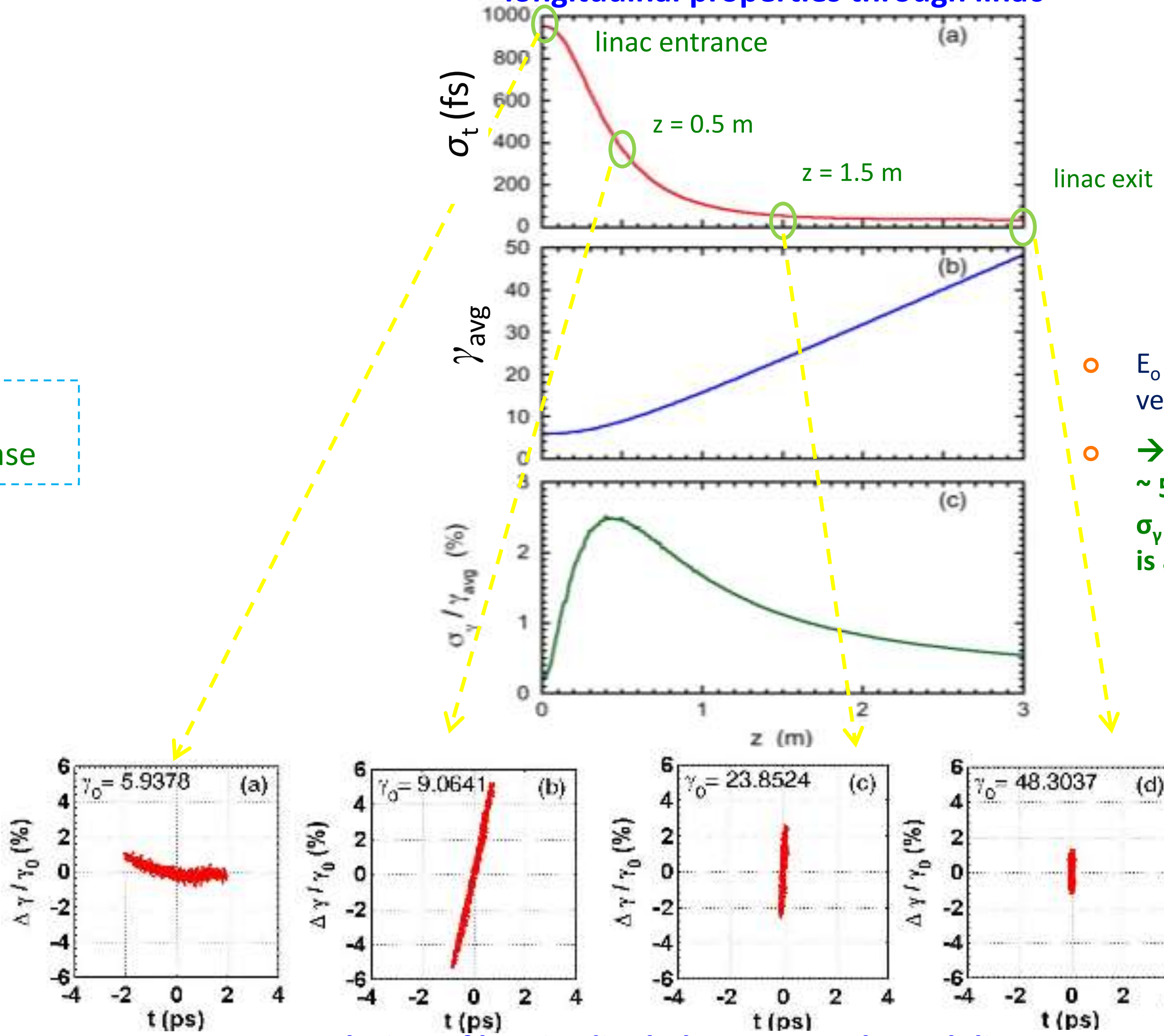


The shortest bunch length is achieved when linac is operated near the rf zero crossing phase



(3 m linac, at E_p= 20 MV/m, injected beam is from α-magnet operated at 4 T/m)

- longitudinal properties through linac-

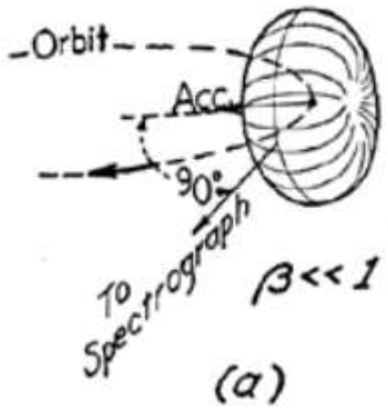


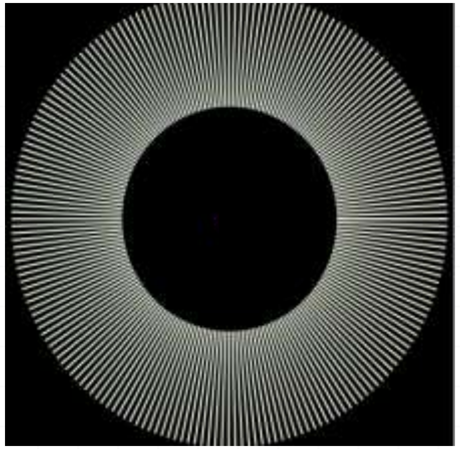
- **Evolution of longitudinal phase space through linac-**

- E_o >10 MV/m, 3 m linac, velocity bunching
- **→ the compressed bunch σ_t ~ 50 fs, Q ~ 30pC, I_p > 250 A, σ_γ / γ ~ 0.5 %, at E < 30 MeV is attainable**

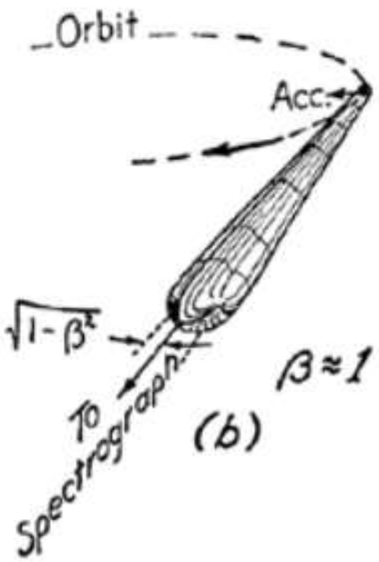
SYNCHROTRON RADIATION FROM RELATIVISTIC ELECTRON

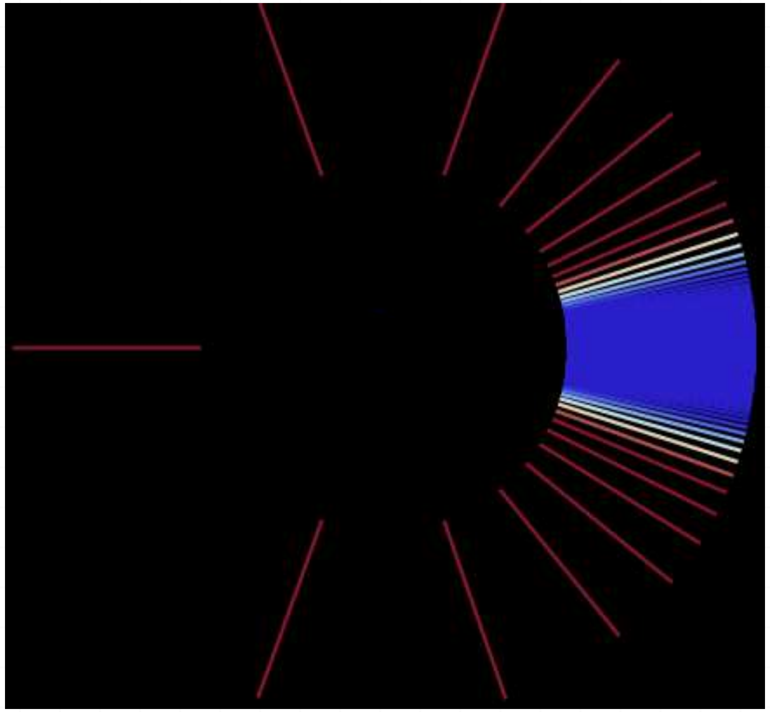
RELATIVISTIC ABERRATION AND DOPPLER EFFECTS





- an evenly distributed light rays incident on an observer at rest





- **Relativistic aberration effects:** → **Excellent collimation of SR light (~ $1/\gamma$)**

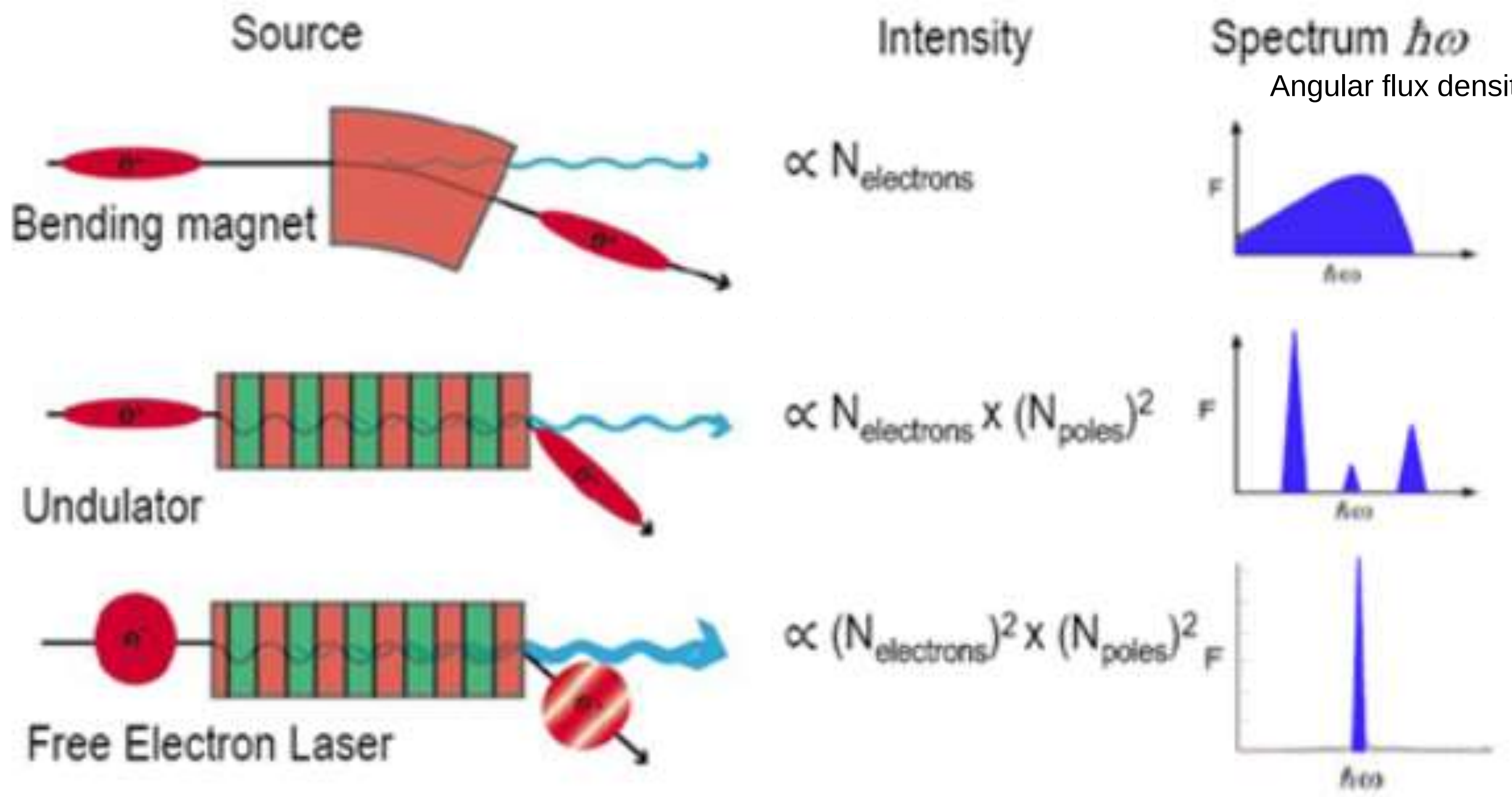
For a moving observer, light rays appear to be tilted in the direction of motion

- **Doppler effect:**

light that comes from the direction of motion is blue-shifted, while light from the opposite direction is red-shifted

[ref] https://demonstrations.wolfram.com/RelativisticAberrationAndDopplerShift/

FORMS OF SYNCHROTRON RADIATION

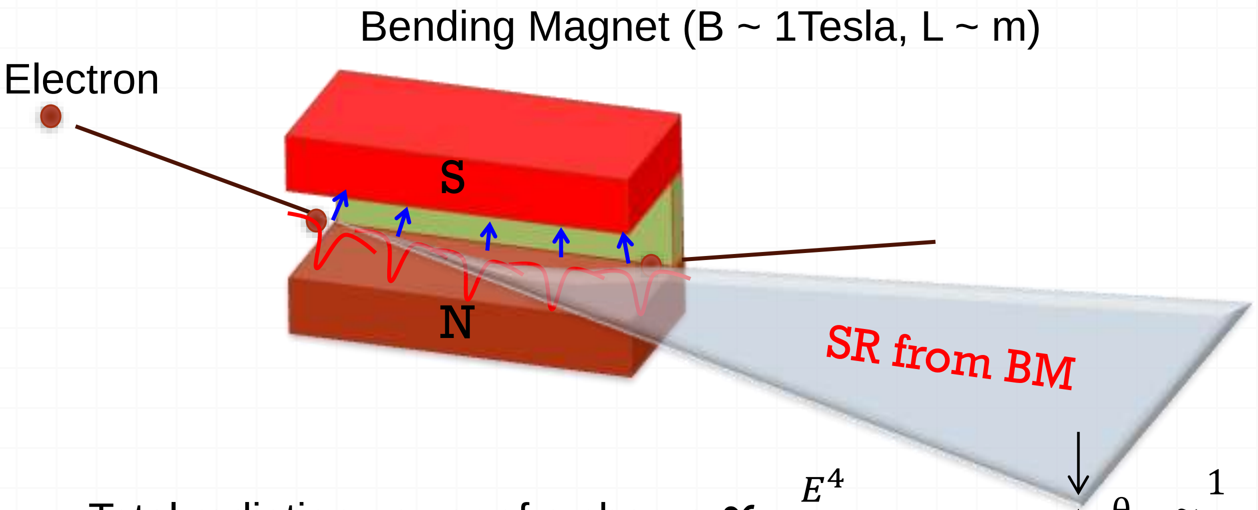


Angular flux density

- constructive interference of E field from the overlapping of e^- moving trajectory
- constructive interference of E field from the correlated electrons in phase

[ref] https://www.bnl.gov/ps/userguide/lectures/Lecture-2-Shaftan.pdf

SYNCHROTRON RADIATION FROM BENDING MAGNET

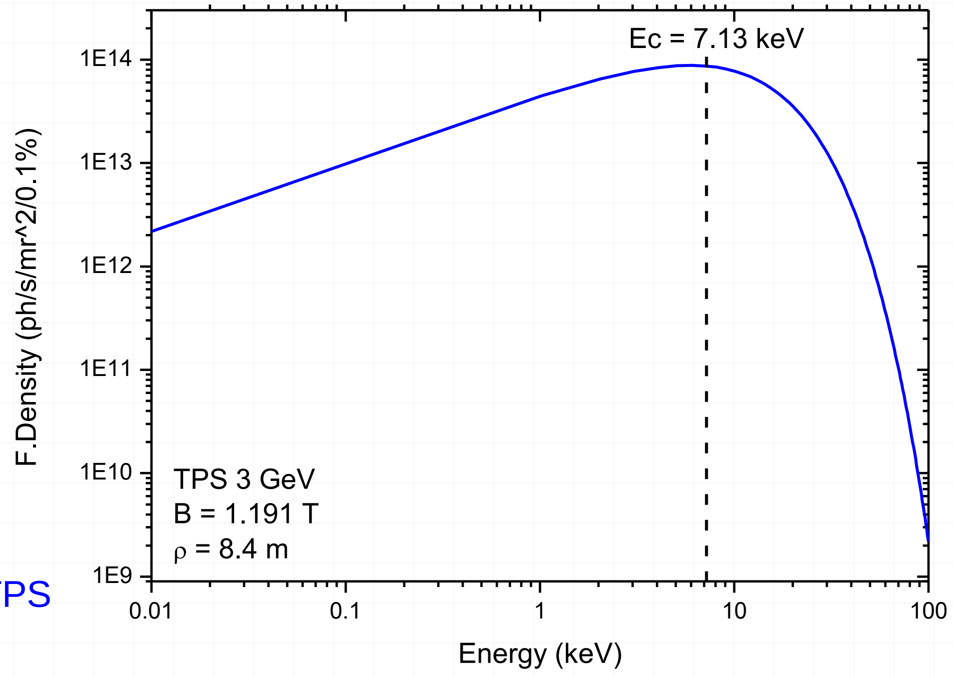


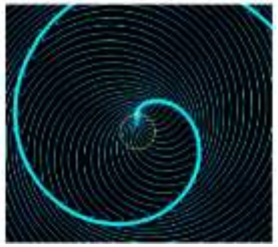
$\theta_{rad} \sim \frac{1}{\gamma}$

±0.17 mrad at TPS

Total radiation power of a charge $\propto \dfrac{E^4}{\rho^2 m_0^4}$

Total radiation power of a beam $\propto I$





at v = 0.9c

- ➢ Field lines condensed as a narrow spiral zone
- → shorter pulse $\propto \frac{1}{\gamma^3}$
- → wider frequency spectrum

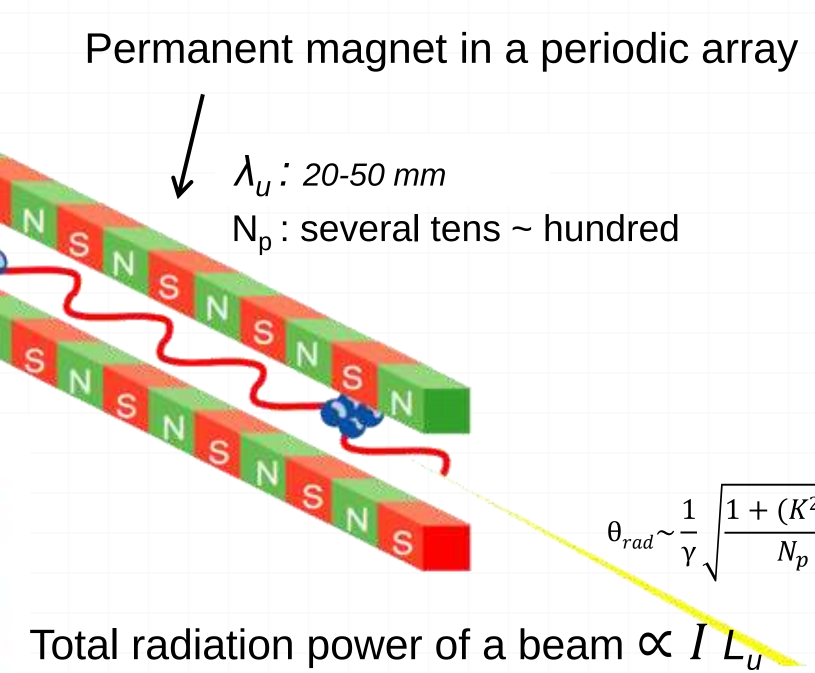
- Ec equally divided the spectrum distribution
- $E_c \sim E_{max}$ with maximum flux density
- An index to compare the bending source

$$\boxed{E_c \propto \frac{\gamma^3}{\rho}}$$

TLS (r= 3.495 m, E= 1.5 GeV)
ε_c ~ 2 keV
TPS (r= 8.4 m, E= 3 GeV)
ε_c ~ 7 keV

[Ref] Radiation 2D, T. Shintake, Real-time animation of synchrotron radiation, NIMA 507, 89-92, 2003.
"Part I: Synchrotron Radiation", lecture notes to NYCU, Ting-Yi Chung, 2022.
Particle Accelerator Physics, Helmut Wiedemann, Springer.

UNDULATOR RADIATION

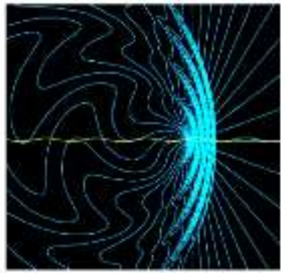


Permanent magnet in a periodic array

λ_u : 20-50 mm

N_p : several tens ~ hundred

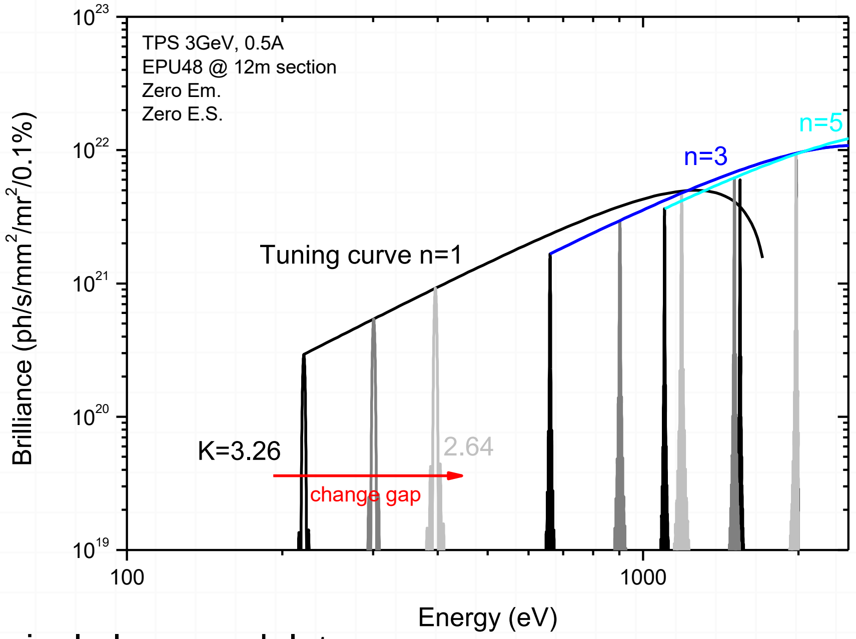
$$\theta_{rad} \sim \frac{1}{\gamma}\sqrt{\frac{1+(K^2/2)}{N_p}}$$



at v = 0.9c, K=1.

Total radiation power of a beam ∝ $I\ L_u$

- Electric fields are composed of N_p periods
- radiation pulse duration ∝ N_p
- FFT → sinc function like spectrum



For typical planar undulator

$$\lambda_n = \boxed{\frac{\lambda_u}{2n\gamma^2}}\left(1+\boxed{\frac{K^2}{2}}+\gamma^2\theta^2\right)$$

λ_u =20 mm、3 GeV (γ ~ 6000)、
λ_1 =0.29 nm

Lorentz contraction & Doppler effect

Adjustable radiation wavelength by tuning K (B field, the undulator gap)

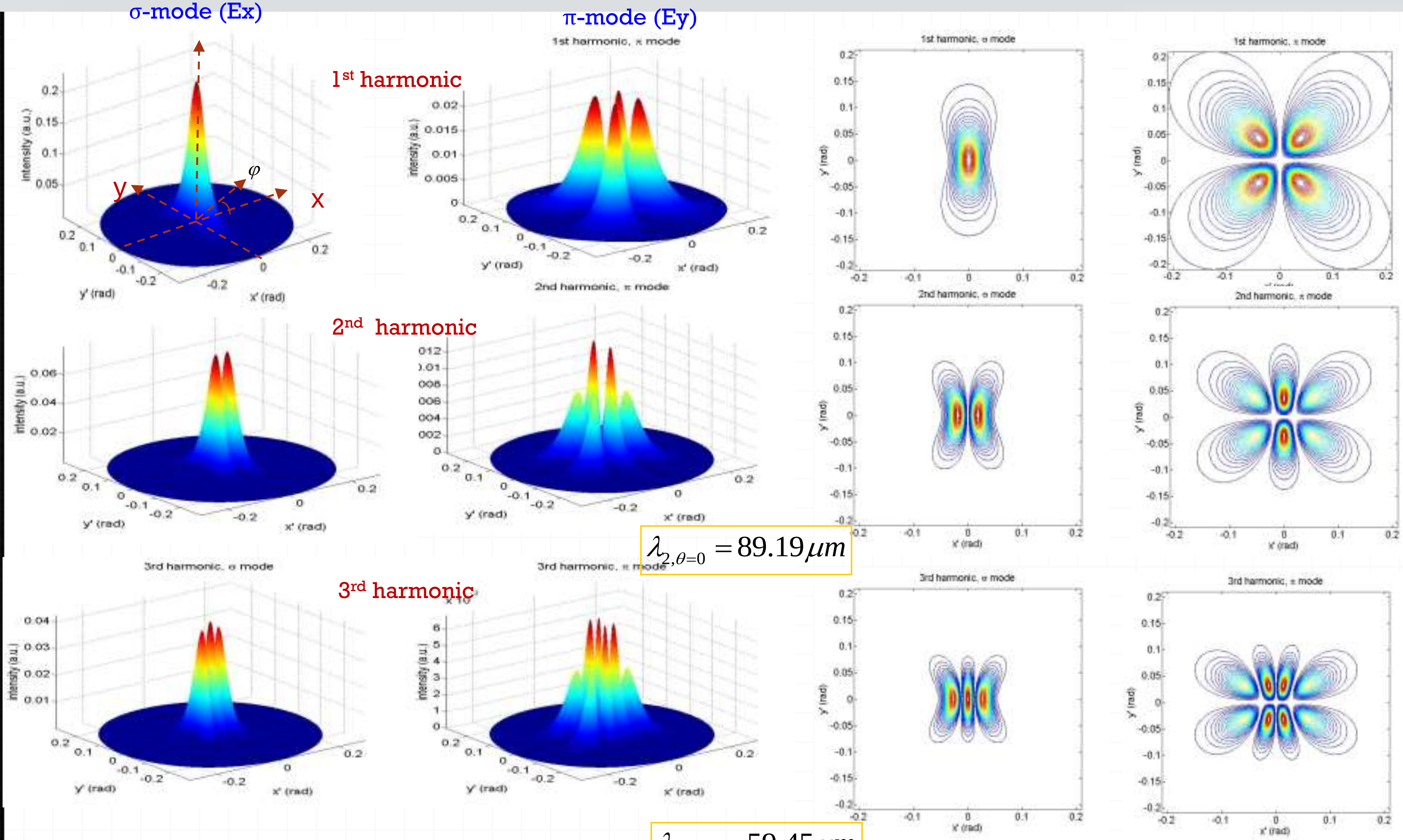
K =0.9337$B[T]\lambda_u[cm]$ = $\gamma\theta$ Deflection parameter
(~ the maximum deflection of moving trajectory)

$$\frac{\Delta\omega_n}{\omega_n} = \boxed{\frac{1}{nN_p}}$$

The spectral bandwidth decreased as the number of undulator periods increased

[Ref] Radiation 2D, T. Shintake, Real-time animation of synchrotron radiation, NIMA 507, 89-92, 2003.
"Part I: Synchrotron Radiation", lecture notes to NYCU, Ting-Yi Chung, 2022.
Particle Accelerator Physics, Helmut Wiedemann, Springer.

SPATIAL AND SPECTRAL CHARACTERISTICS OF UR



$$\lambda_{1,\theta=0} = \frac{\lambda_p}{2\gamma^2}\left(1+\frac{1}{2}K^2\right) = 178.38\mu m$$

$$\lambda_{2,\theta=0} = 89.19\mu m$$

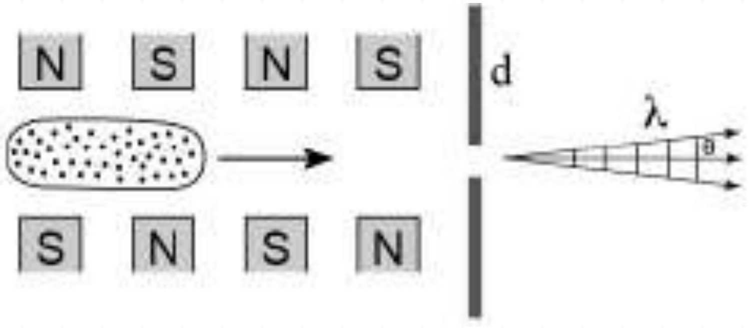
$$\lambda_{3,\theta=0} = 59.45\mu m$$

- Faster fall off in ψ=0,π for the σ mode.
- The opening angle for the radiation is smaller in the deflecting plane than the vertical one.
- No energy distribution in ψ=0,π for the π mode

- ➢ The over all spatial intensity distribution includes a complex set of different radiation lobe depending on frequency, emission angle, and polarization.
- ➢ there is no π-mode along the forward direction (θ=0) while there is a strong forward lobe for the σ-mode (linear polarization)
- ➢ the even harmonics will vanish in the forward direction.
- ➢ i.e. There is only odd harmonics for on-axis observation.

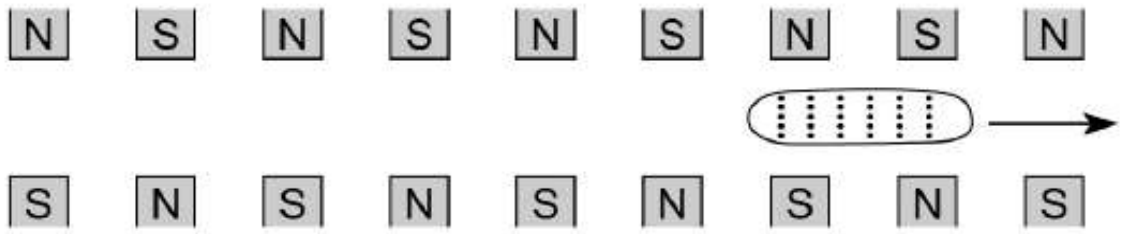
UNDULATOR RADIATION TO FREE ELECTRON LASER

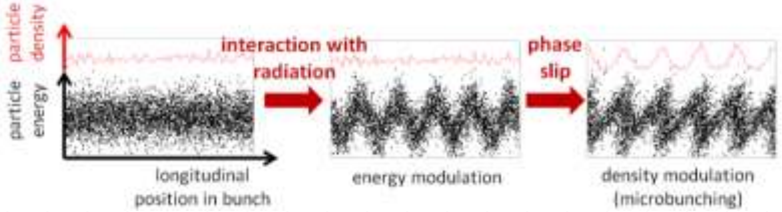
Spontaneous Undulator radiation



- Spatial coherence is somewhat limited (depends on electron beam emittance)
- temporal coherence is bad (long electron bunch in the storage ring, $\sigma_z >> \lambda_\gamma$)
- for uncorrelated electrons, $E_{tot} \propto \sqrt{N_e}$, radiation power $\propto N_e$
- Needs a pinhole to filter the enhance the coherence performance

FEL



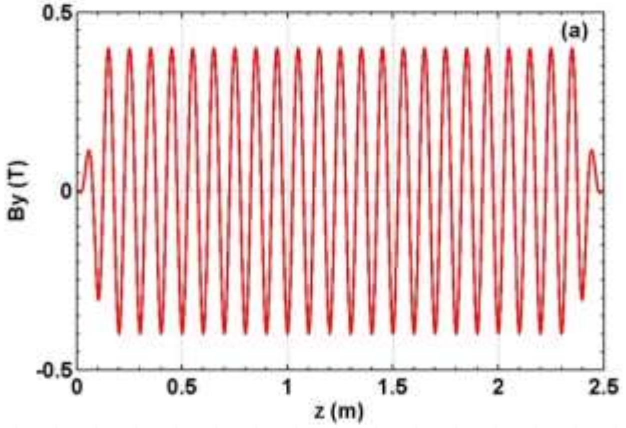


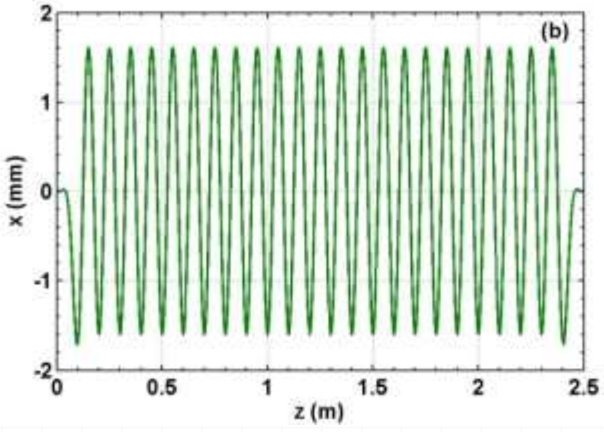
Coherence	Undulator SR	FEL
temporal	~ 10^{-5} %	~ 0.1 % (SASE) ~100 % (seeded)
Horizonal	~ 0.3 %	~ 100 %
vertical	~ 30 %	~ 100 %
total	~ 10^{-8} %	~ 0.1 % (SASE) ~100 % (seeded)

- microbunching of electrons → $E_{tot} \propto N_e$ → Radiation power $\propto N_e^2$
- ~fully spatial and temporal coherence

[Ref] From undulators to free electron lasers, David Attwood, Unversity of Calofornia, Berkley, 2014
A short introduction to free electron laser, Andy Wolski, CERN school, 2012.
Lecture of “Foundation of FEL”, Takashi Tanaka, RIKEN SPring-8 Center, ISBA 2022.

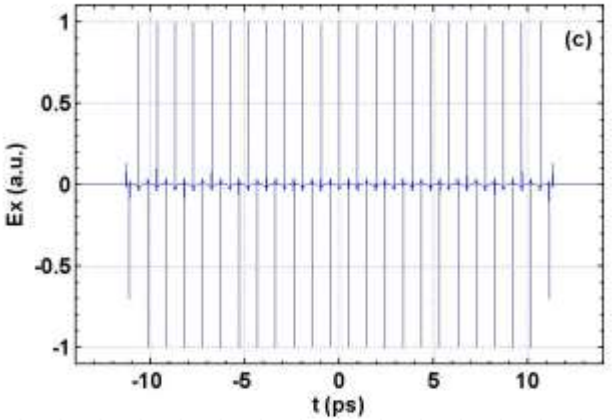
COHERENT UNDULATOR RADIATION





$$\vec{F} = e\vec{v} \times \vec{B} = \gamma m \frac{d\vec{v}}{dt}.$$

$$\begin{cases} \dfrac{dv_x}{dt} = \dfrac{-e}{\gamma m} v_z B_y, \\ \dfrac{dv_y}{dt} = \dfrac{e}{\gamma m} v_z B_x, \\ \dfrac{dv_z}{dt} = \dfrac{e}{\gamma m} v_x B_y. \end{cases}$$

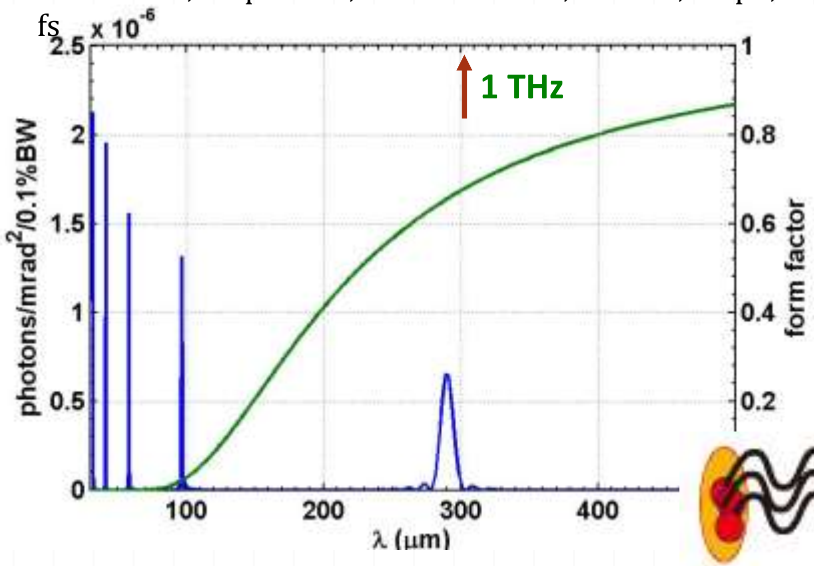


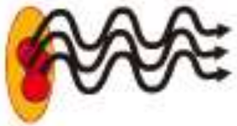
the field of a moving charge in the observed time

$$\vec{E}_{rad}(t) = \frac{q}{4\pi\varepsilon_0 cR}\left[\frac{\hat{n}\times\left[\left(\hat{n}-\vec{\beta}\right)\times\dot{\vec{\beta}}\right]}{\left(1-\hat{n}\cdot\vec{\beta}\right)^3}\right]_{ret},$$

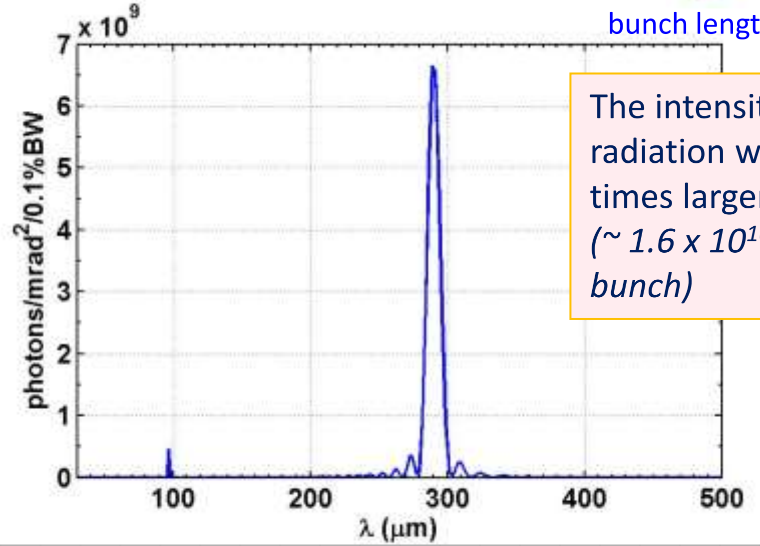
- The undulator radiation spectrum in the forward direction -

period = 0.1 m; 25 periods; Peak B = 0.4 T ; 19 MeV, 20 pC, 100 fs



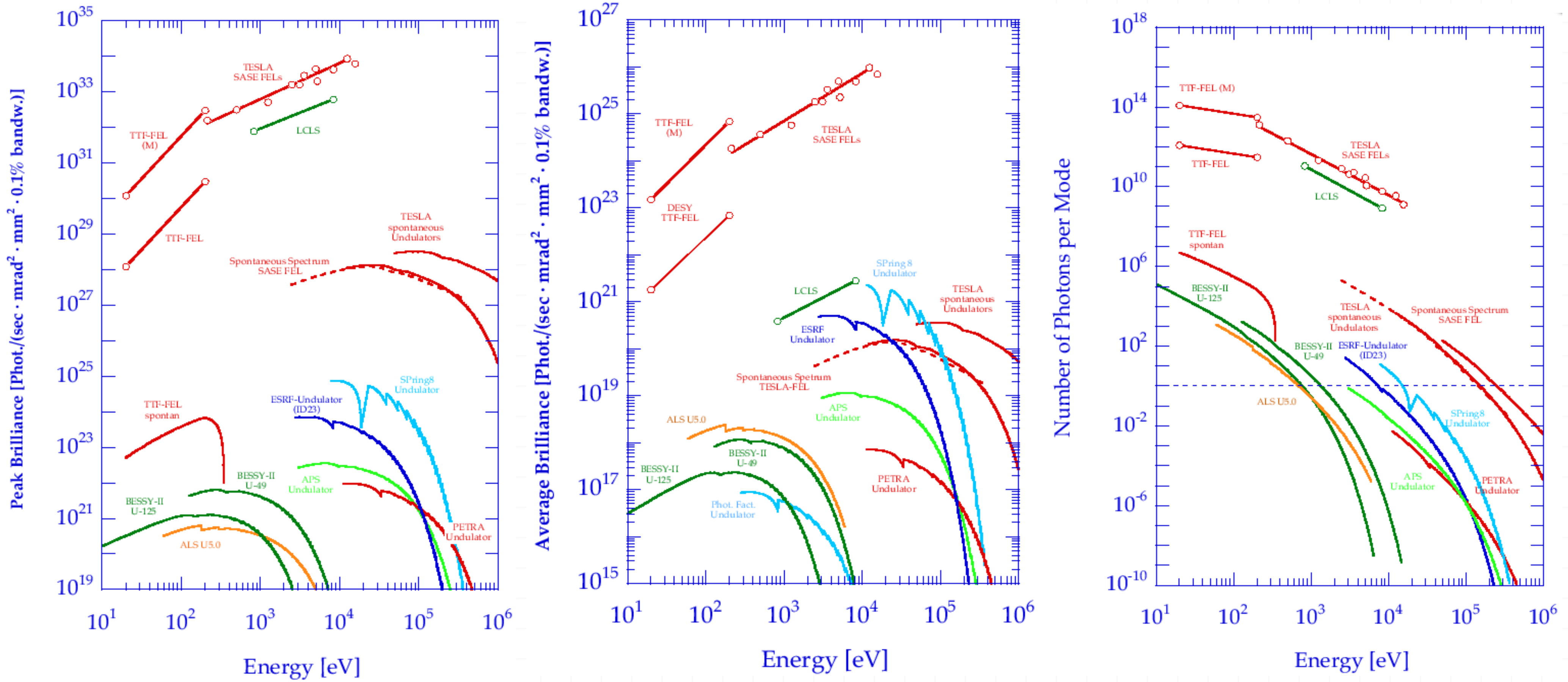


bunch length σ_z < wavelength λ



The intensity of coherent radiation will be about N_e^2 times larger → **CUR**
(~ 1.6 x 10^{16} times for 20 pC bunch)

PERFORMANCES OF SYNCHROTRON RADIATION



$$B = \frac{flux}{(2\pi)^2 \Sigma_x \Sigma_{x'} \Sigma_y \Sigma_{y'}} \left[\frac{photons}{sec - mm^2 - mrad^2 - 0.1\%\mathrm{B.}W.}\right]$$

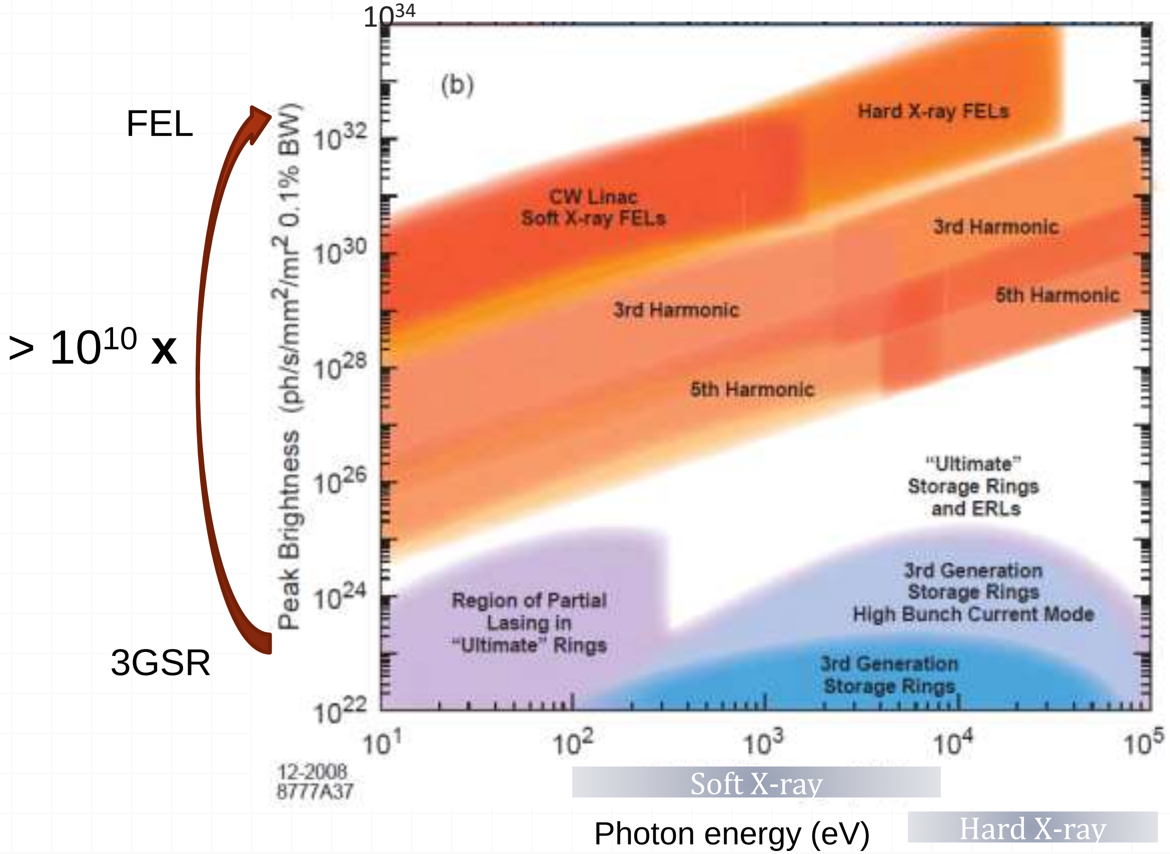
$$B_{avg} = B \times \Delta T \times R_p$$

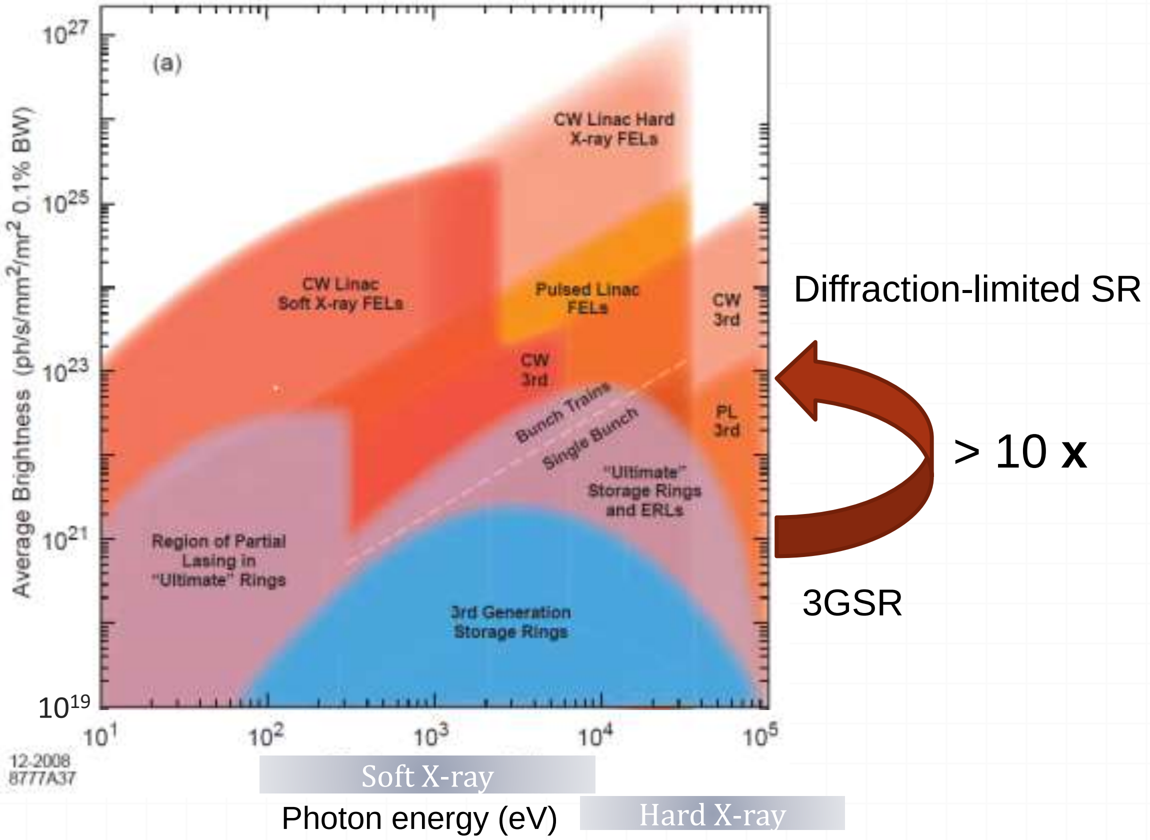
$$spectral\ flux = \frac{\mathrm{N}_{photon}}{\Delta T \cdot \Delta\omega/\omega}$$

$$\Sigma_x = \sqrt{\varepsilon_x \beta_x + (\eta\delta)^2 + \frac{\lambda L_u}{(4\pi)^2}} \quad \Sigma_y = \sqrt{\varepsilon_y \beta_y + \frac{\lambda L_u}{(4\pi)^2}} \quad \Sigma_{x'} = \sqrt{\frac{\varepsilon_x(1+\alpha_x^2)}{\beta_x} + (\eta'\delta)^2 + \frac{\lambda}{L_u}} \quad \Sigma_{y'} = \sqrt{\frac{\varepsilon_y(1+\alpha_y^2)}{\beta_y} + \frac{\lambda}{L_u}}$$

[ref] https://photon-science.desy.de/research/students__teaching/sr_and_fel_basics/fel_basics/tdr_spectral_characteristics/index_eng.html

Science and Technology of Future Light Sources — A White Paper (SLAC-R-917), 2008

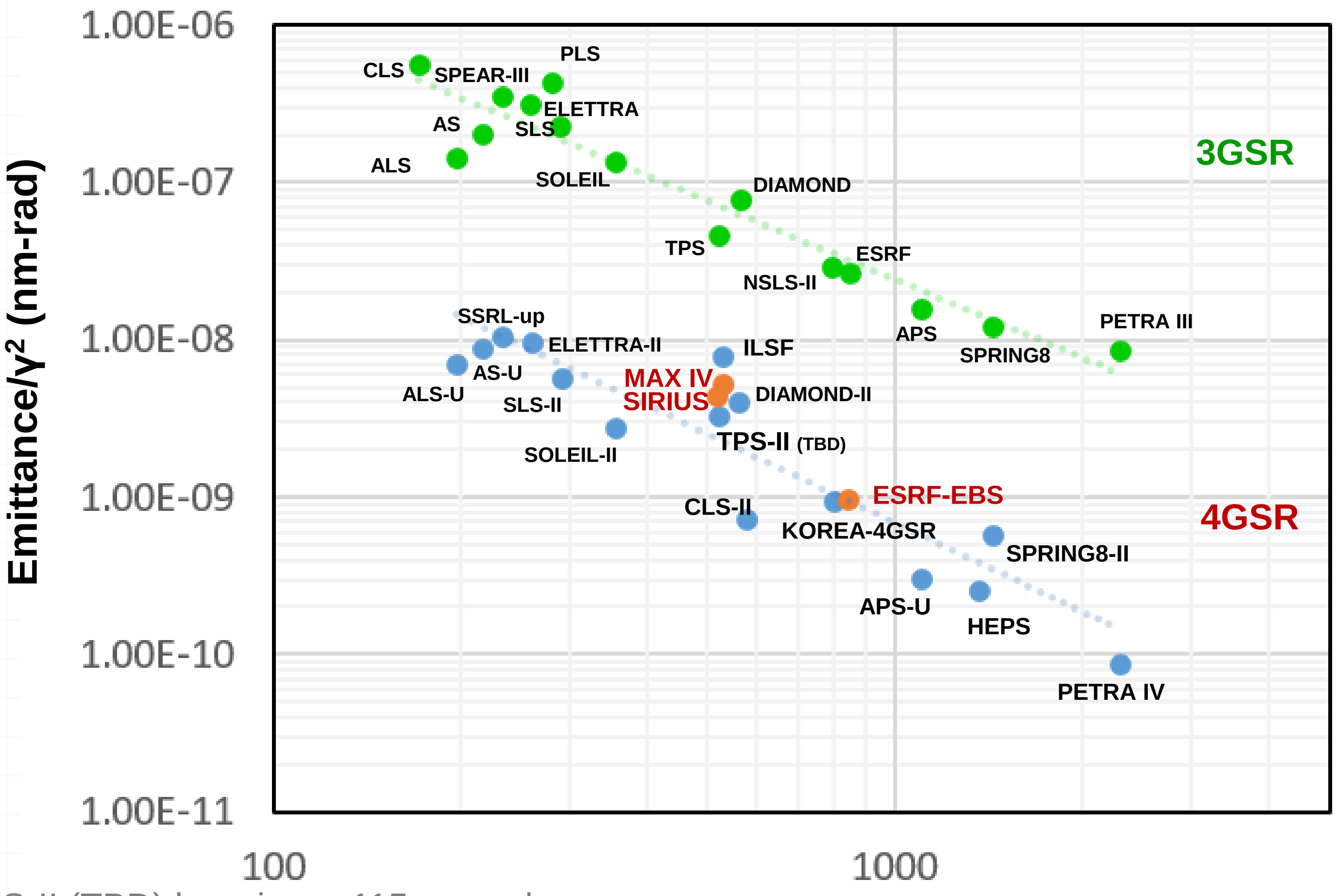


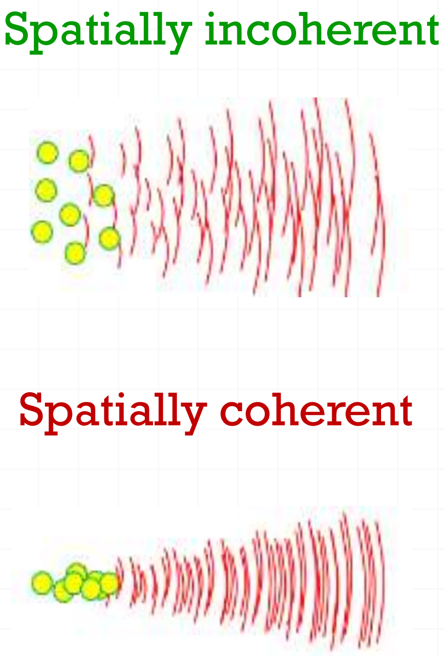


[Ref.] https://www.slac.stanford.edu/pubs/slacreports/reports17/slac-r-917.pdf
Science and Technology of Future Light Sources — A White Paper (SLAC-R-917)

TOWARD DIFFRACTION-LIMITED STORAGE RING LIGHT SOURCES

- Toward smaller electron beam emittance ring to increase the light brightness, spatial coherent flux, and coherent fraction.





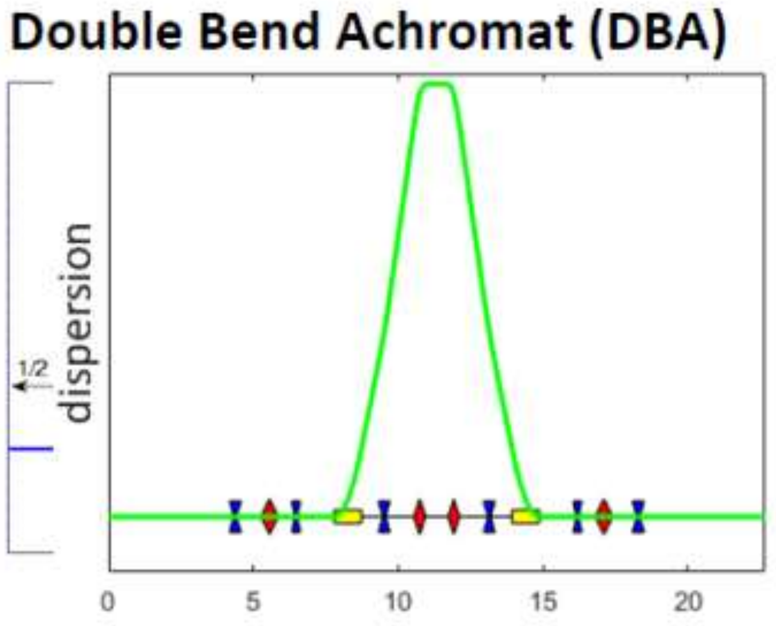
The data of TPS-II (TBD) here is ε = 115 pm-rad,

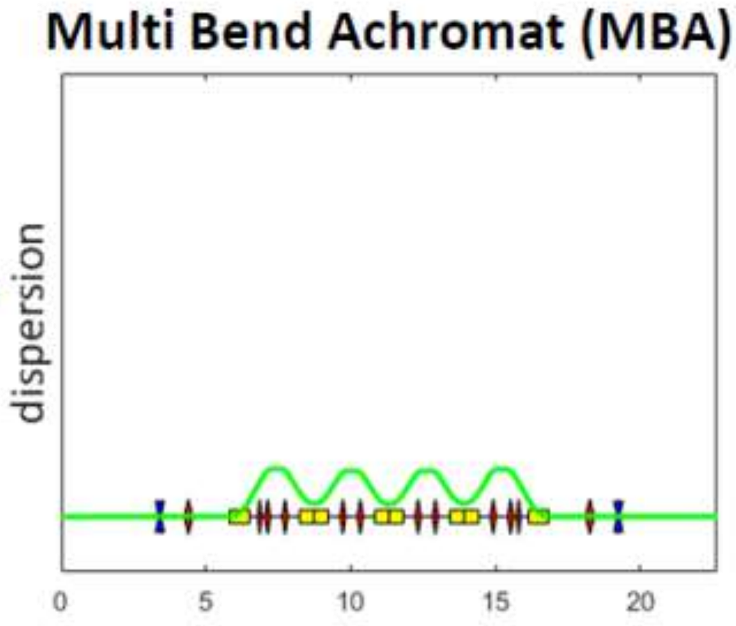
[Ref] #1 Multi-Bend Achromat Lattice Design for the Future of TPS Upgrade, M.S. Chiu et al., Journal of Physics: Conference Series, 1350, 2019, 012033.
#2 Masahiro Katoh, Lecture notes of "Synchrotron Light Source", KEK IINAS 5th International School on Beam Dynamics and Accelerator Technology, 2022/11/23.

DEVELOPING TREND OF LATTICE DESIGN FOR STORAGE RING

$$\varepsilon_0 = C_q \Upsilon^2 \frac{<H/|\rho|^3>}{j_x <1/|\rho|^2>}$$

$$\varepsilon_0 \propto E^2\theta^3$$

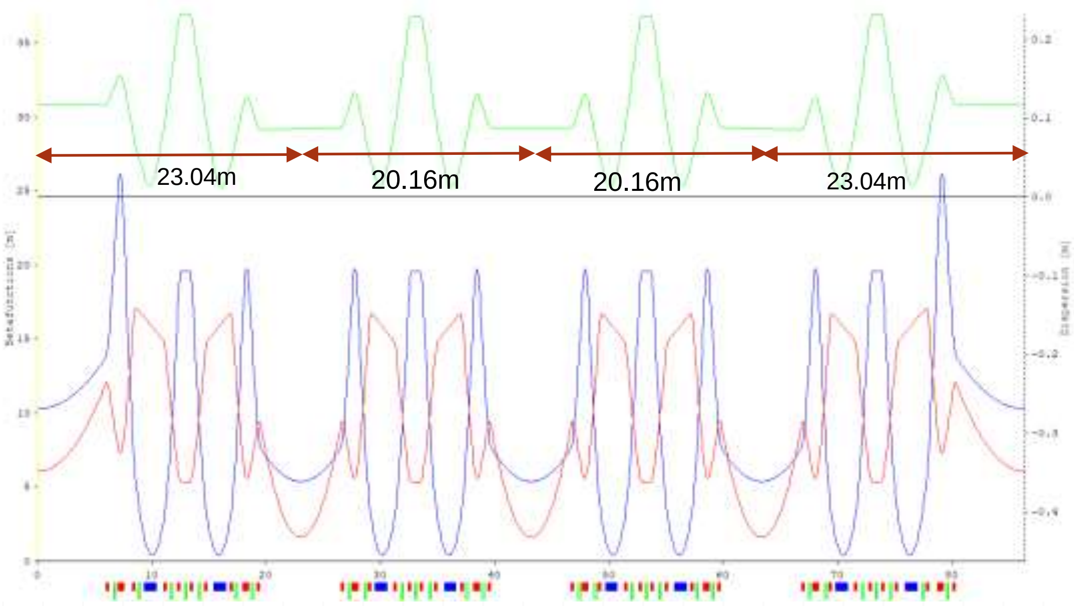




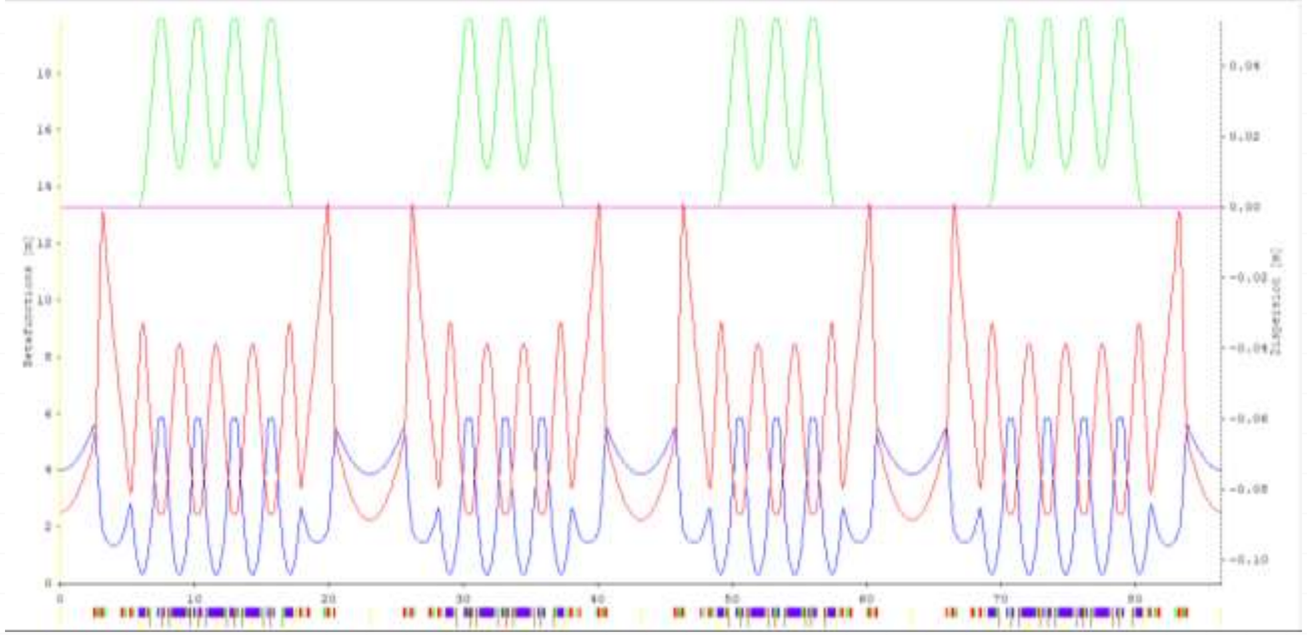
[Ref] Courtesy of R. Bartolini

LATTICE CONFIGURATION FROM TPS TO TPS-II

TPS DBA lattice



TPS-II HOA lattice (TBD)

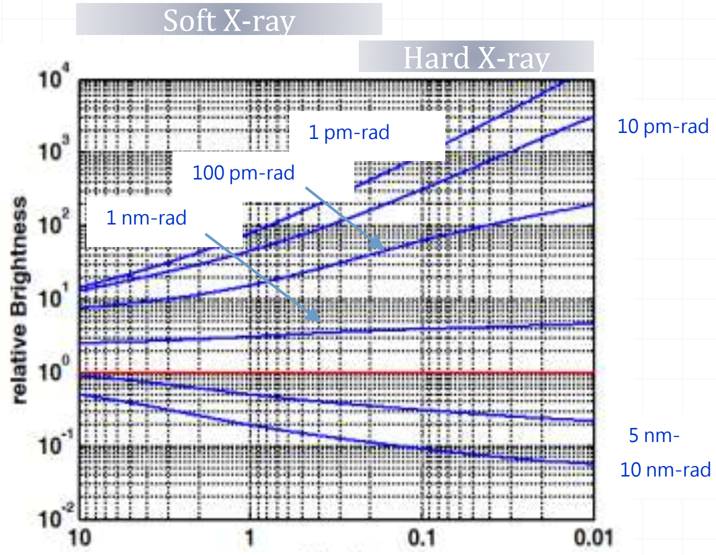


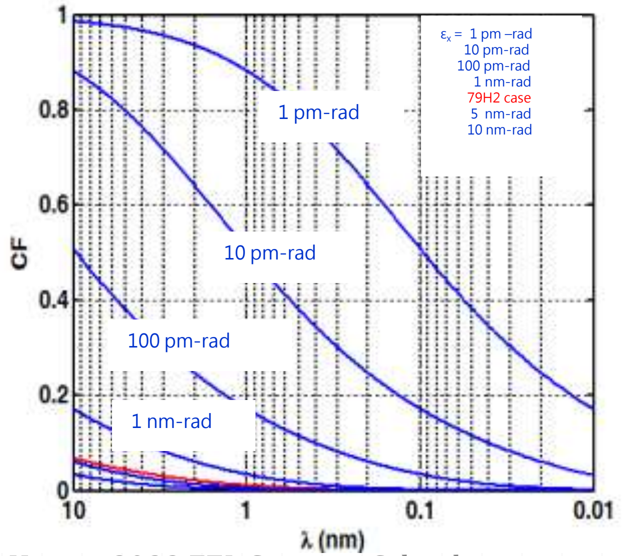
4 DB~A lattice (79H2 case)
518.4 m, 3 GeV
24 cells with 6-fold symmetry
ε = 1.6 pm-rad
α = 2.4 x 10^{-4}
νx = 26.19, νy = 13.25
ξx = -75, ξy = -27

Dipole numbers
48 → 108
ε / 10 times improve

HOA lattice (5-4-4-5 BA)
ε = 162 pm-rad
α = 0.99 x 10^{-4}
ξx = -85, ξy = -62
νx = 56.7, νy = 18.88
βx ~ 4, βy~2.5 @ 5m SS center

IMPROVEMENTS FROM TPS TO TPS-II





	TPS 79H2 case		εx = 100 pm-rad, η =0.088 m		εx = 100 pm-rad, η =0.0 m	
Radiation wavelength	1 nm (1.24 keV)	0.1 nm (12.4 keV)	1 nm (1.24 keV)	0.1 nm (12.4 keV)	1 nm (1.24 keV)	0.1 nm (12.4 keV)
Relative brightness	1	1	3.3	13	16	63
CF	0.0110	0.0005	0.036	0.007	0.172	0.033
Relative CF	1	1	3.3	14.0	15.6	66.0

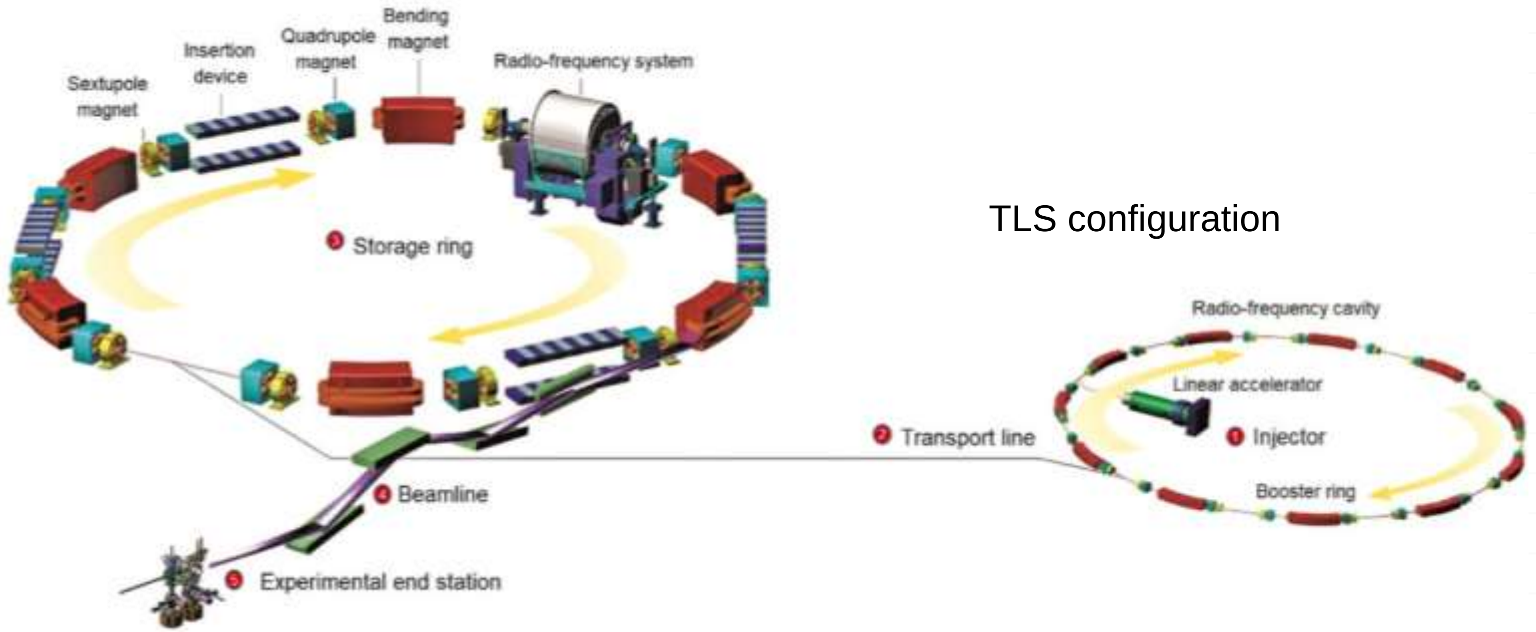
$$Brightness = \frac{flux}{(2\pi)^2\Sigma_x\Sigma_{x'}\Sigma_y\Sigma_{y'}}\left[\frac{photons}{sec-mm^2-mrad^2-0.1\%B.W.}\right]$$

$$spectral\ flux = \frac{N_{photon}}{\Delta T\cdot\Delta\omega/\omega} \qquad CF = \frac{(\sigma_r\sigma_{r'})^2}{\Sigma_x\Sigma_{x'}\Sigma_y\Sigma_{y'}}$$

COMPLEMENTARY AND SYNERGY OF SR AND FEL

SR
(STORAGE RING)

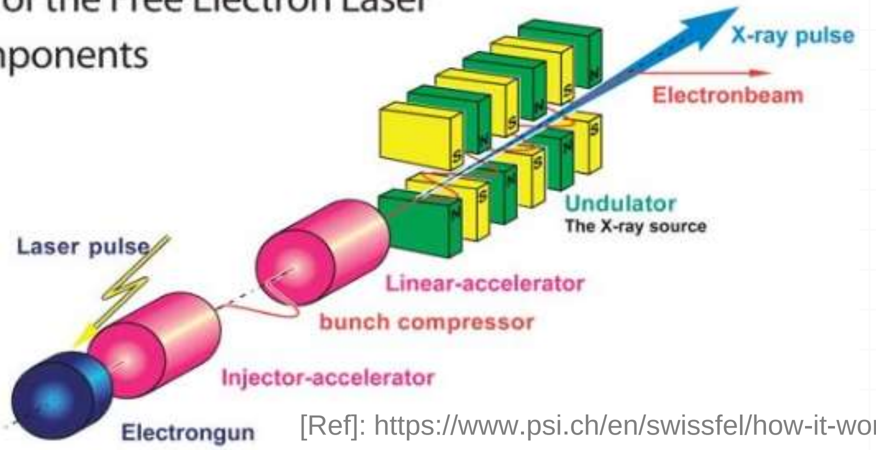
- # of users
- Wider spectrum
- Higher rep. rate
- Stability and flexibility



TLS configuration

Schematic design of the Free Electron Laser with different components

1) Electrongun
2) Injector
3) Accelerator
4) Undulator



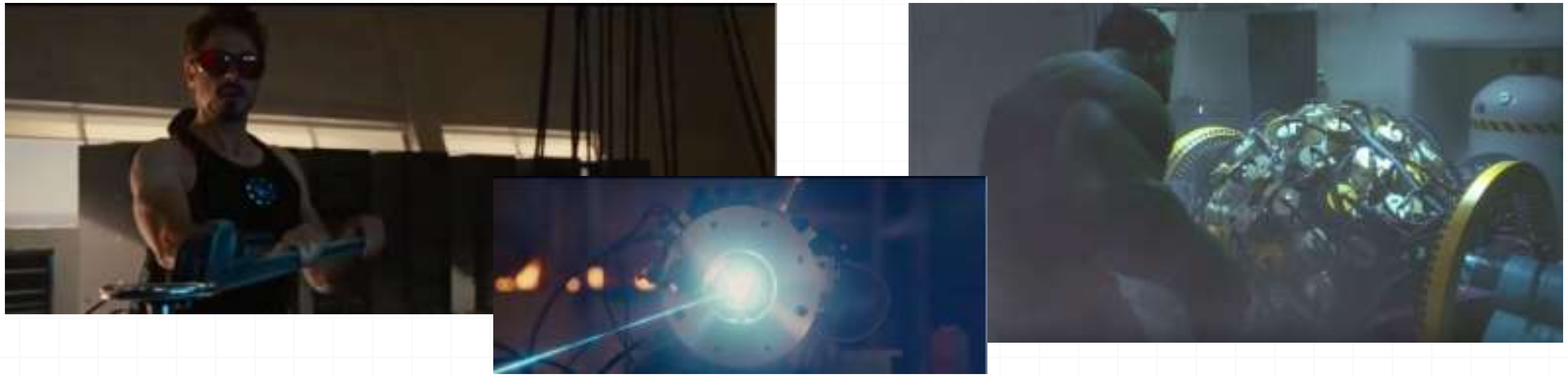
[Ref]: https://www.psi.ch/en/swissfel/how-it-works

FEL
(FREE ELECTRON LASER)

- Higher peak brightness (> 10^{10} x than SR)
- ~ fully spatial coherence
- better temporal coherence (~ 1% for SASE，~100 % for seeded FEL)
- Ultrashort time-resolved related discoveries

CONCLUSION

- Properties of relativistic charged particle
- Some basic concepts of lattice design for a charged particle accelerator
- B for guiding、E for acceleration
- dispersion、chromatic aberration、bunch compressor
- The complementary nature and synergistic potential of SR and FEL sources.
- The development of charged particle accelerator community is entering a new stage.



Charged particle accelerators are far more than that (✪ω✪)

Synchrotron radiation, with its illuminating properties, inspires us to explore the wonders of this beautiful world.

Welcome to Join NSRRC